

not considered. For a polynomial equation, the HART-TRAVIS machine (see *MTAC*, v. 1, p. 350) is described and also T. C. FRY's isograph (see *MTAC*, v. 1, p. 167), with which only the names DIETZOLD-MERCNER are associated by the author.

The geometrical devices part of the book contains chapters on various instruments for the accurate graphing of functions in cartesian or polar coordinates, changing scale and various graphical transformations as well as one on the instruments for drawing conic sections and the more general curves like the cycloids and spirals. Instruments for measuring arc lengths are also given here. In the next part on Differentiators, optical methods for determining tangents are described.

The Integrator part contains a description of the various planimeters, integragraphs and integrometers. This part also contains a section on the differential analyzer. The precise procedure for the use of the PRYTZ planimeter is also given and also for certain related devices. The concluding part on the Harmonic Analyzers (p. 273-290) seems rather brief.

The reviewer feels that a German-English glossary of technical terms would be very helpful for reprints of this type.

The book is a valuable contribution to the literature. Its general plan is well conceived, and the various detailed descriptions and many of the photographs are excellent. It certainly should be available to those who own or operate one of the German machines, both to utilize the device with maximum effectiveness and in case of needed repair.

FRANCIS J. MURRAY

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15. S. VAJDA, "Shortcutting in multiplication on a calculating machine," *Math. Gazette*, v. 31, July 1947, p. 172-173.

16. HENRY G. WEISSENSTEIN, "Calculating machine furnishes shortcut method of computing P.I. of two lines," *Civil Engineering*, v. 17, Sept. 1947, p. 545. 21 × 28.6 cm.

"Computing the coordinates of the point of intersection of two lines is a rather tedious process if done the conventional way using the law of sines. With the help of a calculating machine considerable time can be saved using the method shown in this article."

NOTES

87. GERMAN ALTITUDE AND AZIMUTH TABLES.—DEUTSCHE SEEWARTE, (a) *Höhen und Azimute der Gestirne, deren Abweichung zwischen 30°S und 30°N liegt, für 50° Breite*. (b) *Höhen und Azimute der hellen Fixsterne bis zur dritte Grösse deren Abweichung grösser als 30°N ist, für 50° Breite*. Herausgegeben vom Reichs-Marine-Amt. Berlin, 1916, (a) xxiii, 377 p., (b) xii, 88 p. 20.5 × 30 cm.

(a) This volume is clearly one of a series, but neither the introduction nor preface gives any indication of the extent of that series. The preface of the particular volume under review is dated July, 1916, and that of the exactly similar volume for latitude 70°, September, 1917; there also exists a volume for latitude 55°, preface dated June 1916, which is printed by a photo-lithographic process from manuscript figures. Although the ms. is very good it cannot compare in legibility with printed figures; surprisingly the format is smaller than for the printed volumes, the overall size being 20.5 × 27.5 cm. The preface hints that the manuscript has been reproduced by photography to lessen the chance of errors occurring in the process of letterpress printing; presumably, experience rapidly led to placing legibility higher than freedom from error! In all the volumes users are begged to communicate errors to the compilers.

The main tables comprise the most extensive tables of altitude and azimuth for a given

latitude known to the writers; in the field of solutions of spherical triangles they clearly take a major place and deserve a detailed description. Altitudes are given to 0'.1 and azimuths to 0°.1—the usual precision required for marine navigation—but the unusual, and important, feature is that the interval in declination is 10' and that in hour angle 1^m of time or 15' of arc.

The tables are divided into two parts, the first (and larger) referring to declinations of the "same" name as the latitude and the second to declinations of "opposite" name. The principal argument is hour angle in time, and each part is divided into sections of one hour, easily found by a large marginal index, containing tabulations for declinations 0(10')29°50'. Each pair of facing pages contains the tabulations for two degrees of declination. The left-hand page is concerned exclusively with altitude; with vertical argument 0^m(1^m)59^m—note that the hourly values are not repeated—the altitude is tabulated for twelve values of the declination; no differences or variations are given. On the right of the right-hand page, the azimuth is tabulated for each minute of hour angle and for each half-degree of declination. Whereas for the altitude, there is no repetition, the azimuth is repeated for each even degree of declination; why this is done is by no means clear.

The really interesting feature is the double-entry interpolation tables for the altitude; these are tabulated on the left of the right-hand page and give directly the increment to be applied to the tabular altitude, for each minute of declination (horizontal argument) and for some convenient interval (usually 6°) of hour angle. Generally one such table, based on mean values of the variations, is given for each two degrees of declination and each 10^m of hour angle; near the meridian, when the variation of the difference for 1^m is large but the difference itself is small, tables are given more frequently using a larger interval of 10°, 20°, 30° or even 60°. In the volume for latitude 55°, there is little regularity in the interpolation tables; intervals of 20°, 10°, 5°, 12°, 6°, 4°, 3° are used without apparent plan, the 60° value always being included. The later volumes indicate much more care in the arrangement, the 60° value being omitted to allow of one interpolation table to correspond to 10^m of the main table.

Now the maximum variation of altitude for 1' of declination is 1'.0, on the meridian, and for 6° of hour angle is 1'.5 cos(latitude) on the prime vertical, which for these high latitudes never exceeds 1'.0. A value of the altitude formed by the addition of two direct entry quantities—one from the main table, and one from the interpolation table—will thus never be in error by much more than 0'.8. But this maximum error is erroneously given as 0'.4 in the volume for latitude 55° and 0'.5 in the others; perhaps this assumes that the declination is known only to the nearest minute. In any case, the erroneous statement is made that altitudes correct to 0'.1 can be obtained by mental interpolation in the interpolation tables; this ignores several factors, chief among them being the fact that mean differences are used. Errors up to at least 0'.3 can arise from this cause, especially near the meridian.

Nevertheless, the provision of double entry interpolation tables of this form is an excellent principle and one that should be developed for tables of this nature.

In the case of the azimuth, a small table of nine entries, for 0', 10', 20' of declination and for 0°, 20°, 40° of hour angle, suffices to give corrections to the nearest 0°.1.

Generally, tabulations are given for all altitudes of 2° or greater; the squaring off of pages or half pages involves giving negative altitudes and these are given as low as 3° below the horizon. A thick zigzag rule separates positive from negative values, and signs are also given on all pages where negative values occur.

Fundamental values of the altitude to 0'.01 were computed by the Deutsche Seewarte, using seven-figure logarithms, for every 4^m of hour angle and every degree of declination; these values were then subtabulated, using second differences. The statement that the tabular values of the altitude are correct to 0'.05 is thus optimistic. Comparison with H.O.214 shows that about half the comparable entries differ in the end-figure, and independent computation confirms the accuracy of the H.O.214 figures in spite of their known unreliability (see, for instance, C. H. SMILEY's analysis in MTE 93). A casual examination has, however, failed to find many gross errors, though about one value in every ten pages has already been

corrected in ink in the particular copy under review, and one column of values is wrongly printed.

The general arrangement of the tables is adequate, though it could undoubtedly have been improved by combining the tabulations for altitude and azimuth. The lay-out is far from perfect, the chief criticism being the use of characterless modern face figures, which though large are very difficult to distinguish from each other. The use of rules instead of spaces for horizontal divisions further adds to the crowded appearance of the page. It would be easy to get 60 lines of smaller and well spaced type in the $8\frac{1}{2}$ inches used here, with far greater legibility. In other respects the books are well printed and bound.

Several auxiliary tables are given, comprising the usual corrections to observed altitude for refraction, dip, semidiameter and parallax, as well as mean places of stars with declinations between $\pm 30^\circ$. It is noted that the Sun's semi-diameter is assumed constant at $16'.0$.

H.O.214 takes 24 pages for each degree of latitude; the present volume takes 377 pages. Tabulation at a small interval of latitude would clearly be impossible. The explanation lies in the fact that the tables are designed for use with long intercepts plotted on a stereographic chart; on such a chart all circles (great or small) on the Earth are circles. The centre of the projection is used as an assumed position. The position line determined by an observation of altitude will be tangential to the straight line drawn perpendicular to the intercept (normally regarded as the position line itself) and concave to the sub-stellar point; the departure from the straight line is a simple function of the radius of the circle, i.e. of altitude, and distance from the intercept, which is known to sufficient accuracy by the D.R. position. An approximation to the position line, in the vicinity of the D.R. position, will then be the tangent to the position line at that point; this is constructed by simple geometry on the chart. Each volume contains the table giving the deflection from the straight line—from which table, in fact, it would be easy to draw the circular arc. Specimen charts are shown to be available for latitudes N.45°, N.50°, N.55°, and N.70° and for multiples of $7\frac{1}{2}^\circ$ in longitude. Each chart caters for an area roughly 11° square, so it would seem unnecessary to have them, or the volumes of tables, more frequently than every 10° in latitude. All the examples work from the centre of one or other of these charts and, accordingly, interpolate for hour angle; the scale of the charts is small ($1' = 1$ mm. or $\frac{1}{2}$ mm.), and the examples use $1'$ as the working unit for everything except the calculated altitude from the tables. The most amazing feature of these elaborate tables is that the tabular accuracy of $0'.1$ is not used; carefully designed and used tables to the nearest minute would give adequate accuracy with the declination and observed altitude to the nearest minute only. It would also have been possible to use a stereographic graticule independent of longitude and thus to avoid interpolation for hour angle; but this would involve transfer of the position line to another working chart.

As navigational tables, these can hardly have been very successful; as solutions of a spherical triangle they quite definitely have an interest.

(b) This volume is a companion volume to (a); a similar volume (20.5×27.5 cm.) for latitude 55° exists and is photographically reproduced from manuscript. No knowledge is available of other volumes.

Altitudes, to $0'.1$, and azimuths, to $0^\circ.1$, are tabulated for 22 stars (including *Polaris*), with declinations greater than N.30°, for every minute of hour angle while above the horizon and not within 20° of the zenith; considerations of pagination give rise to altitude greater than 70° and to negative altitudes.

Each star is allotted four pages, each page containing 3 hours of hour angle. For each minute of each hour are tabulated the altitude and azimuth corresponding to the mean declination for 1917.5; for each 10^m of hour angle an interpolation table, based on mean differences, gives the increments to be applied to altitude and azimuth for multiples of 6^s of hour angle. At the foot of each column (i.e. for each hour) are given, for 1922.5, 1927.5 and 1932.5, the corrections to be applied to altitude and azimuth for precession in declination.

The tables are printed in the same style as the main volume and are intended for use in the same manner. The values of the altitude were computed for every four minutes using

six-figure logarithm tables and six-figure natural sines; the azimuths were interpolated (where possible) from "den amerikanischen Azimuttafeln für jede zehnte Minute des Stundenwinkels" (H.O.120), but otherwise calculated.

The most remarkable feature of the tables is the use of 0'.1 as unit of altitude and thereafter the discarding (in the examples) of the advantages of this accuracy; this is borne out by the sketchy method of applying corrections for precession. It would have been simple to expand the correction table to give complete corrections with argument declination of star.

J. B. PARKER & D. H. SADLER

88. HARRY BATEMAN (29 May 1882–21 January 1946).—Harry was a table-maker and a most learned and inspiring guide in the tabular field. Our brief obituary notice was in *MTAC*, v. 2, p. 77 (correction p. 375). This was followed by a tribute from E. T. BELL in *Quarterly of Applied Math.*, v. 4, 1946, p. 105–106, and a "List of Publications of Harry Bateman" by his adopted daughter JOAN MARGARET, p. 106–111, 202 titles, 1902–1946. Eight of these titles refer to problems originally published in the *Educational Times*, London, but the references given are not to this periodical but to volumes of the second series of *Mathematical Questions and Solutions from 'The Educational Times,'* often referred to by *E.T.R.* (Educ. Times Reprint). Such an error is excusable in this case, but it is hardly to be condoned when copied by the contributor of the memoir for the records of the R. Soc. London. Since there are many omissions in Miss Bateman's list (and the other list) of such problems, it may be well to put them on record. First of all, for the following 7 problems in the *Educ. Times* no solutions were ever given; hence these did not appear in *E.T.R.*: 14923 (1901), 15057 and 15255 (1902), 15550 and 15558 (1904), 16239 (1907), and 17104 (1911). Then there were the following items in *E.T.R.*, s. 2, 1902–1912, which Miss B. does not list: 10728, v. 1, p. 108, sol.; 14873, v. 1, p. 85, sol.; 14900, v. 1, p. 117, prop., sol.; 15042, v. 2, p. 77, prop.; 14961, v. 3, p. 62, prop.; 15097, v. 3, p. 28, prop.; 15158, v. 3, p. 120, prop.; 15182, v. 3, p. 108, sol.; 15184, v. 4, p. 38, prop., sol.; 15294, v. 4, p. 94 and v. 17, p. 82, prop.; 15418, v. 6, p. 56, sol.; 15440, v. 6, p. 68, prop. (Miss B. and Dr. Erdélyi incorrectly refer to v. 5 for this); 9388, v. 7, p. 17–18, sol.; 15896, v. 10, p. 66–67, prop., sol.; 15997, v. 11, p. 53–54, prop., note, and v. 15, p. 72, prop.; 16112, v. 12, p. 34, prop., note; 16090, v. 12, p. 97, prop., sol.; 16304, v. 14, p. 48, prop.; 15997, v. 15, p. 72, prop.; 16215, v. 19, p. 51, prop.; 15388, v. 19, p. 54, prop.; 16979, v. 20, p. 34–35, prop., sol., p. 79, prop.; 17119, v. 22, p. 66–67, prop., sol.

In *London Math. Soc., Jn.*, Oct. 1946, issued Aug. 1947, v. 21, p. 300–310, is a memoir by A. ERDÉLYI. A more elaborate memoir—by far the best to the date of its publication—by Erdélyi, with a bibliography of 196 titles (8 *E.T.R.* titles being combined together as no. 1), and a splendid portrait, are given in *R. Soc. London, Obituary Notices of Fellows*, v. 5, Feb. 1947, p. 591–618. Apart from slips and omissions to which we have referred above, our misprint of the name STEINITZ is also copied. There is a brief tribute to Bateman in *Academia Nacional de Ciencias Exactas, Físicas y Naturales de Lima, Peru, Actas*, v. 10, fasc. 1–2, 1947, p. 99. He was elected a Corresponding Member of this Academia, Dec. 28, 1943. In *Amer. Math. Soc., Bull.*, v. 54, Jan. 1948, p. 88–103, Professor F. D. MURNAGHAN, of the Johns Hopkins University, has an interesting memoir, accompanied by a bibli-

ography substantially that of Miss Bateman, with all of its errors. Except for a portrait Professor Murnaghan has informed me that this is not very different from what he is to contribute to the National Academy of Sciences, *Biographical Memoirs*.

R. C. A.

89. THE MAGNITUDE OF HIGHER TERMS OF THE LUCASIAN SEQUENCE 4, 14, 194, . . .—The successive members of this series are connected by the defining relation $s_k = s_{k-1}^2 - 2$. The numbers of digits involved respectively in each of the first ten values of s_k are 1, 2, 3, 5, 10, 19, 37, 74, 147 and 293. While using this sequence in testing the prime or composite character of certain numbers of the form $2^p - 1$, where p is an odd prime, the writer became more and more impressed with the rapid growth in size of s_k as k increases, until finally he yielded to the temptation of calculating the leading figures and more especially the total number of digits in s_{226} and in s_{388} . The first of these two terms was chosen because M_{227} had been investigated by the author, and the second because of its close approach to the power of 2 for which 2^{p-1} as multiplier would move the decimal point in $\log s_{10}$ more than 120 places to the right, that is, beyond the computed approximation to $\log s_{10}$ (*vide infra*). [For $p \leq 257$, $2^p - 1 = M_p$ is called a Mersenne number.¹ Regardless of the limit $p = 257$, M_p is prime when, and only when, $s_{p-1} \equiv 0 \pmod{M_p}$. 227 and 389 are prime.] The details of this investigation will now be presented because they may satisfy the scientific curiosity of other enthusiasts in the same field.

In order to diminish the influence of the subtractive 2 in $s_k = s_{k-1}^2 - 2$ it was convenient to commence with the value of $s_{10}(=a)$. Then the leading terms of the expansion of s_k are found to be

$$(1) \quad s_k = a^b \{ 1 - (2^{k-10})/a^2 + \dots \}, \quad b = 2^{k-10}.$$

It is easy to show that the second term between the braces plays no part in the subsequent calculations because of the inferior degree of approximation set by the arbitrarily chosen limit of 120 decimal places in $\log a$.

In order to compute a^c , $c = 2^{216}$, and a^d , $d = 2^{378}$, it was necessary to find $\ln a$ and convenient to convert to $\log a$. Of the 293 figures in a only the following were used, namely 68 72968 24066 44277 23883 74862 31747 53092 42471 54108 64667 17521 92618 58308 84874 05790 95796 47328 83069 10256 10434 36779 66393 55951 72042. The natural logarithm of this number was obtained by applying the method of initial factoring followed by the addition of negative radix logarithms as explained in detail in the author's book entitled: *Original Tables to 137 Decimal Places of Natural Logarithms of the Form $1 \pm n \cdot 10^{-p}$, . . .*. There were 53 factors of this form in addition to the preparatory factors 10^{-292} and $\frac{1}{6}$. The final factor " $1 \pm c \cdot 10^{-q}$ " had 61 zeros following 1 and preceding +19600 96931 31584 47643 10463 95099 12432 46152 37049 29318 80748 28301. Finally $\ln a = \ln s_{10}$ equalled 674.28244 32255 06154 81602 37298 21679 84334 18147 69391 36843 76414 57795 54327 56149 33219 31575 00001 34886 45211 31809 10303 52158 97494 11743, the increase in the total number of figures presented being due to the addition of $292 \ln 10$. Multiplication by the modulus ($\log e$) then gave $\log s_{10} = 292.83714 43370 80010 66463 76818 65779 91989 91616 19717 85767 99035 37433 53371 72561 53630 91121 52243 44309 03398 12340 09801 71356 22384 20013$.

The first one of the desired results was obtained by multiplying the last number by 2^{216} , that is by 1 05312 29166 85571 86697 91802 76836 70432 31889 50954 00549 11125 43109 77536. Hence $c \log s_{10} = \log s_{226} = 308$ 39350 75581 39495 52047 79655 72677 97216 29131 52823 21013 64722 49007 38462.82923 38824 55508 95824 25948 08561 34247 34163 42090 19894 95461 \dots . The preceding characteristic (increased by one) shows clearly the enormous value of the 226th term of the Lucasian sequence 4, 14, 194, \dots . For present purposes advantage will not be taken of the knowledge of all of the first 55 digits of the mantissa. Suffice it to conclude that $s_{226} = (10^{0.82923388\dots}) \times 10^{308\dots462} = (6.74891\dots) \times 10^{308\dots462}$ and that very roughly the number of figures in s_{226} is 3×10^{67} .

Similarly for the case of s_{388} . The multiplier of $\log s_{10}$ was 2^{378} in its expanded form, that is 6156 56346 81866 37376 91860 00156 47439 65704 37092 61010 22604 18669 20844 41339 40267 96439 15803 34791 02325 76806 88760 35623 48544. Then $\log s_{388} = 2^{378} \log s_{10} = 18$ 02870 46495 37642 27934 36482 96937 59970 26529 12244 19199 46086 61354 33080 80671 26804 26804 16844 29101 89242 85304 79752 21848 58395.65941 3. Therefore $s_{388} = (10^{0.659413\dots}) \times 10^{180\dots395} = (4.56470\dots) \times 10^{180\dots395}$. For s_{388} the number of figures is roughly 1.8×10^{116} .

Obviously $\log s_{10}$ may be multiplied by 2^{p-11} in order to obtain two or more leading figures of s_{p-1} which correspond to the 74 prime numbers beginning with 13 and ending with 401.

If the value of s_{388} were written out in full as my ruled paper regulates, the length of the strip would be about

$$4.6512776 \times 10^{(1.80287\dots)10^{116}-19} \text{ parsecs.}$$

A professional astronomer has recently assured me that the paltry number 2 billion parsecs can be set safely as the present superior limit of observable celestial objects.

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¹ See *MTAC*, v. 1, p. 333, 404; v. 2, p. 94, 341.

90. MATHEMATICAL TABLE MAKERS.—*Mathematical Table Makers. Portraits, Paintings, Busts, Monuments, Bio-Bibliographical Notes.* By R. C. ARCHIBALD. (*The Scripta Mathematica Studies*, no. 3.) New York 33, Yeshiva University, Amsterdam Ave. and 186th St., 1948, vi, 82 p., 20 plates. 16.6 \times 24.7 cm. Cloth \$2.00.

This little book is a thorough revision, rearrangement, and considerable enlargement, with new illustrations, of two articles which appeared in *Scripta Mathematica* in 1946. The following 53 Table Makers are considered (a star indicating an accompanying portrait): *AIREY, ANDING, BABBAGE, BAUSCHINGER, BECKER, BESSEL, BIERENS DE HAAN, BORDA, BROWN (E. W.), BÜRGI, BURRAU, COHN, *COMRIE, *CUNNINGHAM, DASE, *DAVIS, *DICKSON, *DWIGHT, GLAISHER (J.), *GLAISHER (J. W. L.), HOËNE-WROŃSKI, HOPPE, HUTTON, JACOBI, *KEPLER, *KRAÏTCHIK, LALANDE, *LEGENDRE, *LEHMER (D. H.), *LEHMER (D. N.), LODGE, LOHSE, LOMMEL, *LOWAN, MARKOV, MARTIN, *MILLER, *NAPIER, NIELSEN, *PEARSON, PEIRCE, *PETERS, RIVARD, SANG, SHARP, SHEPPARD, STEVIN, STIELTJES, *TALLQVIST, *THOMPSON, TURNER, *UHLER, VIËTE. For each individual there are biographical notes followed by information under three headings: P (sources for a portrait,

painting, etc.) or **PB** (biographical material containing **P**); **B** (references for biographical data); and **T** (list of the individual's tables).

The portraits are distributed in the book as 5 groups of four plates. The portrait of Peters is with Comrie and the astronomer Kruse. The most extensive discussion of **P** is that for Kepler (5 p.), so many of whose alleged published portraits and busts are not of Kepler at all. Of Borda, Bürgi (2), Kepler (2) and Stevin public monuments are listed, as also are medals struck in honor of Kepler and Stevin.

Under **T** the most extensive title list (53) is for Cunningham; Airey (49) and J. W. L. Glaisher (48) come next.

According to countries of birth for the Table Makers the distribution is as follows: Australia (1), Belgium (1), Denmark (2), England (12), Finland (1), France (5), Germany (12), Holland (2), India (1), New Zealand (1), Poland (1), Roumania (1), Russia (2), Scotland (2), Switzerland (1), U. S. A. (8).

By his will Cunningham left to the London Mathematical Society: (i) £1000 for the improvement of the method of factorization of large numbers; (ii) £2000 for the publication of his unpublished mathematical works, and the completion and publication of his mathematical manuscripts; (iii) his library of mathematical books. Of the residue of his estate he left one-twelfth [about £3000] to the London Mathematical Society, and one-twelfth to the British Association, Mathematical Subsection, for preparing new mathematical tables in the theory of numbers. (*Times*, London, May 12, 1928, p. 10; *BAAS, Report 1930*, p. 251-252.)

The tables of D. N. LEHMER listed in Q26 were inadvertently omitted.

91. PORTRAITS OF TABLE MAKERS.—In *Nature*, v. 160, 22 Nov. 1947, p. 721-722, is a report by JAMES T. KENDALL of an "International Congress for Technical Education" held at Darmstadt, Germany, July 31-Aug. 9. Among the 50 foreign visitors were 15 from Great Britain, and among 200 papers read was one by L. J. COMRIE on "Calculating machines and mathematical tables." These papers are later to be published by the Technische Hochschule in Darmstadt. In illustrated reports of the Congress in *Darmstädter Echo*, 2 August 1947, is a reproduction of a photograph of Prof. FRITZ EMDE (b. 13 July 1873) and Dr. Comrie, seated at a table in a restaurant. In a letter dated 26 Aug. 1947, to Dr. Comrie, Prof. Emde stated (1) that Teubner had just received publishing license from the Russians for reprinting the fourth edition of the Jahnke & Emde work, and for a new edition of Emde's Elementary Functions destroyed by air raids in 1944; and (2) that the Russians had ordered resumption of the former bimonthly, *Zeitschrift für angewandte Mathematik und Mechanik*, as a monthly publication.

In *Simon Stevin, Wis- en Natuurkundig Tijdschrift*, v. 25, no. 1, 1946, there is a large folding frontispiece of *Simon Stevin*, a reproduction of a painting, by an unknown artist, in the University Library, Leyden. This periodical is a new one replacing three earlier publications: (a) *Christiaan Huygens* (v. 1-24, 1921-1946), (b) *Wis- en Natuurkundig Tijdschrift* (v. 1-12, 1921-1945, perhaps later numbers); (c) *Mathematica B, Tijdschrift voor allen die de Hoogere Wiskunde beoefenen*, v. 1-13 (1932-1946). This new journal is not the first one so named; more than 40 years earlier appeared *Le "Simon Stevin," Journal des Candidats aux Écoles Spéciales*, Brussels, edited by J. Stevens, v. 1-2, 1905-06, paged continuously, 1-400; a mimeograph print with printed covers.

There are portraits of PAFNUTIĬ L'VOVICH CHEBYSHEV (1821-1894) in *Polnoe Sobranie Sochineniĭ P. L. Chebysheva*, v. 1, 1944; and in *Oeuvres de P. L. Tchebychef*, 2 v. 1899-1907, 3 portraits; see *MTAC*, v. 1, p. 440.

There is a portrait of MIKHAIL BORISOVICH OSTROGRADSKIÏ (1801–1861) in *Les Mathématiques dans les Publications de l'Académie des Sciences 1728–1935. Répertoire Bibliographique*. Moscow, 1936, p. 64; see *MTAC*, v. 1, p. 440; v. 2, p. 252–253.

Of J. H. Lambert there are portraits in his: (a) *Schriften zur Perspektive, herausg. u. eingeleitet von MAX STECK*, Berlin, 1943, Plate I (portrait), Lambert Monument in Mülhausen, Alsace (Plate III), Portrait relief on Monument (Plate IV); and (b) *Mathematische Werke*, v. 1, 1946 (see *MTAC*, v. 2, p. 339).

There are portraits of W. J. ECKERT, director of the department of pure science of the Watson Computing Laboratory, on p. 12 of *IBM Selective Sequence Electronic Calculator*, New York, 1948, and in *Business Machines*, v. 30, March 15, 1948, p. 3. Under his direction tables have been computed.

R. C. A.

92. PORTUGUESE NAVIGATION TABLE.—The table in question is ABEL FONTOURA DA COSTA & FRANCISCO PENTEADO, *Tábuas de Altura e Azimute. Comemorando o 70° aniversário do Club Militar Naval*. (Supplement to *Anais do Club Militar Naval*, November 1936.) Lisbon, Imprensa da Armada, 1936, v. 21 p. 13.2 × 24.3 cm.

These tables are comparable in size and content to those of AGETON, *Dead Reckoning Altitude and Azimuth Table* (H.O.211, RMT 104). The astronomical triangle is divided into two right triangles by a perpendicular from the celestial object upon the meridian; the author acknowledges that his formulae are patterned after Ageton's, "com uma pequena modificação na decomposição do triângulo de posição, de que resultam regras mais simples a aplicar." Actually the changes in the formulae result from calling the length of the perpendicular from the celestial body upon the meridian $90^\circ - \psi$ instead of R , and the declination of the foot of the perpendicular $90^\circ - \gamma$ instead of K . The rules for the use of the tables appear to be essentially equivalent to those advocated by Ageton. It is hard to see how one can say that they are simpler.

The number of pages has been cut in half since tabular values of $10^6 \log \csc x$ and $10^6 \log \sec x$ (called here C and S ; A and B by Ageton) are given for each integral minute of arc rather than each half minute as in Ageton. It is the values of C which are given in heavy type; this is in contrast to Ageton's use of heavy type for B (or S). Values of C and S less than 665 are given to one decimal; this marks an improvement over Ageton's limit of 239.

No warning is given of the difficulties encountered when γ is near zero, Ageton's K near 90° . The larger interval of the argument adopted here makes this more serious.

Upon comparing 600 values of C with the corresponding values of A in Ageton, 34 places were found where the values differed by one unit in the last place. Using VEGA's 7-figure *Logarithmisch-trigonometrisches Handbuch*, FONTOURA & PENTEADO were shown to be correct in 30 of the cases, and the other four were indecisive. Upon examining these four cases again with an 8-figure table, it was found that Fontoura & Penteado were correct in all four cases. This would seem to indicate that the tabular values are definitely superior to those given in Ageton.

The usual auxiliary tables of refraction, dip of horizon, and parallax corrections are provided. The printing and paper are very good and an excellent thumb index is provided.

CHARLES H. SMILEY

EDITORIAL NOTE: There are two errors on p. 19: $86^\circ 10'$, S, for 111487, read 117487; $88^\circ 25'$, S, for 156861, read 155861.