QUERIES—REPLIES

24. TABLES OF $\tan^{-1}(m/n)$ (Q14, v. 1, p. 431; QR18, v. 1, p. 460; QR20, v. 2, p. 62).—In an earlier note by one of us (QR20) methods have been indicated by which the formula

$$\tan^{-1}(p+\lambda) - \tan^{-1}p = \tan^{-1}\frac{\lambda}{1+\lambda p+p^2} = \tfrac{1}{2}\pi - \tan^{-1}\frac{p^2+\lambda p+1}{\lambda}$$

may be used to make a table of values of $\tan^{-1}(m/n)$.

Another systematic use of this formula is suggested by the series

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \cdots + \tan^{-1}\frac{1}{n^2+n+1}$$
$$= \tan^{-1}(n+1) - \tan^{-1}1 = \tan^{-1}\frac{n}{n+2},$$

given, for instance, as an exercise in S. L. Loney's *Plane Trigonometry*, Part 2, 1900, Cambridge University Press, p. 126, or his *Key to Plane Trigonometry*, Part 2, 1912, C. U. Press, p. 91. This is proved by noting that

$$\tan^{-1}(n+1) - \tan^{-1}n = \tan^{-1}\frac{1}{n^2+n+1}.$$

For our purpose this may be generalized and rewritten as

$$\tan^{-1}\left(1+\frac{p}{n}\right) - \tan^{-1}\left(1+\frac{p}{n+1}\right)$$
$$= \tfrac{1}{2}\pi - \tan^{-1}\left(\frac{2n^2}{p} + 2\frac{p+1}{p}n + p + 1\right).$$

Then, with $p = 4$, for example, and $n = 1, 2, 3, \cdots$ we find

$$\begin{aligned}
\tan^{-1}5 - \tan^{-1}3 &= \tfrac{1}{2}\pi - \tan^{-1}8\\
\tan^{-1}3 - \tan^{-1}7/3 &= \tfrac{1}{2}\pi - \tan^{-1}12\\
\tan^{-1}7/3 - \tan^{-1}2 &= \tfrac{1}{2}\pi - \tan^{-1}17\\
\tan^{-1}2 - \tan^{-1}9/5 &= \tfrac{1}{2}\pi - \tan^{-1}23\\
\tan^{-1}9/5 - \tan^{-1}5/3 &= \tfrac{1}{2}\pi - \tan^{-1}30
\end{aligned}$$

and so on. These may be used in succession (equivalent to summing the first few) as a check on the values occurring on the left of the equations; all the values on the right are tabulated, for instance, in the NYMTP *Table of Arc Tan x*. This check is useful because it progresses across columns of values of $\tan^{-1}(m/n)$ for constant n (the method in the earlier note was primarily for going down columns with constant n). The relations can also be used for actual evaluation of entries in the final table, but in this case, to avoid accumulation of errors, they would not be used in succession from the beginning, but backwards or forwards from the nearest value on the left that is given directly in the tables. Thus $\tan^{-1}7/3$ could be found from $\tan^{-1}3$ and $\tan^{-1}12$, or from $\tan^{-1}2$ and $\tan^{-1}17$, both values used being taken from the NYMTP table.

With $p = 6$ and $n = 1, 2, 3, \cdots$ we find

$$\begin{aligned}
\tan^{-1} 7 - \tan^{-1} 4 &= \tfrac{1}{2}\pi - \tan^{-1} 29/3 \\
\tan^{-1} 4 - \tan^{-1} 3 &= \tfrac{1}{2}\pi - \tan^{-1} 13 \\
\tan^{-1} 3 - \tan^{-1} 5/2 &= \tfrac{1}{2}\pi - \tan^{-1} 17 \\
\tan^{-1} 5/2 - \tan^{-1} 11/5 &= \tfrac{1}{2}\pi - \tan^{-1} 65/3 \\
\tan^{-1} 11/5 - \tan^{-1} 2 &= \tfrac{1}{2}\pi - \tan^{-1} 27
\end{aligned}$$

and the relations cannot be used in succession for checking in the same way as for $n = 4$, since some of the values on the right are missing from the tables. It is still possible to work backwards and forwards from NYMTP tabular values as with $n = 4$.

Most early values of p give useful sets of formulae, but for $p = 7, 9, 11, 13, 14, 17 \ldots$ the relations are not of much practical use. Similarly other series may be generalized, but it does not seem useful to give more details.

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CORRIGENDA

P. 215, IB_6 7, *for* $x = .1$, *read* $x = 0, 1$; in line 2 *add* also $2y + \tfrac{1}{2}x^2 - \tfrac{1}{2} - \sum$, to 6D.

P. 255, B_3 13 *for* $2^{\frac{1}{2}}$, *read* $2^{-\frac{1}{2}}$.

P. 256 Transfer item C3 to be before the present first entry in D_1; eliminate and Airey from the heading of C.

P. 257, E 13, *for* 100, *read* 100°.

P. 280, under BACKHAUS 1, *for* v. 19, *read* v. 17.

P. 288, under EULER, *for* 1769, *read* 1748.

P. 301, under OLLENDORFF & SEELIGER, *for* 518, *read* 578.

P. 303, under RODER, *for* [III], *read* [I]; and *for* 10, *read* 19; and add entries in I A_1, and C_1 for $J_n(x)$, $n = 0(1)4$, $x = [0(.1)1(.2)2.8, 5.6; 4D]$.

V. 2, p. 71, l. 18, *for* than ν, *read* than ξ.

P. 85, l. –6, *for* L. J. CUNNINGHAM, *read* L. E. CUNNINGHAM.