Tables of $j_{\nu,s}$—*Continued*

s	$\nu = \frac{2}{3}$	$\nu = -\frac{2}{3}$	$\nu = \frac{3}{4}$	$\nu = -\frac{3}{4}$
8	25.3907144312	23.2959758671	25.5193219366	23.1625059341
9	28.5327278949	26.4380633449	28.6615845511	26.3049025693
10	31.6746579988	29.5800458826	31.8037149655	29.4471278984
11	34.8165272733	32.7219536520	34.9457487864	32.5892313867
12	37.9583508024	35.8638062730	38.0877098894	35.7312451311
13	41.1001390646	39.0056170547	41.2296148817	38.8731908670
14	44.2418995658	42.1473953433	44.3714756721	42.0150838362
15	47.3836378232	45.2891478947	47.5133010223	45.1569350434
16	50.5253579832	48.4308797136	50.6550975224	48.2987526323
17	53.6670632220	51.5725945862	53.7968702239	51.4405427586
18	56.8087560137	54.7142954298	56.9386230642	54.5823101620
19	59.9504383142	57.8559845286	60.0803591576	57.7240585508
20	63.0921116899	60.9976636963	63.2220809997	60.8657908680
21	66.2337774103	64.1393343918	66.3637906137	64.0075094793
22	69.3754365150	67.2809978012	69.5054896572	67.1492163076
23	72.5170898635	70.4226548994	72.6471795012	70.2909129325
24	75.6587381727	73.5643064946	75.7888612893	73.4326006631
25	78.8003820451	76.7059532629	78.9305359832	76.5742805938
26	81.9420219908	79.8475957737	82.0722043975	79.7159536465
27	85.0836584448	82.9892345106	85.2138672265	82.8576206036
28	88.2252917800	86.1308698863	88.3555250659	85.9992821328
29	91.3669223180	89.2725022556	91.4971784295	89.1409388079
30	94.5085503377	92.4141319250	94.6388277630	92.2825911246

NYMTP

150 Nassau Street
New York City

RECENT MATHEMATICAL TABLES

299[A].—*Mathematical Cuneiform Texts*, edited by O. E. NEUGEBAUER and A. J. SACHS. (American Oriental Series, volume 29.) New Haven, Conn., Amer. Oriental So., 329 Sterling Memorial Library, and the Amer. Schools of Oriental Research, 1945. x, 177 p. + 49 plates. $5.00

In Quellen u. Studien zur Geschichte der Mathematik . . ., A. Quellen, v. 3, (MKT I–III) 1935–37, Otto Neugebauer, the outstanding authority on ancient mathematics and astronomy, published a monumental pioneer work on Mathematical Cuneiform Texts, tablets in collections of Berlin, Brussels, Istanbul, Jena, Leyden, London, New Haven, Oxford, Paris, Philadelphia, Strassburg, and Toronto. Nearly 4000 years ago the Babylonians knew how to find the positive roots of quadratic equations; how to solve simultaneous linear and quadratic equations; and how to calculate compound interest; and they knew that the angle in a semi-circle is a right angle, and our familiar relation between the hypotenuse and sides of a right-angled triangle.

The present work in English is a detailed study of about 200 tablets in the United States[1] and nearly a dozen in Europe. New material of great interest is presented.

In all Babylonian mathematical and astronomical work, tables were fundamental, especially tables for multiplication, squares, square roots, cubes and cube roots, and tables of reciprocals. Since 60 was basic in the sexagesimal notation of the Babylonians, if n were a number written in this notation, $\bar{n} = 1/n$ could be expressed in a finite number of terms (e.g. $\bar{9} = 0;\ 6, 40$), or in a never ending series of terms (e.g. $\bar{7} = 0;\ 8, 34, 17, 8, 34, 17 \ldots$, with 8, 34, 17 being repeated indefinitely). The necessary and sufficient condition for the first case is that n must be of the form $n = 2^\alpha 3^\beta 5^\gamma$, where α, β, γ are integers or zero. Such numbers are called regular numbers. In a Louvre tablet dating from about 350 B.C. there are

252 entries in a table of one-place to seventeen-place regular number divisors, and one-place to fourteen-place reciprocals of these numbers.

By all odds the most interesting tablet discussed in the present volume is one at Columbia University, Plimpton 322, dating from 1900–1600 B.C., and containing a table of Pythagorean numbers. If l denote the longer, s the shorter side of a right triangle, and h its hypotenuse, then l, s, and h satisfy the relation

$$l^2 + s^2 = h^2.$$

The values of h and s, always integers, for 15 such Pythagorean triangles are given in two columns of our text; a third column does not give l but h^2/l^2. While the successive numbers in the h and s columns are very irregular, the ratios h^2/l^2 with successive entries decrease almost linearly. The authors discuss the contents of this tablet at great length, and indicate their belief that the following Euclidean relations (*Elements*, book 10, prop. 28, lemma), were known to early Babylonians:

$$l = 2pq, \qquad s = p^2 - q^2, \qquad h = p^2 + q^2,$$

p and q being relatively prime, and $p > q$. Also that the following relation was not only known:

$$h/l = \tfrac{1}{2}(p\bar{q} + q\bar{p}),$$

but was the basis, with known tables of reciprocals, for the discovery of the successive entries of the tablet.

Line 13 of the tablet leads to the 15-fold values of the triangle with sides $l = 4$, $s = 3$, $h = 5$, and in the last line the values for the sides of the resulting triangle, have the common factor 2. All other solutions are relatively prime. For example, line 4 indicates the triangle $l = 13500$, $s = 12709$, $h = 18541$.

The authors point out that final remarks about the character of Babylonian mathematics in MKT III [1937, p. 80] contain the sentence: "Man wird also erwarten können, dass noch eine Art elementarer Zahlentheorie erkennbar wird—etwa so, dass das 'pythagoreisch' der älteren historischen Schule besser 'babylonisch' wird heissen dürfen." "This is fully confirmed," the authors continue, "by the text discussed here. We now have a text of purely number-theoretical character, treating a problem organically developed from other problems already well known and solved by using exactly those tools the development of which is so characteristic for Babylonian numerical methods."

What has been written above is naturally not intended as a review of this very remarkable contribution to scholarship. We were merely drawing attention to certain tables appropriate for consideration in *MTAC*.

R. C. A.

[1] Yale University, 91; University of Pennsylvania 82; Morgan Library 7; Metropolitan Museum 6; Plimpton Library 2; and others at Ann Arbor, Chicago, Ithaca, Princeton and Urbana. The largest and most interesting collection of mathematical tablets at any place in the world is at Yale University.

300[A, K, O].—NYMTP, *Binomial Distribution Functions*, New York, May, 1944, 10 leaves. Hectographed on one side of each sheet. 21.5 × 35.5 cm. A report submitted to the Applied Mathematics Panel National Defense Research Committee. These tables are available only to certain Government agencies and activities.

This report provides values of the function

$$P(n, r, p) = \sum_{k=0}^{r} {}_nC_k p^k q^{n-k}, \qquad p + q = 1,$$

computed over the following ranges of the three variables: n = [5(1)10(5)30(10)50; 6D]; $r = 1(1)(n - 1)$; $p = .01(.01).1(.1).5$.

The function finds special application in connection with the binomial frequency distribution in the theory of statistics. Thus, the probability that an event will happen at least r times in n trials is equal to $P(n, r, p)$, where p is the probability that the event will happen in a single trial, and q the probability that it will fail to happen.

It is of some interest to indicate the relationship of the values given in this report and those in the table of E. C. D. MOLINA in *Poisson's Exponential Binomial Limit* (see *MTAC*, v. 1, p. 18), where one finds the tabulation of the function

$$P(c, a) = \sum_{x=c}^{\infty} \frac{a^x e^{-a}}{x!}.$$

It is well known in the theory of probability that if $n \to \infty$, $p \to 0$, while $a = np$ remains constant, then the binomial function $P(n, r, p)$ will approach the limiting value $1 - P(r + 1, a)$. In applications involving small probabilities it has been the general practice to use the function $P(c, a)$ rather than the exact binomial function because of the analytical advantages of the Poisson formulation. But when p becomes fairly large, let us say greater than 1, then the discrepancy between the limit function and the exact function may become statistically significant. The order of the error may be inferred from the following values: When $n = 50$, $p = .01$, $r = 1, 2, 3$, we have for $P(n, r, p)$ the values: .910565, .986183, .998404, while for $a = 50 \times .01 = .5$, $r = 1, 2, 3$, we find for $1 - P(r + 1, a)$ the corresponding values: .908796, .985612, .998248.

H. T. D.

301[D].—HÉCTOR BALANDRANO, CARLOS CABALLERO, GUILLERMO ESTRADA, JORGE LELO DE LARREA, JOSÉ LICHI, GUSTAVO MONTES, MARIO MORALES, & LUIS MORONES, "Tablas numericas para el calculo de radiadares," Seminario de Radiación, Escuela Superior de Ingeniería Mecánica y Eléctrica, Sección Electromagnética, *Comisión Impulsora y Coordinadora de la Investigación Cientifica. Anuario 1944.* Mexico, 1945, p. 125–247. 16.9 × 23 cm.

These three tables calculated by students were as follows:

T.I, p. 128. $\pi l/\lambda$, for l/λ = [.1(.1)5; 6D]

T.II, p. 129–221. $x = (\pi l/\lambda)(1 - \cos\theta)$, and $x = (\pi l/\lambda)(1 + \cos\theta)$, to 5D, for $l/\lambda = .1(.1)5$, $\theta = 0(1°)180°$.

T.III, p. 222–247. (a) $\sin x$, (b) $\cos x$, (c) $\sin x/x$, (d) $\cos x/x$, $x = .001(.001)1$; to 9D for (a) and (b); to 7 or 8S for (c) and (d).

It is stated, that use was made of the 8D NYMTP, *Tables of Sines and Cosines for Radian Arguments*, 1940, but no reference is made to the 9D NYMTP, *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*, 1939 or 1940; or to tables by J. R. AIREY of $\sin x$ and $\cos x$, for x = [0(·001)1.6; 11D], which appeared in B.A.A.S., *Report*, 1916, p. 59–87, and in BAASMTC, v. 1, 1931, and 1946, p. 8–23. T.III, (c) and (d) seem to be new, although partially covered by K. HAYASHI, *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen*, Berlin, 1930, p. 30 f. at interval .01.

R. C. A.

302[D, L, M].—SO. OF AUTOMOTIVE ENGINEERS, WAR ENGINEERING BOARD, *The Development of Improved Means for Evaluating Effects of Torsional Vibration on Internal Combustion Engine Installations*, New York, 1945, p. 199–207, 345–347. 21.5 × 28 cm. Offset print.

Table 19, p. 199–206, is of $T(x) = \dfrac{180}{x} \tan x$, for x = [0(0°.1)360°; 4–5S] with average Δ.

Table 20, p. 207, is of $\dfrac{\pi}{180} x \tan x$, for x = [0(1°)360°; 4–5S].

On p. 345, are Tables of $K = \int_0^{\frac{1}{2}\pi} \frac{dx}{(1 - \sin^2 \frac{1}{2}\gamma_0 \sin^2 x)^{\frac{1}{2}}}$, of $(2/\pi)K$, and of $[(2/\pi)K]^2$, to 9D, for $\gamma_0 = 0(5°)180°$.

On p. 346–347, are Tables to 7–8S, of γ, $\sin \gamma$, $-2 \cos \gamma_0 \sin \gamma + \frac{1}{2} \sin 2\gamma$, $(2)^{\frac{1}{2}} \sin \gamma\ (\cos \gamma - \cos \gamma_0)^{\frac{1}{2}}$, for $\gamma_0 = 10°(10°)160°$, $\tau = 0(10°)90°$, where $\gamma = 2 \sin^{-1} [\sin \frac{1}{2}\gamma_0 \operatorname{sn} (H\tau, \sin \frac{1}{2}\gamma_0)]$, $H = 2K/\pi$. [$H\tau$ is the integral K with β substituted for $\frac{1}{2}\pi$ in the upper limit; hence $\sin \beta = \operatorname{sn} (H\tau, \sin \frac{1}{2}\gamma_0) = \sin \frac{1}{2}\gamma/\sin \frac{1}{2}\gamma_0$.]

303[D, E, K, L].—BAASMTC, *Mathematical Tables*, v. 1, *Circular & Hyperbolic Functions*, *Exponential & Sine & Cosine Integrals*, *Factorial Function & Allied Functions*, *Hermitian Probability Functions*. Second ed. Cambridge, University Press, for the BAAS, 1946. xii, 72 p. 20.9 × 28 cm. 10 shillings. New York, Macmillan & Co., 60 Fifth Avenue, $2.50.

The first edition of these tables was published fifteen years ago and we have had occasion frequently to refer to them in *MTAC*; e.g., v. 1, p. 15, 72, 73, 109, 323, 363. Errors in this edition listed elsewhere in the present issue, MTE **84**, were reprinted from a sheet procurable on application at the office of the BAAS, Burlington House, London, W.1. These errors were corrected in the stereotype plates used for printing the Mathematical Tables, p. (1)–(72) of the present edition. All other changes are in the introductory material, the 34 pages of the first edition being here reduced to 7 through elimination of descriptions of the functions tabulated, and bibliographical and historical data. Thus some libraries will naturally try to secure the first edition, with its errata sheet, as well as the second.

The tables in the volume are as follows:

I. Multiples of $\frac{1}{2}\pi[1(1)100; 15D]$, computed by L.J.C.
II. Circular sines and cosines, radian arg. [0(.1)50; 15D] c.b. A. T. Doodson, J. R. Airey & L.J.C.
III. Circular sines and cosines, [0(.001)1.6; 11D], c.b. J. R. Airey.
IV. Hyperbolic sines and cosines of πx [0(.0001).01; 15D], δ^2, c.b. L.J.C.
V. Hyperbolic sines and cosines of πx [0(.01)4; 15D], c.b. J. R. Airey.
VI. Hyperbolic sines and cosines of x [0(.1)10; 15D], c.b. J. R. Airey.
VII. Exponential integral [5(.1)15; 10–11S], and Ei $(\pm x) - \ln x$ [0(.1)5; 11D], δ^2, δ^4, δ^6 (some modified), c.b. R. A. Fisher & J. R. Airey.
VIII. Sine and cosine integrals [0(.1)5; 11D], [5(.1)20(.2)40; 10D], δ^2, δ^4, δ^6 (some modified), c.b. R. A. Fisher & J. R. Airey.
IX. Factorial function [0(.01)1; 12D], δ^2, δ^4, (some modified), c.b. R. A. Fisher.
X. Logarithmic factorial integral [0(.01)1; 10D], δ^2 (modified), c.b. G. N. Watson.
XI. Digamma function, $d \ln (x!)/dx$, [0(.01)1, 10(.1)60; 12D].
XII. Trigamma function, $d^2 \ln (x!)/dx^2$, [0(.01)1, 10(.1)60; 12D].
XIII. Tetragamma function, $-d^3 \ln (x!)/dx^3$, [0(.01)1, 10(.1)60; 12D].
XIV. Pentagamma function, $d^4 \ln (x!)/dx^4$, [0(.01)1, 10(.1)60; 10D, 12D] XI–XIV, δ^2, δ^4 (some modified), c.b. A. Lodge & XIV with the cooperation of J. Wishart.
XV. Hermitian probability functions, $Hh_n(x)$, $n = -7(1)+21$, $x = [\geq -7(.1) \leq +6.6; 6-10D]$.
XVI. $Hh_0(x)Hh_2(x)/\{Hh_1(x)\}^2$, $x = [-7(.1) + 5$; varies from 9D to 3D]. XV–XVI δ^2, δ^4 (some modified), c.b. J. R. Airey.

One new table by S. Johnston is given on p. xi. This extends each of the digits in the values of the factorial function, up to 18D; but corresponding changes in the differences are not indicated.

The 8 Constants to 16D in the first edition are now increased to 10 by the addition of $\pi/180$ and $180/\pi$.

The new edition was prepared by J. Henderson, L. M. Milne-Thomson, E. H. Neville, & D. H. Sadler.

This is a most admirable volume which ought to be in every college mathematical library. The gilt title along the back of the present edition is a great improvement over the inked title of the old one.

R. C. A.

304[D, M, P].—WERNER F. VOGEL, *Involutometry and Trigonometry*. Detroit, Mich., Michigan Tool Co., 7171 McNichols Rd., 1945. xii, 321 p. 19.5 × 27.2 cm. $20.00. Procurable only by direct application to the publisher.

This volume has been prepared primarily for the gear and tool engineer. In the words of the author, involutometry "deals with the involute of a circle and with surfaces containing a multitude of involutes." The volume comprises a Main Table and three appendices. The Main Table (p. 1–181) gives the values of the six circular functions and the two "involumetric" functions, $\text{inv}\,\phi = \tan\phi - \text{arc}\,\phi = \text{arc}\,\theta$ and $\text{coinv}\,\phi = \text{inv}\,(90° - \phi)$. These eight functions are tabulated to 7D, for $\phi = 0(0°.01)90°$. Two additional columns give the values in radians of arc ϕ (tabulated to 7D on odd-numbered pages), and the values of θ in degrees and decimals of a degree (tabulated on even-numbered pages). The first differences are tabulated alongside the corresponding functions, except for the column of arc ϕ whose differences are noted at the bottom of the pages.

For purposes of interpolation, a one-page critical table gives the values of $\phi° \cot\phi$ and appropriate values of $\cot\phi$, as well as the values of $\phi° \csc\phi$ and appropriate values of $\csc\phi$ for very small arguments.

Two other involumetric functions defined by the author are $\text{ev}\,\theta = \sec\phi$ and $\text{arc}\,\epsilon = \tan\phi$. The variables ϕ, θ and ϵ are referred to by the gear engineer as the pressure, polar and roll angle of the involute, respectively. Various types of problems involving these angles as well as the radius vector $r = r_b\,\text{ev}\,\theta$ and the radius of curvature $\rho = r_b \tan\phi$, where r_b is the radius of the base circle, may be readily solved with the aid of the Main Table. Several illustrations of such problems are given by the author in Appendix C (p. 298–321). This appendix is devoted to an exposition of the salient points of involutometry such as the generation of the involute of a circle and its geometric properties, the definition of the involumetric functions and a description of curves related to the involute. Among other topics briefly sketched are: *Solid Involutometry* dealing with surfaces containing a multitude of involutes (a typical example is the surface of revolution generated by the rotation of an involute about a given axis) and *Spherical Involutometry* dealing with the involute defined as "the path of any point on the surface of a base cone, when the surface is unwrapped from the base cone and held tightly during this procedure."

Appendix A (p. 189–240) consists of

(a) Various conversion tables (from radians to degrees and decimal fractions of the degree, from degrees and decimal fractions of the degree to radians, etc.)

(b) Polygon tables $\left(\text{values of } \frac{180°}{N}, \sin\frac{180°}{N}, \cos\frac{180°}{N}, 1 - \cos\frac{180°}{N}\right)$ for $N = 1(1)300$. Compare *MTAC*, v. 1, p. 312–313.

(c) Collections of formulae and integrals of trigonometry and involutometry.

(d) A set of involute tracing charts, involute locating charts and cycloid tracing tables.

Appendix B (p. 241–296) consists of a collection of numerical diagrams, tables and charts required in gear calculations.

The tabular material of the six circular functions is based on the tables of BRIGGS in his *Trigonometria Britannica*, Gouda, 1633. Seven-place values of the sine, cosine, tangent and cotangent at interval 0°.001, based on the same source, have also been compiled by J. PETERS (1918) and made available in this country (1942; see *MTAC*, v. 1, p. 12–13). The volume under review possesses the pleasant feature of exhibiting all the circular functions as well as the other four functions above mentioned on two pages facing each other. The usefulness of the volume is further enhanced by the happy choice of types, as well as the excellent charts and diagrams. The author and publishers are to be congratulated on the meticulous planning of this volume which manifests a constant endeavor to place at the disposal of the tool engineer a complete and lucid exposition of the mathematical foundation for his art, as well as an extremely useful table for the solution of his problems.

ARNOLD N. LOWAN

EDITORIAL NOTES: Attention may be also directed to *MTAC*, v. 1, p. 88–92, where there is discussion of an 8-place table, by EARLE BUCKINGHAM, of sin ϕ, cos ϕ, tan ϕ, cot ϕ, $\phi = 0(0^\circ.01)90^\circ$; and inv. ϕ at interval $0^\circ.01$ up to 60°; 12D are given up to $0^\circ.5$, 10 to 1°, 8 to 37°, and 7 to 60°.

Those inquiring as to the originality of Mr. Vogel's work should compare it with J. T. PETERS, *Sechsstellige Werte der Kreis- und Evolventen-Funktionen von Hundertstel zu Hundertstel des Grades nebst einigen Hilfstafeln für die Zahnradtechnik*, Berlin and Bonn, 1937. For example, except for 7-place tables, instead of 6-place tables, p. 2–181 of Vogel are identical in every respect, including order of columns, spacing, face of type, and rules, with p. 2–181 of Peters; the auxiliary interpolation table of p. 182 of Vogel is not expanded at all, but is identical with that of Peters.

In reviewing a previous publication of W. F. Vogel (*MTAC*, v. 1, p. 312) a statement of Peters was interpreted as suggesting that Vogel was a former member of the computing staff of the Rechen-Institut. We are informed that this inference was incorrect.

305[F, Z].—RAPHAEL M. ROBINSON, *Stencils for Solving $x^2 \equiv a$ (mod m)*. Berkeley and Los Angeles, Univ. of California Press, 1940. 272 stencils with a booklet of 14 p.

This set of stencils for solving quadratic congruences appeared in 1940 just when the reviewer's *Guide to Tables in the Theory of Numbers* was in page proof so that it received there only the briefest attention. With the present or pending "reconversion" of many number-theorists from war work it may be of interest to call attention to this tool for the solution of a problem familiar to such computers.

The most general quadratic congruence, as is well known, may be reduced to solving for x the congruence

$$x^2 \equiv a \pmod{m} \qquad (1)$$

or the equivalent diophantine equation

$$x^2 = a + my. \qquad (2)$$

According to Gauss' method of exclusion a convenient set of excluding numbers E is used to determine y by considering (2) modulo E. Those values of y modulo E which make $a + my$ a quadratic non-residue (mod E) are clearly impossible. Thus for each E, y is restricted to a set of about $\frac{1}{2}E$ arithmetic progressions, each with the difference E.

The present stencils offer a rapid and convenient way of finding the one or two values of y which are common to these sets of arithmetic progressions. For each E, the possible values of y are represented by holes punched in a standard IBM card. To find y in (2), and hence x in (1), the computer simply selects nine cards (corresponding to $E = 5, 7, 9, 11, 13, 16, 17, 19, 23$) from the set of 272 cards depending on the values of a and m modulo E. Superposing the cards gives at a glance the values of y common to the cards, and hence those y's for which (2) has a chance of a solution. Each of these is then tried in (2) by comparing the corresponding values of $a + my$ with a table of squares. In this way quadratic congruences with moduli < 3000 are easily solved. The limit is set by the facts that $y < m/4$, and the capacity of the IBM card allows $0 \leq y < 750$. By using the stencils n times, however, congruences for $m < 750(n + 1)^2$ may be solved, as explained in the booklet accompanying the stencils.

This is the second use so far for IBM cards as a coincidence mechanism, the first being that in the revised edition of Lehmer's *Factor Stencils* by J. D. ELDER.[1] In each case the cards are heavily punched in a way never contemplated by orthodox IBM usage, but in the Robinson stencils the holes follow a regular pattern modulo E so that a simple process of repeated offset gang punching and comparing by the reproducer produced and checked the whole set of 2E cards from a single card having simply the multiples of E punched. Besides being a useful tool the set of stencils serves as model for other sets for the solution of a wide variety of similar problems.

D. H. L.

[1] D. N. LEHMER, *Factor Stencils, Revised and extended* by J. D. Elder, Washington, Carnegie Institution, 1939, 27 p. + 2135 stencils.

306[J, L, M].—L. S. GODDARD, "On the summation of certain trigonometric series," Cambridge Phil. So., *Proc.*, v. 41, part 2, Aug. 1945, p. 145–160. 17.3 × 25.6 cm.

The following functions, (a)–(d), are tabulated, $n = [1(1)10; 4D]$:

(a) $-S_n^1(\alpha) = \alpha^{-1}\int_0^\pi \sin^2 nx \cot(x/\alpha)dx$, $\alpha = 1(.25)1.5(.5)3(1)8, \infty$; for $\alpha = \infty$, $-S_n^1(\alpha) = \frac{1}{2}\operatorname{cin}(2\pi n) = \frac{1}{2}\int_0^{2\pi n}\frac{1-\cos x}{x}dx$.

(b) $T_n^1(\alpha)/n = (2\alpha)^{-1}\int_0^\pi (\pi - x)\sin 2nx \cot(x/\alpha)dx - \frac{1}{4}\pi^2\alpha$ (the term $\frac{1}{4}\pi^2\alpha$ being omitted when n is integral but αn is non-integral), $\alpha = 1(.25)1.5, 2, 4, 8, \infty$; for $\alpha = \infty$, $T_n^1(\alpha)/n = \frac{1}{2}\pi\int_0^{2\pi n}\frac{\sin x}{x}dx$.

(c) $-S_n^3(\alpha) = -\alpha^{-2}n^{-2}S_n^1(\alpha) + \theta_3(\alpha)$, $\alpha = 1(.25)1.5(.5)3(1)8$; $\theta_3(\alpha) = \sum_{m=1}^{\infty} m^{-3}\sin^2(\pi m/\alpha)$.

(d) $T_n^3(\alpha) = \alpha^{-2}n^{-2}[T_n^2(\alpha) - S_n^1(\alpha)]$, α as in (c).

There are also tables to 4D of

(e) $-S_n^3(\alpha) = -\pi^2(\alpha-1)/2\alpha^2$; (f) $T_n^2(\alpha) = \pi^2/(4\alpha)$, (g) $\alpha^2\theta_3(\alpha)$, for α as in (c).

Following S. A. SCHELKUNOFF, *Quart. Appl. Math.*, v. 2, 1944, p. 90, Goddard uses the notation $\operatorname{cin} x = \int_0^x \frac{1-\cos t}{t}dt$, p. 157.

R. C. A.

307[K].—H. O. HARTLEY & M. SUMNER, "Tables of the probability integral of the mean deviation in normal samples," *Biometrika*, v. 33, Nov. 1945, p. 252–265. 19.3 × 27.4 cm. These pages are occupied as follows: "The probability integral of the mean deviation, editorial note" by E. S. PEARSON, p. 252–253; "On the distribution of the estimate of mean deviation obtained from samples from a normal population" by H. J. GODWIN, p. 254–256; "Appendix, Note on the calculation of the distribution of the estimate of mean deviation in normal samples" by H. O. HARTLEY, p. 257–258; tables, p. 259–265.

Suppose $x_1, x_2, \cdots, x_n$ are the values of x in a sample from a normal population in which the distribution of x is $(2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$. The mean deviation m of the sample is defined by

$$m = n^{-1}\sum_{i=1}^{n}|x_i - \bar{x}|, \qquad \text{where } \bar{x} = n^{-1}\sum_{i=1}^{n}x_i.$$

GODWIN has shown that the probability element $f_n(m)dm$ for m is given by

$$(1)\quad f_n(m)dm = \frac{n^{\frac{1}{2}}}{2^{\frac{1}{2}(n+1)}\pi^{\frac{1}{2}(n-1)}}\left\{\sum_{k=1}^{n-1}\binom{n}{k}\exp\left[-\frac{m^2n^3}{8k(n-k)}\right]G_{k-1}(\tfrac{1}{2}nm)G_{n-k-1}(\tfrac{1}{2}nm)\right\}dm$$

where $G_r(x)$ is defined by

$$G_r(x) = \int_0^x \exp\left[-\frac{t^2}{2r(r+1)}\right] G_{r-1}(t)dt, \tag{2}$$

and $G_0(x) = 1$.

HARTLEY has tabulated

$$\int_0^m f_n(m)dm \tag{3}$$

for n = [2(1)10; 5D], and for m = 0(.01)3, by a recurrence procedure of numerical quadratures. The starting point of the recurrence was the table of

$$\pi^{-\frac{1}{2}}G_1(x) = \pi^{-\frac{1}{2}}\int_0^x e^{-(\frac{1}{2}t)^2}dt = 2\pi^{-\frac{1}{2}}\int_0^{\frac{1}{2}x} e^{-\alpha^2}d\alpha, \tag{4}$$

given in NYMTP, *Tables of the Probability Integral*, v. 1, 1941. This integral was multiplied by $2\pi^{-\frac{1}{2}}e^{-(\frac{1}{2}x)^2}$ which is also tabulated in the NYMTP *Tables*. Products were formed at interval .05 in x. The function $480\pi^{-1}2^{-\frac{1}{2}}G_2(x)$ was obtained in accordance with (2) by numerical quadrature; the factor 480 was introduced because of the particular method of quadrature used. This process was repeated for r = 3(1)8, thus producing the functions $G_r(x)$. By writing

$$g_r(x) = G_r(x)\cdot\exp\left[-\frac{x^2}{2(r+1)}\right] \tag{5}$$

where $x = \frac{1}{2}nm$, the expression (1) may be rewritten as

$$f_n(m) = n!2^{-\frac{1}{2}(n+1)}\pi^{-\frac{1}{2}(n-1)}\sum_{k=1}^{n}\binom{n}{k} g_{k-1}(x)g_{n-k-1}(x). \tag{6}$$

The functions $g_r(x)$ were obtained by the multiplication indicated by (5), and the computation of the integral (3) was finally made by using expression (6).

A table of values of m_l (lower percentage points), for which

$$\int_0^{m_l} f_n(m)dm = .001, .005, .01, .025, .05, .1,$$

is given for n = 2(1)10; also a table of values of m_u (upper percentage points), for which

$$1 - \int_0^{m_u} f_n(m)dm = .001, .005, .01, .025, .05, .1,$$

is given for n = 2(1)10.

S. S. W.

308[L].—H. BATEMAN, "Some integral equations of potential theory," *J. Appl. Physics*, v. 17, Feb. 1946, p. 91–102. 20 × 26.8 cm.

Incidentally there is a table (p. 98–99) of $P_n(1 - 2e^{-t})$, for n = 1(1)10, t = [1(1)20; 15D], where $P_n(x)$ is the polynomial of Legendre of order n. Quotation: "It is thought that this and similar tables may be useful for the tabulation of functions defined by definite integrals, which are hard to compute directly but have simple Laplace transforms."

309[L].—V. FOK, "Raspredelenie tokov, vozbuzhdaemykh ploskoĭ volnoĭ na poverkhnosti provodnika" [Distribution of currents induced by a plane wave on the surface of a conductor], Akademiia Nauk, SSSR, *Zhurnal Eksperimental'noĭ i Teoreticheskoĭ Fiziki*, v. 15, no. 12, Dec. 1945, p. 700–701. 16.5 × 25.8 cm.

$g(x) = \int e^{(i-1)tx+\frac{1}{3}it^3}dt$, $G(x) = e^{\frac{1}{3}ix^3}g(x)$. Re G, Im G, and $|G|$ are given for x = [− 4.5(.1) − 3.5; 4D], [− 3.4(.1)1; 3D]; arc G is given to the nearest 1″ to −3.5, and to the nearest 1′ thereafter. Further, Re g, Im g, and $|g|$ are given for x = [−1(.1)2.4; 3D], [2.5(.1)4.5; 4D], and arc g is given to the nearest 1′.

310[L].—J. C. Jaeger, "On thermal stresses in circular cylinders," *Phil. Mag.*, s. 7, v. 36, 1945, p. 419–423. 17.1 × 25.5 cm.

Consider the thermal stresses in the cylinder $0 \leq r < a$, initially at constant temperature V, whose surface is kept at zero temperature for $t > 0$. If E is Young's modulus, ν Poisson's ratio, α the coefficient of linear expansion of the solid, v its temperature at radius r, K, ρ, c the thermal conductivity, density, and specific heat of the material of the cylinder, and $\kappa = K/\rho c$; and if, finally σ_r and σ_θ are the radial and tangential stresses, then

$$(1-\nu)\sigma_r/E\alpha V = 2\sum_{s=1}^{\infty} \frac{e^{-\kappa\beta_s^2 t/a^2}}{\beta_s^2 J_1(\beta_s)}\left\{ J_1(\beta_s) - \frac{a}{r}J_1\left(\frac{r\beta_s}{a}\right)\right\}$$

$$(1-\nu)\sigma_\theta/E\alpha V = 2\sum_{s=1}^{\infty} \frac{e^{-\kappa\beta_s^2 t/a^2}}{\beta_s^2 J_1(\beta_s)}\left\{ J_1(\beta_s) + \frac{a}{r}J_1\left(\frac{r\beta_s}{a}\right) - \beta_s J_0\left(\frac{r\beta_s}{a}\right)\right\}$$

where $\pm\beta_s$, $s = 1, 2, \cdots$, are the roots of $J_0(\beta) = 0$. There are tables of (i) $(1-\nu)\sigma_r/E\alpha V$ and (ii) $(1-\nu)\sigma_\theta/E\alpha V$ for $\kappa t/a^2 = [.005(.005).02(.01).1(.05).2(.1)1; 4D]$, $r/a = 0(.1).7(.05).95$; and in the case of (ii) also for $r/a = 1$. On p. 423 are corresponding graphs of (i) and (ii) for $0 \leq \kappa t/a^2 \leq .1$.

311[L].—E. L. Kaplan, "Auxiliary table of complete elliptic integrals," *J. Math. Phys.*, v. 25, Feb. 1946, p. 26–36. 17.4 × 25.4 cm.

The integrals in question are

$$K = \int_0^{\frac{1}{2}\pi} (1 - k^2 \sin^2\phi)^{-\frac{1}{2}} d\phi, \qquad E = \int_0^{\frac{1}{2}\pi} (1 - k^2 \sin^2\phi)^{\frac{1}{2}} d\phi.$$

The table (p. 29–36) gives values of K and E as functions of $\log k'^2 = -\lambda$, for $\lambda = [1(.005)2(.01)6; 10D]$, with $\mu\delta^2$ or $\frac{1}{4}\mu\delta^2$, where $k'^2 = 1 - k^2$.

The only comparable table which facilitates the getting of values of K and E in the neighborhood of their singularity is that of J. R. Airey, "Toroidal functions and the complete elliptic integrals," *Phil. Mag.*, s. 7, v. 19, 1935, p. 180–187. He has tabulated to 10D functions K_1, K_2, E_1, E_2, such that

$$K = K_1 \ln(4/k') - K_2, \qquad E = E_1 \ln(4/k') + E_2.$$

For each K and E there are three quantities to be obtained from tables, and some arithmetical operations to be done besides. The importance of these functions makes it worthwhile to provide a more direct means of getting their values for small values of k'. With the present table the only preliminary to extracting the value desired is the determination of $\log k'^2$. The tabular argument is given as a logarithm with a negative integral part and a positive decimal, in order that the user of the table may be spared the trouble of getting the cologarithm when k'^2 is given as a decimal.

The method of the arithmetico-geometric mean, followed by subtabulation to one-tenth of the original interval, was used for $1 \leq \lambda \leq 2$, while the rest of the table was computed by means of auxiliary functions like Airey's. In both cases the check was by differencing. The use of both methods for $\lambda = 2$ is a check on errors of procedure. From eleven to fourteen decimals were retained.

The most useful general tables of K and E are those of K. Hayashi (*Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen*, Berlin, 1930, p. 72–81) who gives the values for $k^2 = [0(.001)1; 10D]$. When k^2 is near unity, however, it is difficult or impossible to interpolate in these tables—hence the need for some kind of auxiliary table. Since Hayashi's tables in general are notorious for their inaccuracy, the writer checked these by differencing (generally every third one of the fourth differences) either the values themselves or auxiliary functions calculated therefrom. They were also compared with the table of L. M. Milne-Thomson ("Ten-figure table of the complete elliptic integrals," London Math. So., *Proc.* s. 2, v. 33, 1930, p. 162–163), for $k^2 = [0(.01)1; 9D]$, which served only to show that the last figure in Milne-Thomson's table is unreliable. The following errors were found in Hayashi's table (only the four starred values were actually recomputed):

	K				E		
k^2	Decimals	*For*	*Read*	k^2	Decimals	*For*	*Read*
.999	8–9	06	60*	.052	9–10	45	55*
.013	8–9	68	86	.939	9	4	3
.095	0	0	1	.936	9	9	8
.737	9–10	56	65*	.201	3–4	65	86
.493	9	4	9	.732	10	6	8*
				.668	0	0	1
				.669	The first two digits of the argument should be given.		

The eleventh and twelfth decimals given in a portion of the table were not checked, nor were any of the rather inadequate auxiliary tables.

Extracts from introductory text

312[U].—E. E. Benest & E. M. Timberlake, *Astro-Navigation Tables for the Common Tangent Method specially developed and arranged.* Cambridge, Heffer, 1945. 107 p. 21.5 × 27.8 cm. 7s. 6d.

These tables were prepared for use with the common tangent method, a method recommended, as the authors point out, a number of years ago by Radler de Aquino. They do not mention the recent excellent *Astronomical Position Lines Tables* of Juan García (RMT 272) which are intended for use with the same method.

The tables are somewhat similar in content and arrangement to those in Weems' *Line of Position Book* (RMT 298). However instead of determining an altitude and an azimuth for a single assumed position, two altitudes are computed for adjacent integral latitudes and the line of position is drawn as one of the common tangents to two circles with centers at the two assumed positions and with radii equal to their respective altitude intercepts. Six or seven "figures" are given in the logarithmic values (corresponding to 10^5 or 10^6 times five- or six-place logarithms of secants and cosecants with enough zeros placed at the left in each case to provide six or seven "figures"), and it is stated that "In general: 5 figures will give an altitude to an accuracy of 2′, 6 figures will give an altitude to an accuracy of 1′, 6 figures, with interpolation, will give an altitude to an accuracy of 0.′1,"

The astronomical triangle is divided into two right triangles by a perpendicular dropped from the zenith upon the hour circle of the celestial object. K is the latitude of the foot of the perpendicular, or more precisely, it is the angle from the celestial equator to the foot of the perpendicular, measured toward, and if necessary, through the elevated pole. A is usually 10^5 log sec a where a is the length of the perpendicular mentioned above. For $LHA < 11°$, $L < 38°$, and for a few values for $LHA = 11°, 12°$, $A = 10^6$ log sec a and values are given to seven "figures."

The formulae usually employed are: $A = 10^5 \log \sec a$; $B = 10^5 \log \sec (K \sim d)$; $C = 10^5 \log \csc h = A + B$.

The first table (A and K) covers 60 pages. The vertical argument is latitude = 0(1°)90° given in two columns on facing pages, 0 to 45° on the left and 45° to 90° on the right. Three integral values of local hour angle appear on each such pair of pages with the single exception that the first page contains only all of the values corresponding to zero latitude. On page 6 in the explanation, it is erroneously stated, "latitudes 0° to 45° always on the left-hand page, and . . . ." In addition to the usual headings corresponding to local hour angle = 0(1°)90°, there are also other headings on each page corresponding to values of the local hour angle in the other three quadrants. The tabulated quantities in this table are **K** as defined above given to the nearest tenth of a minute of arc in two columns corresponding to the local hour angle in the first or fourth quadrant, and second or third quadrant. **A** is given to six or seven "figures" with the sixth or seventh set off to the right for the convenience of the air navigator who may wish to use only five or six.

In **Table B** which covers 17 pages, the horizontal argument is $K \sim d = 0(1°)90°$, six values to a page except at the end, and the vertical argument is the integral minutes of $K \sim d$. The tabulated quantity is B, $10^5 \log \sec (K \sim d)$ given to six figures with the sixth set off, for $K \sim d$ 10° to 90° and $10^6 \log \sec (K \sim d)$ given to seven figures with the seventh set off, for $K \sim d$ 0 to 10°. In addition, to the right of each column of values of B is a column giving proportional parts (1 to 9 tenths) of the tabular differences, making interpolation very simple. For values of $K \sim d$, 86° to 89° inclusive, special interpolation tables giving 1 to 99 hundredths of the tabular differences are given, but even with these, an uncertainty of as much as 100 units in the last place given may exist in the interpolated value; even so, the uncertainty in the calculated altitude will usually be of the order of a tenth of a minute of arc or less.

Table C covers ten pages and has as its horizontal argument, the integral degrees of altitude $= 0(1°)90°$, 8 values to a page. The vertical argument is C, $10^5 \log \csc h$ given to six figures with the sixth set off, for h 0 to 80°; $10^6 \log \csc h$ given to seven figures with the seventh set off, for 80° to 90°. This arrangement by which logarithms are sometimes multiplied by 10^5 and sometimes by 10^6, and in which sometimes the sixth figure is set apart, sometimes the seventh, appears to be needlessly confusing. It would be better if a few decimal points were used, and if the figures set aside were always the sixth, or the sixth and the seventh.

The final table is one of four pages for transferring position lines in high latitudes of the northern and southern hemispheres. A brief explanation is given of its use.

On page 5 in the explanation, the statement is made that "The plot in Fig. 2 shows the two possible ways of plotting an almost E./W. position line for the body near the meridian," whereas this figure is simply a repetition of Fig. 1 with letters replaced by numbers, and appears not to be the plot the authors had in mind. Such careless errors as have been noted in the explanation, leave one with a feeling that the tables have been hastily prepared and perhaps cannot be relied upon. As a check on the numerical values, those of A given for hour angles 78°–89° inclusive (1080 values) have been compared with a corrected copy of DREISONSTOK (H.O. 208, RMT **103**); 96 errors, each of a single unit in the last place given, were found. If the sample is characteristic of the tables, the numerical values are almost as good as those in COMRIE'S *Hughes' Tables for Sea and Air Navigation* (RMT **115**).

CHARLES H. SMILEY

Brown University

313[U].—GEORGE G. HOEHNE, *Practical Celestial Air Navigation Tables. Volume II, Latitudes* 20° *to* 39° *North inclusive.* Miami, The Navigation Publishing Co., 1943, 256 p. 15 × 24.3 cm. Not available for purchase.

This volume is the only one of a projected set of fourteen to appear. There were to have been nine volumes, each containing altitudes and azimuths for twenty-three navigational stars and covering twenty degrees of latitude in a specified hemisphere, except the last volume which was to cover the regions 80° to 89°, North and South. Five other volumes were to have been provided for the sun, moon, planets and such navigational stars as occurred in the zone of declinations, −30° to +30°; these were to have been arranged for declinations of the same and contrary name to the latitude, and hence could have been used in either hemisphere. Since this volume has been removed from the market because of copyright difficulties (see below), it does not seem likely that the remaining thirteen volumes will ever appear.

The principal tables consist of twenty sections of twelve pages each, one for each of the twenty integral latitudes covered. A thumb index cut into the edge of the volume allows one to find quickly the material corresponding to a particular latitude. The vertical argument is T, Local Hour Angle of the March Equinox (or Local Sidereal Time) [0, 1°, 360°]. Each column is headed by the names of one or two stars. Occasionally a column is interrupted to allow a new star to replace one which has reached a position where it can no longer be satisfactorily observed; the name of the second star appears on the second line at the top of the column and again below where the tabular material is interrupted. Each pair of facing pages carries ten columns and covers 60° of the Local Hour Angle of the March Equinox. The tabulated quantities are altitude plus refraction for an elevation of five thousand feet above sea level, the sum rounded off to the nearest minute, and azimuth (not the usual *azimuth angle*), to the nearest degree.

The Preface indicates that this arrangement is original with the author; according to G. A. PATTERSON of the U. S. Hydrographic Office, it was first suggested by two Americans at about the same time, HOEHNE and HUTCHINGS. It is a very practical arrangement since it brings together on a single pair of facing pages, all of the material likely to be of use to a navigator at a given time and place, and since it eliminates the need for a "star finder" or a similar device. It is true that two entries are required for a given altitude, latitude and star, and one for the star rising and another for it setting. This does not seem very serious when it is realized that there is one less chance for an error of judgment inasmuch as this arrangement allows azimuth to be tabulated instead of azimuth angle. Probably the chief weakness of the tables is that it will be necessary to recompute them from time to time, due to the effects of precession on star positions. Hoehne has partially met this difficulty by two ingenious devices. He has provided a table on the inside of the front cover, "Correction due to Annual Change in Declination to Tabulated Altitude of Stars," and another, Table VII, "Correction to GHA Aries due to Change in Right Ascension of Stars."

Twenty-three stars are listed in this volume, Aldebaran, Alpheratz, Alphecca, Altair, Antares, Arcturus, Betelgeuse, Capella, Caph, Canopus, Deneb, Denebola, Dubhe, Etamin, Fomalhaut, Pollux, Procyon, Rasalague, Regulus, Rigel, Sirius, Spica, Vega. Not every star appears at each latitude; for example, Etamin appears for latitude 31° but not for 35°. Comparing this series of stars with those which appear in the *Astronomical Navigation Tables* (see *MTAC*, v. 1, p. 82 f, hereafter called the *ANT*), of which the American copy is called H.O. 218, it is found that Alphecca, Caph, Denebola, Etamin and Rasalague do not appear in the *ANT*, and that Achernar, Acrux, Peacock and Rigil Kent, which do appear in the *ANT*, are not included in this volume of Hoehne, presumably since it is designed for northern latitudes only.

In the Preface, the following statement occurs:

"The scheme of tabulating the precomputed altitudes and azimuths for integral degrees of Local Hour Angle Aries instead of local hour angle for the stars was conceived and designed by the author and made possible to publish by extracting and interpolating for declination the values of altitude and azimuth from Hydrographic Office Publication 214."

Because of the copyright difficulties mentioned above, it seemed desirable to compare altitudes as given in Hoehne and *ANT* and as they would be interpolated from H.O. 214 (see *MTAC*, v. 1, p. 81 f). Since the values of the altitudes for $d = 45°$, same name as latitude, given in H.O. 214, v. 4, had recently been recomputed, the star Deneb ($d = 45° 04'$) was used as a basis of comparison. All values tabulated in Hoehne for this star for latitudes 31° and 35° were compared with values interpolated in H.O. 214 and corrected for refraction according to the table used in the construction of the *ANT*. Of the 16 values for $L = 31°$ and 12 for $L = 35°$ [$d = 45°$, same name], in H.O. 214 known to be in error by two or more units in the last place given, six give values which, when rounded off to the nearest minute, would be one minute in error. In every one of these cases, Hoehne gave the value given in *ANT*, not the erroneous value obtained from H.O. 214. In fact, no differences were found between Hoehne and the *ANT* in 1052 values compared while in a series of 261 values, 25 discrepancies were found between Hoehne and values interpolated from H.O. 214 for stars found in the *ANT*. On the other hand, 75 altitudes for Rasalague, $L = 35°$N, were computed by H.O. 214 and compared with the corresponding values in Hoehne; there are no altitudes for Rasalague in the *ANT*. There were only three discrepancies in the 75 values, each corresponding to a rounding-off "error" resulting from a difference of only 0ʹ.1 in the interpolated value. This is about what would be expected as a result of the interpolation being carried out by two different persons. The evidence presented above seems to indicate that Hoehne copied most of his altitudes from *ANT* and actually interpolated from H.O. 214 only the relatively few altitudes appearing for the five stars mentioned above which do not appear in the *ANT* and such altitudes of other stars as fall between 5° and 10°. This is serious since the *ANT* are copyrighted and the American copy, H.O. 218, restricted.

It seems most unfortunate that an idea as basically sound and as ingeniously implemented as Hoehne's should have to be shelved. It is to be hoped and expected that a new table, with values interpolated from H.O. 214, or preferably computed afresh, will appear in the near future.

In addition to the tables mentioned above, there are the usual auxiliary tables, arc to time, time to arc, etc.

CHARLES H. SMILEY

314[U].—G. F. MARTELLI, *Martelli's Navigational Tables*, Glasgow, D. M'Gregor & Co., 1946, xi, 50 p. 14.5 × 24.3 cm., 7 shillings and 6 pence.

This is a new edition of a set of tables that has been in use in surface navigation for more than seventy years, principally by British navigators. In 1873, the "Lightning Printing Office," New Orleans, published an edition of *G. F. Martelli's Tables of Logarithms, a simple and accurate method for finding the Apparent Time;* the volume reviewed here has almost the same number of pages and is about the same size. The preface in the 1873 edition, signed by the author, states that "This work was completed in 1853. I now submit it to the seafaring community trusting that its simplicity and accuracy will obtain for it a universal welcome. . . ." It is uncertain whether an earlier edition had been published. The examples worked in the Lightning edition were for 1863 and 1864 and there were letters of commendation for the method signed by Lt. M. F. Maury [1806–1873], U.S.N., and Lt. R. Maxwell, U.S.N., which suggest that these gentlemen had had an opportunity to examine the tables and to try them out. To date no trace of the personal history of G. F. Martelli has been found by the reviewer.

It is interesting to note that, although the tables have been reviewed very unfavorably on several occasions, they have nevertheless been used widely. A "revised edition with additional tables," published by D. M'Gregor and Co. in 1932, still carried examples worked out for 1888. Except for a slight extension of Table 1 (from 71° 35′ to 90° inclusive), improved tables for refraction, correction for sun, planets and stars, and modern examples following current British usage, the 1946 edition is practically identical with that of 1932.

The tables were designed to provide a rapid solution of the spherical triangle based on a slightly modified form of the well-known haversine-cosine formula,

$$\text{hav } z = \text{hav } (L \sim d) + \cos L \cos d \text{ hav } t,$$

where z is the zenith distance of the celestial body; L and d are the latitude of the observer and the declination of the body respectively; t is the meridian angle of the body. Zenith distance (and hence altitude) are said to be determined to the nearest minute of arc.

Table 1, Log. of Lat. and Declination, gives the values of $10^4(.5 + \log \cos x)$ to the nearest integer for $x = 0(1')90°$. In **Table 2, Sum or Difference,** the argument is $x = 0(0'.1)90°$. The tabulated quantity is $10^4(1.2 - 2 \text{ hav } x)$, disguised as a time interval by the use of $0^s.1$ as the unit; thus for $x = 0$, the tabular value would be, not 12000 but $20^m0^s.0$. In Tables 3 and 4, the same device is used. In **Table 3, Angle of Altitude,** the argument is altitude, $h = 0(1')90°$, and the tabulated quantity is $2(10^4) \text{ hav } z$ where z is the zenith distance of the observed body. In **Table 4, Auxiliary Logarithms,** the argument in "minutes and seconds" is $10^4(1.2 + 2 \text{ hav } x)$ for $20^m0^s(0^s.1)36^m59^s$, and the tabulated quantity is $\log(1.08/\text{hav } x)$, to 4D. In **Table 5,** the argument is hour angle, $t = 16^h(5^s)7^h$, and the tabulated quantity is $\log(10.8/\text{hav } t)$ given to 4D.

It will be seen that the haversine-cosine formula can be written

$$(10.8/\text{hav } t) = \sqrt{10}\cos L\sqrt{10}\cos d \frac{1.08}{\text{hav } z - \text{hav } (L \sim d)}$$

and the logarithm of the fraction on the right evaluated by the use of Tables 2, 3, and 4. Table 1 gives the value of log ($\sqrt{10}\cos x$) and Table 5 allows one to find t from the fraction on the left.

The examples of the determination of a position line are carried out in detail for three distinct methods: the longitude method, intercept method, and latitude method. In the first of these, a latitude is assumed and the corresponding longitude on the line of position is computed. The second is the well-known Marcq St. Hilaire method, depending on the difference between a measured altitude and a computed altitude. In the third, a longitude is assumed and the corresponding latitude computed. The azimuth is always computed the same way, using Table 1 only. In each case, the resulting three lines of position, obtained by the three methods, are plotted on a large scale chart and are shown to be essentially the same. Examples are given of the computation of the deviation of a magnetic compass, the amplitude of a rising (or setting) body, the identification of stars, distance and initial course along a great circle and position of the vertex of a great circle.

To determine the probable accuracy of the tabulated values, 1000 values in Table 1 were checked, and 10 errors, each of a unit in the last place due to rounding off, were found. In Table 2, 19 rounding-off errors were found in 500 tabular values, and in Table 3, 16 rounding-off errors were found in 500 values. Since the tables are essentially based on four-place logarithms, it would seem desirable that rounding-off errors be eliminated from the tables, or at the very least, the rounding-off be not done systematically the same way throughout, as appears to be the case in some of the tables.

Remembering that the tables as originally devised did not allow the use of declinations or latitudes greater than 71°34′ and hence avoided negative values in Table 1, one must admire the ingenuity of Martelli in providing tables which have served navigators so long and so well. There would be an advantage, as Browning E. Marean has pointed out, in having an American edition of these tables with explanations designed for use with GHA of sun, moon, planets and star, as tabulated in the *American Nautical Almanac* and *American Air Almanac.*

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EDITORIAL NOTE: According to information supplied to us by the publisher, tables identical with those of Martelli were published by a Mr. POUVREAU, in a French edition of 1885. This is evidently the following work in the Brown University Library: GEORGES POUVREAU, *Nouvelles Tables de Mer pour le calcul de la Hauteur de l'Heure et de l'Azimut.* Paris, Gauthier-Villars, 1885. xvi, 70 p. While it is true that the five tables of Pouvreau are based on the same formulae as those of Martelli, but entirely differently ordered, all of the Pouvreau tables are appreciably more extensive than those of Martelli, published up to 1885, or even later. The Tables 1, 4 and 5, to 4D in Martelli, are to 5D in Pouvreau, whose Table 1 is carried to 90°, instead of to 71° 34′. Martelli's Tables 2 and 3, giving values at interval 0″.1, are at interval 0″.01 in Pouvreau. Indeed it may well be that Pouvreau had no knowledge of Martelli's little volume of tables, which gave no formulae.

Martelli's tables have been discussed as follows in the U. S. Naval Institute, *Proc.*: J. P. JACKSON, "A short and simple method of finding the longitude at sea," v. 46, 1920, p. 739–742; H. V. HOPKINS, "Altitude by Martelli's tables," v. 59, 1933, p. 1171; H. J. RAY, "Altitude by Martelli's tables," v. 59, 1933, p. 1776–1777; H. V. Hopkins, "New Uses for Martelli's tables," v. 60, 1934, p. 651–654; HARRY LEYPOLDT, "Martelli's method," v. 61, 1935, p. 401. Mr. Hopkins, then a boatswain, but now a Lieutenant Commander in the U. S. Coast Guard, contributed also an article, "Martelli's tables," to *Nautical Mag.*, Glasgow, v. 138, 1937, p. 326–329. The 1932 edition of Martelli had only two pages of 1888 explanatory matter, but this was given in four languages, English, French, German and Italian; the same was true of the 1942 edition. But in 1943 the publishers issued a 12-page pamphlet (including the cover) *Supplement to Martelli's Navigational Tables*, and it is stated that "This supplement replaces the introduction and examples (pp. ii to viii) of the 1942 edition." It is entirely in English, incorporated methods and ideas from the articles of Mr. Hopkins, and included his computed extension of Table 1. This material was included in the 1944 and 1946 editions. The typography and paper of the present edition is better than in the earlier ones.

315[U].—P. V. H. WEEMS, (A) *Star Altitude Curves, . . . Latitudes 70° to 90° North*, third ed., and (B) *New Line of Position Tables*, Annapolis, Md., Weems System of Navigation. 1944, i–xiv, 701–716, 801–813 + [5] p. 21.5 × 35 cm. $7.00.

This volume contains star altitude curves for latitudes 70° to 90° North and tables which are suitable for use with any celestial body at any latitude. For use in the polar regions, some combination of this sort is necessary, since there are long intervals when only the sun, moon and one or two bright planets can be used due to the continuous presence of the sun in the sky, and there are other long intervals when the sun does not rise above the horizon.

Because this volume contains two essentially distinct methods, (A) and (B), under a single cover, they will be reviewed here as separate units.

(A) The horizontal scale of each of the charts for latitudes 70° to 80° North is local sidereal time which is given in hours and minutes at one-minute intervals at the top of the page and in degrees and minutes at 15′ intervals at the bottom of the page. The vertical scale is latitude in degrees and minutes at 5′ intervals. In the charts for 80° to 90° North latitude, the scale along the bottom of each page and along the right-hand side is local sidereal time, both hours and minutes, and degrees and minutes, with intervals of 2^m and 30′ respectively. Along the left and at the top are latitude scales, with 5′ intervals and with the pole (90° North latitude) in the upper left-hand corner.

On each page appear equal altitude curves for three stars of markedly different azimuths; those for each star are in a different color, red, green or black. The curves for altitudes measured in integral degrees are heavy and carry numerals indicating the altitude; somewhat lighter curves, one for each 10′ of altitude, are shown in between the heavy ones. The altitudes as shown on the charts are true altitudes increased by refraction (epoch of star positions Jan. 1, 1945); thus altitudes measured with a good bubble octant may be used without correction. Altitudes which have been measured with a marine sextant must be corrected for dip of the horizon before using the curves.

With the altitudes of two or three of the stars shown on the curves, taken within a few minutes of each other, one can enter this volume, sketch in the lines of position on the charts and determine a fix in a very few minutes. Neither a nautical almanac nor an air almanac is necessary if the user times his observations with a timepiece keeping Greenwich sidereal time.

One can easily estimate minutes of arc on all scales appearing on these charts; altitudes measured with a bubble octant aboard a plane are likely to be in error by several minutes. Furthermore most travel in the zone 70° to 90° North latitude will be by plane, so one may say that the star altitude curves in this volume are of adequate accuracy.

On the 16 pages covering latitudes 70° to 80° North, a total of eight stars appear: Vega on 12 pages, Deneb on 11, Arcturus 7, Alpheratz 5, Pollux 5, Capella 4, Dubhe 3 and Caph on one page only. On the 13 pages for 80° to 90° North latitude, only three stars are used, namely: Capella, Alkaid and Deneb. Five of the nine stars chosen are brilliant ones and favorites with navigators: Vega, Arcturus, Capella, Pollux and Deneb. The other four with their stellar magnitudes are Alpheratz 2.2, Caph 2.4, Dubhe 2.0, and Alkaid 1.9; they will be somewhat harder to observe. Fortunately on no page does more than one of these faint stars appear.

Probably the one criticism which is raised most often against the star altitude curves is that the method is graphical. Many persons of strong mathematical background are allergic to graphical methods because of their general lack of satisfactory accuracy. However, as pointed out above, this method, though graphical, provides accuracy somewhat greater than the observations will require. Another criticism to be made of the curves is that there is very little overlap (one degree in the 70°–80° North latitude section) between the charts given on successive pages, or at top and bottom between successive latitude sections (30′). In this connection, it may be pointed out that page III is headed "STAR

ALTITUDE CURVES, Latitude 70° to 80° North," and yet just above DIRECTIONS FOR USE appears the statement, "This edition covers latitudes 0 to 70°30′."

A weakness of the curves is that one must observe at least two of the three stars given for the particular time of the observations and within the time interval for which they appear. For example, if the local sidereal time is 9 hours 10 minutes instead of an estimated 9 hours, and one has observed Caph at latitude 75° North, one will find that the fix will lie outside the chart. The advantage to be gained by the use of a sidereal timepiece will usually be lost, since such instruments are not common.

(B) These tables represent an improved version of the author's *Line of Position Book*, RMT 298; their three principal advantages are improved legibility, increased accuracy, and better arrangement. In particular, all of the data in Table A concerning one latitude are now found on a single page. Since latitude seldom changes as rapidly as local hour angle, even with the breath-taking speeds of today, this represents a real improvement over the arrangement to be found in the *Line of Position Book*. The numerical values of the tabulated quantities appear to be identical with the corresponding values in *Hughes' Tables for Sea and Air Navigation* by L. J. Comrie (RMT 115) and are therefore the most accurate values of this sort in print today.

Table A is a double-entry table with horizontal argument latitude 0 to 89° by integral degrees, five values to a page, and with vertical argument local hour angle, 0(1°)90° on the left side of the page, and 90°(1°)180° on the right side of the page. For local hour angle 0 to 90°, the tabulated quantities are K, A, and Z_1; for local hour angles 90° to 180°, they are 180°-K, A, and Z_1. K and A are as defined in the *Line of Position Book*. K is given to the nearest 0.′1 and values of A less than 665 are given to one decimal, other values of A to the nearest integer. Z_1 is the angle at the zenith subtended by the arc of the hour circle of the celestial object between the elevated pole and the foot of the perpendicular from the zenith on this hour circle; it is given to the nearest 0.′1. Table B is essentially the same as Table B in the *Line of Position Book*, and Weems offers the same formula for determining the altitude, h, as in the older book.

The author names four methods of determining the azimuth, listing them in order of difficulty, the simplest first. They are:

(a) By direct observation with a pelorus or other instrument.
(b) By the use of Rust's modified azimuth diagram, a copy of which is included (see below).
(c) By the use of Table A in obtaining Z_2. One can either replace L, t, and A by h, Z_2, and $B(K \sim d)$ respectively, counting h and B as known and determining Z_2; or replace L, A, Z_1 by h, A, Z_2 respectively, counting h and A as known and determining Z_2. The rule for combining Z_1 and Z_2 to form Z is a trifle complicated.
(d) By the use of the formula $\csc Z = \csc t \sec d/\sec h$ and Table B.

The azimuth diagram of Rust, originally appearing in his *Ex-meridian Altitude, Azimuth and Star-Finding Tables* (N. Y., Wiley, 1908), plate VI, is reproduced here as a single unit on a scale approximately 1.4 times as large. Otherwise the diagram and auxiliary diagram are as described in RMT 298.

Also included in the volume are tables for the correction of observed altitudes for dip of horizon, refraction, and parallax, all well designed for convenient use. Of especial interest are two pages of correction tables for altitudes between 0 and 6°, with special "sub-correction tables" to take care of variations in height above sea level, temperature and pressure. The author warns the user, however, "that refractions at these low altitudes are on rare occasions subject to large and unpredictable variations."

Two pages are devoted to explaining the methods which may be used in adjusting for run between sights or for time elapsed, in the polar regions, including the use of the Greenwich meridian as a basis for describing directions, the so-called G system.

Illyne's Star Chart is presented in a reduced form, opposite a page of material designed to help the navigator identify the principal stars and constellations as they will be seen from the north polar regions of the earth.

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