MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **289** (Theodorsen), **304** (Editorial N.), **311** (Hayashi, Milne-Thomson), **312** (Benest & Timberlake), **313** (Hoehne), **314** (Martelli).

84. BAASMTC, *Mathematical Tables*, volume 1, London, 1931. This v. has long been out of print. It has been announced that the following errors are corrected in the new edition. Compare *MTAC*, v. 1, p. 323, and RMT **303**.

TABLE II (p. 5, 7)

cos 26.1: *for* .56756..., *read* .56755... sin 47.6: *for* .46832..., *read* .45832...

TABLE VII (p. 32)

Ei(5.3): *for* ...031, *read* ...030 Ei(6.7): *for* ...344, *read* ...345
Ei(5.6): *for* ...598, *read* ...597 Ei(8.0): *for* ...954, *read* ...953
Ei(5.9): *for* ...015, *read* ...014 Ei(9.8): *for* ... 35, *read* ... 34
$-\mathrm{Ei}(-7.3)$: *for* ...4446, *read* ...4445

Consequent changes

Corrected end figures of actual differences:

x	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
$\delta^3\mathrm{Ei}(x)$	692	571	295	327	155	484	468	921	585

x	6.6	6.7	6.8	7.9	8.0	8.1	9.7	9.8	9.9
$\delta^3\mathrm{Ei}(x)$	086	208	940	815	887	199	79	20	30

x	7.8	7.9	8.0	8.1	8.2
$\delta^4\mathrm{Ei}(x)$	619	787	240	952	073

x	7.0	7.7	7.8	7.9	8.0	8.1	8.2	8.3
$\delta^6\mathrm{Ei}(x)$	815	855	480	285	259	409	808	391

x	7.2	7.3	7.4
$\delta^3\{-\mathrm{Ei}(-x)\}$	8075	4950	9472

x	7.2	7.3	7.4	7.5
$\delta^4\{-\mathrm{Ei}(-x)\}$	9630	7647	6709	4290

x	7.2	7.3	7.4	7.5	7.6
$\delta^6\{-\mathrm{Ei}(-x)\}$	3852	1045	8519	6346	4426

Corrected values of modified differences:

$\delta^4\mathrm{Ei}(5.6)$ *should read* 299 994 $\delta^4\{-\mathrm{Ei}(-7.1)\}$ *should read* 180 986

Corrected end figures of modified differences:

x	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1
$\delta^4\mathrm{Ei}(x)$	005	898	459	889	337	994	104	863	547	433	834

x	6.5	6.6	6.7	6.8	6.9	9.6	9.7	9.8	9.9	10.0
$\delta^4\mathrm{Ei}(x)$	739	438	227	682	467	75	29	01	81	25

TABLE VIII (p. 35)

Si(8.8): *for* ...21860, *read* ...21861 Ci(5.5): *for* ...29475, *read* ...29476

Consequent changes

Corrected end figures of actual differences:

x	8.7	8.8	8.9
$\delta^3\mathrm{Si}(x)$	303	361	485

x	5.4	5.5	5.6
$\delta^2\mathrm{Ci}(x)$	462	196	704

Corrected end figures of modified differences:

x	8.6	8.7	8.8	8.9	9.0	x	5.3	5.4	5.5	5.6	5.7
$\delta^4 Si(x)$	542	853	146	410	663	$\delta^4 Ci(x)$	505	835	200	685	428

TABLE IX (p. 40)

0.271: *for* ...4465, *read* ...4466

With this change the table, originally described as of slightly less than 12-figure accuracy, becomes accurate to 12 figures throughout.

Consequent changes

Corrected end figures of actual differences:

x	0.26	0.27	0.28
$\delta^2(x!)$	899	391	728

Corrected end figures of modified differences:

x	0.24	0.25	0.26	0.27	0.28	0.29
$\delta^4(x!)$	622	531	605	819	153	631

85. A. FLETCHER, J. C. P. MILLER and L. ROSENHEAD. *An Index of Mathematical Tables*, [FMR *Index*], London, Scientific Computing Service, 1946.

The gratifying review of this book, RMT 233, *MTAC* v. 2, p. 13f, appeared just before the final printing of the work and the opportunity was taken to make three alterations: those indicated in the review (i) to the heading of Art. 5.745 on p. 105, (ii) to Hutton 1775 on p. 404 and (iii) to Kulik 1860 on p. 409.

Three errors have come to our notice:

p. 300. Art. 20.67. Insert ∞ for the upper limit to the integral for $C_1(u)$. This was correct in the proof that was passed for press, and may not be missing from all copies.

p. 381. British Association Mathematical Tables. The statement that the first five volumes are out of print is incorrect. At the time the passage was written, volumes I and II were out of print. Since then, there has been a second edition of volume I, while volume VI has now run out of print. Volumes III, IV and V were very difficult to obtain, mainly owing to war-time conditions, but they were not, in fact, out of print.

p. 387. Under Crelle, *for* 1864, *read* 1857.

J. C. P. MILLER

EDITORIAL NOTE: FMR *Index* was published in England in mid-April and sheets were then sent to McGraw-Hill & Co., New York, for binding, and distribution in the Western Hemisphere. In our review, *MTAC*, v. 2, p. 18, line 16–18, it is not brought out that the *Index* referred to "editions" in 1890, both by Dickstein; one of them, however, was but a small portion of the original of HOËNE-WROŃSKI.

86. A. M. LEGENDRE, *Tafeln der Elliptischen Normalintegrale erster und zweiter Gattung*, hrsg. von FRITZ EMDE, Stuttgart, 1931. This is a facsimile reproduction of Tables VIII and IX, to 10D in Legendre's *Exercices de Calcul Intégral*, Paris, v. 3, 1816, p. 338–416.

In *Z. angew. Math. Mech.*, v. 21, Aug. 1941, p. 254, Emde reports the following six errors in $F(\phi, \theta)$, found by GUSTAV WITT:

Page	ϕ	θ	*For*	*Read*
350(15)	20°	9°	0.34293	0.34923
364(29)	*90°	23°	1.63631	1.63651
368(33)	*85°	27°	1.56840	1.56480
380(45)	*80°	44°	1.58906	1.59806
396(61)	*79°	64°	1.84693	1.84793
416(81)	49°	87°	0.98238	0.98328

Those errors, as well as others below, marked with a star (*) were already reported in N. SAMOĬLOVA-ĨAKHONTOVA, *Tablitsy Ellipticheskikh Integralov*, Moscow and Leningrad, 1935, p. 6, and in *Scripta Math.*, v. 3, 1935, p. 365. All errors both here and below were listed by HEUMAN, April, 1941 (see *MTAC*, v. 1, p. 187; an additional errata sheet by Heuman, dated June 1941, was printed at Stockholm). Seventeen other errors in $F(\phi, \theta)$ are as follows:

Page	ϕ	θ	*For*	*Read*
346(11)	42°	2°	0.73311 099	0.73311 009
350(15)	5°	7°	0.08729 8	0.08726 8
358(23)	32°	20°	0.56174 72	0.56174 52
364(29)	*88°	21°	1.58584	1.58784
366(31)	*35°	30°	0.62003	0.62203
	42°	27°	0.74754	0.74574
368(33)	59°	27°	1.06251 39	1.06251 29
384(49)	51°	46°	0.95158 7	0.95157 7
392(57) and 396(61)	*60°	60°	1.21253 6	1.21259 6
392(57)	*65°	59°	1.34195 7	1.34196 7
	90°	57°	2.08035 816	2.08035 806
404(69)	61°	71°	1.29179	1.29719
410(75)	18°	84°	0.31936 7	0.31939 7
412(77)	*74°	83°	1.92525	1.92515
416(81)	*86°	86°	1.17204 1744	1.17024 9982[1]
	87°	87°	3.45644 5172	3.45667 6096[1]

Samoĭlova-ĩakhontova and Heuman used the 1826 edition of Legendre's tables in his *Traité des Fonctions Elliptiques*, v. 2. There are here at least five errors which do not occur in the 1816 edition, namely in the values for: $\phi = 1°$, $\theta = 14°$; $\phi = 37°$, $\theta = 75°$; $\phi = 4°$, $\theta = 88°$; $\phi = 31°$, $\theta = 90°$. Also in $E(\phi, \theta)$, $\phi = 5°$, $\theta = 10°$. Two of these were noted by K. BOHLIN (1900). FMR *Index* (RMT 233) states, p. 316, "A few of the errors in Legendre 1816 are corrected in Legendre 1826." In the present check we have found that all the errors of Legendre 1816 are in Legendre 1826 and also five new ones, not in Legendre 1816.

There are the following 7 errata in $E(\phi, \theta)$:

Page	ϕ	θ	*For*	*Read*
347(12)	51°	4°	0.88952	0.88962
351(16)	*90°	8°	1.56296	1.56316
363(28)	78°	22°	1.31072	1.31972
377(42)	27°	42°	0.46376	0.46366
379(44) and 383(48)	73°	45°	1.13785 83	1.13785 43
405(70)	32°	79°	0.53101 73	0.53101 13.

Since the K. Pearson facsimile was of Legendre 1826 (Pearson has 1825!, Cambridge, 1934), it is clear from what we have indicated above that there are in it at least 35 serious errors in E and F alone. A facsimile of Legendre 1816 tables is also in L. Potin, *Formules et Tables Numériques*, Paris, 1925. Hence the 30 errors indicated above for the Legendre-Emde tables occur also in these tables.

R. C. A.

[1] These numbers are from the errata list of Heuman who states "The last decimals are uncertain."

87. T. J. STIELTJES, "Table des valeurs des sommes $S_k = \sum_1^\infty n^{-k}$," *Acta Mathem.*, v. 10, 1887, p. 299–302. J. W. L. GLAISHER, "Tables of $1 \pm 2^{-n} + 3^{-n} \pm 4^{-n} + \cdots$ and $1 + 3^{-n} + 5^{-n} + 7^{-n} + \cdots$ to 32 places of decimals," *Quart. J. Math.*, v. 45, 1914, p. 148–150. See also UMT 46.

The Stieltjes table of $S_n = 1 + 2^{-n} + 3^{-n} + \cdots$ is for $n = [2(1)70; 32D]$. Glaisher gave a table of S_n for $n = [2(1)107; 32D]$, but he copied from the table of Stieltjes the values for $n = 2(1)33$.

In the *Anhang* (p. 90) by PETERS & STEIN, of J. T. PETERS, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, there is a table of $S'_n = S_n - 1$ for $n = 2(1)100$. On comparing this table with that of Stieltjes, I found that there were 23 differences in the end-figures of the first 31 values of S_n. Upon checking the values I found that Peters & Stein were correct in every case except one, $n = 25$, so that there were the following 23 errors in the part of the table published by Stieltjes, and later copied by Glaisher. Apart from the 15 cases of unit errors, there were 8 cases of errors of two units in the final digit; S_7, S_{10}, S_{16}, S_{20}, S_{21} were each 2 units in excess of the correct values, but S_{12}, S_{17}, and S_{25} were each 2 units in defect. As to the unit errors there were errors of excess in S_3, S_4, S_6, S_{13}, S_{15}, S_{23}, S_{27}, S_{32}; and errors of defect in S_9, S_{11}, S_{18}, S_{22}, S_{24}, S_{26}, S_{29}. The error noted in S_3 was also checked by D. H. LEHMER, to 100D, *Scripta Mathematica*, v. 4, 1936, p. 293, and by J. W. BRADSHAW, *Amer. Math. Mo.*, v. 51, 1944, p. 390. My last seven calculated figures of S_n to 37D (a) in each of the 23 cases where Stieltjes and Peters & Stein did not agree, (b) in the other 9 values of n, not recalculated by Glaisher, up to and including $n = 33$, are as follows:

S_2	0251892	S_{10}	3190170	S_{18}	6219397	S_{26}	5066307
S_3	4499908	S_{11}	4699365	S_{19}	7951014	S_{27}	0041706
S_4	1679028	S_{12}	7396710	S_{20}	6834493	S_{28}	2040184
S_5	0341681	S_{13}	3573957	S_{21}	6043730	S_{29}	9099454
S_6	9205279	S_{14}	1353337	S_{22}	1867530	S_{30}	7647350
S_7	7967596	S_{15}	6450626	S_{23}	7188823	S_{31}	9233251
S_8	6524653	S_{16}	6367220	S_{24}	2079358	S_{32}	1455976
S_9	4120605	S_{17}	5630292	S_{25}	7050694	S_{33}	2973835

All the data were simultaneously checked by means of the equation

$$\sum_{n=2}^{33} S'_n = 1 - \sum_{n=2}^{\infty} (n-1)^{-1} n^{-33}.$$

The respective members of this equation differed by 3.5×10^{-37} when my 37D approximations to S_n were used in evaluating the left side. This discrepancy is due to the rounding of the data to 37 decimal places.

Glaisher corrected 10 unit errors in final digits of Stieltjes S_n, $n = 39, 42, 43, 46, 47, 56, 57, 65$ in excess; $n = 61, 67$ in defect. The Stieltjes table did not profess to be correct in the final decimal place. The Glaisher table was exactly reprinted in H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 2, Bloomington, Indiana, 1935, p. 244, 218. Hence there are 23 errors, indicated above, in each of the Glaisher and Davis tables. In addition it may be mentioned that I have computed S_n more extensively, $n = 2(2)20$, to 52D, and $n = 3, 5$, to 42D.

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