process. This is erroneous for two reasons. In the first place different degrees of blackness on the manuscript do not show up on the negative in the same proportion. Thus a portion of a figure which looked gray on the manuscript may be entirely missing on the negative. In the second place considerable hand work is done on the negatives. In the lithographic process the retoucher paints over parts of the negative and clears out others. In the case of the line cut the photoengraver "routs" out part of the plate with an implement like a dentist's drill. It is true that with these processes, omissions are more probable than changes, but even these are not impossible.

5. Mechanical Proofreading. The application of the punched card technique has completely revolutionized the methods of proofreading in the Nautical Almanac Office both for tables set in type and for tables printed by methods involving photography; the new methods give revolutionary dependability and economy. The conventional method of having a proofreader assume responsibility for the agreement of the copy and the proof has been abandoned. The proofs, but not the copy, are now given to a card punch operator whose only responsibility is to punch what she sees. A template is used which exposes only one line at a time, so there is no temptation to correct an error by looking up or down the column. These cards are then compared electrically with the cards from which the copy was prepared. This comparison is performed on one of the standard punched card machines at the rate of 6,000 cards per hour. Finally the various columns are differenced automatically on the accounting machine and the differences printed in such a manner that any error would be immediately apparent.

In the case of manuscript prepared on the accounting machine or on the new table printer, the results are so accurate that the line-cut proofs are examined only for general quality and legibility, no attention being paid to the figures themselves. The plate proofs only are subject to the rigorous methods above described.

6. Acknowledgments. The success of various phases of the work has been made possible by the cooperation of many people, including the following: the staff of the Government Printing Office, particularly Mr. R. W. Christie; the staff of the Nautical Almanac Office, especially Mr. G. M. Clemence and Mr. Jack Belzer; Mr. E. W. Gardinor and other engineers of the International Business Machines Corporation; Mr. C. H. Griffith, Mergenthaler Linotype Company.

W. J. E. &

Watson Scientific Computing Laboratory

RALPH F. HAUPT

U. S. Naval Observatory, Washington, D. C.

[1] See W. J. ECKERT, *Punched Card Methods in Scientific Computation*, The Thomas J. Watson Astronomical Computing Bureau, 1940, p. 106.
[2] See W. J. ECKERT, "Air Almanacs," *Sky and Telescope*, v. 4, Nov. 1944, p. 12.

RECENT MATHEMATICAL TABLES

337[A, B, C, D, N, O].—CLEMENTE BONFIGLI a. *Manuale Logaritmico Completo del Tecnico ad uso degli Ingegneri, Architetti, Geometri, Periti, ecc. con cinque cifre decimali Tavole logaritmiche dei numeri; Tavole log. delle Funzioni di angoli sessagesimali, centesimali e sessadecimali; Tavole* *aritmetiche e di trasformazione angolare; Tavole dei val. naturali delle funzioni di angoli sessagesimali e centesimali; Tavole speciali per il calcolo diretto dei prodotti a sen b e a cos b; Tavole dei coefficienti finanziari, a 8 decimali, e loro logaritmi; Tavole per il picchettamento delle curve circolari.* Milan, Hoepli, 1945. xvi, 374 p. 12.8 × 19.6 cm. 250 lire, bound in boards.

b. *Nuove Tavole Logaritmiche con cinque decimali (per i sistemi: sessagesimale, centesimale, sessadecimale), Tavole Aritmetiche, Tavole Finanziarie, Valori Naturali, ad uso dei Licei Scientifici e Classici.* Milan, Hoepli, 1944. xvi, 230 p. 12.8 × 19.4 cm. 120 lire, paper covers.

a: Tables, p. 1–337; explanations for the use of the tables p. 340–373. **b**: Tables, p. 1–197; explanations, p. 199–229. Several of the tables in this volume are the same as those in **a**.

338[B].—L. ÎA. NEĬSHULER, "Ob optimal'nykh slitnykh Tablit͡sakh kvadratov i kubov" (On optimal coalescent tables of squares and cubes), Akad. N., SSSR, Moscow, (*Dok.*) *C.R.*, n.s., v. 47, 1945, p. 478–481 + folding plate. Russian ed. v. 47, p. 462–465. 16.8 × 25.9 cm. See *MTAC*, v. 1, p. 7, 67, 68.

Without questioning the author's right to coin a word or a phrase when introducing a new concept, we shall consider with the author a table *coalescent*, when both the argument and the tabular function are separated into several sets of figures and these sets are located in various parts of the page, leaving it to the user of the table to collect the sets and to annex them to each other in the proper manner. The author refers to several publications in which he described his method of constructing coalescent tables, with the latest previous application to multiplication tables, and now extends his method to tables of squares and cubes.

The folding plate, accompanying the article, contains outlines of seven tables. Tables (A), (B), and (C) give three forms of coalescent tables for squares of 5-digit numbers. Table (D), consisting of two subdivisions, D(a) and D(b), with an auxiliary table (G), furnishes an outline of a table for squares of 6-digit numbers. Table (H) deals with cubes of 4-digit numbers.

In order to give an idea of the method pursued by the author we shall confine ourselves to the description of T. (A), this being the simplest in form. It furnishes the headings and the numerical results of the first two lines of page 1 of his table, but as his page is too wide for *MTAC*, we shall print his line of headings in two parts:

part I	00	1 6 — 3 8	2 7 — 2 7	3 8 — 1 6

part II	4 9 — 5	5 — 4 9	99

Each page thus consists of five vertical sections, corresponding to the first figure of the argument: (i) 1 or 6, (ii) 2 or 7, (iii) 3 or 8, (iv) 4 or 9, and (v) 5. (Each couple, 1 and 6, etc., is followed by a "dash," the purpose of which will be explained presently.) The 5-digit argument is divided into three sets, of 1, 2, and 2 figures resp. As we have already seen, the first figure is placed in the heading line. The second set (2d and 3d figures) of the argument are placed in the top corners: those running from 00 to 49 being in the left corner, and from 50 to 99 in the right corner. Thus the complete table will require 50 p., and p. 1 has 00 in the left top corner, and 99 in the right top corner. Finally, the third set (the 4th and 5th figures) of the argument are contained in the first and last columns, 01 to 99, and 99 to 01 respectively.

Here we reproduce the first and fifth sections of the table, with the two horizontal lines of figures, given by the author in the outline:

00	1	6	—	3	8		5	—	4	9	99
01	1000	36001	20001	15999	80998		25001	00001	24999	99998	99
02	1000	36002	40004	15998	80996		25002	09004*	24998	99996	98

* erratum for 00004

While the 5-digit argument is separated into 3 sets, its 9 or 10-digit square is separated into 2 sets only, (1) the right 5 figures and (2) the left 4 or 5 figures. The right 5 figures are entered in the middle column under the "dash"; while the left 4 or 5 figures are entered in the column headed by the first figure of the argument, to the left or to the right of the middle col., depending upon whether the 2d set of the argument, in the top corner, is to the left or to the right.

For instance,

$1\,0001^2 = 1000\ 20001$ $\qquad 3\,9999^2 = 15999\ 20001$
$6\,0001^2 = 36001\ 20001$ $\qquad 3\,9998^2 = 15998\ 40004$
$6\,0002^2 = 36002\ 40004$ $\qquad 4\,9998^2 = 24998\ 00004.$

For the other tables, (B) to (H), which are more complicated, there is an additional feature: the author uses in one and the same table figures of different sizes, as ordinary, large and small, and of different types, as ordinary, bold and italics. In the latter two classes, a bold figure requires the increase of a certain figure in the preceding set by 1, and an italic figure a similar increase by 2. In view of this, one can easily understand how difficult it is for the printer and the proof reader to produce a table of that kind without errors, and the subsequent difficulty for the user of the table to obtain the correct result from the table.

As an illustration, the following is a list of errata found by the reviewer while reading the art., without an attempt to make the list complete. Some of the misprints appear in the Russian edition only, some in the English only, and some in both.

T. (A), section 5, col. "dash," l. 02, *for* 09004, *read* 00004	(R. and E.)
T. (B), last sec., col. 2, l. 1.6, *for* 152, *read* 153	(R. and E.)
T. (B), last sec., col. 8, l. 3.8, *for* 716, *read* 816	(E.)
T. (B), last sec., col. 1, l. 4.9, *for* 990, *read* 990	(R. and E.)
T. (B), last sec., col. 9, l. 4.9, *for* 655, *read* 655	(R. and E.)
T. (C), last sec., col. 6, l. 09, *for* 9126, *read* 9216	(R.)
T. (Da), last sec., col. 1, l. 4.9, *for* 9950, *read* 9950	(R. and E.)
T. (Da), last sec., col. 7, l. 4.9, *for* 4901, *read* 4901	(R. and E.)
T. (Db), col. 4, l. 6, *for* 2329, *read* 2349	(E.)
T. (Db), col. 6, l. 3, *for* 3336, *read* 4336	(R.)
T. (H), col. 5, l. 9, *for* 500779, *read* 500749	(R.)

It may be well to conclude this review by pointing out defects in the English text, which are due to faulty translation from the original Russian. The word *cipher* does not have the accepted meaning of *zero* (0), but is a translation of the Russian word *t͡sifra* (French *chiffre*) and means figure, digit, numeral. A second instance, perhaps even more serious, due either to a hasty reading or to misunderstanding of the Russian text, is the following: p. 464, sec. 2, l. 2, *for* the first, or the second, or the third cipher, *read* the first, second and third figures; similarly, l. 3, 4, *for* the fourth, or the second, or the third cipher, *read* the fourth, second and third figures.

S. A. J.

339[A, B].—NBSMTP, *Tables of Fractional Powers*. New York, Columbia University Press, 1946, xxx, 486 p. 18.6 × 26.5 cm. Reproduced by a photo-offset process. $7.50.

This new volume, produced by the prolific computing staff of the NBSMTP, under the technical directorship of Dr. A. N. LOWAN, is destined to be one of the most useful yet produced by this group. Nearly 500 of its pages are devoted to tabulations of the two functions A^x and x^a over ranges of the variable x for a selected set of values of A and a. The foreword is written by F. BERNSTEIN of New York University who makes the following pertinent observations: "The table of fractional poweis presented here for the first time may be regarded, in a sense, as a generalization of BARLOW's tables, and it would be impossible to enumerate the many fields in which the present tables will be found useful. Suffice it to say that the algebraic and even the transcendental counterpart of numerous linear problems can now be attacked numerically with the prospect of success."

The first part of the volume, some 278 pages, contains five tables which give 15D values of the function A^x for the following values of A and x:

Table 1. $A = 2(1)9$ and $x = .001(.001).01(.01).99$.

Table 2. $A = 10$, and $x = .001(.001)1$.

Table 3. $A = \pi$, and $x = .001(.001)1$. The following special values of x are also included: $\pm x = \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, and $1(1)12$.

Table 4. $A = .01(.01).99$, and $x = .001(.001).01(.01).99$.

Table 5. $A = .001\,P$, where P is any prime between 100 and 1000, and $x = .001(.001).01(.01).99$.

The second part of the volume, some 208 pages, is devoted to six additional tables which give the values of x^a over the ranges indicated below. The first five tables, **Table 6** to **Table 10** inclusive, are computed to 15D, but the last, **Table 11**, is to 7D. The ranges over which the tabulations are made are as follows:

Table 6. $a = \pm\frac{1}{2}$ and $x = 0(.01)9.99$.

Table 7. $a = \pm\frac{1}{3}$ and $x = 0(.01)10$.

Table 8. $a = \pm\frac{2}{3}$ and $x = 0(.01)10$.

Table 9. $a = \pm\frac{1}{4}$ and $x = 0(.01)9.99$.

Table 10. $a = \pm\frac{3}{4}$ and $x = 0(.01)9.99$.

Table 11. $a = .01(.01).99$ and $x = 0(.01).99$. Values are also included over a variable range: $x = .001(.001)m$, where m never exceeds .3.

It will appear from this description that we have in this volume a set of tables which may be adapted for a wide range of uses. In the first place the familiar square and cube roots, together with their reciprocals, are found in Tables 6 and 7. However, these are not likely to replace the familiar BARLOW-COMRIE tables, except where more than nine significant figures are required, or where reciprocals are needed. The latter include square roots of x and $10x$ over the range $x = 1(1)12500$, a total of 25000 entries. The new tables give only a thousand square roots.

The present work may also be used as a brief table of antilogarithms, since Table 2 gives the values of 10^x. Here again only a thousand values are recorded, whereas in the classical tables of J. DODSON 100000 entries are given. But to quote the introduction: "Some similar tables with a smaller interval between the arguments are available, but the best of them are scarce."

One is pleased to find in the present volume tables of π^x over a generous range of values of x. Powers of π, integral, reciprocal, and fractional, have been computed in the past in some abundance and to high decimal approximation, but for the most part these are scattered in many places and are difficult to find. Hence it is very pleasing to have these values now readily accessible.

Interpolation in the present tables will be found to be difficult, since no differences are given. It is clear, however, that when values are tabulated to 15D at interval .01 a complete differencing would have required the printing of differences of high order. Nevertheless the publication of first differences would have served a useful purpose since the error in linear interpolation for A^x will be less than $.7h^2 (\log A)^2$, where h is the tabular interval. Throughout most of the values of A the accuracy thus achieved is from 4 to 6D. Quadratic interpolation by means of the 3-point Lagrangean interpolation coefficients has an error less than $.8h^3 (\log A)^3$. Throughout most of the range of A, quadratic interpolation would thus give an accuracy varying from 6 to 9D. Similar formulae are given for the errors of linear and quadratic interpolation in the tables of x^a.

The methods of computation and the means to insure accuracy are described in the Introduction. Thus the construction of the values given in Part I was achieved from the key values $A^{0.001}$, which were computed to 20 places by means of the formula,

$$A^x = 1 + \frac{x \ln A}{1!} + \frac{(x \ln A)^2}{2!} + \frac{(x \ln A)^3}{3!} + \cdots,$$

where $\ln A$ was taken from WOLFRAM's table. For this computation seven terms of the series were found sufficient. Powers of the key values yielded the desired tabular entries.

In the computation of cube roots a method of successive approximation was used by means of the formula: $x^{\frac{1}{3}} = p_1 + h$, where p_1 is the 6D value taken from BARLOW's *Tables.* An estimate of h is obtained from the expansion $x = p_1^3 + 3p_1^2h + \cdots$, and the corrected value again employed in place of p_1. It may be of interest to observe that the author of this review, in making similar calculations, discovered that more rapid progress could be made by computing $y = \frac{1}{3} \ln x$ and then evaluating $x^{\frac{1}{3}}$ by means of the NBSMTP tables of e^y. The difficulty of inversion from existing tables of common logarithms to more than 10D, and the lack of antilogarithm tables to 15D, preclude the use of the more usual method of computation.

The volume under review also provides an extensive bibliography of tables of fractional powers. Seventy-six contributions to the subject are listed.

H. T. D.

340[B].—LUDWIG ZIMMERMANN, *Vollständige Tafeln der Quadrate aller Zahlen bis 100009*, third edition. Berlin, Wichmann, 1938. Lithoprinted by Edwards Bros., Ann Arbor, Mich., 1946, xix + 187 p. 19 × 25 cm. $2.80. Compare MTE **89**, *MTAC*, v. 2, p. 180, note on p. 444; the first and third editions are practically identical.

This table is the largest table of squares in print. As the title indicates, the main table gives the exact values of the first 100009 squares. The table is similar in its arrangement to the usual table of 7-place logarithms, the final digits of the 5-figure arguments being used as column headings. Eight digits are given in the body of the table; the leading two digits are printed in the upper left hand corner of the page and in the left column when these digits change. The extreme right column is headed d and is to be used in computing the difference of the tabulated squares. In fact the difference $\Delta(N^2)$ is merely $1 + 2t + 10d$, where t is the last digit of N.

This table should not be used for finding squares of isolated 5-digit numbers, unless one has no computing machine. Depending on the type of machine, the computer can calculate and copy down the square of such a number in from one half to three quarters of the time required to read it out of the table. The reviewer has found the table quite handy as an auxiliary to a desk calculator in finding square roots to 10 significant figures. The first five figures of $N^{\frac{1}{2}}$ can be read from the table and the last five can be read from the machine while it automatically divides N by this approximation to $N^{\frac{1}{2}}$.

To the computer who uses no calculating machine the table is of course a great help in finding squares, square roots, and products (by the method of quarter squares). Discussing the latter use of the table the author asserts that, by using this table together with a 5-figure natural trigonometric table, problems in trigonometry may be solved more easily than by using 6-place logarithms. The examples in the introduction all suppose that the user is without a calculating machine.

When one considers the rather crowded printing on each page and the fact that many such users may not need the full ten digits of a square one wonders whether possibly another arrangement suggested by the rather obscure table of GOSSART[1] might not be preferable. In this arrangement the first (rather than the last) digits of the 5-figure argument would be used as column headings. Thus page 154 might be laid out in larger type somewhat as follows:

	1	2	3	4	5	6	7	8	9	
	3	**8**	**14**	**23**	**34**	**46**	**61**	**78**	**97**	
8500	4225	1225	8225	5225	2225	9225	6225	3225	0225	**0000**
8501	4228	1230	8232	5234	2236	9238	6240	3242	0244	**7001**
8502	4232	1236	8240	5244	2248	9252	6256	3260	0264	**4004**
....	....	....	....	....	....	....	....	....	....	....
8549	4406	1504	8602	5700	2798	9896	6994	4092	1190	**5401**

Thus the first six significant figures of the square of 68502 are 469252. If the exact value of this square is needed one merely annexes the four digits in the extreme right column to obtain

$$68502^2 = 4692524004.$$

This arrangement would require about 2250 printed figures per page instead of the 4200 actually used.

There are two other two-page tables as follows:

Table II (p. 184–185) giving the first 1000 cubes.
Table III (p. 186–187) giving the first hundred k-th powers for $k = 4(1)9$. This table contains the following two errata[2] unfortunately carried over into the present edition: 27^9, *for* 7625597484387, *read* 7625597484987; 77^6, *for* 208422390089, *read* 208422380089. The name Zimmermann is misspelled on the back of the volume. The quality of the lithoprinting is high. Every entry is clear and uniform in spite of the heavy character of the page.

D. H. L.

[1] ALEXANDRE GOSSART, *Table des Carrés de 1 à 100 Millions* . . . Paris, 1865, 86 p. There are also copies of this book with the title beginning *Table des Carrés de un à cent Millions*.

[2] These errata are given by J. C. P. MILLER, in BAASMTC, *Mathematical Tables*, v. 9, *Table of Powers*. Cambridge 1940, p. xii.

341[B, C, D].—VÁCLAV ELZNIC & MILOSLAV VALOUCH, *Geoma. Pětimístné Tabulky hodnot a Logaritmů goniometrických funkcí v setinném dělení, Tabulky geodetické a katastrální* [Five-place tables of values and logarithms of trigonometric functions in the centesimal angle division, geodesic, and surveying tables], Prague, Czechoslovakia, Jednota Českých Matematiků a Fysiků [Society of Czech Mathematicians and Physicists], 1944. 319 p. 16.7 × 25.4 cm. Paper bound 178 Czech crowns.

Before the recent war this Czech Society, which was founded nearly 85 years ago, was the most affluent mathematical society in the world, and also one of the very largest. It owned its own building and printing presses, and did an enormous publication business especially in texts used in schools throughout Czechoslovakia. Besides various mathematical periodicals published during the past 75 years it sent forth a score of treatises on advanced topics in the fields of mathematics and physics.

The main tables of the present volume for geodesists and surveyors are natural trigonometric functions with centesimal divisions of the quadrant as argument, the use of which beginning late in the eighteenth century has been already set forth in *MTAC*, v. 1, p. 33–39. This centesimal unit has made no headway in America, but was widely used in European countries during the recent war, for example, J. T. PETERS, *Sechsstellige Werte d. trigonometrischen Funktionen von Tausendstel zu Tausendstel des Neugrades*, of which there were at least 7 editions, 1938–1943.

The tables in the volume under review are as follows:

P. 11, $x = a \cot a$(6S), and $\cot a$(4S), for 213 values of a from 0 to $2^g.009$.
P. 12–27, $\cot a$, $a = [2^g(0^g.001)9^g; 6S]$, Δ, p.p.
P. 28–127, $\sin a$, $\tan a$, $\cot a$, $\cos a$, K, for $a = [0(0^g.01)50^g; 5D]$, Δ, p.p., where $K = 1 + \sin a + \cos a$.
P. 128–147, N^2 for $N = [0(.001)10.009; 4D]$, p.p.
P. 148–169, $\log N$, $N = [0(1)11009; 5D]$, p.p. Also S, T, $\log \sin a$, $\log \tan a$, $a = [0(0^g.01)1^g.1; 5D]$.
P. 170–181, $\log \sin a$, $\log \tan a$, $\log K$, $a = [0(0^g.001)3^g; 5D]$, Δ.
P. 182–275, $\log \sin a$, $\log \tan a$, $\log \cot a$, $\log \cos a$, $\log K$, $a = [3^g(0^g.01)50^g; 5D]$, Δ, p.p.
P. 276, conversion tables of degrees to radians and radians to degrees.
P. 277, tables for centesimal measure $\sin \epsilon/(\sin 1'' \cdot 10^6)$.
P. 278–281, tables and geodesic constants of the Bessel and international ellipsoids.
P. 282–284, tables and examples for the solutions of the principal geodesic problems.
P. 285, tables for correction of the curvature of the earth and refraction in trigonometric measurement of heights.
P. 286, interpolation coefficients for Newton's formula, Δ.
P. 287, interpolation coefficients for Bessel's formula, Δ, with second difference table.
P. 288–292, formulae in trigonometry.
P. 292–294, series, constants, interpolation.
P. 295–297, measures of length in various countries, surfaces, areas, etc.
P. 298, constants and a table of arcs subtended by central angles $a = [0(10')10^g; 10D]$. In the table of constants the thirtieth decimal place in the value for π^{-2} should be increased by unity. There are also several errors on p. 4, lines 6, 10, 12, and −4.

R. C. A.

342[B, C, F].—R. de Marchin, *Tables Numériques, à l'usage de l'enseignement moyen et normal, des Écoles Industrielles, des candidats aux Écoles Spéciales des Universités.* Brussels, A. Manteau, 1942. ii, 36 p. 16 × 24.2 cm.

This little volume by a professor in the Athénée at Herstal, near Liége, is intended primarily for use in places indicated by the title. It presents some tables not often found in such booklets of other countries. The tables are as follows:

1. Log N, p. 6–9, 5D table, with p.p., for $N = 100(.1)200(1)999$. On p. 10 there is a serious error; the values of N, 119 and 118 are interchanged. Ln N, p. 10–15, 5D table, with p.p., for $N = 100(.1)400(1)999$.
2. N^2, $N = 1(1)999$, p. 16–17.
3. Tables for finding N^2, $N < 10^6$, p. 18.
4. $N^{\frac{1}{2}}$, $N^{\frac{1}{3}}$, for $N = [1(1)100; 10D]$, p. 19.
5. N^3, $N = 1(1)999$, p. 20–21.
6. N^p, $p = 4(1)9$, $N = 1(1)50$; $p = 4(1)8$, $N = 51(1)99$, p. 22–24.
7. Inverse table of Euler's totient function $\phi(N)$ giving solutions x of $\phi(x) = N$ for $N \leqslant 100$, p. 25. The author states that one may consult discussions of this "Gauss" function in
   Philippens-Van de Werke, *Arithmétique*, ninth ed., Namur, Wesmael, 1939, p. 181.
   Herbiet-Horwart, *Traité d'Arithmétique*, Namur, Wesmael, 1933, p. 205, 216.
   Horwart, *Questions d'Arithmologie*, Namur, Wesmael, 1940, p. 212.
   Schons, *Traité d'Arithmétique*, Namur, La Procure, 1938, p. 236.
   Goss, *Traité d'Arithmétique*, Tongres, Michiels, 1934, p. 141.
8. Table of the number of digits in the periodic and (or) nonperiodic parts of the decimal expansions of the reciprocals of $N = 2(1)50$, p. 25.

9. Table of prime numbers <100110, p. 26–33.
At the bottom of each of seven of the pages is the statement of a theorem in the theory of numbers, for example:

(a) If the prime number p does not divide a, it divides $a^{p-1} - 1$ (Fermat)
(b) If p is prime to a, and if a^{p-1} is the least power of a which equals a multiple of p plus 1, p is prime. (Lucas)
(c) For p to be a prime it is necessary and sufficient that the sum $(p - 1)! + 1$ be divisible by p. (Wilson)
(d) If a and b are prime to one another, b divides the expression $a^{\phi(b)} - 1$, ϕ defined as above. (Euler)

10. Frequency tables of the 9592 prime numbers $<10^5$, (a) by hundreds; (b) by thousands. In (c) is a table of the number of hundreds containing n primes, n varying from 1 hundred containing 25 primes down to 3 hundreds containing 3 primes. p. 34.
11. Table of the composite numbers $<10^4$ whose smallest divisor (13 to 97) is >11, p. 35–36.
12. Table of primitive rational right triangles $a^2 = b^2 + c^2$, arranged according to $a < 1000$, p. 36. The author adds, for properties of the sides of a primitive triangle consult:

HORWART, *Questions d'Arithmologie*, Namur, Wesmael, 1940, p. 125.
HERBIET-HORWART, *Traité d'Arithmétique*, Namur, Wesmael, 1933, p. 193–194.
SCHONS, *Traité d'Arithmétique*, Namur, La Procure, 1938, p. 225.

R. C. A.

343[D].—CH. FÉVROT, "Caractéristique théorique d'un projecteur à miroir parabolique à source cylindrique axiale," *Revue d'Optique*, v. 23, Oct.–Dec. 1944 (publ. Apr. 1946), p. 264–267. 15.9 × 24 cm.

We have here a table and supplementary graphs. The table (p. 266) gives the values of $2\sin^{-1}[2k(1 + m^2)]^{-1}$ in degrees and fractions of a degree (tenths or hundredths), for $m = .1(.1)1(.2)3$, and for $k = .05(.025).1(.05).3(.1)1(.2)1.6$. [The second argument $k = .15$ is incorrect and should be .5; furthermore, under $k = .15$, 180° opposite $m = 5$ is also incorrect.] The table is completed by figures (p. 264, 265, 267), which do not give the angle but $m \times 2\sin^{-1}[2k(1 + m^2)]^{-1}$, as functions of m for the different k.

R. C. A.

344[D, R].—VÁCLAV ELZNIC, (**a**) *Sintacos 10. Desetimístné Tabulky hodnot goniometrických funkcí sin, tg, cos, pro setinné dělení kvadrantu*, [Sintacos 10. Ten-place tables of values of the trigonometric functions sin, tan, cos, for centesimal division of the quadrant]. Prague, 1941, 112 p. 15.5 × 22.6 cm. Paper covers 98 Czech crowns.
(**b**) *Osmimístné Tabulky, přirozených hodnot goniometrických funkcí sin, cos, tg a Tabulky Geodetické pro úhlové dělení šedesátinné.* [Eight-place tables of values of the trigonometric functions sin, cos, tan, and geodetic tables, for sexagesimal angle division.] Prague, 1940, 64 p. + interpolation table card, and errata slip. 18.8 × 27 cm. Boards, 54 Czech crowns. These volumes are publications of the Jednota Českých Matematiků a Fysiků [Society of Czech Mathematicians and Physicists]. Compare RMT **341**.

(**a**) The main table, p. 6–105 is of $\sin x$, $\cos x$, for $x = [0(0^g.01)100^g; 10D]$, Δ^2; also $\tan x$, 0–50^g, $\cot x$, 50^g–100^g, each at interval $0^g.01$, 10D, Δ^2. On p. 106–112 are reprints of Andoyer's 20D tables of $\sin x$ and $\cos x$, each with v^8, and $\tan x$ with v^{12}, $x = 0(1^g)50^g$. Of previous tables of this type and interval was the 8D table, Δ, of H. C. C. ROUSSILHE & BRANDICOURT, *Tables à 8 Décimales des Valeurs naturelles des Sinus, Cosinus, Tangentes . . .*, new ed., Paris, 1933. But no previous printed table of greater than 8D was as extensive throughout the quadrant as the one under review.

(b) In this v. **T. II**, the principal table, p. 7–51, is of sin x, tan x, cos x, and $K = 1 + \sin x + \cos x$, for $x = [0(1')45°; 8D]$, Δ^2, with columns for $\Delta 1''$ and $\Delta 1'$ for each entry. Each page of this table contains at the left a table of the variations in direction per km., a^*, b^*, with Δ.

T. I, p. 6, is of cos x, for $x = [0(10'')1°; 10D]$, Δ^2.

T. III, p. 52, is a 15D conversion table for arcs into radians, $x = 0(1°)120°$; $0(1')60'$; $0(1'')60''$.

T. IV–V, p. 53–54, are for conversion of (i) sexagesimal units into time, (ii) degrees into centesimal units, and centesimal units into sexagesimal.

T. VI–VIII, p. 55–60, geodesic; **T. IX–XI**, p. 60–63, cartographic; **T. XII** orthometric projection; **T. XIII** terrestrial curvature and refraction; **T. XIV** spherical excess; **T. XV–XVI** pendulum; **T. XVII** variation in attractions of the earth for varying altitudes. On one side of the card, inserted loose in the v., is a table, **App. A** for second difference correction; on the other side, **App. B**, a table of $\Delta/60$, for $\Delta = [0(.1)60;5D]$.

In the table of constants on p. 5 there are three errors; the thirtieth decimal places of the values for π, M and e should each be increased by a unit.

R. C. A.

345[F].—Lord Cherwell, "Note on the distribution of the intervals between prime numbers," *Quart. Jn. Math.*, Oxford s., v. 17, 1946, p. 46–62.

This article gives data on the distribution of pairs of primes differing by a given amount. These data are compared with approximate formulae derived by probability methods. In particular, **Table 2** gives the number of primes p with $250000 < p < 255000$ for which $p + k$ is a prime for $k = 2(2)98$. **Table 4** gives the number of primes whose difference is k, in the ranges 500–2000, 2000–10000, 10000–30000, 90000–110000, 220000–255000 with $k = 2(2)62$. Glaisher's[1] counts of the twin primes in the first 100000 numbers of the n-th million for $n = 2, 3, 7, 8, 9$ are reproduced in **Table 1(a)**. There are also data on the distribution of triplets of primes p, $p + 2$, $p + 6$ and p, $p + 4$, $p + 6$ for the 5 ranges mentioned above and also for $m \cdot 10^6 < p < (m + 1) \cdot 10^6$ with $m = 1(1)9$.

The author was apparently unaware of a paper by Sutton[2] (also employing a probability argument) giving data on the distribution of twin primes up to 800,000. Unfortunately the two sets of data do not agree in the few places where they are comparable. Thus Table 4 gives 198 as the number of twin primes between 90000 and 110000 whereas the correct number is 204, as given by Sutton. The probability of missing a few primes in such a count is not small, as the reviewer has found by very little experience, and errors of omission are apt to be about as large as the discrepancies between the actual count and the conjectured approximate formulae being tested.

D. H. L.

[1] J. W. L. Glaisher, "An enumeration of prime-pairs," *Mess. Math.*, v. 8, p. 28–33, 1878. See also BAAS, *Report, 1878*, p. 470–471.

[2] C. S. Sutton, "An investigation of the average distribution of twin prime numbers," *Jn. Math. Physics*, v. 16, 1937, p. 1–42.

346[F].—Albert Delfeld, "Table des solutions de la congruence $X^4 + 1 \equiv 0 \pmod{p}$ pour $300000 < p < 350000$," Institut Grand-ducal Luxembourg, Section des Sciences, *Archives*, v. 16, 1946, p. 65–70.

This table is an extension of three previous tables of Cunningham[1] ($p < 10^5$), Hoppenot[2] ($10^5 < p < 2 \cdot 10^5$) and Gloden[3] ($2 \cdot 10^5 < p < 3 \cdot 10^5$), all giving the two least positive solutions of the congruence $X^4 \equiv -1 \pmod{p}$.

The method of solution, based on the two quadratic partitions of p: $p = x^2 + y^2 = 2z^2 + t^2$, is described in *MTAC*, v. 2, p. 71. This table contains entries corresponding to the 980 primes p of the form $8n + 1$ between the limits $3 \cdot 10^6$ and $3.5 \cdot 10^6$. The author refers to a manuscript table by Gloden for $3.5 \cdot 10^5 < p < 5 \cdot 10^6$. No doubt this upper limit is a misprint for $5 \cdot 10^5$.

D. H. L.

[1] A. J. C. CUNNINGHAM, *Binomial Factorisations*, v. 1. London, 1923, p. 23–33; v. 4, London, 1923, p. 19–37.
[2] S. HOPPENOT, *Table des Solutions de la Congruence* $X^4 \equiv -1$ (mod N) *pour* 100000 $< N <$ 200000, (Librairie du Sphinx). Brussels, 1935, 18 p.
[3] A. GLODEN, "Table des solutions de la congruence $X^4 + 1 \equiv 0$ (mod p) pour $2 \cdot 10^5 < p < 3 \cdot 10^5$," *Mathematica* (Timişoara), v. 21, 1945, p. 45–65.

347[F].—EDWARD B. ESCOTT, "Amicable numbers," *Scripta Math.*, v. 12, 1946, p. 61–72. 16.8 × 24.6 cm.

This is a table described in UMT **9**, *MTAC*, v. 1, p. 95–96, with introductory text, partially quoted from Mr. Escott's *MTAC* note.

348[F].—A. GLODEN, "Table de factorisation des nombres $X^4 + 1$ dans l'intervalle $1000 < x \leqslant 3000$," Institut Grand-ducal Luxembourg, *Section des Sciences*, *Archives*, v. 16, 1946, p. 71–88.

This table is a by-product of tables of solutions of the congruence $x^4 + 1 \equiv 0$ (mod p), referred to in RMT **346**, which extend (if one includes a manuscript table of the author) to $p < 500000$. Although this upper limit serves to factorize all numbers $x^4 + 1$ only up to $x = 708$ when x is even and $x = 833$ when x is odd, nevertheless a large number of numbers $x^4 + 1$ beyond this range have relatively small factors and hence succumb to this method. The present table for $1000 < x \leqslant 3000$ is an extension of a previous table of CUNNINGHAM.[1] Of its 2000 entries, 1417 are completely factorized, 124 partially factorized, and the remaining 459 are of unknown character. All unknown factors exceed 500000. In order to discover more of these factors the author announces his intention of extending the congruence table to $p < 650000$. One eventually reaches a point of diminishing returns by this method since it becomes easier to attack each number to be factored directly. Just where this point is, depends upon the factorization methods at one's disposal. The table contains only one prime entry: $\frac{1}{2}(1089^4 + 1) = \frac{1}{2}(33^8 + 1) = 703204309121$, due to N. G. W. H. BEEGER.

D. H. L.

[1] A. J. C. CUNNINGHAM, *Binomial Factorisations*, v. 1. London, 1923, p. 113–119. See also p. 139 for 38 cases of $y^4 + 1$ with $y > 1000$. Erratum: $y = 2518$, *for* 461801, *read* 4681801.

349[G].—H. DAVENPORT, "The product of n homogeneous linear forms," Akad. v. Wetens., Amsterdam, *Proc.*, v. 49, 1946, p. 827. 18.1 × 26 cm.

$x^{-a} + (x-1)^{-a} = 1$, $f(a) = (ax + x^{-a})/(a + 1)$. There are tables of x, x^{-a}, $f(a)$, for $a = [.5(.1)3; 3D]$.

350[I, O].—I. J. SCHOENBERG, "Contributions to the problem of approximation of equidistant data by analytic functions. Part B. On the problem of osculatory interpolation. A second class of analytic approximation formulae," *Quart. Appl. Math.*, v. 4, July, 1946, p. 112–141. The tables, p. 136–141, are by Mrs. MILDRED YOUNG. 17.7 × 25.4 cm. See *MTAC*, v. 2, p. 167–169, for a review of Part A; also Corrigenda p. 228.

The numbering of the formulae in this review continues that of the previous one. Schoenberg's notation D^m, C^μ, E^k, s will be used to designate a broken polynomial base function of degree m, with μ continuous derivatives, which reproduces every polynomial of degree $\leqslant k$ exactly, and is of span s.

In this paper Schoenberg extends his theory to the case where his polynomial base functions are not spline functions, but have some lower order of continuity. As before, there are two main parts: a first part deals with the broken polynomials themselves, while the second deals with analytic base functions derived from them by considerations based on heat flow. These two chapters are preceded by one containing some lemmas whose motivation is not clear until one has got far into the paper.

The underlying motif of the first part of the paper may be described as follows. If $g(u)$ is the characteristic function (in the sense of Fourier analysis) of a base function $L(x)$ satisfying Schoenberg's restrictions, then Schoenberg proved in Part A that a necessary and sufficient condition that $L(x)$ "preserves the degree $k - 1$" [i.e. transforms every polynomial of degree $\leqslant (k - 1)$ into one of the same degree with the same leading coefficient] is that $g(0) = 1$ and for every integer $n \neq 0$, $g(u)$ has a zero of order at least k at $u = 2\pi n$. Then if $\psi_k(u)$ is defined by (10), with $t = 0$, it follows that

$$g(u) = \omega(u)\psi_k(u), \tag{17}$$

where $\omega(u)$ is regular in a horizontal strip containing the real axis. Special interest attaches to the case where $L(x)$ is an "ordinary" interpolation formula. There are three types of these which Schoenberg specially considers.

(1) (Schoenberg's second type). Here $\omega(u)$ is periodic (with period 2π). In that case $\omega(u) = [\phi_k(u)]^{-1}$, where ϕ_k is defined by (11) with $t = 0$. The resulting interpolation formula is D^{k-1}, C^{k-2}, E^{k-1} but is of infinite span if $k \geqslant 3$. This is a spline; compare (12).

(2) (Schoenberg's first type). Here

$$\omega(u) = \omega_1(u) + \omega_2(u)\psi_{k+1}(u)/\psi_k(u) \tag{18}$$

where $\omega_1(u)$ is of period 2π. Instead of taking ω_2 with period 2π, Schoenberg shows that there exist even functions ω_1 and ω_2 with $\omega_1(u + 2\pi) = \omega_1(u)$, $\omega_2(u + 2\pi) = -\omega_2(u)$, such that the resulting $L(x)$ is D^k, C^{k-2}, E^{k-1}, and $s = 2k - 2$ if k is even and $2k - 1$ if k is odd. This settles a conjecture of Greville's as to the existence of ordinary interpolation formulae of such character.

(3) (Schoenberg's third type). Here $\omega(u)$ is a polynomial in u^2 consisting of the Maclaurin expansion of $[\phi_k(u)]^{-1}$ as far as the term in $u^{2\mu-2}$, where μ = the integral part of $\frac{1}{2}(k + 1)$. The resulting formula is the k-point central difference (i.e. Lagrangean) formula.

Of these three types the first and third are useless as ordinary formulae, but they give rise to smoothing formulae by truncating the $\omega(u)$. Thus in the first case it is shown that $\omega(u)$ can be expanded as a power series in $s = \sin^2 \frac{1}{2}u$ with positive coefficients. If the first $m - 1$ terms of this series are taken, where $0 < 2m < k + 2$, we get a formula, designated $L_{k,m}(x)$ which is D^{k-1}, C^{k-2}, E^p, $s = k + 2m - 2$, where $p = \min(2m - 1, k - 1)$. In the third type if the Maclaurin series is stopped with the term in u^{2m-2} with $m < \mu$ we get a smoothing formula of type D^{k-1}, C^{k-2m}, E^{2m-1}, $s = k$.

In the second part of the paper Schoenberg considers formulae $L_{k,m}(x, t)$ which are derived from the $\psi_k(x, t)$ given by (10) in the same way as the $L_{k,m}(x)$ were derived from $\psi_k(x)$ above. It is shown that $L_{k,m}(x, t)$ is an analytic interpolation formula which is exact for the degree $\min(2m - 1, k - 1)$ and preserves the degree $k - 1$. The smoothing properties of $L_{k,m}(x, t)$ are, however, frankly left to depend on a conjecture. It is stated that $L_{k,m}$ damps out as $x \to \infty$ like $e^{-c^2x^2}$; whereas the L_k in Part A damps out only like e^{-c^2x}.

In the tables three functions $L_{4,2}(x, \frac{1}{2})$, $L_{4,2}(x, \frac{1}{4})$, and $L_{6,3}(x, \frac{1}{2})$ are tabulated to 8D, together with their first and second derivatives to 7D. The interval in x is .1, and the tables are arranged so that the values for x differing by an integer are in the same column. This facilitates subtabulation. The range of the tables varies; it includes the entire range where the function concerned is not zero to 8D—the approximate average is from -5 to $+5$.

As the author states explicitly the tables are only examples to illustrate the method. For practical use, with varying requirements as to smoothness, accuracy, etc., a more complete set of tables would be necessary.

HASKELL B. CURRY

Pennsylvania State Coll.

351[K, L, M].—A. J. BARKLEY ROSSER, *Theory and Application of* $\int_0^z e^{-x^2}dx$ *and* $\int_0^z e^{-p^2y^2}dy\int_0^y e^{-x^2}dx$. *Part I. Methods of Computation. Final Report* OSRD 5861, series B, no. 2.1 from the Alleghany Ballistics Laboratory, division 3, section H, Office of Scientific Research and Development, National Defense Research Committee, xii, 212 p. Nov. 1945. Offset print. 20.1 × 26.6 cm. **B.** EDWARD M. COOK & J. B. ROSSER, *Basic Computational Data* used at ABL in the *Preparation of the Numerical Tables in ABL Final Report, OSRD no. 5861, Part I.* 44 leaves, offset print on one side of each leaf. Nov. 1945. 20.1 × 26.6 cm. These publications are available only to certain Government agencies and activities.

A. The need for numerical values of the error function $\int_0^z e^{-x^2}dx$ for complex values of z arises in many scientific fields, especially statistics, optics, heat conduction, electro-chemical diffusion, and the propagation of electromagnetic waves. The present report consists mainly of a collection and extensive development of the known methods for calculating both the error function, i.e. the single integral, and the double integral $G(z, p) = \int_0^z e^{-p^2y^2}dy\int_0^y e^{-x^2}dx$, where z and p are any two complex numbers. At the end, tables are given for auxiliary functions to aid in the computations of both the single and double integral.

The first 105 pages treat the single integral $\int_0^z e^{-x^2}dx$ (which is called either "error function" or "error integral"), or alternatively, the sometimes more convenient function $F(z) = e^{z^2}\int_z^\infty e^{-x^2}dx$, and these pages comprise the more fundamental part of the report. The introduction establishes the notation and more obvious properties of the error function, (e.g. such as are seen by completing the square) and gives four other forms of $F(z)$. Then there follows a short discussion of the error function along the real, imaginary, or 45° rays, (the last being essentially Fresnel integrals), and the two direct power series expansions, i.e. the usual Maclaurin series and a series obtained by integrating by parts. In section 4. a number of fundamental inequalities are proved, in order to be used later, e.g. the simplest inequalities being those for z in the first quadrant, $|F(z)| \leqslant \frac{1}{2}\pi^{\frac{1}{2}}$, and for z^2 in the right half plane, $|F(z)| \leqslant 1/|2z|$. The next section, on computation by use of exponentials, is based upon Poisson's formula which expresses an infinite sum in terms of an infinite sum of cosine transforms. The equivalent of a special case of Poisson's formula is proved from first principles, and then it is applied to a function of exponential type, to obtain an equation between two infinite series of exponential terms. It is shown in several ways that it is possible to choose the parameters so that undesired terms, which usually include all of the left or right member of the equation, become negligible, and an expression for the error integral is obtainable as the sum of essentially exponential functions. In section 6. there is a discussion of the usual asymptotic series, in reciprocal powers, for $F(z)$, and upper bounds are found for the remainder in the more difficult case when z^2 is not in the first quadrant, (when z^2 is in the first quadrant, it is easily shown that the remainder is less in absolute value than the first term neglected). The section on the asymptotic series closes with a list of the first twenty terms of the series, as powers of $\frac{1}{2}z^{-2}$. Section 7. discusses properties of polynomials $A_N(z)$ and $B_N(z)$ which are the respective denominators and numerators of the Nth convergent in the continued fraction expansion for $F(z)$, and lists $A_N(z)$ and $B_N(z)$ as functions of z for $N = 1(1)16$. Here $A_N(z) = (-i)^N H_N(iz)$, where H_N denotes the Hermite polynomial of order N. Section 8. is devoted to the continued fraction for $F(z)$, namely $\frac{1}{2z+}\,\frac{2}{2z+}\,\frac{4}{2z+}\,\frac{6}{2z+}\cdots$, and a proof is given of the fundamental theorem $F(z) = \frac{B_N(z)}{A_N(z)} + \frac{1}{A_N(z)}\frac{d^NF(z)}{dz^N}$. A detailed discussion of the remainder in approximating $F(z)$ by $\frac{B_N(z)}{A_N(z)}$ leads to a number of practical upper bounds for $\left|\frac{d^N F(z)}{dz^N}\right|$, and to the theorem that the continued fraction converges to $F(z)$ for z^2 in the right half plane. (The authors suspect convergence everywhere except on the imaginary axis, but are unable to prove it.) More involved expressions for the remainder are found when z^2 is in the second quadrant, and the section closes with a note on Laplace's form of the continued fraction. The next three sections are concerned with general asymptotic formulae and two approximate quadratic equations for $F(z)$, which are admitted to be of limited use. The final recommendation regarding the single integral gives preference to the asymptotic series and continued fraction, both for their simplicity and accuracy.

The treatment of the single integral might have been enhanced by noting that much greater accuracy by means of the asymptotic series for $F(z)$ could have been obtained by expanding the remainder in a number of ways. Thus there is a method described in J. R. AIREY, "The 'converging factor' in asymptotic series and the calculation of Bessel, Laguerre and other functions," *Phil. Mag.* s. 7, v. 24, 1937, p. 521–552. Another method, which came to the reviewer's attention because it was used extensively by several members of the NBSMTP, is to write the integrand of the remainder as an exponential and to integrate by parts. Thirdly, the remainder appears to have a rather simple expression by employing for $\int_z^\infty e^{-x^2}x^{-m}dx$ a continued fraction that is given in a manuscript of H. T. DAVIS (available in microfilm) and which appears worthy of study. This continued fraction, which does not seem to be widely known, is

$$\int_z^\infty e^{-x^2}x^{-m}dx = \frac{e^{-z^2}}{z^m}\left[\frac{1}{2z+}\,\frac{m+1}{z+}\,\frac{2}{2z+}\,\frac{m+3}{z+}\,\frac{4}{2z+}\,\frac{m+5}{z+\cdots}\right].$$

The rest of the theoretical material, p. 106–173, is devoted to the double integral $G(z, p)$. For the special case when $p = i$, the methods used for the single integral are generalized. Thus sections 16. and 19.–23. concern themselves more specially with $p = i$ and apply methods similar to the power series, Poisson formula, asymptotic series, continued fraction, and approximate quadratic equation. For the more difficult case of a general p, there are developed, in sections 13.–17. and 24., six different types of series of the form

$$\psi + X\sum_{N=0}^{\infty} K_N Y^N \phi_N(Z),$$

where ψ, X, Y, Z are functions of z and p, the K_N's are constants and the ϕ_N's are various functions of Z. Furthermore, for each set of ϕ_N's it is possible to employ more than one set of values of the ψ, X, Y, Z and K_N's. The validity of these various series is for restricted sets of z and p, and the convenience extends to an even more limited range of z and p; but that is compensated by the number of different series from which to choose. Also, in section 18., a rather elaborate method is described for calculating $G(z, p)$, which is analogous to the Poisson formula approach in section 5. Many of the methods for computing $G(z, p)$ require that values of the single integral be available. In all these methods, scrupulous attention is devoted to the estimation of the accuracy, usually by supplying an upper bound for the remainder. As a by-product of the study of $G(z, p)$, means are provided for evaluating four different types of integrals, namely

$$\int_0^\infty \frac{e^{-x}dx}{(x+z^2)^{N+1\frac{1}{2}}},\qquad \int_0^\infty \frac{e^{-x}dx}{(x+z^2)^{\frac{1}{2}}(x+2z^2)^{N+1}},\qquad \int_0^\infty \frac{e^{-x}dx}{(x+z^2)^{N+\frac{1}{2}}(x+2z^2)},$$

and

$$\int_0^w y^{2N}F(y, c)dy.$$

In the appendix, p. 174–212, there are tabulated the following functions:

Table 1: $Rr(u) = \frac{1}{2}\cos\frac{1}{2}\pi u^2 - \frac{1}{2}\sin\frac{1}{2}\pi u^2 + C(u)\sin\frac{1}{2}\pi u^2 - S(u)\cos\frac{1}{2}\pi u^2$,
$Ri(u) = \frac{1}{2}\cos\frac{1}{2}\pi u^2 + \frac{1}{2}\sin\frac{1}{2}\pi u^2 - C(u)\cos\frac{1}{2}\pi u^2 - S(u)\sin\frac{1}{2}\pi u^2$,

(where $C(u)$ and $S(u)$ are the Fresnel integrals defined by

$$C(u) = \int_0^u \cos \tfrac{1}{2}\pi x^2 dx \quad \text{and} \quad S(u) = \int_0^u \sin \tfrac{1}{2}\pi x^2 dx,$$

$$Rr^2(u) + Ri^2(u), \quad \text{and} \quad \int_0^u Rr(x)dx.$$

All four of these functions are given to 12D, for $u = -.06(.02)3.5(.05)5$ and the first three are given also for $u = 5.05(.05)5.15$.

Table 2: $e^{-w^2}\int_0^w e^{y^2}dy$, $e^{w^2}\int_w^\infty e^{-y^2}dy$, $\int_0^w e^{-y^2}dy \int_0^y e^{x^2}dx$, and $\int_0^w e^{y^2}dy \int_y^\infty e^{-x^2}dx$.

All four functions are given to 10D. The first is for $w = -.2(.05)4(.1)6.5(.5)12.5$. The second is for $w = -.2(.05)3.8(.1)6.3$. The third and fourth are for $w = -.2(.05)+3.5(.1)6$. Sections 25. and 26. contain a discussion of the above mentioned tables which includes means of computation and checking. Section 27. discusses the method of interpolation, which is supplemented by the graphic representation of the error incurred in 2- to 6- or 7-point Lagrangean interpolation. At the end, 16 references are cited.

In evaluating this book, it is fitting to say that Dr. Rosser and his collaborators are to be highly commended for a rather thorough as well as rigorous investigation of the calculation of the single and generalized double error integral. This work is both a useful tool and a convenient reference source for all future computers of the error integrals. Its subject matter is both stimulating and suggestive of approaches to much more general types of integrals where the integrands are functions of the exponential type.

In connection with **A**, some preliminary computational data are tabulated separately in **B**.

Most of this report, p. 2–36, gives the first 30 or so coefficients in the Taylor expansions of $Rr(u)$, $Ri(u)$, $Rr^2(u) + Ri^2(u)$, $\int_0^u Rr(x)dx$, and $\int_0^u dx \int_0^x Rr(y)dy$ about the points $u = \tfrac{1}{2}(\tfrac{1}{2})3$, (about the point 0, coefficients are given for certain smooth parts of these functions, which are defined in the text). Most coefficients are given to 20D or more. On p. 41–44 are some 30 Taylor coefficients of $e^{-w^2}\int_0^w e^{y^2}dy$ about $w = [2(1)5; 15D]$. On p. 37 the following functions are tabulated: $Rr(u)$, $Ri(u)$, $Rr^2(u) + Ri^2(u)$, for $u = [5(.05)6.15; 13D]$. On p. 40, there are values of $e^{-w^2}\int_0^w e^{y^2}dy$ for $w = .05(.05).6, 1(.5)4, 5, 6, 6.5$, almost all to 15D, and of $e^{w^2}\int_w^\infty e^{-y^2}dy$ for $w = 3, 3.8, 4.1, 4.7, 5, 5.5, 6$, to 13D or more. On p. 38–39, the approximating polynomials $Q(n, u)$ and $P(n, u)$, where $Rr(u) + iRi(u) \cong \frac{Q(n, u)}{P(n, u)}$, are listed for $n = 4(4)28$ as functions of u and π.

HERBERT E. SALZER

NBSMTP

352[L].—J. COSSAR & A. ERDÉLYI, *Dictionary of Laplace Transforms.* Admiralty Computing Service, Department of Scientific Research and Experiment, London. Part 3B, no. SRE/ACS 108, June 1946, leaves VII 30–VII 70 + leaf "Index of notations" + t.p. leaf. 20.2 × 33.1 cm. Also *Corrections and Additions to SRE/ACS* 53, 68, *and* 71, June 1946, 9 leaves. All leaves are mimeographed on one side only. These publications are available only to certain Government agencies and activities.

This is the concluding part of the *Dictionary* (started in 1944) the earlier parts of which were reviewed in *MTAC*, v. 1, p. 424–425 and v. 2, p. 76. Part 3A contained the beginning of Section VII, with a Table of inverse Laplace Transforms, that is, a classification, according to $\phi(p)$ for the Elementary Functions, Gamma and Related Functions, and Functions defined by Integrals. Part 3B, the concluding part of Section VII, is the classification $\phi(p)$ for Bessel Functions, Confluent Hypergeometric Functions, Legendre Functions, and Hypergeometric Functions, etc. Practically all the corrections and additions are for

Section VI, Table of Laplace Transforms. We are told that some of these entries were discovered by comparing parts of the published *Dictionary* with N. W. McLACHLAN & P. HUMBERT, *Formulaire pour le Calcul Symbolique*, Paris, 1941, and that among correspondents H.B. "made the greatest single contribution to this list."

R. C. A.

353[L].—ROBERT C. HERMAN & CHARLES F. MEYER, "The thermoluminescence and conductivity of phosphors," *Jn. Appl. Physics*, v. 17, Sept. 1946, p. 748. 19.8 × 26.6 cm.

The article includes a table of $-Ei(-x) = \int_x^\infty e^{-u}du/u$, for $x = [15(.1)20; 6S]$, employed in the calculation of glow intensity curves, etc. It bridges a gap between tables of NBSMTP and BAASMTC,[1] for $0 < x \leqslant 15$, and of AKAHIRA,[2] for $20 \leqslant x \leqslant 50$. The table was calculated from the asymptotic expansion

$$(1) \qquad -Ei(-x) = e^{-x}[x^{-1} - x^{-2} + 2!x^{-3} - 3!x^{-4} + 4!x^{-5} - \cdots],$$

and second (central) differences are included under the heading Δ^2.

Recalculation by (1), of errors discovered by differencing, led to the following corrections:

At $x = 15.9$, *for* 7.38008×10^{-9}, *read* 7.38256×10^{-9};
$x = 19.1$, *for* 2.52729×10^{-10}, *read* 2.52793×10^{-10}.

There are corresponding errors in the differences in addition to the following:

At $x = 17.9$, *for* $.10031 \times 10^{-10}$, *read* $.09931 \times 10^{-10}$;
$x = 18.0$, *for* $.08843 \times 10^{-10}$, *read* $.08943 \times 10^{-10}$.

Modified second differences should have been provided.

E. L. KAPLAN

2025 Fendall St. S. E.
Washington 20, D. C.

[1] NBSMTP, *Tables of Sine, Cosine and Exponential Integrals*, 2 v., 1940. BAASMTC, *Mathematical Tables*, v. 1, *Circular & Hyperbolic Functions, Exponential & Sine & Cosine Integrals* . . ., second ed., 1946.

[2] TAKEO AKAHIRA, *Tables of e^{-x}/x and $\int_x^\infty e^{-u}du/u$ from $x = 20$ to $x = 50$*, Institute of Phys. and Chem. Research, *Sci. Papers*, Tokyo, *Table no. 3*, 1929.

354[L].—TOKIHARU OKAYA, "Numerical tables of Tchebychef's q-functions and their integrated and derivated functions," Nippon Suugaku-buturigakkwai *Kizi*, Tokyo, *Proc.*, s. 3, v. 23, 1941, p. 788–799. 18.1 × 25.6 cm.

The functions referred to in the title are

$$q_v(n, z) = v!2^{-v}\sum_{k=0}^{v}\binom{v+k}{k}\binom{v-n}{v-k}\binom{z}{k}$$

and were introduced by CHEBYSHEV[1] to approximate a given function $f(z)$ defined for $0 \leqslant z \leqslant n - 1$ by the linear combination

$$(1) \qquad F_k(n, z) = a_0 + a_1q_1(n, z) + \cdots + a_{k-1}q_{k-1}(n, z).$$

This choice of the functions q minimizes the quantity

$$\sum_{r=0}^{n-1} \{f(r) - F_k(n, r)\}^2$$

and the coefficients $a_0, a_1, \cdots, a_{k-1}$ are determined by

$$S_v(n)a_v = \sum_{r=0}^{n-1} f(r)q_v(n, r),$$

where

$$S_v(n) = \sum_{r=0}^{n-1} \{q_v(n, r)\}^2.$$

The integral or derivative of $F_k(n, z)$ is an approximation to the integral or derivative of $f(x)$ and either can be obtained at once from (1).

The author refers to previous tables[2] of these functions, and notes that they contain misprints, but does not actually list them. The present tables (p. 790–798) give the exact (rational) values of I. $q_v(n, r)$ and of IV. its integral, of III. its derivative, and of II. $S_v(n)/u_v(n)$ for $n = 5(1)20$, $r = 0(1)19$ and $v = 1(1)6$; $u_v(n)$ is a certain numerical coefficient. T. I was prepared in collaboration with ZIRÔ YAMAUTI.

D. H. L.

[1] P. L. CHEBYSHEV, Akad. N., Leningrad, *Mémoires*, s. 7, v. 1, 1859, p. 1–24; and *Oeuvres*, St. Petersburg, v. 1, 1899, p. 473–498.

[2] (a) CHARLES JORDAN, London Math. Soc., *Proc.*, s. 2, v. 20, 1922, p. 322–325. The tables I–V are for $q_v(n, r)$, and table VI for $S_v(n)$, $v = 1(1)5$, $n = 2(1)20$, $r = 0(1)11$. (b) T. OKAYA [Methods of calculation, probability and statistics] (in Japanese), 1940. EDITORIAL NOTE: We have not been able to check this item whether it was an article in a periodical, or a book, by Okaya.

355[L].—JOSEPH B. ROSENBACH, EDWIN A. WHITMAN, & DAVID MOSKOVITZ, *Mathematical Tables*. Boston, Ginn, 1937. xl, 208 p., frontispiece, and duplicated p. 207–208 in pocket. 14.3 × 21 cm. Second edition, corrected and enlarged, 1943. xl, 212 p. $1.75.

T. XXVII, p. 183: $\Gamma(n)$, n = [1.01(.01)2; 5D].
T. XXVIII, p. 183: $\log \Gamma(n)$, n = [1.01(.01)2; 5D].
T. XXIX, p. 184–185: $n^{-\frac{1}{2}}$, $[n(n-1)]^{-\frac{1}{2}}$ each to 8D; $.67449(n-1)^{-\frac{1}{2}}$, $.67449[n(n-1)]^{-\frac{1}{2}}$, $.84535[n(n-1)]^{-\frac{1}{2}}$, $.84535[n(n-1)^{\frac{1}{2}}]^{-1}$ each to 5D; and all tables for $n = 2(1)100$.
T. XXX, p. 186: $(2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$, for x = [0(.01)5; 5D].
T. XXXI, p. 187: $(2\pi)^{-\frac{1}{2}}\int_0^x e^{-\frac{1}{2}t^2}dt$, for x = [0(.01)5; 5D].
T. XXXII, p. 188: $n!$ to five figures, and $\log n!$ to 5D, for $n = 1(1)60$.
T. XXXIII, p. 188: B_n to five figures, and $\log B_n$ to 5D, for $n = 1(1)60$.
T. XXXIV, p. 189: K and E for θ = [0(1°)70°(30′)80°(12′)89°(6′)89°30′(3′)90°; 4D].
T. XXXV–XXXVI, p. 190–197: $F(k, \phi)$, and $E(k, \phi)$, for θ = 5°(5°)90°, ϕ = [1°(1°)90°; 4D].
T. XXXVII, p. 198–199: $P_n(x)$, for $n = 1(1)7$, x = [0(.01)1; 4D].
T. XXXVIII, p. 200–201: $P_n(\theta)$, for $n = 1(1)7$, θ = [0(1°)90°; 4D].
For T. XXXIX–XLI, see *N65*.
T. XLII, p. 204: $\log x$ and $\log(1 + 0.1^n x)$, for $n = 1(1)11$, and x = [1(1)9; 20D].
T. XLIII, p. 204: kM and k/M, for k = [1(1)9; 24D].
T. XLIV, p. 205: $\log p$, p prime and <1000, to 20D.
T. XLVI, p. 207–210 (in new edition): Table of haversines. P. 211–212 (old edition p. 207–208): Proportional parts (also in pocket of new edition).

The changes in the first edition of the *Tables* were as follows:

P. 191, $F(75°, 37°)$, *for* 0.6913, *read* 0.6919.
P. 193, $F(60°, 60°)$, *for* 1.2125, *read* 1.2126.
P. 200, 53 corrections of $P_6(\theta)$ and $P_7(\theta)$.
P. 201, 2 corrections of $P_2(\theta)$, 2 of $P_4(\theta)$, 1 of $P_5(\theta)$, 16 of $P_6(\theta)$, 18 of $P_7(\theta)$. Pages 200–201 now agree with the corresponding tables in JAHNKE & EMDE (1945).
P. 203, $J_0(3)$, and fourth, fifth and eighth zeros of $J_1(x)$ are corrected.
P. 206, the digit in the thirty-second decimal-place approximation for π was correctly 0 in the first edition and incorrectly changed to 6 in the second edition.

R. C. A.

356[L, S].—F. B. PIDDUCK, *Currents in Aerials and High-Frequency Networks.* Oxford, Clarendon Press, 1946. iv, 97 p. 14 × 22.1 cm. 8s. 6d.

The tables include the following:

P. 70, $(1, s)$ for $s = [0(1)23; 5D]$, where $(n, s) = \int_0^\pi e^{in\pi y} y^s dy$;
$(1, 23) = -35061200000 + 4410840000i$.
P. 86–91, Ci x, Si x for $x = [0(.01)10(.1)29.9; 5D]$.
P. 70, 96–97; in the treatment of aerials parallel to the earth the function $E_n^s(x) = C_n^s(x) + iS_n^s(x) = \int_0^\pi e^{in\pi y} J_0(xy) y^s dy$ was introduced. For large values of x the function is expressible in terms of Lommel functions. There are tables of C_1°, S_1°, $C_1^{\circ\prime}$, $S_1^{\circ\prime}$, $C_1^1(x)$, $S_1^1(x)$, $\pi C_1^\circ(x) + S_1^{\circ\prime}(x) - C_1^1(x)$, $(x^2 - 1)^{\frac{1}{2}}$, for $x = [0(.05)2(.2)10; 4D]$.
P. 92–95, sin x and cos x, for $x = [0(.01)7.99; 5D]$.

357[M].—MURLAN S. CORRINGTON, "Table of the integral $2\pi^{-1}\int_0^x \tanh^{-1} t\, dt/t$," *R. C. A. Review*, v. 7, Sept. 1946, p. 432–437. 14.9 × 22.7 cm. Compare UMT **51**, p. 184.

The integral $B(x) = \pi^{-1} \int_0^x \ln \left| \frac{1+t}{1-t} \right| dt/t = 2\pi^{-1} \int_0^x \tanh^{-1} t\, dt/t$ is involved in the computation of either the real or imaginary component of a minimum-phase-shift network, having the component given.[1] $B(x)$ is tabulated for $x = [0(.01).97(.005).99(.002)1; 5D]$ rounded off from computations to 8D [see *MTAC*, v. 2, p. 184]. The methods of computation and checking are explained in detail. After the difference test indicated that the computed values were all within one unit in the eighth decimal place, the values were rounded off to 5D. It is therefore hoped that the table is free from error.

Extracts from introductory text

[1] HENDRIK W. BODE, *Network Analysis and Feedback Amplifier Design*, New York, 1945, chapters 14–15.

358[M].—HARVARD UNIVERSITY, Computation Laboratory Reports for the U. S. Bureau of Ships: No. 2, June, 1944, *Evaluation of the Function* $S(b, h) = \int_0^h \sin (x^2 + b^2)^{\frac{1}{2}} dx/(x^2 + b^2)^{\frac{1}{2}}$; by H. H. AIKEN & R. V. D. CAMPBELL. 5 leaves mimeographed on one side of each. No. 10, Oct. 1944, *Evaluation of the Function* $C(b, h) = \int_0^h \cos (x^2 + b^2)^{\frac{1}{2}} dx/(x^2 + b^2)^{\frac{1}{2}}$, by H. H. AIKEN, H. A. ARNOLD, R. V. D. CAMPBELL, & R. R. SEEBER. 9 leaves mimeographed on one side of each. 20.3 × 26.7 cm. These tables are available only to certain Government agencies and activities.

These integrals arise in considerations connected with coupled antennae. Their values are tabulated for $h = .5(.5)6.5$, $b = [0(.1)6.3; 5D]$. In the table of $S(b, h)$ all arithmetic work was carried to 15D. Hence all tabular entries should be accurate. In the table of $C(b, h)$ an error of less than 3×10^{-5} is assured for all results, and in most cases it is less than 10^{-5}.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **338** (Nelshuler), **340** (Zimmermann), **341** (Elznic & Valouch), **342** (Marchin), **343** (Févrot), **344** (Elznic), **345** (Cherwell), **346** (Delfeld), **348** (Cunningham), **352** (Cossar & Erdélyi), **353** (Herman & Meyer), **354** (Jordan, Okaya), **355** (Rosenbach, Whitman & Moskovitz), N **63** (Leibniz), **64** (Womersley), **65** (Rosenbach, Whitman & Moskovitz), **66** (Crommelin, Meares).