61[F].—ALBERT GLODEN, *Tables of the Decimal Endings of Cubes, Fourth Powers, and Eighth Powers together with the Linear Forms of the Corresponding Roots.* 6 + 7 + 3 leaves, 21 × 29.7 cm. Typed manuscripts in the possession of the author (rue Jean Jaurès 11, Luxembourg), and of the Brown University Library.

These tables give 1-, 2- and 3-digit endings of cubes, fourth and eighth powers (for the decimal system), arranged in order of magnitude, as well as 4-digit endings of fourth powers. With each such ending are listed all numbers whose corresponding power ends in this way. Thus with the fourth power ending $\cdots 3041$ the author gives the entry

$$3041 \qquad 77{,}361$$

This means that all numbers whose fourth power ends in 3041 are given by

$$1250n \pm 77, \qquad 1250n \pm 361 \qquad (n = 0, \pm 1, \pm 2, \cdots).$$

These tables are extensions to higher powers of corresponding tables, such as those of CUNNINGHAM,[1] concerning squares. The lists of power endings are useful in showing at a glance that a given number is not a power of degree 3, 4, or 8. The rest of the information is useful in setting up exclusion procedures in dealing with diophantine equations of these degrees.

D. H. L.

[1] A. J. C. CUNNINGHAM, *Quadratic and Linear Tables.* London, 1927, p. 89–92.

62[F].—LUIGI POLETTI, *Atlante di centomila numeri primi di ordine quadratico entro cinque miliardi.* Manuscript in possession of the author, Via Cairoli 1, Pontremoli, Italy.

For many years Poletti has been investigating the distribution of primes of the form $ax^2 + bx + c$ for as many as 366 different values of (a, b, c), and now has a list of 116683 primes $> 10^7$ and $\leqslant 5101683361$. The principal forms considered are those for which $(a, b, c) = (1, 1, 1), (1, 1, -1), (2, 2, 1), (2, 2, -1), (1, 1, 17), (6, 6, 31), (3, 3, 1), (3, 3, -1),$ $(1, 21, 1)$ and

$$x^2 + x + 41 \qquad\qquad x^2 + x + 72491$$
$$x^2 + x + 19421 \qquad\qquad x^2 + x + 146452961.$$
$$x^2 + x + 27941$$

These last five were chosen for study on account of their apparently high density of primes among the numbers they represent. The first of these is due to EULER and admits of no prime factor < 41. The second was suggested by D. H. L. and admits of no factor < 47. The third and fourth are due to N. G. W. H. BEEGER and also admit of no factor < 47. The last is due to D. H. L. and admits of no factor < 109. (See *Sphinx* v. 6, 1936, p. 212–214; v. 7. 1937, p. 40; v. 9, 1939, p. 83–85.) These 5 series have been pushed to high limits x, in fact to $x = 55102, 32147, 16356, 16345$ and 70400 respectively. The corresponding numbers of primes found are 18667, 11473, 6897, 7016 and 27858. The percentages of primes in these five cases are thus .3388, .3569, .4216, .4292 and .3957. It is interesting to note that it has never been proved that any one of these quadratic progressions contains infinitely many primes.

D. H. L.

AUTOMATIC COMPUTING MACHINERY

This new Section will deal with matters pertaining to large-scale automatically-sequenced computing machinery. The wartime need for ultra high-speed calculations has caused a development of the field which may well have a profound effect upon methods of classification and compilation of data and of numerical computation. As a result of this activity, there now exist in the United States four types of large-scale computing machines: (1) the differential analyzer of the Massachusetts Institute of Technology, (2) an electromechanical computing device (the IBM Sequence Controlled Calculator) of the Harvard Computation Laboratory, (3) relay-type computing machines of the Bell Telephone Laboratories, and (4) an electronic computing machine (ENIAC) of the Ballistic Research Laboratory, Aberdeen Proving Ground, Md.

After the introductory article, the material in this Section falls under the general headings of TECHNICAL DEVELOPMENTS, DISCUSSIONS, BIBLIOGRAPHY, and NEWS. The necessary editing is provided by the staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. CANNON, 418 South Building, National Bureau of Standards, Washington 25, D. C.

THE ZUSE COMPUTER[1]

General Description. The Zuse computer is a relay-type digital machine. At present electro-magnetic telephone relays are used except in the storage unit, where special mechanical relays have been introduced. The machine operates upon numbers in dyadic representation and carries out automatically the ordinary algebraic operations, including division and extraction of square roots. Automatic handling of algebraic sign and of decimal point are provided. Numerical input is limited to the use of a manual keyboard, and output to a visual display panel. An automatically-sequenced program can be introduced by means of punched tape; however it must be followed through in a rigidly prescribed manner.[2] Only directions as part of the program, and not numerical values, can be entered by means of the tape. The machine will handle numbers of five significant figures, within a range of magnitude of $10^{\pm 20}$. Multiplying time is of the order of one second. Projected storage capacity is for 1024 numerical quantities, but only 16 cells have been constructed to date.

The machine is constructed of rather rough homemade components, so that it may be assumed that it occupies more space, and must operate more slowly, than would be necessary for a more carefully engineered model. The manual controls together with the visual display panel are built into a unit of approximately the shape and size of a small upright piano. The relay circuits are contained in four large cases roughly 6′ × 3′ × 1′. The storage component, with space provided for the projected 1024 cells, is contained in two boxes less than a yard square and 15″ high. Zuse estimates about 1000 cells to the cubic yard. In addition to these parts there are an electric motor for operating the mechanical components, a drum commutator for operating the relays, a tape reader, and a tape punch.

Numerical Input and Output. The machine receives five-digit quantities in decimal notation. These are entered by successively pressing a combination of the 10 digit keys and the key for the decimal point. In addition, the decimal point may be shifted 6, 12, or 18 places to the right or left by pressing one of the keys 10^{+6} or 10^{-6} one, two or three times. The numbers so entered appear immediately in the form of illuminated lamps on the display panel. All numbers are entered thus in semilogarithmic decimal form and are automatically converted into dyadic representation. Twenty-two dyadic places are retained, which corresponds to something over six significant figures in decimal notation.

In operation all quantities are taken in dyadic semilogarithmic form with now exist in the United States four types of large-scale computing machines: (1) the differential analyzer of the Massachusetts Institute of Technology, (2) an electromechanical computing device (the IBM Sequence Controlled Calculator) of the Harvard Computation Laboratory, (3) relay-type computing machines of the Bell Telephone Laboratories, and (4) an electronic computing machine (ENIAC) of the Ballistic Research Laboratory, Aberdeen Proving Ground, Md.

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In operation all quantities are taken in dyadic semilogarithmic form with algebraic sign. Decimal point position and algebraic sign are handled automatically in all calculations.

Output is obtained by a visual display panel comprising seven columns of ten lamps each for the significant figures, with auxiliary lamps to indicate position of the decimal point, algebraic sign, and other special quantities. Any quantity in the machine may be called for on the display panel at any time, and is automatically converted into decimal notation before presentation.

No means of numerical input or output other than the keyboard and the display panel are provided.

Special Algebraic Devices. A special key is provided for zero, the reason being given that this quantity is unsuited for expression in semilogarithmic form. In addition to the special symbol for zero, a symbol, $\ll$, is provided on the display panel, signifying "of too small a magnitude for the scope of the machine." This would appear, for example, as the result of subtracting any number from itself. It is difficult to see the value of this arrangement, since it was admitted that the symbol "0," for exactly zero, could never be obtained as the result of a computation.

Three expressions for infinity are employed. Of these, $+\infty$ and $-\infty$ indicate a result which is positive or negative but numerically beyond the scope of the machine; the third, $\pm\infty$, denotes a large quantity which is ambiguous as to sign, e.g., the result of dividing 1 by 0 or by $\ll$. An additional symbol, 0/0, signifies an indeterminate result, e.g., the result of dividing one very small quantity by another.

The algebraic combination of these various signs for infinite, infinitesimal, and indeterminate quantities is handled automatically in the same fashion as for ordinary numbers; no special attention is demanded of the operator, nor in preparation of an automatic program. Thus, for example, $(+\infty) - (+\infty)$ automatically yields 0/0, etc.

One key is provided for the minus sign, $-$, and corresponding indicators for $+$ and $-$ on the display panel.

Likewise a key and indicator are provided for the imaginary unit, $i = \sqrt{-1}$. However, operation with imaginary quantities is limited to the two rules that when the square root of a negative number is called for, the symbol i will register, and that when two pure imaginaries are multiplied together a minus sign will register. Beyond this, the machine is not equipped for automatic computation with complex numbers.

Arithmetical Unit. The arithmetical unit contains two special storage cells. When numbers have been read into one or both of these cells, a key may be pressed directing that the two numbers be added, subtracted, multiplied or divided; or, in the case of a single number, that the negative, the square root, the double, the half, the 10 fold, the 10th part of the first of these numbers be produced, or that the number be multiplied by π. Provision is planned for squaring, taking the reciprocal, and the maximum or minimum of two quantities, but these operations are not yet available. The result of an operation is in turn stored in the arithmetical unit. By pressing an appropriate key, the result of the operation can be made to appear upon the display panel as fast as it is computed.

The arithmetical operations are carried out by relay circuits. The conversion from decimal to dyadic representation is carried out by the relays as are used for addition, together with some auxiliary relays for controlling the operation. Information regarding the actual circuits employed was withheld; however, it was stated that the more complicated operations of division and extraction of square roots are carried through by the procedures used in hand computation.

Storage. The storage is planned to contain 1024 cells, each of which will hold a single numerical quantity with its algebraic sign and decimal position. For this purpose mechanical relays are used. These are of a special design, of which the details were not revealed. One layer of 16 cells is now in operation.

The relays appear to consist of a number of thin strips of metal lying between two plates of glass, not more than a quarter of an inch apart. Strips running in one direction through this layer represent the 16 cells, each containing one numerical quantity, while those in the perpendicular direction represent the individual digits, etc., of each cell.

Motion of the strips is controlled by electromagnetic relays that engage or disengage individual strips with an arm providing mechanical impulses at regular intervals. It would appear that motion of a particular strip representing a chosen cell exposes other moving parts, corresponding to each digit position in the cell, to the motion communicated by the various transverse digit strips. The whole mechanism appeared quite compact and simple; however, detailed examination or description was denied.

There is no doubt that this type of storage is more compact than electromagnetic relays. One layer of 16 cells is roughly two feet square and ¼″ thick, to which must be added a few inches for the protruding ends and the operating mechanism. This storage was demonstrated and appeared satisfactory with regard to accuracy and speed of operation; however it was stated that some difficulty had resulted from the necessity of using low-quality metal.

In operation a number may be directed into any cell of the storage from either the input keyboard or the computer. The storage does not operate as an accumulator, and it is not necessary to clear a cell before use. If a second number is directed into a cell in which a number is already stored the first number will be completely lost and replaced by the second. Once a number is entered, it remains in the storage until replaced by another. A number in any cell can be read into the computer, or onto the display panel, at will and as often as is required.

Control Unit. The timing of the basic operations of the machine is controlled by an electric motor, which drives a cylindrical commutator operating the relays and also provides the mechanical drive for the storage relays, thus assuring synchronization. The actual speed of operation was stated to be about half what could ultimately be expected. This delay was attributed to difficulties with the electrical circuits rather than to the electromagnetic or mechanical relays themselves.

All operations and transfers are initiated by pressing a control key, and the completion of an operation is indicated by a special lamp on the display panel. Operating time was stated to be about one multiplication per second. This was confirmed in the demonstration. Extraction of a square root was timed at slightly over 5 seconds. Compared with these operating times, and with the time required for manual input and control, the time required for transfer from one part of the machine to another is negligible. It was stated that the mechanical relays had about the same speed of operation as the electromagnetic ones.

The timing of the sequence of arithmetical operations is controlled by the operator. When the signal flashes, indicating that one operation is complete, a key must be pressed to initiate the next operation. When the machine operates under automatic control, the completion of one operation causes the tape reader to advance and initiate the next.

Automatic Program. The machine may be operated by a punched film. This is ordinary celluloid moving picture film about an inch wide. An order may be punched on to the film in the form of some combination out of 8 holes, occupying two lines across the film. Numerical values cannot be entered by means of a tape, but only directions to transfer numbers between the storage and the computer, or to perform algebraic operations on the quantities in the computing unit. Before an automatic program can begin, all the initial values must be entered manually by way of the keyboard into the proper cells of the storage. After the calculation is complete, or at any time the operator chooses to intervene, the quantities in the storage may be read from the visual display panel.

The tape is prepared by directing the machine through the course of the calculation in exactly the same way, and possibly at the same time, as for non-automatic operation. The program can be of any length compatible with the storage capacity. An important limitation upon programming is that the machine must adhere to a prescribed linear course of operation. It cannot at any point choose between two subsequent programs on the basis of results already obtained; nor can it be directed to repeat automatically sub-programs within the same total program. A further incidental inconvenience is that even such constants as the number 1 must be entered into the storage along with the initial values, or else obtained as the result of trivial calculations. The speed of the machine for automatic operation is somewhat, but not a great deal, faster than for manual operation.

Present Status of Machine. There exists at present only one model of the Zuse machine. As has been indicated, this is incomplete both in mathematical and electrical design, and with regard to the engineering of the components. The machine is the property of Dipl. Ing. Konrad Zuse and is now housed in the cellar of a farm building in the village of Hopferau near Füssen, in southernmost Germany. Zuse has with him one assistant by the name of Stucken.

Improvements are being undertaken in the electrical circuits, as well as the completion of some of the projected algebraic operations which have not yet been installed, and the construction of more storage cells. This work is being carried out under rather primitive conditions and with inadequate material.

Zuse plans ultimately to convert the machine entirely to mechanical relays. On the basis of his experience with the mechanical relays now in use in the storage, and with earlier entirely mechanical machines, he is convinced that this will offer no serious difficulty once materials become available. It is his hope that after perfecting the design of a compact mechanical relay machine, he will be able to undertake commercial production in quantity.

So far as can be ascertained, Zuse is carrying out his work independently of any interest or assistance from outside sources.

ROGER C. LYNDON

Office of the Assistant Attaché
for Research, American Embassy,
London, England

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[1] This Report was prepared for the Office of Naval Research by Dr. Lyndon of the London branch office. The Zuse computer, invented by Konrad Zuse, is of interest in that it represents the work of German scientists in the field of large-scale computing devices. By 1943 Zuse had established his own firm for the construction of the computers he had invented. Two special-purpose aerodynamic machines developed during the war apparently proved successful. A general-purpose algebraic computer was also completed and had passed its test runs prior to the cessation of hostilities. Although these computers are interesting because of their similarity with wartime developments in the United States, it is clear that they are of limited utility in their present state of development. See also BIBLIOGRAPHY, Z-I, no. 3.

[2] ZUSE states in a letter, received since the completion of this article, that it is now possible to enter constants by means of the punched tape; he also points out that with four tape readers and two tape punches, as called for in the design, the machine will be able to repeat programs and to choose between alternative sub-programs. He states further that when the designed wirings are complete, the machine will be fully equipped to handle problems involving complex quantities.

TECHNICAL DEVELOPMENTS

The Selectron—A Tube for Selective Electrostatic Storage

(See frontispiece plate)

We are engaged at the RCA Laboratories in the development of a storage tube for the inner memory of electronic digital computers. This work is a part of our collaboration with the Institute for Advanced Study in the development of a universal electronic computer. The present note describes briefly the principle of operation of the tube, which is still in its experimental stage. It is a summary of a paper presented at the "Symposium of Large Scale Calculating Machinery" at Harvard University on January 8, 1947; see *MTAC*, v. 2, p. 229–238.

The necessity of an inner memory in electronic digital computers has been realized by all designers. The high computing speed possible with electronic devices becomes useful only when sufficient intermediary results can be memorized rapidly to allow the automatic handling of long sequences of accurate computations which would be impractically lengthy by any other slower means. An ideal inner memory organ for a digital computer should be able to register in as short a writing time as possible any selected one of as many as possible on-off signals and be able to deliver unequivocally the result of this registration after an arbitrarily long or short storing time with the smallest possible delay following the reading call.

The selectron is a vacuum tube designed in an attempt to meet these ideal requirements. In it, the signals are represented by electrostatic charges forcefully stored on small areas of an insulating surface. The tube comprises an electron source which bombards the entire storing surface. The insulator can be a circular cylinder coaxial with a standard thermionic cathode. Between the cathode and the storing surface there are two orthogonal sets of spaced parallel metallic bars, which form a checkerboard of windows creating the corresponding small storing elements on the insulating surface. In the cylindrical structure one set is formed by rings and the other by straight bars spaced angularly around the cylinder. These bars are insulated from each other, making it possible to apply positive or negative potentials to them and thereby to stop the flow of electrons through all windows except a desired one. Between the selecting structure and the insulating surface is the collector, a grid-like unipotential electrode. The insulating surface is backed by a metal plate called the capacity plate. The operation of the tube consists of assigning selectively an element of surface for each incoming signal, storing the signal-information on that element, and subsequently detecting the stored information identified by its previously assigned location. The selecting and storing mechanisms will now be described separately.

Consider one of the sets of selecting parallel bars. Electrons will pass between two adjacent bars when they are both at the same positive potential with respect to the cathode. On the other hand, if both bars are substantially negative, the electrons will be blocked by a negative potential barrier in front or in the gate formed by wires. When one bar is positive and the adjacent one negative, electrons will also be stopped, provided the geometry of the bars and voltage levels are properly chosen. It is clear, therefore, that if another set of parallel bars is placed at right angles behind the first, electrons will pass through a window limited by two pairs of bars only if all four bars are positive. For a large checkerboard of windows, the number of control voltages and consequently the number of leads to be sealed through the vacuum envelope would be very large if each bar had to be controlled separately. However, this is not necessary, because the fact that a coincidence of both limiting bars is necessary for the opening of a gate makes it possible to connect internally the bars of any one set in groups and control only the potential of a relatively small number of groups. Single positive wires surrounded by negative ones do not open any gates and, therefore, can be connected to the wires of the selected open gate. There are many connection systems solving the combinatorial problem of how to group the elements of each row such that each group contains one element neighboring with an element of each of the other groups, once and once only. As an example, in a system used in some experimental tubes, there are 64 selecting bars in each direction, connected in 16 groups of 4 each. These groups are divided into two families identified by 1, 2, 3, 4, 5, 6, 7, 8 and 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′. The enumeration of the bars according to the group to which they belong is as follows: 1, 1′, 2, 2′, 3, 1′, 4, 2′, 5, 1′, 6, 2′, 7, 1′, 8, 2′; 1, 3′, 2, 4′, 3, 3′, 4, 4′, 5, 3′, 6, 4′, 7, 3′, 8, 4′; 1, 5′, 2, 6′, 3, 5′, 4, 6′, 5, 5′, 6, 6′, 7, 5′, 8, 6′; 1, 7′, 2, 8′, 3, 7′, 4, 8′, 5, 7′, 6, 8′, 7, 7′, 8, 8′, from which it is apparent that each non-primed group has an element neighboring with an element of every primed group once and once only. In this example, $16 + 16 = 32$ sealed leads control $64 \times 64 = 4096$ elements. More efficient combinatorial systems are possible, particularly with several successive sets of bars in each direction. Anyhow, the number of necessary seals in the indicated system for which N leads control $(\frac{1}{4}N)^4$ elements is relatively so small that it presents no technological limitation even for many elements (e.g., 128 seals can control 1,048,576 elements).

The storage mechanism is based on the fact that an insulating surface exposed to electron bombardment will assume naturally one or the other of two stable equilibrium potentials for which the net electron current will be zero, the cathode potential for which electrons cannot reach the surface for lack of energy or the potential of the collector for which the primary and resulting secondary electron currents are exactly equal. These equilibria are stable because any potential deviation, as could occur from imperfect insulation, for example, results in stabilizing electron currents of a direction proper to bring the surface back to equilibrium. The potential of the collector, the electrode determining the potential gradient at the target surface, may be several hundred volts. It must be sufficiently high for the intrinsic secondary emission ratio to be greater than one. When all the surface of the insulator is bombarded, some elements of surface can be stably maintained at the high collector potential, while others are simultaneously maintained at the low cathode potential. This is an ideal condition for the quiescent state of the memory tube in its stand-by condition. It can be obtained by the simple expedient of making positive all the selecting bars and thereby opening all the windows. The pattern of the equilibrium potentials is "written" into the tube, one element at a time, by closing momentarily all windows except a chosen one and overpowering the electron current locking mechanism remaining on the corresponding element by a displacement current resulting from a voltage pulse applied to the backing capacity plate. The polarity of the pulse is made to depend on the on-off signal assigned to that element and determines to which of the two stable potentials the element will be driven. The "reading," also one element at a time, is obtained by closing momentarily all windows except the one identified by its previously assigned location in the tube and detecting at which of the two potentials the element finds itself. This detection can be by means of a displacement current or with special targets, by a direct electronic current. Another method, convenient for monitoring in any case, consists of coating the insulator with a cathode-luminescent material and making the backing capacity plate semitransparent. Clearly, light will be produced by electron impact for the high but not the low equilibrium potential. This signal can easily be detected and amplified by a multiplier photo-tube viewing the whole storing surface. It is apparent that this method of storage provides for an indefinite storing time, for writing without previous erasing, and repeatable readings.

The tentative engineering characteristics of the selectron tube which we are engaged in developing are: Size from 3 to 4 inches in diameter, 4 to 6 inches long, 50 lead stem, capacity of several thousand elements; and writing and reading times of about 30 microseconds. A greater storage capacity can be compounded by using a number of tubes. It is convenient to use as many selectrons as there are basic binary places in the computer, or in nonbinary machines as many as on-off signal channels, and to connect the selecting control leads in parallel and operate all writing and reading channels simultaneously.

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DISCUSSIONS

Should Automatic Computers be Large or Small?

The extremely high speeds of electronic devices are being utilized in machines for the automatic solution of long and complicated numerical problems. This application is of very recent origin, and we are still in process of exploring its possibilities, and of organizing electronic devices into useful tools of applied mathematics.

In planning any large computer of this nature, it must be kept in mind that the automatic computer is a labor saving device. It does nothing that enough human beings with paper and pencil could not do, given sufficient time. Basically, therefore, the design must be determined by economic considerations, and these considerations force the designer to compromise between the speed and utility of the machine, on the one hand, and the cost of construction and maintenance on the other.

In other words, the designer of a new computing machine must decide whether he will build a large machine or a small one, or perhaps a group of small machines. In view of the diversity of uses to which computers will be put, it is unlikely that the optimum size will be the same in all cases. Certain influences operate in general, however, and it is hoped that this note will help the designers of new computing devices to recognize and evaluate the effects of such influences.

In certain parts of the computation process, it is possible to make a fairly even exchange between over-all speed and number of elements. Thus, an electron path can be opened and closed at the rate of 100,000 times per second, to use a conservative figure. We can imagine a computer in which the addition of two complete numbers is performed in one or two intervals of 1/100,000 second each by the use of a large number of parallel paths. In contrast, we can imagine a computer in which a few paths are used over and over to perform the addition. Very roughly, the number of additions per second is proportional to the number of paths operating simultaneously, and hence to the amount of equipment involved.

An automatic computer, however, contains more than the combinational circuits involved in addition and similar operations. One of its major functions is the storage of numerical and control data. In this part of the system, we cannot imagine means for effecting an even exchange of speed and simplicity. The amount of stored information is a function of the problem being solved and is practically independent of the speed with which the storage mechanism operates. Since this is so, it becomes necessary to supply practically as much storage capacity in a slow machine as in a fast one if the same problems are to be treated. This fact, in itself, is a strong argument in favor of large and fast computers. On the other hand, if the calculating mechanism is to have a relatively low over-all speed, then delays in response of the storage mechanism are less important, and slower storage methods can be tolerated.

On the whole, it may be said without too great exaggeration, that the storage device should be chosen or developed to fit the requirements set up for a given computer, and the calculating system then chosen to operate with that particular means of storage.

Another important element of the automatic computer which affects the compromise is the input-output mechanism. In most existing computers, much of this part of the system is mechanical and relatively slow. In large computers, it has been necessary to supply a multiplicity of key boards and typewriters in order to handle the volume of data required by the machine.

The last major element of an automatic computer that has a bearing on the optimum size of the computer is the switching or path-selecting mechanism. In general, the faster the computation and the greater the number of parallel paths, the more complicated is the switching problem. If a small number of calculating elements is used, then the switching becomes relatively simple, in the sense that few switches are required. The number of switches is roughly proportional to the number of parallel data paths and hence to the speed and size of the computer.

Summarizing the equipment requirements for large and small computers, we find that the calculating, switching, and input-output equipments are roughly proportional to the speed of computation. To the extent that such proportionality holds, economic consideration of these elements does not influence the optimum size of computer one way or the other.

The cost of storage, on the other hand, does affect the economy of the computer. It appears that one should use the largest computer compatible with a chosen type of storage unit.

So far, economics of the computers favor the largest possible machine. However, as the machines increase in size other factors make their appearance. Besides the obvious fact that one reaches the limit of profitable work, there is the traffic factor. The larger the machine, the more administrative detail there is in routing problems through it. Priorities must be established; a large staff of operators and maintenance people must be supervised and made to cooperate smoothly and without interference. Problems must be assembled from a large number of sources and the results distributed.

For all these reasons, there are unavoidable delays in handling the flow of data through a large computer.

Another factor that favors smaller units is the increase of maintenance trouble with increasing size of unit. The breakdown of any one part of a machine makes the entire computer unusable until the part has been repaired or replaced. As the number of parts increases, the chance that some part will fail in a given period increases, and hence the proportion of "down" time increases with the size of the machine, other things being equal.

Finally, there are advantages to be gained in close cooperation between the operator of a computer and the person or group with which the problems arise; or better still, the originator of a problem should have a machine available for his own use. Ordinarily, the larger the computer, the more users there must be to support it, economically, and as a general thing automatic computers can be assigned to individuals only if the computer is relatively inexpensive. Obviously there will be exceptions to this general rule.

In conclusion, then, the writer's opinion is that automatic computers should be designed in the smallest units consistent with the problems to be handled. While the larger mechanisms intended for group operation are easier to design economically, the advantages of the small group or one-man users seem to outweigh the purely mechanical considerations. It is recognized, of course, that "large" and "small" are relative terms, and that, because of the problems to be solved, a single group of users working on an "irreducible" problem may require a machine that is physically very large; but if the problem is reducible to a set of practically independent problems, then a group of smaller machines seems desirable.

G. R. STIBITZ

BIBLIOGRAPHY, Z–I

1. GEORGE R. STIBITZ, *Relay Computers.* Prepared for the National Defense Research Committee, February 1945, v, 70 leaves and appendix, 21.7 × 27.8 cm., printed from manuscript by the photo-offset process.

This report, consisting of 9 chapters and an appendix, treats the performance of numerical operations by relay calculating mechanisms.

In Chapter 1 the author states that the purpose of his memorandum is two-fold: to acquaint those who have computing problems with the potentialities and limitations of relay calculators, and to acquaint those who are familiar with relay circuit designs with the special requirements of the relay computer. No attempt is made to cover the details of actual relay circuit design or lay-out. Continuous and digital computers are discussed from the standpoint of accuracy, operator attention required, construction and maintenance, flexibility, and contrast in the design problems of the new types.

Chapter 2, entitled Computational Processes, includes a definition of numerical computing and a classification of computing processes with corresponding requirements on computing systems. The requirement which is of perhaps the most interest to a reader is that a computing machine must be intelligent enough to make what are called decisions. It is frequently necessary for computing machines to determine from the basis of the results of a computation step which of alternative steps shall be followed. Such decisions are necessary, for instance, in certain step-by-step numerical integrations where successive approximations are computed until an error term has been reduced to a predetermined reliable value.

Chapter 3, entitled Relay Computing Elements, treats the relay as a computing element. The relay is essentially an electrically operated switch. Relays of the type used in the computing mechanism described in the report have two and only two stable positions—the off position and the on position. If the off position is represented by the symbol 0 and the on position by the symbol 1 and if numbers are represented by means of the symbols 0 and 1, the positions of relays in a relay network can be used to represent them. The manipulation of numbers in relay computers is discussed; the use of relays to store numbers and to perform the operations of addition, multiplication, division and square rooting is treated in a general manner.

Chapter 4, entitled Theory of Automatic Checks, contains a discussion of the reliability of relay computing elements and an analysis of "troubles" (a trouble being defined to be a condition not considered in the circuit design) and checking circuits as aids to maintenance. Trouble conditions pertaining to relay computers may be grouped as troubles involving leads and troubles involving contacts. Trouble conditions occur when there is either abnormally high resistance or abnormally low resistance between different points in the system. On the one hand, there might be abnormal conditions existing between two leads or between a lead and some other point in the system. On the other, an abnormally low resistance might occur across relay contacts at a time when the contacts should be open, or abnormally high resistance at a time when the contacts should be closed. The latter condition is much more likely to occur than the former because of the presence of dirt between the surfaces and the contact. Practicable checking circuits are discussed including their use as aids to maintenance of the computing machine.

Chapter 5, entitled The Design Problem-Functional Design Step, consists of a functional analysis of computing machine systems. In the author's words, "it has been found convenient in designing computing devices for complicated operations to follow a general pattern reminiscent of the human nervous system in the sense that the calculator is arranged to have a number of levels of nerve centers with differing orders of intelligence, ranging from a kind of simple reflex at the bottom of the scale up to an over-all control which integrates and guides the entire operation." A specific computing device, namely a relay interpolator, is used as an example in this Chapter. This interpolator is designed to accept the values of an arbitrary function for integral values of the argument, to compute interpolated values of the function using an interpolating polynomial of degree 3 and to punch the function values in telegraph tape in coded form. The control levels required in such a computing device are listed and a functional schematic is drawn for it.

Chapter 6 is entitled The Design Problem—Translator Design Step. By translator design step is meant the translation of a functional schematic, or block diagram, into the more detailed relay circuitry. An interesting and instructive systematic approach to this problem attributed to Dr. C. E. Shannon is discussed. This approach consists in the application of Boolean algebra to the design of relay circuits to possess given functional ability. The elements of this algebra are given and the conversion of the algebra into relay circuits is treated.

Chapter 7, entitled Radix Notation, gives representations of numbers that are convenient when relays are used. It is pointed out that binary representation of numbers unfortunately does not convert easily to and from a decimal notation and that consequently its use in relay machines is limited. A hybrid representation, in which the bases 2 and 5 are used and which is called the bi-quinary notation by the author, is adopted by him for use in relay computers. This scheme of representation consists of using the base 2 to tell if the decimal digit lies between 0 and 4 or between 5 and 9 and in using the base 5 then to tell which digit it is. The elementary arithmetical operations upon numbers expressed in the bi-quinary notation are discussed very completely in the remainder of the chapter.

Chapter 8, entitled Schematic Diagrams, consists of a discussion of physical and functional schematics and gives some conventions used in such schematics. As illustrations four figures are given. Some advantages and disadvantages of the conventions used are listed.

Chapter 9, entitled Relay Complexes, consists of further discussion of the computer nerve centers discussed in Chapter 5. A relay complex is defined to be an intermediary nerve center controlling a frequently used operation. Relay complexes capable of performing the operations of addition, subtraction and multiplication are discussed. Alternative types of complexes for performing the same function are compared. The details of a nonchecking binary relay adder are given and adders in other bases are discussed. It is pointed out that the combination of a binary adder with a quinary adder constitutes a decimal adder. This is the combination used in the relay interpolator mentioned in the preceding chapter. Relay complexes for storing information on tape and for searching through the stored data for particular desired numbers are discussed.

The appendix is particularly interesting. It is stated that the relay interpolator which was designed to calculate automatically by a cubic formula a set of interpolated values between given data points will solve an astonishing variety of problems. These problems include interpolation, smoothing of data, integration, differentiation, solution of linear differential equations of the first order, solution of linear differential equation with constant coefficients of higher order, harmonic analysis, and the extraction of roots of polynomials. The appendix purports to be an investigation of the underlying principles that lead to so diversified a set of results. This mathematical treatment of the function of the interpolator is one of the most interesting portions of the report.

The flavor of this report by George R. Stibitz has undoubtedly not been retained in this review. As a treatment of underlying mathematical theory of numerical computation by relays, of the general principles of the design of relay networks for computing purposes, and of a discussion of reliability of computing machines from the engineering viewpoint, the report is among the most instructive and interesting that have been written on numerical computers.

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2. NATIONAL DEFENSE RESEARCH COMMITTEE, *Description of the ENIAC and Comments on Electronic Digital Computing Machines*, prepared by J. P. ECKERT, JR., J. W. MAUCHLY, H. H. GOLDSTINE and J. G. BRAINERD of the Moore School of Electrical Engineering, University of Pennsylvania. Report dated November 1945. Printed from manuscript, viii, 58 leaves and 11 drawings, 21.7 × 27.8 cm., by the photo-offset process. See also *MTAC*, v. 2, p. 97–110.

This report, consisting of six chapters and a sizable appendix in three parts, gives a description of the ENIAC, the first general-purpose automatically-sequenced electronic digital computing machine.

As stated in the Introduction, the speed of the ENIAC (Electronic Numerical Integrator and Calculator) greatly exceeds that of any nonelectronic machine, and its accuracy is in general superior to that of any nondigital machine, such as a differential analyzer. The ENIAC is extremely flexible and is not fundamentally restricted to any given class of problems. However, there are problems for which its speed is limited by the input and output devices. In such cases it is not possible to derive the full benefit of its high computing speed. Although the ENIAC carries out its entire computing schedule automatically, the sequence to be followed must be set up manually beforehand. However, as the intended use of the ENIAC is to compute large families of solutions all based on the same program of operations, the time spent in setting up the problem can usually be disregarded.

A second electronic digital machine, the EDVAC, is now being planned. Completely automatic, it will be of larger capacity than the ENIAC and will have a somewhat higher computing speed. Despite these features, it will require considerably less equipment than the ENIAC, since the electronic components will be used in a quite different and much more efficient way.

Chapter 1 gives an interesting account of some of the computing problems that led to the organization of a project for the design and construction of the ENIAC. The most pressing need at that time was for a computing device capable of making firing tables quickly to keep pace with the development of gun-projectile propellent combinations. The nature of the problems which these machines bring within the range of computation and the real need for speed in carrying out such calculations are discussed.

Also included in Chapter 1 is a brief history of the growth of the ENIAC project. Early in the spring of 1943, Captain Herman H. Goldstine of the Ballistic Research Laboratory and Colonel PAUL N. GILLON of the Office of the Chief of Ordnance became interested in the utilization of an electronic computer for the preparation of firing and bombing tables. At Goldstine's request, J. W. Mauchly and J. P. Eckert, Jr., wrote a tentative technical outline of a machine capable of numerical integration of trajectories and of handling other problems of similar complexity. This material was included by J. G. BRAINERD in a report that formed the basis of a contract between the University of Pennsylvania and the Government to develop an electronic device along these lines. Eckert was chief engineer on the project, Mauchly acted as principal consultant, and Goldstine was appointed resident technical representative of the Ordnance Department.

In Chapter 2 the point is made that, in order to attain the speed, accuracy, and flexibility required for the solution of problems such as those discussed in Chapter 1, electronic digital machines must be used.

A description of the ENIAC in Chapter 3 gives the reader a rather complete account of its general features and also provides him with some information on the way in which the various units can be used. The important function of control, various kinds of memory or storage facilities, the arithmetic units, and the input and output devices are considered. (Specific explanations of some arithmetic and programming techniques will be found in the appendices; there is also an appendix giving constructional data.)

The latter chapters of the report are concerned with a discussion of some general principles which seem pertinent to computing machine design and which have been used in formulating the plans for the EDVAC.

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3. K. Zuse, *Calculator for Technical and Scientific Calculations Designed According to a Theoretical Plan.* 59 p. 21.6 × 28 cm. Some pages will not reproduce well. Distributed by the Office of the Publication Board, Department of Commerce, Washington 25, D. C. $4.00 (photostat), $2.00 (microfilm).

This report is interesting as an indication of the recent thinking in Germany concerning automatically-sequenced computing machines. It consists of an introduction; brief sections on algebraic calculating devices, logical calculating devices, and the construction of computing devices; and three rather verbose appendices amplifying each of the sections.

In the Introduction the author states that his report is concerned with machines capable of logical inference. Calculating is defined as the deduction of conclusions from given assertions by the application of a prescribed set of rules. He states that calculating thus defined is based on the fundamental logical concepts of conjunction (A and B), disjunction (A or B), and negation (not A) and that it can be expressed throughout, from assertion to conclusion, in a language of "yes-no" values. Numerical computation then becomes a special case of calculating as defined. Calculating machine elements therefore need only be capable of distinguishing between two contrasting states—for example, two states of voltage, of current, or of physical position. It follows that electrical or mechanical relays are inherently suitable for use as the basic elements in calculating machines.

Algebraic and logical calculating devices are also discussed in the Introduction. It is apparent that the two are one and the same, namely, a simple relay computing machine, capable of performing as high as fifty basic arithmetic operations a minute. The binary representation of numbers constitutes the "yes-no" series. It is claimed that the machine can convert automatically decimal numbers. The device can be sequenced by hand manipulation of a keyboard or automatically by perforated tape, and can store both original data or intermediate results internally, operating at the appropriate times upon the stored numbers. The machine can execute automatically order sequences of any length, but it is incapable of modifying orders, that is, of selecting an order routine according to the results of previous computations. Zuse states, however, that the method of constructing devices capable of deducing the order sequences required for the solution of given problems is clear. He foresees the use of machines of this nature to do elementary thinking and thus free man for deeper reasoning and philosophical pursuits.

Section 1 and Appendix 1 list the operations considered basic for an algebraic calculating machine and present the order sequences for the solution of various simple problems. Appendix 2 treats a more complicated problem—the determination of the zeros of a third order determinant having complex elements that are functions of two variables. This determinant occurs in the flutter analysis of plane airfoils with three degrees of freedom.

Zuse calls fundamental the operations of addition, subtraction, multiplication, division, and extraction of the square root. He mentions additional operations that a calculating machine should be capable of performing, ranging from extracting the sign of a number to selecting the smaller or larger of two unequal numbers. He states that the performance of the algebraic calculating machine can be extended to include operations on complex numbers, the use of trigonometric and hyperbolic functions, the determination of the roots of algebraic equations of the third and higher degrees, interpolation, and the numerical integration of differential equations.

Section 3 and appendix 3 enumerate applications of a logical calculating machine. These include the derivation of all mathematical theorems that follow from a given set of axioms, the performance of combinatorial reasoning, exercises in Boolean logic, the preparation of order sequences for computing machines, the simplification of literal algebraic expressions, and studies in aerodynamics, heat flow, optics and probability.

Zuse's report is rather general. More detailed information on the actual construction and use of the machines discussed would have been of interest to many readers. For example, the coding of the numbers and the order sequences as well as the speeds required for the performance of the fundamental operations of addition, multiplication, division, and extraction of the square root might well have been included. Also in reading the report one becomes curious about the types of relays used as basic computing elements.

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4. Harvard University, Computation Laboratory, *Annals* v. 1: *A Manual of Operation for the Automatic Sequence Controlled Calculator.* Cambridge, Mass., Harvard University Press, 1940, xiv, 561 p. and 9 plates. 20 × 26.7 cm. $10.00.

This volume was reviewed by Professor D. H. Lehmer in *MTAC*, v. 2, p. 185–187. The review states: "This volume gives the first really scientific account of the Automatic Sequence Controlled Calculator, the first of the large all-purpose digital calculators developed during the war." The reviewer gives an accurate and comprehensive account of the contents of this manual of operation for the large-scale electro-mechanical computing machine developed jointly by Harvard University and the International Business Machines Corporation.

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News

American Institute of Electrical Engineers.—At the Winter Meeting held in New York City from January 20 to 31, 1947, the following series of talks on electronic digital computing machines were given:

John von Neumann of The Institute for Advanced Studies pointed out the inherent error caused by (1) random noise in the analogy machine and (2) the inability of digital computers to carry numbers to their full places. He emphasized that the digital technique is intrinsically the better method of keeping the error small. The two greatest difficulties of the machine, said Dr. von Neumann, are those of programming problems for the machine and of getting data from physical measurements on which the machine can operate to produce meaningful answers.

The use at Harvard University of the IBM automatic sequence-controlled calculator was described by H. H. Aiken, who likewise called attention to the problem of the prearranged controlled sequences that must be worked out by trained mathematicians before the machine can proceed. Once these program orders have been set down for a given type of problem, many similar problems can be solved merely by substituting the appropriate numerical values in the original set of program orders.

Julian Bigelow of Princeton, in a discussion of the thinking that goes into the design of a computer, stated that, far from the popular misconception of the solution on these machines of problems heretofore unsolvable, mathematicians must think through beforehand all manipulations which the computer executes for an ultimate solution. Hence, the first consideration in making efficient use of computers is to simplify the programming. Mr. Bigelow called attention specifically to the external magnetic-tape memory, permitting storage of a great many numbers in a very small space. This was contrasted with the internal electronic memory, which has the desirable feature of being instantly readable at any point.

An all-electronic computer that can be adapted to the specific problems of industry as well as those of pure and applied science, engineering, and statistical studies was described by J. W. Forrester of the Massachusetts Institute of Technology. New features include the transmission of 40 digits of a number simultaneously over parallel busses (introducing the circuit problem of precisely gating each train of pulses to the proper unit of the computer), electronic storage on a dielectric plot, and the possibility of manual programming for locating faults in the machine.

T. K. SHARPLESS of the University of Pennsylvania described the ENIAC—a machine developed for the specific purpose of preparing ballistic firing tables; see *MTAC*, v. 2, p. 97–110. He also mentioned the general purpose computer EDVAC, which has greater memory facilities, although fewer tubes. The EDVAC, however, does not perform operations quite so fast, because it uses time sequence rather than the spatial division of the ENIAC. The coding is done by magnetic tape prepared at slow speed and run through the calculator at high speed.

A relay-operated computing device produced by Bell Telephone Laboratories was described and illustrated by S. B. WILLIAMS, consulting engineer. Controlled by a punched tape, the machine adds, subtracts, multiplies, divides, extracts square roots of real or complex numbers, and prints results on teletypewriters. If the computation called for by the control tape is not completed in the allotted time, the machine stops and signals a fault.

The problem of accuracy of calculations performed by automatic computers was outlined by JOHN MAUCHLY of the Electronic Control Company. Inaccuracies produced by rounding off in digital machines introduces errors whose magnitudes are difficult to estimate. The machines themselves can be used to determine their truncation errors by solving the problem in several ways.

The Actuarial Society of America (393 7th Avenue, New York).—The Council at its meeting in February authorized the President, E. W. MARSHALL, Vice President, Providence Mutual Life Insurance Company, Philadelphia, to appoint a committee to explore the possibilities of electronic sequence-controlled calculators. The committee appointed consists of:

MALVIN E. DAVIS (Committee Chairman), Actuary, Metropolitan Life Insurance Company, 1 Madison Avenue, New York;
PEARCE SHEPHERD, Vice President and Associate Actuary, Prudential Insurance Company of America, Newark, N. J.; and
WILLIAM P. BARBER, JR., Secretary, Connecticut Mutual Life Insurance Company, Hartford, Connecticut.

This Committee has had several meetings. On June 25 it met at the Harvard Computation Laboratory in Cambridge.

At the meeting of the Society at the Hotel Commodore in New York, on May 8, 1947, EDMUND C. BERKELEY presented a paper entitled "Electronic machinery for handling information, and its uses in insurance."

The Life Office Management Association.—In March, the President, HORACE W. FOSKETT, Vice President of the Equitable Life Insurance Company of Des Moines, Iowa, appointed a committee of the Life Office Management Association on electronic sequence-controlled calculators. This committee consists of:

CHARLES H. YARDLEY (Committee Chairman), 2nd Vice President and Comptroller, Penn Mutual Life Insurance Company, Philadelphia, Pa.;
LESTER H. VAN NESS, Supervisor, Planning Division, Acadia Mutual Life Insurance Company, Washington, D. C.;
BURGH S. JOHNSON, Treasurer, Guardian Life Insurance Company, New York;
R. A. MANGINI, Manager, Planning Division, John Hancock Mutual Life Insurance Company, Boston, Mass.;
FRANK L. ROWLAND, Executive Secretary, Life Office Management Association (ex officio);
H. W. FOSKETT, Vice President, Equitable Life Insurance Company, Des Moines, Iowa (ex officio);
E. C. BERKELEY (Committee Secretary), Chief Research Consultant, Prudential Insurance Company of America, Newark, N. J.

The first meeting of this committee was on May 9, 1947. The second meeting was on June 25 at the Harvard Computation Laboratory, at the invitation of Professor HOWARD H. AIKEN.

Institute of Radio Engineers.—Tuesday afternoon, March 4, 1947, was devoted to discussions of Electronic Digital Computers at the 1947 National Convention of the IRE. The meeting was presided over by **Harry Diamond** of the National Bureau of Standards, Washington, D. C. Speakers were as follows:

J. W. **Forrester** of MIT discussed "The electronic digital computer" (with mention of early attempts and existing systems), a general block diagram of a modern proposed computer, and an outline of the fundamental computer operations.

S. N. **Alexander** of the National Bureau of Standards in his talk on "Input mechanisms for electronic digital computers" established criteria for acceptable input mechanisms. He also discussed recently developed input systems and special materials used in them.

H. H. **Goldstine** of the Institute for Advanced Study, Princeton, New Jersey next talked on "Electronic computing" and demonstrated how arithmetical operations as well as switching of numbers and control of computation can be realized by means of vacuum-tube circuits.

"The Selectron—A tube for selective electrostatic storage" was described by J. A. Rajchman, RCA Laboratories Division, Princeton, New Jersey. See under TECHNICAL DEVELOPMENTS of this issue.

P. **Crawford**, Special Devices Division, Office of Naval Research, Washington, D. C., terminated the discussions with a talk on "Applications of electronic digital computers," which included comments on the future relation of analogue and digital computers, and also on the possible engineering application of electronic digital computers to automatic process and factory control, traffic control, and business calculations.

Because of the great interest in the computer program these talks were repeated the same day from 5 to 7 p.m.

OTHER AIDS TO COMPUTATION

See also our introductory article "Film Slide Rule," and QR **30**.

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