to .1, and of real refractive index varying from 1.44 to 1.55. For the higher values of the extinction coefficient, the total scattering coefficient may be in error by 1%. When the extinction coefficient is zero, approximate values of the scattering coefficient will be obtained for transparent materials of refractive index as low as 1.33 and as high as 1.65 by substituting into the polynominal expression for $K(m; \alpha)$.

The number concentration, n, or the particle size, can be obtained when either is known by measuring the transmission at a known wavelength; or both can be obtained by measuring the transmission at two or more wavelengths. The transmission, $T = e^{-K\pi r^2 nl}$, where $K\pi r^2 n$ is the "absorption" coefficient as usually defined, i.e., $e^{-K\pi r^2 nl}$ is the fraction of light scattered by transparent particles or scattered and absorbed by absorbing particles per unit distance in the suspension.

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MECHANICAL AIDS TO COMPUTATION

See also the two introductory articles of this issue, as well as RMT **305**.

22. DE FOREST ON ELECTRICAL COMPUTERS.—To the *Yale Scientific Monthly*, published by the Senior class of the Sheffield Scientific School, Yale University, LEE DE FOREST (1873–) contributed an article ("A Wheatstone bridge for solving numerical equations," v. 3, 1897, p. 200–206),[1] explaining an idea of his for the electrical solution of equations. After describing a scheme in which the operator achieved his solution by adjusting the sliders on resistance potentiometers so as to balance a Wheatstone bridge, he pointed to the possibility of an "automatic balancer." He believed that "a relay type galvanometer driving or reversing some electrically governed mechanism might be devised, which would keep this length shifting until balance was attained."

If not the first, this is at least a very early description of the self-centering servomechanism as a computing device, the basic unit of some of the most important military computers of the recent war, including the electrical gun director for the control of antiaircraft fire.

DeForest may be pardoned for his qualifying remark—"it could not be very accurately done"—since, presumably, he did not yet know that he was going to invent the three-element thermionic valve, which did so much to make high precision possible.

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[1] EDITORIAL NOTE: This machine is one which escaped the attention of J. S. FRAME in his survey article "Machines for solving algebraic equations," *MTAC*, v. 1, p. 337–353. He did, however, deal with the machines of G. B. GRANT, C. V. BOYS, L. L. C. LALANNE referred to by DeForest in his introductory paragraphs. DeForest adds the new remark, that A. W. PHILLIPS (1844–1915) of Yale University was in 1879 an independent inventor of Lalanne's machine. Phillips was dean of the Graduate School 1895–1911, and the joint author of a number of texts in elementary mathematics.

NOTES

56. APPROXIMATIONS TO π.—In R. So. London, *Trans.*, 1841, p. 281–283, WILLIAM RUTHERFORD (1798?–1871) gave a value of π to 208D, which was correct to 152D, and derived from the Euler formula (1764)

$$(1) \qquad \frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}.$$

With a formula, apparently due to L. K. SCHULZ VON STRASSNITZKY (1803–1852),

$$\frac{\pi}{4} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}, \tag{2}$$

ZACHARIAS DASE (1824–1861) computed correctly to 200D, *J. f. d. reine u. angew. Math.*, v. 27, 1844, p. 198.

Three years later a value computed by THOMAS CLAUSEN (1801–1885), and correct to 248D, was published in *Astron. Nach.*, v. 25, 1847, col. 207f. Clausen used the formula of MACHIN (1706)

$$\frac{\pi}{4} = 4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}, \tag{3}$$

and a formula of EULER (1779) *Opera Omnia*, s. 1, v. 16_2, 1935, p. 26; also used by VEGA (1794)

$$\frac{\pi}{4} = 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}. \tag{4}$$

In *Phil. Mag.*, s. 4, v. 5, 1853, p. 214, Rutherford indicated the value of π calculated to 440D by means of Machin's formula (3). Using the same formula, WILLIAM SHANKS (1812–1882) gave full details of his work in his *Contributions to Mathematics comprising chiefly the Rectification of the Circle to 607 Places of Decimals*, London, 1853. These calculations were correct to only 459D. Rutherford had, however, published (p. 214) Shanks' value to 530D, which is correct to 500D, at least, as may be checked by the independent computation (climaxing two previous publications) of RICHTER (d. 1854) in *Archiv Math. Phys.*, v. 25, 1855, p. 472, reprinted from *Elbinger Anzeiger*, no. 85, 18 Oct. 1854.

Until lately no one but Shanks seems to have published any original computed value of π beyond 500D. In R. So. London, *Proc.*, v. 21, 1873, p. 319, Shanks gave a value to 707D but this was correct to only 459D; there were errors also in three following places. In the next volume of the *Proc.*, 22, p. 45, Shanks gave an amended value to 707D in which there was an error in the 326th decimal place, followed by an unexplained change of a 7 into a 1, in the 680th decimal place. The 707D have appeared in print correctly (so far as Shanks found them) in at least seven places:

(i) *Z. f. math. u. naturw. Unterricht*, v. 26, 1895, p. 263;

(ii) *L'Intermédiaire d. Mathématiciens*, v. 2, 1895, p. 389;

(iii–vi) G. PEANO, *Formulaire de Mathématique*, Turin, v. 2, no. 3, 1899, p. 143; v. 3, 1901, p. 176; v. 4, *Formulaire Mathématique*, 1902, p. 234; also v. 5, *Formulario Mathematico*, 1908, p. 256;

(vii) J. T. PETERS, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, *Anhang*, p. 1.

For the first time, so far as we know, the Shanks' computation of π beyond the 500th decimal place has recently been checked, with interesting results, by Mr. D. F. FERGUSON, of the Royal Naval College, Eaton, Chester, England. In *Nature*, v. 157, 16 Mar. 1946, p. 342, he tells us that he used a formula, discovered by his colleague R. W. MORRIS,

$$\frac{\pi}{4} = 3 \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{20} + \tan^{-1}\frac{1}{1985}, \tag{5}$$

and found that Shanks' value was incorrect, beginning with the 528th decimal place. He then gives both values from the 521st to the 540th decimal places. Mr. Ferguson has kindly supplied us with his calculated values from the 526th through the 620th decimal place, as follows:

S:	39501	60924	48077	23094	36285	53096	62027	55693	97986	95022
F:	39494	63952	24737	19070	21798	60943	70277	05392	17176	29317
S:	24749	96206	07497	03041	23668	86199	51100	89202	38377...	
F:	67523	84674	81846	76694	05132	00056	81271	45263	56082...	

J. W. WRENCH, JR. informed us that the relation (5) is not new, by any means; that it was given, for example, by S. L. LONEY, in his *Plane Trigonometry*, Cambridge, 1893, p. 277, and again by CARL STØRMER in *Archiv f. Mathem. og Naturv.*, v. 19, no. 3, 1896, p. 70, of his monograph to which we have already referred (*MTAC*, v. 2, p. 28), "Sur l'application de la théorie des nombres entiers complexes à la solution en nombres rationnels $x_1x_2 \cdots x_nc_1c_2 \cdots c_n$, k de l'équation $c_1 \operatorname{arctg} x_1 + c_2 \operatorname{arctg} x_2 + \cdots + c_n \operatorname{arctg} x_n = k\pi/4$." Here are tables of 102 three-term (left-hand member) equations, $k \neq 0$.

R. C. A.

57. GARRARD'S TABLES.—Brown University has recently acquired a copy of WILLIAM GARRARD, *Copious Trigonometrical Tables showing the results in all cases of Plane Trigonometry, By Inspection. Intended to complete the Requisite Tables to the Nautical Almanack; and as a Necessary Companion to the Theodolite*, London, 1789, 303 p. 15.3 × 24.4 cm. Garrard states that he was "late assistant observer at the Royal Observatory, Greenwich." Table, I, the largest (225 p.), a navigation Traverse Table, for every hypotenuse, d (distance) = 1(1)300, of a plane right triangle are given, to 2D, the value of the base (L = latitude) and altitude (D = departure) for each angle 0(10′)90°. Table II (22 p.) is a similar table for every angle $0(\frac{1}{4})7\frac{3}{4}$ points. Table III (26 p.), for d = 1(1)10 are given to 5D, the value of L and D, for 0(10′)90°. T. IV (10 p.) is "an improved table of Meridional Parts for more accurately solving the cases of Mercator's Sailing." Then there are 3 p. of "Errata to the Traverse Tables in the last edition of Robertson's *Navigation*." This reference is doubtless to the enlarged fourth edition of JOHN ROBERTSON, *Elements of Navigation*, "carefully revised and corrected by WILLIAM WALES, Master of the Royal Mathematical School, Christ's Hospital, London." The traverse tables in v. 2, London, 1780, p. 81–130, cover the material of Garrard's T. I–II, for d = 1(1)120, and are possibly due to William Wales (1734?–1798), fellow of the Royal Society, which sent him in 1769 to observe at Hudson Bay the transit of Venus, and author of various works including a restoration of the Determinate Section of Apollonius of Perga (London, 1772). For further information about

Wales, see C. HUTTON, *Phil. and Math. Dict.*, second ed. 1815; E. I. CARLYLE, *Dict. Nat. Biog.*, v. 59, 1899; *Math. Gazette*, v. 14, 1929, p. 388; and *Mechanics' Mag.*, v. 60, 1854, p. 436–437. Garrard's T. IV of Meridional Parts is for each minute of the quadrant to 2D; Robertson's table, p. 215–224, for the same range, is to 1D. John Robertson (1712–1776) became a fellow of the Royal Society in 1741, and was its Librarian for the last few years of his life.

R. C. A.

58. TABLES OF POWERS N^p. Among many tables of this type, p non-integral, attention may be directed to one which seems to be comparatively unknown to mathematicians. It was calculated by L. A. BARRY and published in JOHN GOODMAN, *Mechanics Applied to Engineering*, London, New York & Toronto, Longmans, v. 2, 1927 [new impression 1941; reprint 1943], p. 450–469, 12.0 × 18.4 cm. The table gives to 4S, without differences, values of N^p where $N = 1.1(.1)30$ and $p = 1.1(.1)4$. As a one-volume work, Goodman's book goes back to the last century; but the second volume, which consists chiefly of worked examples, was first published in 1927.

Another, though smaller, table in which p takes a range of fractional values is that given in JAHNKE & EMDE, *Tables of Functions* (1933 edition and Dover reprints), and repeated in F. EMDE, *Tables of Elementary Functions*, 1940, p. 8 (see *MTAC*, v. 1, p. 384). This gives N^p to 3–4D, without differences, for $N = .1, .5(.1)1(.2)3(.5)5, 10$ and $p = .05(.05)1$. On errors in this table, see *MTAC*, v. 2, p. 47.

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QUERIES

18. ADDITIONS TO THE RUDOLPHINE TABLES.—Kepler's last great work, *Tabulæ Rudolphinæ, Quibus Astronomicæ Scientiæ, Temporum longinquitate collapsae Restauratio continetur* was published at Ulm in 1627, and there were later editions including one in English. To this original work three additions were later made, namely: (a) in 1629, 8 pages (p. 121–128), by Kepler, of "Sportula Genethliacis missa de Tabularum Rudolphi usu in computationibus astrologicis: cum modo dirigendi novo et naturali"; (b) A celebrated "Mappa Mundi Universalis," dated, 1630, but mysteriously dedicated to Kaiser Leopold, who did not commence to reign until 1658; (c) An Appendix, 46 p., by Kepler's son-in-law, J. BARTSCH, published at Sagan in 1630. In Providence, R. I. there are three copies of Kepler's work all including (a), and two of them also including (b). In MAX CASPAR, *Bibliographia Kepleriana* (Munich, 1936), where libraries having Kepler's works are listed, are the names of 9 German libraries owning copies of Kepler's work with (c). Where are copies with (c) in other countries? Tables in this Appendix, as well as in the Rudolphine Tables, are of value as exhibiting the spread of Napier's ideas on the continent, and as the basis of the final edition of a volume containing material due to Kepler and Bartsch, and published at Strassburg in 1700.

R. C. A.