QUERIES

26. SOME CLOTHOID OR EULER SPIRAL TABLES.—The curve is defined by the equations $x = \int_0^s \sin(\frac{1}{2}t^2/a^2)dt$, $y = \int_0^s \cos(\frac{1}{2}t^2/a^2)dt$, and with asymptotic points at $(\frac{1}{2}a\pi^{\frac{1}{2}}, \frac{1}{2}a\pi^{\frac{1}{2}})$, $(-\frac{1}{2}a\pi^{\frac{1}{2}}, -\frac{1}{2}a\pi^{\frac{1}{2}})$. The intrinsic equation of the curve $Rs = a^2$, shows that the radius of curvature of any point of the curve is inversely proportional to the length of arc to that point. All of these results were found by Euler (1744, 1781) who imagined half of the curve with its infinite number of whorls, and hence the name Clothoid given by Cesàro (1886). Euler studied the curve in connection with the solution of a problem of an elastic spring. The similar problem of an elastic lamina was considered earlier (1694) by JACQUES BERNOULLI, but there is no indication that he had any conception of the real form of the curve. These integrals became of importance in optics after Fresnel's diffraction study (1818). Since Cornu plotted the curve accurately (1874) the spiral is sometimes called Cornu's spiral. I gave a summary of history and bibliography of the curve in *Amer. Math. Mo.*, v. 25, 1918, p. 276–282 (the name "Peters" here should be replaced by "Gilbert"). One of the first published papers of D. N. LEHMER was "Cornu's spiral as a transition curve," *California Jn. Technology*, v. 3, 1904, p. 71–82. There are here seven tables to enable the engineer to lay out the curve from a point with curvature zero up to that of an appropriate circular arc, then back through a Clothoid arc to another straight line. Among many engineering treatments of the Clothoid see A. L. HIGGINS, *The Transition Spiral and its Introduction to Railway Curves*, London, 1921; there are various tables p. 100–107.

If $a^{-2} = \pi$, the equation of the Clothoid may be written

$$x = S(u) = \tfrac{1}{2}\int_0^x J_{\frac{1}{4}}(t)dt, \qquad y = C(u) = \tfrac{1}{2}\int_0^x J_{-\frac{1}{4}}(t)dt$$

that is, Fresnel integrals, of which there are many tables.

In a book published in 1943 by the Herbert Wichmann-Verlag, Berlin-Grunewald I find the following book listed: SCHÜRBA, *Klothoiden-Absteckta-feln. Anleitung zu Entwurf, Berechnung und Absteckung*, 143 p., "grossoktav-format," 47 figures. Elsewhere the date of publication is given as 1942. Who was Schürba? Where may a copy of this book be consulted? Are there other books of this kind?

R. C. A.

QUERIES—REPLIES

34. AN ENGEL TABLE (Q6, v. 1, p. 131, QR7, v. 1, p. 170–171).—Some information has been already recorded regarding the following table of ERNST ENGEL: *Zehnstellige Tafeln der Sinus, Cosinus und Tangenten für die dezimale Teilung des Nonagesimalgrades. Generaldirektion des Grund-steuerkatasters (Österr. Triangulierungs- und Kalkulbureau)*. Vienna, Bundes-vermessungsamt, 1920, 95 p. 19.6 × 30 cm. Brown University has recently acquired a copy of this work, which seems to be a fact worth noting, since there does not appear to be any other copy in the United States; nor do I know of any copy in England.

R. C. A.