where $d_m = b_1a_{m,1} + b_2a_{m,2} + \cdots + b_{m-1}a_{m,m-1}$ is obtained by an accumulation of the products of the $(m-1)$ b's, which we know, by the corresponding a's in the row of coefficients of x^m.

Thus we can calculate as many of the coefficients of the inverse series as we wish by this method, being careful only to take a sheet of paper which is large enough, i.e., having $(r+1)$ rows and $(r+1)$ columns for r coefficients. To recapitulate, this method permits us to calculate the coefficients of the inverse power series systematically and on one page. Furthermore in the calculations it requires only accumulations of products with the exception of r divisions.

Now consider the inversion problem where the coefficient of x is zero. If $z = d_nx^n + d_{n+1}x^{n+1} + \cdots$, we may, by the method described in 2, obtain

$$z^{1/n} = a_{1,1}x + a_{2,1}x^2 + \cdots,$$

where $a_{1,1} = d_n^{1/n} \neq 0$. Then we may obtain x as a power series in $z^{1/n}$.

H. CHERNOFF

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[1] FRANZ KAMBER, "Formules exprimant les valeurs des coefficients des séries de puissances inverses," *Acta Math.*, v. 78, 1946, p. 193–204.

[2] The case $a = 0$ is easily handled by factoring out x^n where a_n is the first non-zero coefficient.

EDITORIAL NOTE: A "movable strip" is extensively used by actuaries in their insurance and annuity calculations, in connection with their "commutation" columns. In actuarial literature there are frequent references to this "movable strip"; e.g., GEORGE KING, *Institute of Actuaries' Text Book*, part II, second ed., 1902, p. 392–393, 402.

RECENT MATHEMATICAL TABLES

For other RMT see ACM: Bibliography (Stibitz, NDRC, Zuse); OAC: Bibliography; N75 (Horton) and 79 (Katz); QR30.

425[A].—A. ADRIAN, *Barème Forestier. Cubage des Bois abattus des Bois en grume d'après la Circonférence et le Diamètre et des Bois Équarris. Débit et Équarrissage des Bois. Cubage et Estimation des Bois sur Pied. Conversion du Volume réel.* Paris, Éditions Berger-Levrault, 54th thousand, 1944. iv, 214 p. 11.3 × 17.5 cm.

T. 1, p. 5–97 gives the volume in cubic meters, to 3D, of round wood of circumference $c = 25(1)300$ centimeters and length $l = .25(.25)16$ meters.

T. 2, p. 99–131, gives similar results for diameter $d = 5(1)100$ centimeters.

T. 3, p. 132–179, is for volumes of squared wood, $l = .25, .33, .5, .66, .75, 1(1)20$ meters, and cross sections $5 \times 5(1)11$ up to $50 \times 50(1)55$, 100 centimeters.

T. 4, p. 180–185, by three different methods of "squaring" round wood, from $c = 32$, $d = 10$, $(7 \times 7, 8 \times 8, 8 \times 9)$ to $c = 300$, $d = 95\frac{1}{2}(67 \times 68, 75 \times 75, 90 \times 91)$.

Miscellaneous small tables p. 190–212.

426[A].—R. C. MORRIS, "Table of multiples of the square root of three," *Electrical World*, v. 125, June 8, 1946, p. 108–109. 21.6 × 28.8 cm.

This is a table of $N\sqrt{3}$, where $\sqrt{3}$ is taken as 1.73205; $N = [1(.01)9.99; 4D]$. In MARCEL BOLL, *Tables Numériques Universelles*, Paris, 1947, p. 184–185, are 6D tables of $Nx^{\frac{1}{2}}/D$ for $x = 2, 3, 5$, N or $D = 1(1)10$.

427. [A, B, C, D, E, M].—E. S. ALLEN, *Six-Place Tables. A Selection of Tables of Squares, Cubes, Square Roots, Cube Roots, Fifth Roots and Powers, Circumferences and Areas of Circles, Common Logarithms of Numbers and of the Trigonometric Functions, The Natural Trigonometric Functions, Natural Logarithms, Exponential and Hyperbolic Functions, and Integrals. With Explanatory Notes.* Seventh ed., New York, McGraw-Hill, 1947. xxiii, 232 p. 10.7 × 17.7 cm. $1.75. Compare RMT **49, 184**.

The seventh edition is 51 pages larger than the sixth, which was reviewed, *MTAC*, v. 1, p. 348. This is due to the expansions of T. X, Natural Logarithms (by 17 p.), and T. XI, Exponential and Hyperbolic Functions (by 34 p.), from 4D to 6D, with finer arguments. T. XIV, Mathematical Constants, has been slightly extended.

428[A–F, H, L–N].—MARCEL BOLL, *Tables Numériques Universelles des Laboratoires et Bureaux d'Études.* Paris, Dunod, 1947, iv, 882 p. 18.5 × 27.1. Bound in boards 3200 francs (about $27).

This well-printed, well-arranged, and excellently indexed volume contains more than 200 tables grouped under the following six general headings. A, "Arithmétique" and algebra, p. 7–218, where the tables are denoted by the numbers An, $n = 1(1)40$; **T**, Trigonometry, p. 219–386, $n = 1(1)38$; **E**, Exponentials, p. 387–536, $n = 1(1)44$; **P**, Probabilities, p. 537–686, $n = 1(1)40$; **C**, Complex numbers, p. 687–746, $n = 1(1)19$; **U**, Units, constants, p. 747–854, $n = 1(1)36$. Graphs of the functions discussed are numerous; there are no less than 122. The page numbers are in large black-face type at the bottom of each page and above every page of every table are page references to text or graphs or reliefs dealing with the function tabulated. All the printed numerals in the tables are of uniform block, black-face type. Black-face arrows at the bottom of each right-hand page indicate if the table is continued on the next page.

On p. 6, there is a general Bibliography with 39 titles. To the uninitiated a number of these titles might be thought to refer to comparatively recent works. For example, "Edouard Barbette, Sommes des $p^{\text{ièmes}}$ puissances (Paris, 1942)." But this work appeared originally in 1910, and the fuller title is *Les sommes de $p^{\text{ièmes}}$ Puissances Distinctes égales à une $p^{\text{ième}}$ Puissance: suivie d'une Table des 5000 premiers Nombres Triangulaires.* Liége and Paris. Or consider also the entry "J. Claudel, Tables des Carrés, Cubes, Longueurs de Circonférences, . . . (Paris, 1939)"; the original of this work appeared 89 years earlier, with somewhat different title. Claudel died in 1880. Four of the titles without dates (the only ones) are all German works, by Crelle, Peters, Landolt-Börnstein, and Schrön. The title of the Crelle work is given as "Produits des nombres de 1 à 1.000 par les nombres de 1 à 1.000 (Berlin)." Of course no book of Crelle with such a title was ever published; the possibilities of Crelle's *Rechentafeln* or *Calculating Tables* are thus set forth. So also for Peters' *Neue Rechentafeln für Multiplikation und Division* . . . (1909) the title of which is listed as "Produits des nombres de 1 à 10.000 par les nombres de 1 à 100 (Berlin)."

If all previously published tables in the volume were adapted from the bibliographic sources listed, it is obvious that many tables are new; and indeed the author states that more than a third of the pages in the volume are filled with such previously unpublished material. As the general title-survey given above suggests, the tables are mainly of an elementary nature—the more advanced ones being of two complete elliptic integrals, of Fresnel integrals, of sine, cosine, and exponential integrals, of Legendre polynomials, and of Bessel functions. But the elementary tables are often given because they are useful in various non-elementary applied fields, for example: Equation of Van der Waals (p. 204–206); corrections of relativity (p. 207–209); equation of paramagnetism, Langevin, 1905 (p. 472) greatly inferior to the table in EMDE, *Tables of Elem. Functions*, 1940, p. 123; functions of Planck, 1901 (p. 493–502) supplementing the table in EMDE, *Tables of Elem. Functions*, 1940, p. 117–119; curves of Einstein and Debye (p. 503–512); curves of Gauss and Galton (p.

580–632); curves of Poisson (p. 671–686). There is emphasis on tables useful for the physicist and chemist.

Some random indications of tables included in the volume are as follows: (A1), circumferences of circles of diameter $n = [1(1)1000;\ 6S]$; (A2), Surfaces and volumes of spheres of diameter $n = [1(1)100;\ 7S]$; (A11), triangular coordinates, triangles of J. W. GIBBS (1876), and BAKHUIS ROOZEBOOM (1894); (A19), $(a/b)^{1/2}$ and $(b/a)^{1/2}$, $a = 1(1)100$, $b = [2(2)30(5)100;\ 4D]$; (A20), $(a - 1/b)^2$, $a = .1(.1)10$, $b = .1(.02).3(.05)1$; (A25), $\sqrt{ab}$; (A26), $M = 2ab/(a + b)$; (A33), molecular refraction, $(x^2 - 1)/(x^2 + 2)$; (A34), molecular polarization, $(x - 1)/(x + 2)$; (A40), simple approximations of some incommensurable numbers ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, e, $\log e$, π, $\sqrt{\pi}$, $\sqrt{2\pi}$, π^2) with indications of the errors of the approximations; (T2), remarkable expressions (about 40) expressed as functions of π; (T16), $\sin^2 A$, $\cos^2 A$, $\cos^3 A$, $\cos^4 A$, $\cos^{1/2} A$, $\cos A \cos^{1/2} A$, $(1 - \tan A)/(1 + \tan A)$, $\sin A \cos A$, $\sin A \cos^2 A$, $\sin A \cos^3 A$, $\sin A \cos^{1/2} A$, $\sin A \cos^{-1/2} A$, somewhat like tables in the other French volume of over 800 pages of formulae and tables by L. POTIN, Paris, 1925, p. 407–417; (E 12), $u = x^x$, $v = x^{-x}$, u', v', $U = \int_0^X x^x dx$, $V = \int_0^X x^{-x} dx$; (E 20), $\log \log x$; (E 26), $x + \frac{1}{2}\sinh 2x$, $\sinh x/x$, $\tanh(\frac{1}{2}x)/\frac{1}{2}x$; (E 30), $\sinh x \cos x$, $\cosh x \sin x$, $\cosh x \cos x$, $\sinh x \sin x$; (E 42), $y = e^{-2\pi zx}\sinh[2\pi(z^2 - 1)^{1/2}x + \psi]/\sinh \psi$, where $\sinh \psi = (z^2 - 1)^{1/2}$; (P 31c), $(1 - p^\alpha)/[(1 - p)p^\alpha]$; (C 7–8), roots of equations of the third degree as in JAHNKE & EMDE, and also by another method; (U 12), $y = (e^{2x} - 1)^{-1/2}$; (U 22), $h = v^2/2g$, h in cm., v in cm/s, $g = 980.9$ cm./sec^2.

We have now given some suggestions as to the wealth of material in this volume, which, if accurate, might often be useful in very varied fields of work. The alphabetical index (p. 859–868) leads quickly to material in the volume which may be sought. There is also a list of the graphs and reliefs (p. 869–871) and a list of the tables in order (p. 873–882). The pages (856–857) headed "Interpolation précise des tables" do not make a favorable impression. Wholly random checking of a few of the tables (see below) shows that they are highly unreliable, displaying not only defective proof reading, but also carelessness and inadequate checking of basic calculations. Hence the reliability of no table in the volume should be assumed without careful checking. It looks as if Hayashi's throne has been lost to a Frenchman.

Marcel Boll (1886–) is also the author of: (i) *La Chance et les Jeux de Hasard* . . ., Paris, 1936, 386 p. (ii) *Le Mystère des Nombres et des Formes* . . ., Paris 1941; fourth ed., 1946, 330 p. (iii) *Les Étapes des Mathématiques*, Paris, third ed., 1944, 128 p.

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P. 134, $x = 7$, $p = 56$, *not* 46; $x = 8$, $p = 40$, *not* 38; $x = 14$, $y = 48$, *not* 38, and $p = 112$, *not* 102; $x = 15$, $z = 113$, *not* 115, $p = 240$, *not* 142; $x = 22$, $p = 264$, *not* 164; $x = 23$, $p = 552$, *not* 352; $x = 26$, $p = 364$, *not* 398.

P. 226, T. 2, l. 10, col. 2, *for* 0.063602, *read* 0.063662.

P. 234–239, T. 8, 6-place table of the six trigonometric functions, 0(1°)360°: In csc 5°, sec 85°, sec 95°, csc 175°, csc 185°, sec 265°, sec 275°, csc 355°, *for* end-figures 513, *read* 713; in sin 21° and 7 other angles, *for* end-figure 3, *read* 8; in csc 29° and 7 other angles, *for* end-figures 53, *read* 65; in sec 65° and sec 245°, *for* end-figures 62, *read* 02.

P. 310, 18°, $\sin^2 \alpha$, *for* 0.095496, *read* 0.095492; 19°, $\sin^2 \alpha$, *for* 0.106092, *read* 0.105995; 21°, $\sin^2 \alpha$, *for* 0.128425, *read* 0.128428.

P. 312, 28°, $\cos^2 \alpha$, *for* 0.779614, *read* 0.779596; 39°, $\cos^2 \alpha$, *for* 0.603953, *read* 0.603956; 41°, $\cos^2 \alpha$, *for* 0.569584, *read* 0.569587; 41° 30′, $\sin^2 \alpha$, *for* 0.439043, *read* 0.439065; $\cos^2 \alpha$, *for* 0.560985, *read* 0.560935; 48°30′, $\sin^2 \alpha$, *for* 0.560985, *read* 0.560935; $\cos^2 \alpha$, *for* 0.439043, *read* 0.439065; 49°, $\sin^2 \alpha$, *for* 0.569584, *read* 0.569587; 51°, $\sin^2 \alpha$, *for* 0.603953, *read* 0.603956; 62°, $\sin^2 \alpha$, *for* 0.779614, *read* 0.779596; 63°30′, $\cos^2 \alpha$, *for* 0.199090, *read* 0.199092; 67°30′, $\cos^2 \alpha$, *for* 0.146417, *read* 0.146447; 69°, $\cos^2 \alpha$, *for* 0.128425, *read* 0.128428; 71°, $\cos^2 \alpha$, *for* 0.106082, *read* 0.105995; 72°, $\cos^2 \alpha$, *for* 0.095496, *read* 0.095492; 87°, $\cos^2 \alpha$, *for* 0.002727, *read* 0.002739.

P. 318, 40°, col. 1, *for* 0.58740, *read* 0.58682; the same error in a whole line of multiples.
P. 319, 50°, col. 1, *for* 0.41260, *read* 0.41318; the same error in a whole line of multiples.
P. 320, 29°, col. 1, *for* 0.84895, *read* 0.84805, the same for p. 321, 61°, col. 1.

P. 400, $N = 36$, $n = 1$, *for* 606, *read* 666; $N = 8$, $n = 2$, *for* 240, *read* 204.

P. 634, in the first 50 entries of the 7D table of $H(x)$, attributed to DEMORGAN 1845, there are 36 end-figure errors, 25 unit errors, 10 2-unit errors, and 1 90-unit error ($x = .08$). All of these errors except the last one (where Boll substituted 871 for DeMorgan's correct 781) are also in DeMorgan's *An Essay on Probabilities, Cabinet Cyclopædia*, 1838, p. xxxiv.

P. 786, $t = -197$, *for* 0.27868, *read* 0.27878; $t = -196$, *for* 0.28248, *read* 0.28245.

S. A. J.

429[B, P].—L. VUAGNAT, "Courbes de raccordement," *Bull. Technique de la Suisse romande*, v. 73, 10 May, 1947, p. 127–128. 23 × 31.4 cm.

In this paper on railway transition curves, p. 128, there is a table of $C_2 = n^4(2n^2 - 6n + 5)$ for $n = [.08(.001).999; 6D]$, Δ. There is also a brief table of $C_1 = n^3(6n^2 - 15n + 10)$, $n = [.05(.05)1; 3D]$. The values of the general formulae for C_1 and C_2 are each given incorrectly in this paper.

430[C, D].—CARL CHRISTIAN BRUHNS (1830–1881), **a.** *Nuovo Manuale Logaritmico-trigonometrico con sette decimali*. Twenty-second stereotyped ed., Novi Ligure, Società editrice Novese, [1941], xxiv, 610 p. Preface to the Italian ed. by TITO FRANZINI, 15.7 × 24.3 cm. The Library of Congress spells the first name Karl.

b. *A New Manual of Logarithms to Seven Places of Decimals*. Revised ed. Chicago, Ill., Charles T. Powner Co., P.O. Box 796, 1942, xxiv, 610, 3 p. 15.6 × 24 cm. $2.00.

First of all it is to be noted that one of the bases of Bruhns' work was *Logarithmisch-trigonometrisches Handbuch* . . . Leipzig, Tauchnitz, 1847, xxxvi, 388 p. 15.8 × 24 cm. by HEINRICH GOTTLIEB KÖHLER (1779–1849). The second rev. stereotyped ed. was in 1848, the third in 1856, the fourth in 1855, the fifth in 1857, the eighth in 1862, the thirteenth in 1876, the fourteenth in 1880, the fifteenth in 1886, and the sixteenth in 1898. There were also Italian editions in which the author's name appears as E. T. Köhler since Heinrich Gottlieb = Enrico Teofilo.

The first editions both in German and in English of the work by Bruhns appeared in 1870: *Neues logarithmisch-trigonometrisches Handbuch auf sieben Decimalen*. Leipzig, xxiv, 610 p. Some extracts from the preface (written Aug. 1869) are as follows: "Köhler's Handbook of Logarithms, which has hitherto been published by Tauchnitz, and which will still be published by them, has always found a very favourable reception from the public both on account of its arrangement and its exactness. However Bremiker's edition of Vega's seven-figure logarithms [1852] extended and improved, well known and frequently used, and Schrön's logarithmic tables [1860], are preferable for many elaborate astronomical calculations. Bremiker gives in the trigonometrical tables the logarithms of the Sine and Tangent for the first 5 degrees to every second and the logarithms of the Sine, Tangent, Cotangent, Cosine from 0 to 45° (and therewith it is self-evident of the whole quadrant) for every 10 seconds; whilst Schrön has added to the last an extensive Interpolation Table."

"The publishers did not wish to be behindhand with their Handbook of logarithms and when they became aware that I was willing to undertake the necessary labour of preparing one—they determined to preserve Köhler's in its present form . . . and desired me to prepare an entirely new Manual. . . . The logarithms of numbers from 1 to 108000 as they are in Köhler have been reduced to the extent of from 1 to 100000. . . . The logarithms of the first 6 degrees of the trigonometrical functions, Sine, Cosine, Tangent and Cotangent have been given to every second [in Köhler only to every 10″], with the addition of the differences and where the space would allow of it, of the proportional parts. . . . The remaining 39 degrees of the trigonometrical functions are given to every 10 seconds, whilst in Köhler from the 9th degree they are only given to every minute. . . . We have omitted the Addition and Subtraction logarithms (Gauss's) . . . and we have also omitted the goniometrical and trigonometrical formulae contained in Köhler, as well as the other tables which though useful are not so frequently wanted; so that the present work with the exception of some few small additional Tables consists merely of the logarithms of numbers and of the trigonometrical logarithms."

The second English, German (Henderson has 1880), and Italian editions were in 1881, and the second French edition in 1880; apparently all of these language editions were, after 1881, given simultaneously successive edition numbers. The third edition was in 1889, the fourth in 1894, the seventh in 1906, the eighth in 1909, and the ninth in 1910. An eleventh English edition in 1913 had on the title-page both the names of Tauchnitz, Leipzig, and Lemcke & Büchner, New York. In this edition we find the following: "For the first edition, the publisher fixed one Friedrichsd'or as a prize for finding a typographical error, and since 1870 there have only been found 6 errata. In this edition these misprints are corrected." These were already corrected in the second edition.

The eleventh French edition, 1913, bore the title: *Nouveau Manuel de Logarithmes à Sept Décimales pour les Nombres et les Fonctions Trigonométriques*. A twelfth English edition was published by Van Nostrand, New York, in 1919, a thirteenth in 1922, and a sixteenth English edition by Regan Publ. Corp., Chicago in 1929. What edition Powner copied has not been determined. He issued copies also dated 1936, 1939, and 1941. The three pages added at the end, in the 1942 Powner print, white on black, gave "Table for converting minutes and seconds into decimals of a degree," "Decimal equivalents of an inch and corresponding logarithms"; "Trigonometric lines," and "Trigonometric functions of all angles."

Reporting on Bruhns' work Glaisher wrote, "On the whole, this is one of the most convenient and complete (considering the number of proportional-part tables) logarithmic tables for the general computer that we have met with; the figures have heads and tails; and the pages are light and clear."

R. C. A.

431[C, D].—MARIO O. GONZÁLEZ, *Nuevas Tablas de Logaritmos y de Funciones Naturales*, Havana, Cuba, Editorial Selecta, O'Reilly 357, 1945. xxxii, 98 p. Bound in Boards $1.00.

This volume contains the following five tables, mostly to five places: **T. 1**, log N, $N = 1000(1)10{,}009$, with P.P.; **T. 2**, logarithms of trigonometric functions sine, tangent, cotangent, cosine with Δ and P.P.:, at interval 1′; **T. 3**, Log S, log T at interval 1′ for 0 to 3°; **T. 4**, Natural trigonometric functions, at interval 1′, 5D for all functions 0–45°, except sin, cos, tan 6D for 0–2°, and cot 4D for 0–6°; **T. 5**, (a) n^2, (b) $n^{1/2}$, (c) $(10n)^{1/2}$, (d) n^3, (e) $n^{1/3}$, (f) $(10n)^{1/3}$, (g) $(100n)^{1/3}$, for $n = 1(1)100$; (b) and (e) are to 6D, (c) and (f) to 6 or 7S.

432[D, F, L, R].—J. H. LAMBERT, *Mathematische Werke, I. Band: Arithmetik, Algebra, und Analysis*. Ed. by ANDREAS SPEISER. Zürich, Füssli, 1946, Portrait + xxxii, 358 p. 15.5 × 22.9 cm.

JOHANN HEINRICH LAMBERT (1728–1777), German physicist, mathematician and astronomer, was born in Mülhausen, Alsace, and largely self-taught. During 1748–1756 he was tutor to children of Count de Salis at Coire, Switzerland; in 1753 he was elected a member of the Swiss Society of Basle and later contributed several memoirs to *Acta Helvetica*. During 1756–1758 he traveled, sojourning at universities of Germany, Holland, France and Italy. In 1761–1763 he spent some time again at Coire and Zürich but from 1764 to the end of his life he was almost wholly at Berlin where he received many favors from Frederick the Great and was in 1765 elected a fellow of the Royal Academy of Sciences. At this time Euler and Lagrange were also active in Berlin. The table-maker J. C. SCHULZE (1749–1790) was a pupil of Lambert.

In 1761 Lambert proved that π was irrational, although as early as 1689 J. C. STURM, in his *Mathesis Enucleata*, stated (p. 181) "Area circuli est quadrato diametri incommensurabilis." He introduced hyperbolic functions into trigonometry and published the first table of such functions (1770). He made geometrical discoveries of value, and theorems concerning conic sections bear his name (see C. TAYLOR, *Introd. to the Ancient and Modern Geometry of Conics*, Cambridge, 1881). Astronomy was enriched by his investigations.

From what has been indicated above it is not inappropriate that a Swiss publisher and a Swiss scholar should collaborate in bringing out first volumes of Lambert's publications in the fields of "Arithmetik," Algebra and Analysis. Later volumes are planned to care for his writings in Applied Mathematics, Astronomy and Physics, and in Philosophy and Logic. We shall now refer only to the tabular material in four places of the present volume. All of this material appeared originally in Lambert's *Beyträge zum Gebrauche der Mathematik und deren Anwendung*. Berlin, 3 v., 1765–1772. The first three portions were in the second v., 1770, and the last in the third, 1772.

I, *Vorschlag, die Theiler der Zahlen in Tabellen zu bringen* p. 117–132. In 1767 HENDRIK ANJEMA's *Table of Divisors of all the Natural Numbers from 1 to 10000* (vi, 302 p.) was published. This table gave every divisor for each number, even 1 and the number itself. Since Lambert felt that a very small table would readily produce all the essential facts of this volume, he here gives his substitute (omitting all numbers divisible by 2, 3, 5) on a single folded sheet 34 × 51 cm. in size for the printed part. This is arranged in 9 facsimile pages (124–132) in the Speiser edition. Speiser also notes the 29 errors in Lambert's table which J. WOLFRAM sent to Lambert in a letter of 3 August 1772, printed in *Deutscher gelehrter Briefwechsel Joh. Heinr. Lamberts*, ed. by JOHANN (III) BERNOULLI, v. 4, 1784, p. 448. In this table all the prime factors are sometimes given for the non-prime numbers. In the year 1770 Lambert published also *Zusätze zu den logarithmischen und trigonometrischen Tabellen.* Table I in this collection gives the smallest factor of all numbers less than 102000 and not divisible by 2, 3, or 5. Concerning this volume LAGRANGE wrote as follows to D'Alembert on 4 April 1771: "J'y joindrai aussi un Ouvrage de M. Lambert qui a paru l'année passée, et qui n'est qu'une collection de différentes Tables numériques qui peuvent être très-utiles dans plusieurs occasions; c'est moi qui lui en ai donné l'idée et qui l'ai excité à l'exécuter." [LAGRANGE, *Oeuvres*, v. 13, 1882, p. 195. To the word "passée" the editor, J. A. SERRET, has added the following absurd footnote: "*Observations trigonométriques.* Lu à l'Académie de Berlin en 1768 et imprimé (p. 327–356) dans le volume portant la date de cette année, qui ne parut qu'en 1770."]

In ANTON FELKEL's Latin edition of Lambert's *Zusätze* (Lisbon 1798), T. I. has been elaborated by the indication of many extra factors, partly through the use of 36 letters standing for prime numbers up to 173, a scheme similar to that used by Felkel in his factor table of 1776–1777 (see *Scripta Mathematica*, v. 4, 1936, p. 336–337).

II, *Algebraische Formeln für die Sinus von drey zu drey Graden*, p. 189–193. On p. 190–191 is a table of sin $n3°$, for $n = 1(1)29$, expressed as functions involving only the square roots of numbers. Speiser corrects one error in the last entry. The table is reprinted in *Zusätze* p. 137–138, and in the 1798 edition, p. 125–126.

III, *Vorläufige Kenntnisse für die, so die Quadratur und Rectification des Circuls suchen.* The table on p. 204–205 contains 27 entries of various ratios, continually closer approximations to the ratio of the diameter to the circumference of a circle. These values (1:3, 7:22, 106:333, 113:355, ⋯) are the successive convergents of the regular continued fraction for π^{-1} which Lambert gives. In the second last term of this fraction Lambert made a numerical slip (pointed out by Wolfram in the letter of 3 August 1772, referred to above) which vitiated the accuracy of his last two ratios. As Speiser notes, these should have been

4448 54677 02853:13975 52185 26789
13630 81215 70117:42822 45933 49304.

The last ratio gives π^{-1} correct to 29 decimal places. In his edition of *Archimedes, Huygens, Lambert, Legendre. Vier Abhandlungen über die Kreismessung*, Leipzig, 1892, Rudio failed to observe the errors noted above and hence made a misleading statement. The first 16 of the entries of Lambert's table are in his *Zusätze*, p. 145, and in the 1798 edition, p. 133.

Both Speiser and Rudio are guilty of a curious oversight, in failing to refer to the table which JOHN WALLIS gave, 85 years before Lambert, in his *A Treatise of Algebra . . .*, London, 1685, p. 51–55; (see D. H. LEHMER, "Euclid's alogrithm for large numbers," *Amer. Math. Mo.*, v. 45, 1938, p. 227–233). Not only do we here find the whole of the table of Lambert, and without any error, but 7 terms more, of which six are correct; the last correct ratio (the 33rd) is

842 46858 74265 13207:2646 69312 51393 04345;

from this, π may be determined correctly to 38D. It may be determined to 95D from the 91st ratio given by D. H. L. in the article indicated above.

IV, *Rectification elliptischer Bogen durch unendliche Reihen*, p. 312–325. The treatment of this problem is illustrated by a problem in geodesy. On p. 324 is a facsimile reproduction of a table giving for each degree of latitude the polar distance with its first difference which is approximately the length of one degree of longitude at that latitude. The earth is assumed to be a spheroid whose meridian section is an ellipse whose axes are in the ratio 230/229, an assumption attributed to NEWTON. Distances are given to the nearest klafter, an archaic French unit of length equal to 0.3875 rod. Unfortunately the whole table is slightly wrong, each polar distance being too large by about .047 percent, as noted by Speiser, on account of a small error in the formula on which the table is based.

R. C. A. & D. H. L.

433[E, M].—L. LANDWEBER & M. H. PROTTER, "The shape and tension of a light flexible cable in a uniform current," *Jn. Appl. Mechanics*, v. 14, June, 1947, p. A123–A124.

$$\tau = e^{(1/45)\cot\phi} = e^{u/45},\ u = \cot\phi;$$
$$\xi = \int_\phi^{\frac{1}{2}\pi} \tau \cot\phi \csc\phi d\phi = \int_0^u u e^{u/45} du/(1+u^2)^{\frac{1}{2}};$$
$$\eta = \int_\phi^{\frac{1}{2}\pi} \tau \csc\phi d\phi = \int_0^u e^{u/45} du/(1+u^2)^{\frac{1}{2}};\ \sigma = 45(\tau - 1).$$

The tables are for $\phi = 1°(0°.1)12°.9$, $167°.1(0°.1)179°$, τ mostly to 4S; ξ mostly to 3–4S; $\pm\eta$ mostly 4S; $\pm\sigma$, 3–4S, η and σ — for $90° < \phi < 180°$. Also for $\phi = 10°(1°)170°$, all the 4 functions to 4D, except some terminal values to 5D.

434[F].—D. H. LEHMER, "On the factors of $2^n \pm 1$," Amer. Math. So., *Bull.*, v. 53, Feb. 1947, p. 164–167, 15.1 × 24 cm.

Professor Lehmer gives factors of $2^n - 1$, in 32 cases, for values of n from 113 to 489, and of $2^n + 1$ in 44 cases, for values of n from 91 to 500. This list for $n \leqslant 500$ was intended to supplement the fundamental table of CUNNINGHAM & WOODALL[1] and the addenda to this list found by KRAÏTCHIK.[2] It is believed that all factors under 10^6 have now been found, and that any other factors of $2^n - 1$ for $n \leqslant 300$, or of $2^n + 1$ for $n \leqslant 150$, lie beyond 4538800.

Eight complete factorizations, n varying from 91 to 170, are given; the fifth of these for $2^{128} + 1$ has been already noted in MTE **107**. The first and eighth correct errors in Kraïtchik and in Cunningham & Woodall.

Eleven of the new factors given by Lehmer pertain to Mersenne numbers $2^p - 1$, p a prime not greater than 257. These factors are included in the range $p = 113$ (now completely factored) to $p = 233$. Of the 55 Mersenne numbers 12 are prime (p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127), 14 are composite and completely factored, for 9 two or more prime factors are known, for 8 only one prime factor is known, 11 are composite but no factor known, and in one case ($p = 193$) the character is unknown. As indicated above, any other factor now discovered for a Mersenne number must be greater than 4538800.

Professor H. S. UHLER completed the proof that M_{199} was composite on July 27, 1946 (Amer. Math. Soc. *Bull.*, v. 53, 1947, p. 163–164); and that M_{227} was composite on June 4, 1947; see also *MTAC*, v. 1, p. 333 (M_{157}), 404 (M_{167}), v. 2, p. 94 (M_{229}). In the article here reviewed D. H. L. checked the last two results at which Uhler had arrived, by showing that M_{167} had the factor 2349023 and M_{229} the factor 1504073.

R. C. A.

[1] A. J. C. CUNNINGHAM & H. J. WOODALL, *Factorisation of* ($y^n \pm 1$), London, 1925.
[2] M. KRAÏTCHIK, (a) *Recherches sur la Théorie des Nombres*, v. 2, Paris, 1929; (b) "Factorisation de $2^n \pm 1$," *Sphinx*, v. 8, 1938, p. 148–150.

435[F].—Nils Pipping, "Goldbachsche Spaltungen der geraden Zahlen x für x = 60000–99998," Åbo, Finland, Akademi, *Acta, Math. et Phys.*, v. 12, no. 11, 1940, 18 p. 16 × 23.7 cm.

This table is an extension, for the range indicated in the title, of a previous table[1] for the range $x < 60000$. Its purpose is to verify the unproved Goldbach conjecture that every even number x greater than 4 is the sum of two odd primes. For the present range the conjecture is true with plenty to spare. Of the 20000 even numbers x in this range the author finds that 15315 of them are representable in such a way that the largest possible prime is involved: that is, one of the primes q in $x = p + q$ can be taken as the greatest prime not exceeding $x - 3$. The remaining 4685 numbers x are listed in the table together with the least prime m_x for which $x - m_x$ is also a prime. The largest m_x occurs at $x = 63274$ where m_x has the value 293.

D. H. L.

[1] N. Pipping, "Die Goldbachsche Vermutung und der Goldbach-Vinogradowsche Satz," Åbo, Finland, Akademi, *Acta, Math. et Phys.*, v. 11, no. 4, 1938, 25 p.

436[F].—Ernst S. Selmer & Gunnar Nesheim, "Tafel der Zwillingsprimzahlen bis 200.000," K. Norske Videnskabers Selskab, Trondhjem, *Forhandlinger*, v. 15, 1942, p. 95–98.

This is a short table of those values of n for which $6n + 1$ and $6n - 1$ are both primes less than 200,000. Since all prime pairs except (3, 5) are of this form we have in effect a table of prime pairs under 200,000. The number of these less than 100,000 was found to be 1224 which is in agreement with the count by Glaisher.[1] The number of prime pairs in the second 100,000 was found to be 936, which differs from the count 935 made by Sutton,[2] and Hardy & Littlewood.[3] The present list is based on the list of primes by Lehmer.[4]

D. H. L.

[1] J. W. L. Glaisher "On certain enumerations of primes," BAAS, *Report, 1878*, p. 470–471.
[2] C. S. Sutton, "An investigation of the average distribution of twin prime numbers," *Jn. Math. and Phys.*, v. 16, 1937, p. 41–42. See RMT **345**.
[3] G. H. Hardy & J. E. Littlewood, "Partitio numerorum III: On the expression of a number as a sum of primes," *Acta Math.*, v. 44, 1923, p. 44.
[4] D. N. Lehmer, *List of Primes from 1 to 10 006 721*. Washington, 1914.

437[F].—Ernst S. Selmer & Gunnar Nesheim, "Die Goldbachschen Zwillingsdarstellungen der durch 6 teilbaren Zahlen 196.302–196.596," K. Norske Videnskabers Selskab, Trondhjem, *Forhandlinger*, v. 15, 1942, p. 107–110.

This note contains a table (p. 108) giving the number of representations of $6n$ as a sum of two primes in which each prime is one of a pair of twin primes, for each integer of the form $6n$ between 196301 and 196597. Besides the actual count of such representations, approximate values are given as computed from the following formula of Stäckel

$$4.1532 \cdot [\pi(6n)]^4 (6n)^{-3} \prod_p \frac{p-2}{p-4} \prod_q \frac{q-3}{q-4}$$

where $\pi(x)$ is the number of all primes $\leqslant x$ and, in the two products, p ranges over the prime factors > 3 of n, while q ranges over the odd prime factors of $3n \pm 1$. The ratio of this approximation to the exact count is also tabulated. This ratio ranges from .83 to 1.25 but has an average of 1.001. The approximate and exact values are compared graphically. The exact count was based on the authors' previous table (RMT **436**) of prime pairs.

D. H. L.

438[F].—Dov YARDEN,[1] "Luach Mispare Fibonatsi" [Table of Fibonacci numbers], *Riveon Lematematika Lelimud Vlemechkar* [Quarterly Journal of Mathematics for Study and Research], Jerusalem, Palestine, v. 1, no. 2, Sept. 1946, p. 35–37. 21.6 × 33.6 cm. The text of this periodical, edited by Dov Yarden for graduate students of the University of Jerusalem, is entirely in Hebrew, and mimeographed on only one side of each sheet. The cover title-page is printed.

This table gives the two Fibonacci sequences U_n, V_n for $n = 0(1)128$. These famous numbers are defined by the recurrence formulae

$$U_n = U_{n-1} + U_{n-2}, \quad U_0 = 0, \quad U_1 = 1$$
$$V_n = V_{n-1} + V_{n-2} \quad V_0 = 2, \quad V_1 = 1$$

and have an extensive literature. Besides the mere values of these numbers the table gives their factorization into primes. These are complete as far as U_{66} and V_{61}. Beyond these points many entries have large factors enclosed in parentheses, indicating numbers of unknown composition, while many others are completely factored. The author was unaware of the work of POULET and KRAÏTCHIK summarized in Kraïtchik's table.[2] However certain small factors appear in Yarden's table which are not given in that of Kraïtchik as follows:

n	Factor of V_n	n	Factor of V_n
94	563	114	227
101	809	119	239
103	619	124	743
106	1483	127	509
112	223·449		

Addenda to the present table are promised for a future issue.

D. H. L.

EDITORIAL NOTES: Primitive factors in U_n, V_n, that is, the product of those factors which have not occurred previously, are underlined. The statement concerning $n = 106$ was in a communication from the author to the reviewer. There are the following errors in the table: U_{87}, the factor 514229 should not be underlined, since it occurs previously in U_{29}; U_{122} first factor, for first figure 3, *read* 2; V_{67}, *for* 9343, *read* 9349; V_{117}, *for* 67861, *read* 79·859; V_{120}, last factor, *for* first figure 3, *read* 2.

[1] Library of Congress transliteration is here, and later, employed. In *Scripta Mathematica*, v. 11, 1945, the name occurs as Dov Juzuk.

[2] M. KRAÏTCHIK, *Recherches sur la Théorie des Nombres*. v. 1, Paris, 1924, p. 77–81.

439[F].—D. YARDEN, "Luach Tsiyune-hehofa'a Besidrath Fibonatsi" [Table of the ranks of apparition in Fibonacci's sequence], *Riveon Lematematika*, v. 1, no. 3, Dec. 1946, p. 54. 21.6 × 33.6 cm. Compare RMT **438**.

The notion of the rank of apparition of a prime in Fibonacci's sequence

$$U_0 = 0, \quad U_1 = 1, \quad U_2 = 1, \quad U_3 = 2, \quad \cdots, \quad U_{n+1} = U_n + U_{n-1}$$

is the counterpart of the exponent of a number with respect to a prime modulus; it is simply the least positive subscript n for which U_n is divisible by the given prime p. The author denotes this function of p by $a(p)$. Thus $a(2) = 3$. By a theorem of Lucas, $a(p)$, for p an odd prime, is some (unpredictable) divisor of $p - \epsilon$, where $\epsilon = (5/p)$ is Legendre's symbol and has the value

$$\epsilon = \begin{cases} +1 & \text{if} \quad p = 10k \pm 1 \\ 0 & \text{if} \quad p = 5 \\ -1 & \text{if} \quad p = 10k \pm 3 \end{cases}$$

The present note gives a table of $a(p)$ for all primes $p \leq 1511$. Besides $a(p)$ the number $p - \epsilon$ is given in factored form. In those cases where $a(p) < p - \epsilon$, the factors of the "residue index" $(p - \epsilon)/a(p)$ are underlined. For every prime p the Fibonacci numbers are periodic, modulo p. The proper period is $4a(p)$ if $a(p)$ is odd, $2a(p)$ if $a(p)$ is divisible by 4, and $a(p)$

otherwise. This appears to be the first table of its kind. It gives indirectly all small prime factors of all Fibonacci numbers, where by "small" we mean $\leqslant$ 1511.

D. H. L.

EDITORIAL NOTE: Comparing this table with a short one for $p < 1000$ in M. KRAÏTCHIK, *Recherches sur la Théorie des Nombres*, v. 1, Paris, 1924, p. 55, we note three errors in Kraïtchik: p. 269, *for* $\gamma = 1$, *read* $\gamma = 4$; p. 499, *for* $\gamma = 2$, *read* $\gamma = 1$; p. 743, *for* $\gamma = 1$, *read* $\gamma = 3$. In Yarden's table, column of p's, there is a misprint of 367 for 467.

440[L].—HARVARD UNIVERSITY, Computation Laboratory, *Annals*, v. 5: *Tables of Bessel Functions of the First Kind of Orders Four, Five, and Six;* v. 6: *Tables of Bessel Functions of the First Kind of Orders Seven, Eight, and Nine.* By the staff of the Laboratory, Professor H. H. AIKEN, Technical Director, Cambridge, Mass., Harvard Univ. Press, 1947, xii, 650 p. and viii, 646 p. 19.5 × 26.7 cm. $10.00 + $10.00. Compare *MTAC*, v. 2, 176f, 185f, 261f. The offset printing continues to be of outstanding excellence.

The tabulation of these functions was undertaken at the request of the Bureau of Ships, in behalf of the Naval Research Laboratory, and continued under the cognizance of the Bureau of Ordnance. The tables were computed by recurrence from the values of $J_2(x)$ and $J_3(x)$ previously published in v. 4 of the *Annals*. In v. 5, the preface is by H. H. AIKEN and the pages ix–xi, on "Interpolation in the tables," with two numerical examples, are by R. M. BLOCH. The range of the parameter is $x = [.012(.001)25(.01)99.99; 10D]$. Since to 10D the values of $J_7(x)$ are zero for $x < .229$, it is with this value that the tabulation of v. 6 commences. Up to $J_7(.267)$ the values are all the same, .00000 00001.

The results in these volumes are almost entirely new. Among published tables, to at least 10D and for $n = 4(1)9$, are those by MEISSEL, for $x = [0(1)24; 18D]$; by AIREY, for $x = [6.5(.5)16; 10D]$; and by HAYASHI, for 15 values of x to at least 21D.

R. C. A.

441[L].—H. G. HAY, assisted by Miss N. GAMBLE and approved by G. G. MACFARLANE, *Five-Figure Table of the Function* $\int_0^\infty e^{-zy}\mathrm{Ai}^2(y - j_1)dy$ *in the Complex Plane.* (Great Britain, The Mathematical Group, Telecommunications Research Establishment (TRE), Malvern, Worcestershire), no. T 2047, November 1946, 22 p. 20.2 × 33 cm. Mimeographed. Compare *MTAC*, v. 2, p. 41, and RMT **444**. This publication is not generally available.

The need for these tables arose in connection with the determination of the eigenvalues of the wave equation for centimetric wave propagation in the atmosphere. The wave equation is

$$\frac{d^2U}{ds^2} + [s + \sum_{n=1}^{r} A_n e^{-\alpha_n s} + D_1]U = 0$$

in which the constants A_n and α_n are known. The equations from which the eigenvalue D_1 is found involve the function $F(z) = \int_0^\infty e^{-zy}Ai^2(y - j_1)dy$, and its first derivative, where $-j_1(= -2.3381)$ is the first zero of the Airy function Ai(y), as defined by MILLER[1] and tabulated in the complex plane by P. M. Woodward & Mrs. A. M. Woodward.[2] The function has also been discussed in the same connection by C. L. PEKERIS.[3] It satisfies the differential equation

$$dF/dz = (2z)^{-1}[\mathrm{Ai}'(-j_1)]^2 - [(2z)^{-1} + j_1 + \tfrac{1}{4}z^2]F,$$

from which the derivative can be calculated when $F(z)$ is known. Asymptotic formulae for F and F' can be used to extend the table to $|z| > 4$, by the methods given by PEKERIS.[3]

The tables giving values of the real and imaginary parts of $F(z)$ and $F'(z)$, to 5D, 16 p., cover the region $x = 0(.2)4$, $y = 0(.2)3.2$, in the upper half of the complex plane. The values of $\frac{1}{4}(\Delta_x^2 - \Delta_y^2)$ and $\Delta_x^2\Delta_y^2$ are placed beneath each function value, as in the Woodward tables, for applying their method of bivariate interpolation for a function of a complex variable.

In the computation of the tables each function value was calculated to six figures and the power series expansions around two enclosing origins were used as a first check. At this stage the error did not exceed five units in the last decimal place. The values were then rounded off by the Woodward method, which probably reduces error to less than a unit in the sixth decimal place. The tables presented in five-figure form are hoped to be correct to the number of figures shown. An error of one unit in the fifth figure may occur where the sixth figure is in the region of five.

Extracts from the introductory text

[1] J. C. P. MILLER, *The Airy Integral*, Cambridge, 1946 (see *MTAC*, RMT, **413**), and *MTAC*, v. 1, no. 7, p. 236.
[2] P. M. WOODWARD & Mrs. A. M. WOODWARD, "Four-figure tables of the Airy function in the complex plane," *Phil. Mag.*, s.7, v. 37, p. 236–261 (see *MTAC*, RMT **420**).
[3] C. L. PEKERIS, "Perturbation theory of the normal modes for an exponential *M*-curve in non-standard propagation of microwaves," *Jn. Applied Physics*, v. 17, 1946, p. 678.

442[L].—RAY S. HOYT, "Probability functions for the modulus and angle of the normal complex variate," *Bell System Technical Jn.*, v. 26, April 1947, p. 318–359. 15 × 22.8 cm.

There is a table on p. 346, of $Q(R) = 2\int_0^R t^{\frac{1}{2}} e^{-t} I_0(bt) d\lambda$, $t = \lambda^2/(1 - b^2)$, for $R = .2(.2).8$, and of $Q^*(R) = 2u^{\frac{1}{2}}\lambda\int_0^r \lambda^{-3} e^{-u} I_0(bu) d\lambda$, $u = 1/[\lambda^2(1 - b^2)]$, for $R = r^{-1} = 1.6, 2$. For each of these integrals $b = 0, .3(.1)1, .95$. Corresponding to each of the values of R there are in the table four rows under the various values of b. In the first row of any set of four rows are the values of $Q(R)$ or $Q^*(R) = e^{-R^2} +$ values of $P_{b,0}$, given in a table on p. 53 of a paper by Hoyt in *Bell System Technical Journal*, v. 12, 1933. In the second row are the values computed from formulae indicated above, to 4 or 5D. The third row of each set of four rows gives the deviations of the second row from the first row; and the fourth row expresses these deviations as percentages of the values in the first row.

Extracts from the text

443[L].—L. INFELD, V. G. SMITH & W. Z. CHIEN, "On some series of Bessel functions," *Jn. Math. Phys.*, v. 26, April 1947, p. 22–28.

"In a study of radiation from a cylindrical antenna in a rectangular wave guide we were confronted with the series $\sum_{m=0}^{\infty}(-1)^m Y_0(mx)$." Let $S = \sum_{m=1}^{\infty}(-1)^m Y_0(m\pi x)$. On p. 27 is a table of S, $x = [0(.2)3; 10D]$ rounded off from 13D calculations. There are also three 10D tables of six entries each to make possible interpolation near a discontinuity; these tables are of the functions (i) $S + \pi^{-1}\ln x$, (ii) $S + 2\pi^{-1}(1 - x^2)^{-\frac{1}{2}}$, (iii) $S + 2\pi^{-1}(9 - x^2)^{-\frac{1}{2}}$. On p. 26 are graphs of these four functions.

444[L].—G. G. MACFARLANE, *The Application of a Variational Method to the Calculation of Radio Wave Propagation Curves for an arbitrary Refractive Index Profile in the Atmosphere.* (Great Britain, The Mathematical Group, TRE, Malvern, Worces., no. T. 2048.) December, 1946. 15 p. + 5 plates of figures. 20.2 × 33 cm. This publication is not generally available.

The basis of discussion here is the differential equation already noted in RMT **441**. On p. 14 are two small tables. Denoting the zeros of the Airy function by j_r, then for $r = 1(1)10$ are given the values of j_r to 5D and of $\mathrm{Ai}'(-j_r)$ and $[\mathrm{Ai}'(-j_r)]^2$, each to 6D. The other table, to 6D, is of $P_{rs} = \mathrm{Ai}'(-j_r)\mathrm{Ai}'(-j_s)/(j_r - j_s)^2$, $r > s$, $P_{rr} = (j_r/3)$ $[\mathrm{Ai}'(-j_r)]^2$, $r = 1(1)5$, $s = 1(1)5$.

445[L].—F. W. J. Olver, "Note on a paper of H. Bateman," *Jn. Appl. Phys.*, v. 17, Dec. 1946, p. 1127. 19 × 26.1 cm. See RMT **308**, v. 2, p. 126.

The text of the Note: "In a paper [*Jn. Appl. Physics*, v. 17, 1946, p. 91–102] entitled 'Some Integral Equations of Potential Theory' the late Professor Bateman includes a table of $P_n(1 - 2e^{-t})$, to 15D, for $n = 1(1)10$, $t = 1(1)20$. A number of errors were noticed in this table and subsequent investigation showed the need for complete recomputation. This has been done and the new table, in which the values given are correct to within 0.52 units of the fifteenth decimal, is appended."

"Attention is also drawn to the erroneous value given [Reference 1, p. 99] for $P_4(u)$, where $u = 1 - 2e^{-\pi}$; the last four decimals given should read . . . 7662 instead of . . . 5162."

446[L].—Max Eric Reissner, "Stresses and small displacements of shallow spherical shells. II," *Jn. Math. Physics*, v. 25, Jan. 1947, p. 279–300. 17.4 × 25.1 cm. Table on p. 298.

For $\lambda = .1, .2, .5, 1(1)10$, are given 4S values for $Wr(\lambda)$, $Wu(\lambda)$, C_1 and $C_2(k = 0$, $k = \infty$ and $\nu = \frac{1}{3})$, and 3D values for $f_1(\lambda)$ and $f_2(\lambda)$, $(k = 0, k = \infty$ and $\nu = \frac{1}{3})$, also with values for $\lambda = 0$.

$Wr(\lambda) = \text{ber}\,\lambda\,\text{kei}'\,\lambda - \text{ber}'\,\lambda\,\text{kei}\,\lambda$, $Wu(\lambda) = \text{bei}\,\lambda\,\text{kei}'\,\lambda - \text{bei}'\,\lambda\,\text{kei}\,\lambda - \frac{1}{2}\lambda$,
$f_1(\lambda) = (8/\lambda^2)[\frac{1}{4}\pi + \text{kei}\,\lambda + C_1(\text{ber}\,\lambda - 1) + C_2\,\text{bei}\,\lambda]$,
$f_2(\lambda) = 2[-\text{ker}\,\lambda + C_1\,\text{bei}\,\lambda - C_2\,\text{ber}\,\lambda]$.
For $k = 0$, $C_1 = [Vu(\lambda) + \text{bei}'\,\lambda/\lambda]/Vb(\lambda)$, $-C_2 = [Vr(\lambda) + \text{ber}'\,\lambda/\lambda]/Vb(\lambda)$;
For $k = \infty, \nu = \frac{1}{3}$, $C_1 = \{\frac{2}{3}[Vu(\lambda) + \text{bei}'\,\lambda/\lambda] - \lambda Wu(\lambda) - \frac{1}{2}\}/[\frac{2}{3}Vb(\lambda) - \lambda Wb(\lambda)]$,
$-C_2 = \{\frac{2}{3}[Vr(\lambda) + \text{ber}'\,\lambda/\lambda] - \lambda Wr(\lambda)\}/[\frac{2}{3}Vb(\lambda) - \lambda Wb(\lambda)]$;
$Vb(\lambda) = (\text{ber}'\,\lambda)^2 + (\text{bei}'\,\lambda)^2$, $Vr(\lambda) = \text{ber}'\,\lambda\,\text{ker}'\,\lambda + \text{bei}'\,\lambda\,\text{kei}'\,\lambda$,
$Vu(\lambda) = \text{bei}'\,\lambda\,\text{ker}'\,\lambda - \text{ber}'\,\lambda\,\text{kei}'\,\lambda$, $Wb(\lambda) = \text{ber}\,\lambda\,\text{bei}'\,\lambda - \text{bei}\,\lambda\,\text{ber}'\,\lambda$.

The calculations were "carried out with two more decimals than are listed in the table." The author informed us that c_1, c_2 in the table should be C_1, C_2.

447[L, M].—H. B. Dwight, *Tables of Integrals and Other Mathematical Data.* Revised ed., New York, Macmillan, 1947. x, 250 p. 13.4 × 20.3 cm. $2.50.

The first edition of these tables has been already reviewed in *MTAC*, v. 1, p. 190–191, and errata therein listed, p. 195–196. Although 28 pages have been here added, the main numbering of different items remains unchanged except that a probability integral table (no. 1045) has been introduced. Otherwise the numerical tables in the Appendix are unchanged, except for corrections. All the errors which we previously noted have been expunged. The considerable number of items added in other parts of the volume includes groups of integrals involving $(a^2 + bx + c)^{\frac{1}{2}}$, $(a + b\sin x)^{-1}$, and $(a + b\cos x)^{-1}$, and also material on inverse functions of complex quantities and on Bessel functions. To the 37 volumes listed under "References" in the first edition, 30 new works have been added, but none dated later than 1945. Scattered throughout the book are constant directions for consultation of this list. For example, on p. 177: "For tables of $K_0(x)$ and $K_1(x)$, see Ref. 50, p. 266, and Ref. 12, p. 313. Tables of $e^xK_0(x)$ and $e^xK_1(x)$, Ref. 13. Tables of $(2/\pi)K_0(x)$ and $(2/\pi)K_1(x)$, Ref. 17." There are some other literature lists in footnotes; the old one mentioning *Reports* of B.A.A.S. still has (p. 241) 1916, "p. 109," when I fancy that p. 122 is intended. On p. 129, T. 1045 is referred to as T. 1035. In 808.4 *read* $-J_n'(x)$; in 812.4, $-N_n'(x)$. The revised edition of this excellent work may be heartily recommended.

R. C. A.

448[U].—Great Britain, Nautical Almanac Office, *Astronomical Navigation Tables*, v. *Q*, *Latitudes N70°–N79°*. (*British Air Publication* no. 1618.) London, His Majesty's Stationery Office, [1944], iv, 341 p. 16.3 × 24.6 cm.

This volume is the fifteenth of a series, the first fourteen of which were reproduced by photolithography in the United States as *H. O. 218*. It is understood that this volume is not to be reproduced as a volume of *H. O. 218*.

The latter have been reviewed earlier (*MTAC*, v. 1, p. 82f.) and the reader is referred there for the general description of the tables, methods of use, etc. This review will concentrate on the points at which this volume differs from the earlier ones.

Each of the first fourteen volumes covered only five degrees of latitude, but could be used in the appropriate belt either north or south of the equator. Only twelve bright stars are offered in this volume instead of twenty-two, namely Aldebaran, Alpheratz, Altair, Arcturus, Betelgeuse, Capella, Deneb, Dubhe, Pollux, Procyon, Regulus, and Vega. The tabular material for these stars covers only the northern latitude 69°30′ to 79°30′, but the general tables for sun, moon and planets, declinations 0(1°)23°, can be used in either hemisphere.

The lower limit of altitude tabulated is 1° for declinations up to 16°, 10° for other declinations. This may be compared to the uniform lower limit of 10° in the earlier volumes. When one recalls that, for weeks at a time in the polar regions, the sun may not reach an altitude as great as 10°, one senses the importance of this change. It is to be hoped that the refraction corrections were also adjusted for the polar regions, since at low altitudes, the difference in refraction corrections between temperate and polar temperatures may amount to more than 2′.

The star tables shortly to appear as Hydrographic Office *Publication No. 249* will serve the same purpose as the star tables in this volume and will have the added advantage that they will be useful over a wider range of latitudes. There will still remain a need for a Hydrographic Office publication to cover the sun, moon and planets for aerial navigation in polar latitudes. The volumes of *H. O. 214* (*MTAC*, v. 1, p. 81f) cover the latitudes and the declinations but were designed primarily for surface navigation. Aerial navigation needs a single book of smaller bulk and weight and faster to use than the volumes of *H. O. 214* required for polar travel.

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449[V].—A. HORACE WILLIAMS KING, *Manning Formula Tables for Solving Hydraulic Problems*. v. 1, *Flow in Pipes* . . .; v. 2, *Flow in Open Channels* . . ., New York and London, McGraw-Hill, 1937–1939. xi, 351 p.; xiv, 379 p. 15 × 22.5 cm.

B. SHERMAN M. WOODWARD & CHESLEY J. POSEY, *Hydraulics of Steady Flow in Open Channels*, New York, Wiley, 1941; Tables, p. 8–14. 14.7 × 23 cm.

C. JOSE S. GANDOLFO, "Calculo de canales en movimiento uniforme. Conversión de las tablas de Woodward y Posey en adimensionales," *Revista de la Administración Nacional del Agua*, Buenos Aires, Argentina, v. 9, Dec. 1945, p. 449–458. 19.5 × 28.3 cm.

The first algebraic expression concerning the flow of water in pipes or open channels was published in 1757 by ANTOINE DE CHEZY (1718–1798). It is the well-known Chezy formula, $V = C\sqrt{Rs}$, where V is the mean velocity of flow, R is the hydraulic radius, or ratio of the area of the cross-section to the length of the wetted perimeter of the cross-section, s is the slope of the hydraulic gradient and C is a numerical coefficient which varies with the roughness of the channel lining and with the hydraulic radius. Many experimenters have proposed formulae for the evaluation of the coefficient C.

In 1869, GANGUILLET & KUTTER, Swiss engineers, after a great deal of experimentation and measurement on both natural and artificial channels, suggested a rather complicated formula in which not only n, a coefficient of roughness, and R, but also s appeared. They also gave a list of various types of channel linings, with the appropriate value of n in each case. Their formula for the determination of C has had and is still having considerable use, both in the United States and abroad, although subsequent investigators do not agree that the value of the slope has any bearing. In 1890, ROBERT MANNING, an Irish engineer, one time president of the Institution of Civil Engineers of Ireland, presented a formula[1] for the determination of C in terms of R and n, with values of n as proposed by Ganguillet and Kutter. His formula is

$$C = R^{1/6}/n$$

and this value inserted in the Chezy formula gives:

$$V = R^{\frac{2}{3}}s^{\frac{1}{2}}/n$$

in metric units, or

$$v = 1.486R^{\frac{2}{3}}s^{\frac{1}{2}}/n$$

for use with units of feet and seconds.

The commonly used form of the Manning formula is:

$$Q = 1.486aR^{\frac{2}{3}}s^{\frac{1}{2}}/n,$$

where Q is the discharge in cubic feet per second and a is the cross-sectional area of the channel in square feet. This formula, as applied to both pipes and open channels, enjoys wide-spread use. Various tables have been prepared to facilitate its use.

A. For a circular pipe, flowing full, the hydraulic radius, or R, is equal to the area of the circular cross-section divided by its circumference, or $d/4$, where d is the internal diameter of the pipe. Replacing R in the Manning formula by $d/4$ and solving the equation for d, there results:

$$d_i = \left(\frac{1630\,Qn}{s^{\frac{1}{2}}}\right)^{3/8},$$

where d_i is the internal diameter of the pipe in inches. The tables in v. 1, p. 1–351, give values of d_i for various combinations of Q, n and s. Q is given in cubic feet per second and also in equivalent gallons per minute or million gallons per day. With any three of the quantities in the formula known, the fourth one can be found from the tables.

Q = .001(.0005).005(.001).02(.005).05(.01).2(.5)1(.1)5(.5)20(1)100(5)200(10)500(25)1000 (50)2000(100)5000(250)10000,
s = [.00002(.00002).0002(.00005).0005(.0001).001(.0002).003(.0005).005(.001).01(.002) .02(.005).05; 2–4S],
n = .01(.001).02.

For trapezoidal channels, including those with rectangular and triangular sections, having a depth of flow D, a bottom width b and side slopes of z horizontal to 1 vertical, for circular channels having a depth of flow D and a diameter d, and for parabolic channels having a depth of flow D and a top width T, the Manning formula may be put in the form

$$Q = KD^{8/3}s^{\frac{1}{2}}/n,$$

where K is a function of z and the ratio of D to b for trapezoidal sections, a function of the ratio of D to d for circular sections and a function of the ratio D to T for parabolic sections. These respective values of K are given in Tables A, C and E of v. 2.

Table A, p. 352–365, D/b = [.001(.001).5(.01)2, ∞; 3S], z = 0(¼)1(½)3, 4.
Table C, p. 378, D/d = .01(.01)1.
Table E, p. 379, D/T = .01(.01)1.

The Manning formula may also be written:

$$Q = K'(b,\, d \text{ or } T)^{8/3}s^{\frac{1}{2}}/n$$

for trapezoidal, circular or parabolic channels, respectively, where K' is a different function of the same variables which combine to form K. These respective values of K' are given in Tables B, D and F of v. 2. Table B does not, however, contain values of K' for triangular channels.

Table B, p. 365–377, D/b = .001(.001).5(.01)2, z = 0(¼)1(½)3, 4
Table D, p. 378, D/d = .01(.01)1
Table F, p. 379, D/T = .01(.01)1.

Tables B, D and F are really superfluous as they simply provide alternate methods for securing the same results that tables A, C and E give.

The Manning formula may be put in the general form

$$Q = FL^{8/3}s^{1/2}/n,$$

where F is either K or K' and L is a linear dimension, either D, b, d or T. The main table in v. 2, p. 1–351, gives a solution of this equation for L, transformed to the form

$$L = \left(\frac{Qn}{F}\right)^{3/8}\left(\frac{1}{s}\right)^{3/16};$$

Qn/F = .00005(.00005).0002(.0001).0004(.0002).001(.0005).005(.001).01(.002).02(.005) .08(.01).3(.02).7(.05)2(.1)5(.2)10(.5)20(1)40(2)80(5)150(10)300(20)600(50)1500(100) 3000(200)5000(500)10000(1000)20000(2000)40000(5000)100000,
s = [0(.000001).0001(.00001) 001(.0001).01(.001).1(.01)1; 3–4S].

In H. W. King, *Handbook of Hydraulics*, third ed., second impression, 1939, p. 331–358, (10 × 16.7 cm.) there are tables of (a) K and K' for trapezoidal channels; (b) values of $1/K'^2$ in the formula $s = \left(\frac{Qn}{K'b^{8/3}}\right)^2$, for trapezoidal channels; (c) values of K and K' for circular and parabolic channels.

B. The tables here are less detailed and of somewhat different range.

Table 102A, K' for trapezoidal sections, p. 8–9,
D/b = .02(.01).5(.02)1(.05)2(.1)3(.2)4(.5)5; z = [0(¼)1½(½)3, 4; 3S].
Similar to Table B in **A.**

Table 102B, K for trapezoidal sections, p. 10–11,
D/b = .01(.01).5(.02)1(.05)2(.1)3(.2)4(.5)5, z = 0(¼)1½(½)3, 4.
Similar to Table A in **A.**

Table 103, for circular sections, p. 12, d/D = .01(.01)1.

In the fourth and fifth columns of this table are the values of K and K' which are given, respectively, in Tables C and D of **A.** In the second and third columns are shown, respectively, values of the ratio of the area of flow to the square of the diameter of the section and of the ratio of the hydraulic radius to the diameter of the section.

Tables 104A and 104B for special sections, p. 13–14,
D/r = .02(.02)1(.05)2(.1)3, z = ½(¼)1(½)2.

For special round-bottomed channels, of radius r, with sides tangent to the bottoms and having slopes of z horizontal to 1 vertical, and with depth of flow D, the Manning formula may be put in the forms:

$$Q = Kr^{8/3}s^{1/2}/n, \quad \text{and} \quad Q = K'D^{8/3}s^{1/2}/n,$$

where K and K' are functions of z and of the ratio of D to r. Values of K and K' are given in Tables 104A and 104B, respectively.

C. These tables are rather trivial, being tables of **B**, with tabular values of K and K' divided by 1.486, for use with metric units.

Table No. 1 for trapezoidal section,
D/b = .01(.01).5(.02)1(.05)2(.1)3(.2)4(.5)5, z = 0(¼)1½(½)3, 4.

Table No. 2 for trapezoidal sections,
D/b = .02(.01).5(.02)1(.05)2(.1)3(.2)4(.5)5, z = 0(¼)1½(½)3, 4.

Table No. 3 for circular sections,
D/d = .01(.01)1.

Table No. 4 for special sections,
D/r = .02(.02)1(.05)2(.1)3, z = ½(¼)1(½)2.

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[1] R. Manning, "Flow of water in open channels and pipes," Institute of Civil Engineers of Ireland, *Trans.*, v. 20, 1890.

Editorial Note: There is a "Nomograph for solving Manning's formula," by Paul McH. Albert, in *Water Works & Sewerage*, v. 92, 1945, p. R278–279.