

A Bell Telephone Laboratories' Computing Machine—II

This is the second of two articles by Dr. ALT describing a Bell Telephone Laboratories' machine. The first article, which appeared in the January issue of *MTAC*, treated the machine number storage, calculation, and input and output. Sections 2, 3, 4 of this first article are referred to in the second.

5. Control

By "control" is meant the process of causing the machine to perform the desired operations. This definition overlaps in part the concept of input discussed in the preceding section. The input consists of numbers and orders, and the input of orders is part of the control of the machine. In this section we shall consider the remaining aspects of the control, namely, what kinds of orders the machine can receive and how they are represented.

a. Calculator Orders.—The most frequently used orders are those which cause the machine to perform the ordinary arithmetic calculations of addition, subtraction, multiplication, division, and extraction of square root. Each of these orders contains the designations of two registers, the locations of the two numbers on which the operation is to be performed (in the case of square roots, only one such register), and of one further register, the location to which the result of the operation is to be transmitted. These locations are referred to as the first, second, and third "registrations." In the language of some of the more recent machine designers, they would be called "addresses."

Thus the control of this machine is accomplished, for the most part, by means of three-address orders. In contrast, the "Mark I" at Harvard¹ uses a two-address system. Of the projected electronic machines, one uses a four-address system and another a single-address system. In systems of less than three addresses several orders are necessary to accomplish arithmetic operations. For instance, for multiplication there would be one order stating that the number at location *a* should be used as a multiplicand. The next order states that the number at location *b* should be used as a multiplier. A third order designates location *c* to receive the product. In such a system, the orders are shorter and there are fewer types of orders, but more orders are required in any given program. It is not easy to see that any system is decidedly superior to the others.

In addition to the designation of locations, arithmetic orders for our machine contain an indication of the kind of operation to be performed, and indications as to whether the numbers stored in the first and second registrations are to be cleared (deleted) after the operation has been performed, or whether they are to be retained for future use.

The register to which the result is transferred must be clear at the time the operation is performed. If, erroneously, an attempt is made to transfer a number into a register which already contains a number, the machine will alarm. It will likewise alarm if one of the registers designated in the first two registrations of an order fails to contain a number at the time the operation starts.

As mentioned in section 4b, orders are expressed in a code whose elements are the first twenty letters of the alphabet; these are represented on

tape by combinations of three holes out of the possible six holes. Each order consists of several letters. (Occasionally, arithmetic symbols are used in place of letters. For instance, the code for addition is the letter P, but for convenience we sometimes write + instead. The same three-hole code on the tape represents both symbols.)

Each letter or code has different meanings depending on its position within an order. For the purpose of coding and decoding, the code letters of an order are divided into groups in such a way that the meaning of a code letter is uniquely determined by the group in which it stands. For example, suppose that we want the machine to carry out a subtraction. Suppose that the minuend is located in register A (the fifteen storing registers of the machine are designated by the letters A through O), that the subtrahend is located in register C, that it is intended to store the difference in register B. Suppose also that the subtrahend will be needed for further computations and should therefore be held in register C, whereas the minuend is to be cleared after this subtraction. The order which accomplishes this operation is

$$AC - CH = B \overline{EO}.$$

The letters in this order belong to the following groups:

$$\begin{array}{ccccccccc} A & C & - & C & H & = & B & \overline{EO} \\ 1 & 2 & 3A & 4 & 5 & 5B & 6 & 7 \end{array}$$

The groups are denoted by 1, 1A, 1B, 2, 2A, etc. The numbering is somewhat arbitrary and need not be explained in detail. Each order begins with group 1. The letters A to O in this group designate the corresponding storing registers and indicate that the number located in the designated register should be used as the first argument of the operation prescribed by the order. Letters P to T in this group have special meanings some of which will be mentioned later. In the above example, the letter A in group 1 indicates that the first argument of our operation (the minuend) is to be taken from register A.

The code in group 2 refers to clearing or holding the number in the register; the letter C, as in the above example, designates clearing, the letter H designates holding, and other letters have related meanings.

Group 3 is not used in this particular order (nor are groups 1A, 1B, 2A or 2B). The minus sign in group 3A indicates subtraction. (The letter O could have been used instead.)

Letters in groups 4 and 5 have a meaning corresponding to that of groups 1 and 2 but referring to the second argument. In our example, the C in group 4 indicates that the subtrahend is to be taken from register C, and the H in group 5 indicates that register C should not be cleared after this operation.

In group 5B the only code ever to appear is the sign =. It has no functional meaning but acts merely as an indicator of the fact that the first part of the order is completed. It enables the machine to overlap the reading of orders with calculation. It also serves as a check against operators' errors in coding, since the machine will stop if the equality sign does not appear at the proper place in the tapes.

In group 6, the letters from A to O designate the register to which the

result of the operation is to be transferred. The remaining letters are used for special purposes.

The symbol \overline{EO} in group 7 denotes *the end of the order*. It is written here as two letters for typographical reasons, but on the tape it is represented by a single code. This code, like the equality sign mentioned before, has no functional meaning but serves to check and synchronize the operation of the machine. Every order, regardless of its contents, ends with the symbol \overline{EO} or with one of two similar symbols which have the additional function of directing the control to the beginning of the next problem (or section of a problem) or back to the beginning of a repetitive cycle of computations.

All orders for arithmetic operations—addition, subtraction, multiplication, division, square root—are written in analogy to the above example, except that on square roots the second argument is omitted, e.g., $AC\sqrt{\quad} = B\overline{EO}$ orders that the root of the number stored in register A be extracted, register A be cleared, and the root stored in register B.

Slight modifications of these orders, using some of the groups 1A, 1B, etc., which are not used in the above example, will have the effect of changing the sign or taking the absolute amount of one or both of the arguments, and of cancelling the rounding of the result or the shifting off of initial zeros (see section 3).

b. Transfer Orders.—Transfer orders accomplish the transfer of a number from one register to another. The first register may be cleared or held as desired. For example, the order $AC \equiv B\overline{EO}$ results in transferring the number previously stored in register A into register B, and in clearing register A (as indicated by the letter C). It is necessary that previous to the receipt of this order register A contain some number and register B be clear; if either of these conditions is not met, the machine will give an alarm. The symbol \equiv could have been replaced by the letter S.

c. Tape Orders.—On receipt of a tape order, the machine forms a four-digit page or block number and moves one of the table tapes forward or backward until the desired page or block is found. If this be a page number, nothing further happens until another tape order calls for a block. If a block has been called for, the machine, after finding the block, automatically transfers the first number in this block into the table register, where it is used (as described in 4f) as if it were in a storage register.

The tape order indicates which table tape is to be used, whether a page or block number is to be found, and in which way the page or block number is to be formed. Either of these numbers may be formed in one of four different ways: (1) by taking the first four digits of a number stored in any register; (2) by taking the first three digits of a number stored in one register, and the first digit of a number stored in another register, and using these four digits, in this order, as a four-digit block or page number; (3) by taking the first two digits from two numbers; or (4) by taking the first digit of one number and the first three of another. The last three cases are particularly useful in connection with functions of two variables, i.e., double-entry tables. The tape order also indicates whether the registers which furnish the page or block number are to be cleared or held.

d. Printing and Perforating.—The control of the printing of answers and of perforating numbers on the storage tape is effected by the second

part of a calculator or transfer order, rather than by a separate order. For example, the order

$$A C - B H = Q P A G F \overline{EO}$$

indicates to the machine that the difference of the numbers in registers A and B is to be calculated and printed. The five letters after the equality sign indicate whether the result is to be printed, perforated or both; whether the recording (printing or perforating) should be in the usual decimal notation or in one of the abbreviated notations which are used in the machine; and in the former case, whether the sign of the number is to be recorded or omitted, how many decimal places are to be recorded before and after the decimal point, and how many blank spaces should be left on the printing paper after the number. Other printing orders direct the perforating of page and block numbers on storage tape. Routine orders can only direct the printing of numbers. Alphabetical information is printed under the direction of a SWP (switch to printer) section of the problem tape.

e. Discrimination.—By discrimination is meant the process of selecting one of several possible courses of computation, depending on the outcome of a previous computation. In this machine there are two types of discrimination. The first and simpler type is similar to that used in most other machines. It is limited to two alternatives, between which the decision is made depending either (a) on the sign of a previously computed number or (b) on whether such a number is equal to or different from zero. Possible alternatives are (a) continuing the computation with the next order on the routine tape, (b) moving to the beginning of the same or some other section of the routine tape, (c) going on to a new problem or to a new section of the same problem, (d) stopping and sounding an alarm. For instance, a discrimination order might run like this: "Look at the number stored in register A. If the number is positive, go to section 2 of this routine tape; if it is zero or negative, go to section 4 instead." Or: "Subtract the numbers in registers A and B. If the difference is zero (within the seven-place accuracy of the machine), proceed with the next order; if it is different from zero, give the alarm."

Discrimination of this type is effected by the second part of a calculator on transfer order, rather than by a separate order. For instance, the order

$$A C \equiv P R B \overline{EO}$$

instructs the machine to proceed either to the next order or to the beginning of section 2, depending on whether the sign of the number stored in register A is $-$ or $+$.

The machine is also equipped for a more complicated type of discrimination. In this, the sign of a number is transferred to a special organ of the machine, called the discriminator. The signs so transferred are counted, and their number is compared with a previously-set upper limit. For instance, suppose that the discriminator has been pre-set to count up to three plus signs and up to six minus signs. (The operator may choose any combination not exceeding four plus signs and seven minus signs.) An order might read as follows: "Take the number stored in register A and transfer its sign to the discriminator. If the sign is $-$ and the number of minus signs trans-

ferred to the discriminator so far, including this one, is less than 6, proceed to the next routine order on the tape. If this is the sixth sign to be transferred to the discriminator, this problem is finished; go on to the start of the next problem. If the sign is +, and less than three plus signs have been transferred to the discriminator so far, go to section 3 of this routine tape. If this is the third plus sign, give the alarm." In this process, the count of signs will be released (set back to zero) under several conditions: (a) whenever the pre-set limit is reached, (b) the count of minus signs is released whenever a plus sign is recorded (i.e., only consecutive minus signs are counted), (c) the count of plus signs is released whenever a pre-set number of minus signs have been recorded since the last plus sign (i.e., sequences of plus signs need not be consecutive, but they must not be interrupted by more than a certain maximum number of minus signs). This limit for intervening minus signs may be chosen different from the limit for consecutive minus signs mentioned before, and must not exceed five.

This discrimination feature is rather complicated, and it would have been possible to achieve the same effect by a succession of simple orders rather than by building a special machine component. However, it is bought at a surprisingly low price. The discriminator contains only 71 relays.

One of the many possible uses of the count of signs in the discriminator is as follows. Suppose we wish to integrate a differential equation by a step-by-step method in which each step is iterated several times until the result of two successive steps agrees within a prescribed limit of accuracy. Suppose furthermore that we wish to adjust the length of the integration step in such a way as to reduce the computing time to a minimum. If too many iterations are needed at each step, it may be preferable to change to a shorter step; if in several successive steps the first iteration is correct, it may be economical to lengthen the step. To accomplish this, the machine can be instructed to compute, at the end of each iteration except the first, the absolute value of the difference between the results of the last and the preceding iteration. From this value, the desired limit of accuracy is then subtracted, and the sign of the difference is referred to the discriminator. A positive sign indicates that the desired accuracy has not yet been reached; a negative sign indicates that the result is sufficiently accurate and that the next integration step may be started. The instructions to the discriminator are: on receipt of a plus sign before the pre-set limit of such signs is reached, go back to the beginning of the routine tape (i.e., perform another iteration); on receipt of a minus sign before the limit is reached, go on to the next section of the problem (i.e., to the next integration step); on reaching the limit of plus signs (i.e., if too many iterations are needed for the desired accuracy), go to a section of the routine tape which will cause the length of the integration step to be decreased; on reaching the limit of minus signs (i.e., if for several successive steps the first iteration has given the desired accuracy), go to another section of the routine tape which will cause the length of the integration step to be increased.

f. Miscellaneous Orders.—It is hardly necessary to enumerate all the different types of orders which may be given to the machine. There are orders to clear a register, orders to the typewriter to feed a new line, orders to the discriminator to set up the desired limits for the count of signs,

orders to insure that the proper tapes have been loaded into all tape transmitters before beginning a problem, orders to decide whether, at the start of a new problem or of a new section of a problem, the numbers then in the machine should be held or cleared and whether numbers stored on the storage tape should be held available for future use or moved outside the machine.

There are in the machine eight "sign registers" capable of storing only a plus or minus sign. An order may be given to transfer from a storing register to a sign register, in which case the sign register will record the sign of the number previously held in the storage register. If that number was zero, a negative sign will appear in the sign register unless special provisions are made to the contrary. An order calling for transfer from a sign register to a storage register will cause the latter to be set to + 1 or - 1. Also, one or both of the variables occurring in a calculator order may be taken from a sign register; the result is always the same as if one of the numbers + 1 or - 1 had been used.

It is possible to replace one (but not both) of the variables in a calculator order by a constant which is put on the routine tape. This is indicated by the letter Q, followed by the desired constant. For instance, the order

$$\begin{array}{cccccccc} A & H & \times & Q & + & 4000000 & +01 & \overline{NC} & C & = & B & \overline{EO} \\ 1 & 2 & 3A & 4 & & & & & 5 & 5B & 6 & 7 \end{array}$$

in which the grouping of the code letters is indicated by the numbers written below them, causes the machine to multiply the number in register A by the constant factor 4 and put the product into register B. The number 4 is written in the form in which it will be stored in the machine; i.e., sign of number, seven digits, and "exponent." In ordinary notation this corresponds to + .4000000 10^{+01} . The symbol \overline{NC} or *number check* indicates the end of the number.

The machine is equipped with a register called the "BTL Register," which, in addition to serving as a storing register, is capable of performing a number of special operations in connection with the forming of "block numbers" and with the computation of trigonometric and logarithmic functions. These operations usually involve some form of isolating and shifting of individual digits of a number. For instance, if a number is referred to the BTL register with orders to compute its antilogarithm, the register will automatically separate the characteristic from the mantissa, use the latter as the argument for computing the antilogarithm, and use the former for obtaining the exponent of the result. Similar operations are performed in computing the logarithm or a trigonometric function of a number. The operations involved in forming a page or block number, as described in section 5c, are also performed by means of the BTL register. (The letters "BTL" stand for block-forming, trigonometric, and logarithmic operations.) While these features were provided in the machine for their special purposes, they have been used by the operators of the machine for a variety of other purposes which involve manipulation of individual digits of a number.

The BTL register is closely associated with the so-called permanent function tables in the machine. These are wirings which make it possible to obtain very quickly the value of certain functions for any desired value of the argument. The functions so available include $\sin x$, $\cos x$, $\tan^{-1} x$, and on the machine of the Ordnance Department, $\log x$ and 10^x .

6. Operational Characteristics

a. Arrangement of Machine Components.—As was said before, two machines of the kind with which we are dealing are in existence at present. Each of these two installations is called a "computing system" and consists of two "computers" and associated equipment.

A computer contains all the components needed for dealing with a problem, with the exception of tape-reading devices (tape transmitters). Each computer contains:

Fifteen storage registers.

Eight sign registers.

A calculator, capable of performing addition, subtraction, multiplication, division and square root extraction.

A routine control, the organ which accepts orders from the routine tapes (and also from the SWR (switch to routine) sections of the problem tape), directs the carrying out of these orders by the computer, and directs the movement of the tapes from which the orders are received. It includes the routine register, an organ similar to a storing register, which receives numbers read from the routine tape (see section 5f above).

A BTL register (see section 5f above).

A table register and table control. The former receives and stores numbers read from table tapes, the latter directs the movement of the table tapes so as to find any desired location in them.

A problem register and problem control. The former receives and stores numbers read from CCS (computing cycle section) sections of the problem tape; the latter directs the movement of the problem tape (except in SWR sections) and directs the machine to act in accordance with various special symbols perforated on this tape.

A printer register and printer control. The former receives numbers from various machine components for the purpose of having them printed or perforated on the storage tape, and the latter directs the process of printing or perforating.

A discriminator (see section 5e above).

A recorder table, containing a printer (teletype page printer), a reperforator, a tape transmitter—the latter two for the storage tape—and a "distributor," which has the task of translating numbers from the relay representation used in the machine to the teletype code which is acceptable to the printer and reperforator.

In addition to the two computers, each installation contains the following:

Permanent tables, as described in section 5f above. These are accessible to both computers. The electrical connection between a computer and a permanent table is performed automatically under the direction of the table control.

Several (three or four) position tables.

Relay equipment connecting the position tables to the computers ("Position Connectors").

Auxiliary equipment, not electrically connected with the main machine.

The position tables are the input organs of the machine. Each of them holds twelve tape transmitters, one for the problem tape, five for routine tapes, and six for table tapes.

In general, a problem is solved on one computer, and the corresponding tapes are loaded into one position table. For large problems it is possible to use two computers and two position tables. In either case, each of the two computers is connected to one position table, and one or two position tables are unused. Into these spare positions the tapes for a problem can be loaded by the operating personnel while the machine is occupied with other problems. The "Position Connector" is so arranged that whenever one computer has finished the problem on which it has been working, it disconnects itself automatically from the position to which it had been connected, and connects itself to a loaded spare position. Thus the transition from one problem to the next is accomplished without any loss of machine time.

In addition to the very considerable saving in set-up time, setting problems up on spare position tables makes it possible to run the machine over night without anyone in attendance. The usual procedure is to load the machine up with problems in the evening and to come back in the morning to find the work done.

The auxiliary equipment consists of several hand perforators, on which the operating personnel produces the tapes to be used in the machine, and one or two "Tape Processors." These have the dual functions of translating tapes from the simple code used on the hand perforator to the more complicated but shorter code acceptable to the machine, and of comparing two tapes independently hand-perforated by two operators so as to check against errors. They can be used for other purposes, such as making duplicates of tapes, making minor corrections in tapes, making printed copies of the contents of tapes, etc.

b. Reliability.—It has been mentioned before that reliability is the most outstanding operating characteristic of this machine. During the first four months of its operation, the machine of the Ordnance Department gave one wrong answer due to a mistake in the permanent wiring and a few wrong answers due to operating or maintenance personnel interfering with the operation of the machine. Not a single wrong answer was obtained as a result of transient machine trouble, such as relays failing to give a contact.

The reason for this performance is not that transient trouble does not occur—it does, in fact, occur more frequently than on many other machines—but that it is rendered harmless by the elaborate checking system built into the machine.

The checking system has been already mentioned several times, especially in sections 2d and 4b. Those instances are not exhaustive. One or more checks are provided for almost every operation of the machine. As an example, let us consider the process of reading a code from a tape. As mentioned in section 4b, the tape transmitter moves the tape until a code is exactly above the sensing fingers. Then the tape is stopped, the fingers come up, and those fingers which find holes in the tape penetrate them, thereby closing certain electrical contacts and breaking certain others. The electric paths thus closed set up the corresponding "finger relays." There are in all six finger relays, corresponding to the possible six holes in the tape.

A circuit, previously set up, checks whether the right number of finger relays is up, i.e., whether the code just read consists of the expected number of holes. The machine then translates the code; i.e., it operates a binary and quinary relay in case of a number, or a single relay corresponding to the code read in case of an order. It checks the translation by making sure that one and only one relay in a proper group is up. It then sets up the "steering relays" which will direct the translation of the next code into the proper relay group (for example, when reading numbers, the steering relays direct the first code into the bi-quinary relays representing the first digit, the second code into the relays representing the second digit, etc.). The steering relays which were used for the code just read are dropped out. Relays are set up which will determine the proper number of holes expected in the next code. These and, in some cases, other steps are performed approximately in this order.

If any one in this series of steps fails, the machine stops and gives an alarm. The stoppage may be due to equipment failure (the most frequent being dirt between contact points of a relay, which prevents a circuit from being closed), an extra hole in a tape caused by long wear, an operator's error in perforating an improper code in a tape, or the like.

The system of checks and safeguards just described does not apply only to the reading of numbers or orders from tape but is typical of all operations of the machine. For instance, checks are performed at the end of each addition, and at the end of each of the partial additions of which multiplication is composed, to make sure that the proper number of relays is operated. Before any number is transferred into a register, a "down check" is made to insure that all relays of the register are in their released state. Before a number is taken out of a register, an "up check" insures that at least one binary and one quinary relay for each digit are operated. A permanent check is kept against "over-registration" in all registers, so that the machine is stopped instantly when more than two relays come up in the same digit at any time. Down checks are taken before registering an order in the routine control, table control, etc. These and many other checks have the common feature of stopping the machine instantly when a failure occurs. If the machine were allowed to proceed, wrong answers might result; by stopping it, not only are wrong answers prevented from arising but the task of finding the source of the trouble is made enormously easier.

If a trouble is encountered, one of two things happens depending on the setting of a key. One setting, designed for daytime operation in the presence of operating personnel, causes the machine to stop and sound an alarm. The other setting, designed for unattended operation, causes it to act as if a section of the problem had been finished: the control is returned from the routine tape to the problem tape, and the latter is moved to the beginning of the next (minor or major) section. If no more sections are provided on the problem tape, the computer is disconnected from the position and connected to some other loaded position. This arrangement makes it possible to obtain the fullest use of machine time during unattended hours.

c. Locating and Remedying Machine Trouble.—Each computer contains a panel of about 400 lamps which indicate the progress of the problem and

the numbers stored in various places. When a machine stoppage occurs, a quick scanning of this control panel indicates the nature of the trouble and the computational step in which the trouble has occurred. Since each step involves only a relatively small number of relays, it is easy to find the first relay which has failed to come up or has come up through error. It remains to investigate the circuit which energizes that relay. This circuit is likely to be long. Nevertheless, if it is one of the more frequently used circuits, the maintenance man probably knows its course and can check systematically the several dozen contact points through which this circuit flows. If it is an unfamiliar circuit, he may have to consult the circuit diagrams. In the former case, locating and remedying the trouble is accomplished in a few minutes; in the latter case it may take as long as two or three hours. From past experience, the average time required is ten minutes.

As for the frequency of stoppages, the experience gained during the initial period after completion of the machines cannot be considered as typical. During this period, various unfavorable conditions led to failures with a frequency ranging up to ten or twenty per day. An extrapolation of the experience gained from the smaller relay-type machines of the Bell Telephone Laboratories leads to the expectation that the frequency of machine failures will eventually be at most one per day, and possibly as low as one or two per week.

The number of stoppages caused by operators—usually by improperly perforated tapes—is expected to be higher than that of machine failures. Depending on the amount of tape-making called for by various problems, the frequency of such errors is expected to vary somewhere between one and ten per day. These estimates are naturally tentative. Operator errors are usually easier to trace and to remedy than machine failures. Cases in which the machine is stopped for more than five minutes because of such an error should be infrequent.

d. Personnel Requirements. Coding.—A two-computer installation requires for full efficient operation the services of one mathematician in charge, one maintenance man, and approximately four operators. Depending on the types of problems handled and similar circumstances, it might be advisable to have a substitute maintenance man and some clerical help, as well as additional machine operators.

The duties of the machine operators include not only the physical operations such as pressing the start keys, loading tapes into the machine, or hand-perforating of tapes (in fact, these duties occupy only a minor fraction of their time), but mainly the composition of problems, i.e., the translation of the mathematical formulae which govern the problem into the codes which are read by the machine. Depending on their mathematical ability, the operators can assume a large part of the duties of the mathematician in charge, which consist in dealing with such mathematical questions as the selection of appropriate methods of numerical analysis, selection of limits of accuracy, control of the rounding errors occurring in a problem, and the like.

The coding of problems (i.e., the translation mentioned before) is a simple and straightforward process. It is hardly possible to give any estimate of the time required for it, since it is inevitably interwoven with the mathe-

mathematical preparation of the problem. If the desired course of the computation were exactly known in advance, the operator could code the problem as fast as he (or she) can write—about ten seconds per order, or less than an hour for most problems. But, in practice, the operator is concerned with numerous questions of digital accuracy and the accumulation of rounding errors, with efficient ways of storing intermediate results so as to minimize computing time, with choices between several possible numerical methods to achieve the same result, with scale factors occasionally needed to keep all numbers within the capacity of the machine, and with the proper arrangement of the output of the machine. Including all these elements, which constitute the mathematical preparation of the problem, the time required for preparing a problem has ranged from half a day to a month and may occasionally be even longer in the case of complicated problems. Certain features of the Bell Laboratories' machine, such as the floating decimal point, tend to facilitate the task of the operator or mathematician in comparison with other machines.

The training of operators is surprisingly easy. A person with mathematical experience equivalent to an M.A. degree, or a B.A. degree and several years of experience in computing, requires between one and three months of training to become fully conversant with the machine and competent in its operation, and perhaps another three months to become fast and efficient. The ease of training personnel is an important advantage of the machine.

On the other hand, the training of maintenance personnel for this machine is not easy. The circuits are complicated, and yet it is desirable that the maintenance men know most of them by heart. A period of from six to twelve months seems to be needed for fully adequate training.

e. Speed.—Enough has been said in the preceding pages to arrive at an appreciation of the speed or lack of speed of this machine. Of all the machines which have been developed in recent years, possibly with the exception of the Harvard "Mark I," this is the slowest.

The calculator will perform an addition in 0.3 second, a multiplication in 1 second, a division or square root in perhaps five seconds on the average (the exact computing time depends on the digits occurring in the operation). However, the routine control requires about two seconds to read the order for any of these operations. Thus the calculator remains idle part of the time, and the reading of orders becomes the controlling element in determining the time requirements of a problem.

Printing or perforating a number takes about three seconds. However, while the printer control carries out this operation, the routine can go ahead with the reading and execution of the next order. Thus, unless several printing or perforating orders follow each other (which can usually be avoided with a little care in programming), the computing time is not increased by the slowness of these operations.

"Hunting" for a section of the routine tape cannot be overlapped by reading of routine orders and therefore adds to the total computing time. By careful programming, the amount of hunting can usually be reduced to a point where it constitutes only a small fraction of the total computing time.

The speed of hunting is 2 inches per second, or about $\frac{1}{2}$ second for each order which is skipped in hunting.

Finding a desired location on a table tape is potentially the most time-consuming operation of the machine. It can be overlapped by reading of routine orders, computing, printing, or any other operation not involving the table register and table control. In composing a problem, every effort is made to give tape orders as early as possible, so as to allow ample time for the machine to hunt for the desired location on the table tape. In most of the problems which have so far been done on the machine, it has been possible to overlap tape hunting completely with other operations so that no loss in machine time resulted. There are, however, problems where this is not feasible and where a considerable part of the total machine time is spent in tape hunting. This condition could have been remedied to a large extent if each computer had been equipped with three storage tapes instead of only one.

With the exception of problems requiring unusually long hunts, then, the computing time is about two seconds per routine order. In the problems handled so far, a little more than half of all the routine orders referred to genuine arithmetic operations, and the remainder to tape reading, printing, number transfer, clearing of registers, discrimination, and other operations not usually considered as separate steps in computing. This statement "a little more than half" is an important characteristic of this machine; the proportion may be quite different on other machines. Furthermore, on some machines it may be necessary to insert arithmetic operations for the purpose of checking the results obtained; on this machine, because of its reliability, this is hardly ever done. Some machines, as explained in section 5a, require several orders for each arithmetic operation.

Thus, in general, the computing time needed for solving a problem on our machine can be estimated by allowing two seconds for each routine order, or about $3\frac{1}{2}$ to 4 seconds for each arithmetic operation. If we prefer to count only the number of multiplications in a problem (the number of divisions and square roots is usually negligible), we may make use of the fact that usually a little more than half of all arithmetic operations in a problem are multiplications. We then have to allow between 6 and 8 seconds for each multiplication.

In comparison with hand computing, the speed of this machine is about five times that of a human computer equipped with a good (fully automatic) desk machine. Since the machine can usually handle two problems simultaneously, its output, hour for hour, is equivalent to that of ten human computers. This ratio is further increased if the machine is left running at night. At this rate it will accomplish the work of thirty computers and will require a crew of about five (we do not count the mathematician in charge since his services would be needed to the same extent for supervising the work of the human computers). The comparison may be further shifted in favor of the machine by the fact that, while the machine loses some time because of stoppages, human computers lose a great deal more because of tiring and similar factors. Also, the work of human computers is far more subject to error than that of the machine; and finally, the machine delivers its output in printed form, which obviates the necessity of much copying, typewriting, and proofreading.

7. Problems for the Machine.

a. Types of Problems Handled.—The list of problems given here is not exhaustive. Rather, it contains a selection of problems which have been carried out or programmed and which are typical of what the machine is capable of doing.

(1) *Systems of linear equations.* Several methods of solution have been tried out on the machine. In all of them, elimination is used to arrive at a first approximation of the solution, and if this is not close enough (because of the cumulative effects of rounding errors), iterations are used to find better approximations. Systems up to 13 equations can be handled efficiently on a single computer. The computing time for the elimination process for 13 equations is about $3\frac{1}{2}$ hours. Division of matrices of the same order can be accomplished by a repetition of the elimination process, taking advantage of the fact that many steps in the computation are common to all repetitions and need not be carried out each time. If the order of the system of equations (or of the matrices) is greater than 13, it becomes necessary to treat the problem on the two computers combined. This entails an increase in computing time, so that 13 equations, solved as a two-computer problem, require about 8 hours of computing time. The time required for systems of higher order varies approximately as the cube of the order. This puts a practical limitation on the size of systems to be solved. A more stringent limitation is set by the seven-digit accuracy of the machine. It is believed that this will limit the process used, even if used iteratively, to about 20 or 30 unknowns.

(2) *Algebraic equations with complex coefficients.* A routine has been set up (but not yet tested) for such problems. The method used is that of "false position."

(3) *Systems of ordinary differential equations.* These have so far furnished the main portion of problems for the machine. Both PICARD's method and step-by-step methods have been tried, and the latter have so far been found more efficient. As an example, in a system of order five each step required about three minutes. This will, of course, vary greatly with the complexity of the system, the length of step, etc. The machine can be directed to change the length of step, as has been mentioned above (section 5e).

With regard to complexity of such problems, the machine can handle systems which are beyond the capacity of existing differential analyzers and of the ENIAC. In fact, there is no reason why systems of very high order could not be handled on this machine.

(4) *Partial differential equations.* Two problems of this kind have been treated so far. In both cases, the region in which the solution was sought was a semi-infinite strip in the plane. The first was a relatively simple equation, describing the magnetization of a thin layer under the influence of a known field at the boundaries of the layer. This known field varies in time but is, at any moment, constant over the boundary. The boundary conditions, and therefore the solution, are symmetric with respect to the central plane of the layer, so that the computation had to be carried out for only half of the layer. The numerical integration was carried out, using step-by-step procedure, for a grid containing two points in the direction of x (across half of the layer) and several hundred points in the direction of t (time). The computing time was about a day, or about one minute per grid

point. The problem was repeated with a grid of five points in the x direction and well over a thousand in the t direction, with a correspondingly longer computing time. The second problem consisted of a more complicated system of equations, and the grid used contained sixteen points in one direction and a very large number of points in the other. The computing time on this problem was about two minutes per point. It is believed that partial differential equations in three independent variables are within the capacity of the machine.

(5) *Computation of functions.* This has been done in a variety of cases. Quadratures, infinite series which are broken off after a finite number of terms, and recursive formulae are the usual methods. They have been applied to the computation of convolution integrals, of Fourier summations and power series, and of a table of the binomial distribution

$$B_x = \binom{n}{x} p^x (1-p)^{n-x}$$

and

$$S_x = \sum_{i=0}^x B_i$$

for $x = 0(1)n$; $p = .01(.01).50$; and $n = 50(5)100$.

(6) *The machine is useful for differencing and for interpolation;* in particular it has been used for third order (Lagrangean) interpolation with unequal intervals, a process which is very tedious by hand computing methods. Other fields of useful application include transformation of coordinates in space, such as occur in photographic surveying, and more involved transformations such as the determination of the location of a point when the sums of distances from certain fixed points are known.

b. Efficient Distribution of Problems Among Large Computing Machines.—We shall attempt to state the requirements which a problem must fulfill in order that its solution on this machine may be more efficient than on other large machines. Such statements will necessarily be vague and tentative.

A necessary condition in order that a problem may be handled efficiently on any large machine, in preference to computing by hand and with desk machines, is that the input of orders and numbers be small in comparison with the number of operations performed. Otherwise the time required to put the problem on the machine becomes too long. As far as numbers are concerned, this requirement means that each number introduced into the machine should be used repeatedly, or at least that intermediate results derived from the original inputs should be used to a large extent. Concerning orders, it may mean either that many problems of the same type (with the same input of orders) are to be solved, or that the problem contains a repetitive nucleus, as is the case in step-by-step methods of integration or in iterative or recursive processes.

The allocation of problems between the several large machines in existence is based on the capacities and time requirements of these machines. With respect to capacity, the usual limiting factor is the capacity to store

numbers; occasionally, the capacity to handle complicated instructions or the digital accuracy have to be considered. With respect to time requirements, we have to distinguish between (a) set-up time off the machine, (b) set-up time on the machine, and (c) computing time.

Our machine excels in storage capacity (especially if the storage tape is used), in the ability to handle complicated instructions, and in the fact that its set-up time "on the machine" is zero. That is to say, all of the preparation necessary for a problem can be done while the machine is doing something else. Its worst feature is the long computing time required. With respect to set-up time off the machine (the time required for coding and producing of the required tapes, punched cards, or the like, as well as handling of any auxiliary equipment separate from the machine, such as removable plugboards on IBM machines and the ENIAC or the tape processor on our machine), none of the existing or projected future machines seems to have a distinct advantage over any other.

The advantage of the Bell Laboratories' machine in requiring no set-up time on the machine is important in problems for which many solutions are desired intermittently; for instance, evaluation of experimental data which should be made as quickly as possible after the data have come in, or research problems which will be run a number of times with small modifications in the input, the changes to be made each time depending on the previous answers. Problems of this type will preferably be done on this machine unless they are so long that the computing time becomes prohibitive.

The next-shortest set-up time on the machine is found on the IBM relay multipliers, where setting up consists merely in inserting two previously wired plug boards and setting a few switches, and can be accomplished in less than ten minutes. These machines are several times faster than the Bell Laboratories' machine and can therefore be used to good advantage in cases where the computing time is an important consideration. However, they are severely limited in the complexity of problems which they can handle, somewhat limited in storage capacity, and greatly handicapped by lack of reliability. The Harvard "Mark I" and the Dahlgren "Mark II," have set-up times of less than an hour, combined with great flexibility and considerable storage capacity. Their computing speed is somewhat between that of the IBM relay multipliers and that of the Bell Laboratories' machine. The ENIAC, with a speed perhaps a thousand times greater than that of the Bell Laboratories' machine, has the longest set-up time, of the order of one day. It will be used preferably for problems which can stay on the machine for a long time without requiring any change in the set-up, that is, where the same computing routine is used on a varying numerical input many times in succession.

None of the projected future machines is planned with the automatic set-up feature of the Bell Laboratories' machine (nor are any of these machines contemplated for unattended night operation). However, the set-up times will be moderate, amounting perhaps to a few minutes in each case.

The Bell Laboratories' machine, in addition to being efficient for problems which require frequent setting up, is at present our only means for dealing with problems exceeding the storage capacity of other machines. It is also unexcelled in flexibility (i.e., the ability to handle complicated

routines), although the Dahlgren "Mark II" is approximately equal to it in this respect.

To sum up: This Bell Laboratories' machine and the ENIAC represent two extremes, with the other existing machines fitting in between. The ENIAC is adapted to very long problems, provided they are not too complicated and do not need too much storage capacity. The Bell Laboratories' machine will handle the most complicated problems, requiring considerable number storage, provided they are not too long. The ENIAC prefers continuous runs, the Bell Laboratories' machine does not mind "on-and-off" problems. None of the existing machines will handle problems which are long and require a great deal of number storage. Future machines are expected to fill this gap.

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¹ This machine was built and largely designed by the IBM.

RECENT MATHEMATICAL TABLES

476[A, B].—PETER BARLOW, *Tables des Carrés-Cubes, Racines Carrées, Racines Cubiques et Inverses de tous les nombres entiers de 1 jusqu'à 10.000*. Paris et Liège, Béranger, 1946, iv, 200 p. 10.7 × 17 cm. Bound in boards. 120 francs.

This is, apparently, a facsimile reproduction of one of the stereotyped editions of Barlow's tables edited by AUGUSTUS DE MORGAN, with headings of the columns in French instead of the English originals. See *MTAC*, v. 1, p. 16-17, 26, 100, 169, 356-357; v. 2, p. 85.

477[A, B, C].—EMILIO CAZZOLA, *Tavole Grafiche dei Logaritmi a 6 decimali con interpolazione ottica sulla 6^a cifra Mantisse dei Logaritmi e dei cologaritmi Immediate per numeri da 1 a 100.000 Antilogaritmi Immediati per mantisse da 00.000 a 99.999*. (Brevetto N. 415.030.) *Tavole Aritmetiche e Numeriche ad uso degli Ingegneri, Matematici, Fisici, Chimici, Geometri, Periti, ecc. nonché delle Scuole Medie Superiori (Licei ed Istituti tecnici)*. Milan, Hoepli, 1947, xix, 90 p. 18.5 × 26.1 cm. Stiff paper covers, 1000 lire.

The principal table of the volume (p. 1-63) gives $\log N, N = 1(1)100\ 000$ to 6D. For $N = 1(1)999$ the mantissae are given in the usual manner. But for $N > 999$, numbers of more than 4 digits are found to correspond to a certain mark or point on a vertical scale; opposite this mark on an adjacent mantissa scale the six-place mantissa may then be read off, the first four digits being given and the last two read off from the scale. Adjacent to each printed four-figure mantissa is given the corresponding cologarithm decimal part. Thus the author claims that in his 63-page table, 8 columns to the page, one has a 6D logarithmic and antilogarithmic table, without interpolation, for 5-figure numbers. In the sixth digit, however, there is, at times, decided uncertainty, to the extent, perhaps, of 4 units. PIERO CALDIROLA, professor of physics at the University of Pavia, praises this work, in an introductory note, as one originally conceived by the author. It may be the first six-place table of the kind, but as long ago as 1925, a five-place table of exactly this same type (but without cologarithms) was published in New York, Macmillan, by ADRIEN LACROIX & CHARLES L. RAGOT. Their work was entitled *A Graphic Table combining Logarithms and Anti-logarithms Giving directly without Interpolation the Logarithm to five places of all five-place numbers and the numbers to five places corresponding to all five-place logarithms, also a*