# MECHANICAL AIDS TO COMPUTATION

In *Nature*, v. 158, 12 Oct. 1946, p. 500–506, is an interesting illustrated article by Professor D. R. Hartree, of the University of Cambridge, entitled "The ENIAC an electronic computing machine." (Dr. Goldstine's name is repeatedly misspelled; see *MTAC*, v. 2, p. 97–110.)

JACK LADERMAN & MILTON ABRAMOWITZ are the authors of an article "Application of machines to differencing of tables," in Amer. Statistical Assoc., *Jn.*, v. 41, June 1946, p. 233–237.

26[Z].—CLIFFORD E. BERRY, DOYLE E. WILCOX, SIBYL M. ROCK & H. W. WASHBURN, "A computer for solving linear simultaneous equations," *J. Appl. Physics*, v. 17, Apr. 1946, p. 262–272. 19.8 × 26.8 cm.

This paper is a description of the recent commercial model computer, brought out by the Consolidated Engineering Corporation of Pasadena, California, for solving twelve simultaneous linear equations. The mathematical theory of the machine based on the well-known Gauss-Seidel method of iteration is described briefly together with questions of convergence. References are given to recent articles in which necessary and sufficient conditions for convergence of the process are given. These are too complicated to be of much practical use, so the authors suggest two empirical rules for arranging the given equations for a satisfactory application of the method.

(1) Arrange the matrix of coefficients $A = (a_{ij})$ so that diagonal elements are large and other elements are small.

(2) If (1) is not feasible try to arrange $A$ so that for every $i \neq j$, $|a_{ij}a_{ji}| < |a_{ii}a_{jj}|$. When (1) and (2) fail, another method, to be described in detail elsewhere, is suggested in which systems of equations in 2, 3, 4, $\cdots$, $n$ unknowns are solved in turn.

The electrical analogue of a set of linear equations is, of course, a composite of circuits for adding and multiplying electrical quantities. In the present machine AC voltages, proportional to the sum and product of given voltages, are produced by simple circuits involving variable resistances. In operating the computer these circuits are balanced by observing a small cathode ray null indicator. The potentiometers corresponding to the coefficient of each one of the 12 equations are laid out along one element of a cylindrical switching turret. By rotating this cylinder by hand through successive multiples of 30°, about its horizontal axis, each one of the 12 sets of 13 potentiometers is made available, in turn, for setting at a horizontal window. Once these 156 potentiometers are set, the machine has been given the problem. According to the authors' examples this operation requires about 39 minutes for a 12 × 12 problem. Of course, if a series of such problems with the same matrix $A$ is to be done (as for instance in the inversion of a matrix) it will be necessary to reset only the 12 constant terms each time. This requires only 3 minutes.

The next operation consists in adjusting the potentiometers corresponding to the unknowns $x_1, x_2, \cdots, x_{12}$. With the cylinder in its first position, the $x_1$ potentiometer is adjusted until the null indicator shows that a balance has been obtained. This means that the first equation is satisfied for this value of $x_1$ and the (arbitrary) other values $x_2, \cdots, x_{12}$. The cylinder is now rotated to position 2 and the $x_2$ potentiometer adjusted for balance. This process continues for perhaps several revolutions of the cylinder (depending upon the rapidity of the convergence of the process) until no further adjustments are required. The values of the $x$'s at this time constitute the computer's answer to the problem.

Four examples are given, each employing 4 decimal places and showing the time required and the accuracy obtained in each case. The computer deals with numbers between + 1 and − 1 and can be set to within .1 percent. Such accuracy in the $x$'s, of course, cannot be guaranteed since this will depend on what the authors rather vaguely call the "leverage" of the system, a quantity practically determined by the size of the determinant of $A$. In the rather favorable looking example 1 the maximum error is .0004. The computer appears to be of an order of magnitude faster than desk calculators for this type of work. The time required to work Example 1 is given as 44 minutes of which only 2 were spent in adjusting the $x$'s. This compares favorably with the 5-hour estimate given for desk machine work.

In problems where more accuracy is required, as in certain statistical work for example, it is possible to use first the computer and then to improve the solution thus obtained by using a desk calculator. In this way the electrical computer should be quite useful in any one of the host of problems which can be reduced to systems of linear equations.

D. H. L.

27[Z].—G. S. MERRILL, "Slide-disk calculator," *General Electric Rev.*, v. 49, no. 6, June 1946, p. 30–33. 22.3 × 29.5 cm.

This note is a well illustrated description of an ingenious combination of a sliding rule and a rotating disk for the computation of the standard deviation

$$\sigma = [x_1^2 + x_2^2 + \cdots + x_n^2]^{\frac{1}{2}} n^{-\frac{1}{2}}$$

of the set of deviates $x_1, x_2, \cdots, x_n$ from the arithmetic mean of $n$ values. The quantities

$$x_1, (x_1^2 + x_2^2)^{\frac{1}{2}}, (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}, \cdots$$

are built up in succession geometrically by successive applications of the Pythagorean theorem. An alignment chart is then used to multiply the final square root by $n^{-\frac{1}{2}}$. The sliding rule permits one to set the device with the origin at the arithmetic mean of the values concerned so that one need not really calculate the deviates themselves. No statement is made as to the accuracy obtainable. This depends somewhat on the order in which the $n$ values are introduced. In general the accuracy should be comparable with that obtained from an ordinary slide rule. Even a much cruder device would be useful for class room demonstration purposes.

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## NOTES

63. EARLY TABLES OF FACTORIALS.—While tables of $N!$, by various authors, 1926 to 1936, $N = 20$ to 30, are reported in FMR, *Index*, p. 34, the following three very much earlier appearances of such tables may be recorded:

(a) G. G. LEIBNIZ, *Dissertatio de Arte Combinatoria*. Leipzig, 1666, p. 57. Table of $N!$ for $N = 1(1)24$ (erratum $N = 15$, *for* 1307*8*74368000, *read* 1307*6*74368000). Also, with the same erratum, in *Leibnizens mathematische Schriften*, ed. by C. I. GERHARDT, part 2, v. 1, Halle, 1858, p. 61.

(b) JOHN WALLIS, *A Treatise of Algebra. . . .* London, 1685, Part IV, p. 116. Table, without error, up to $N = 24$.

(c) JAMES DODSON, *The Calculator: being, correct and necessary Tables for Computation. Adapted to Science, Business and Pleasure*. London, 1747. P. 44, Table XXIII, up to $N = 30$.

S. A. J.

64. ERRATUM:—May I call attention to a rather amusing type of error made by J. R. WOMERSLEY in his article "Scientific computing in Great Britain," *MTAC*, v. 2, p. 114? In illustrating the very great power of the $h^2$-process in deferred approach to the limit, he states that its application to the determination of $\pi$ from the perimeters of the inscribed square and