were extended by the writer up to $n = 20$. Incidentally this also gives the numerical values of the coefficients of $C_n(x)$, range $[-1, 1]$, for n even, up to $n = 40$. For the coefficients of $C_n(x)$, range $[-1, 1]$, up to $n = 20$, see JONES, MILLER, CONN, & PANKHURST, R. Soc. Edinb., *Proc.*, v. 62A, p. 190 (*MTAC*, v. 2, p. 262). For x^n in terms of $C_m(x)$, for either the range $[-1, 1]$ or $[0, 1]$, only binomial coefficients are needed (readily available up to $n = 50$ in J. W. L. GLAISHER, *Mess. Math.*, v. 47, 1917, p. 97–107).

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AUTOMATIC COMPUTING MACHINERY

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TECHNICAL DEVELOPMENTS

The leading article of this issue of *MTAC*, "A Bell Telephone Laboratories' Computing Machine—II," by Dr. FRANZ L. ALT, is our current contribution under this heading.

DISCUSSIONS

Applications of Large-Scale High-Speed Computing Machines to Statistical Work

This discussion is essentially the reproduction of a talk given by Mr. J. L. MCPHERSON of the Census Bureau, Washington, D. C. See under NEWS.

The construction and use of high-speed computing machines is a comparatively new art. This art is old enough, however, to have developed some specialized meanings for certain words. The terms defined in the following short glossary are used in their technical sense in this discussion.

1. Memory:	A device into which code can be entered, and then abstracted at a later time.
2. Word:	A group of digits (usually the equivalent of 10 or 12 decimal digits) stored in coded form in a single memory position.
3. Memory Position:	One of N possible positions which a word may occupy in the memory.
4. Message:	A group of words, usually that group of words required to describe one statistical observation.
5. Instruction:	A word directing the machine to perform a particular operation.
6. Program:	A series of instructions directing a sequence of operations.

The very newness of the art makes it rather difficult to talk about statistical applications for high-speed computing machines. At present, there are but a few such machines in existence. To date, these machines have been used on problems not particularly representative of the statistician's work. Therefore, it must be kept in mind that these remarks refer to proposed machines. The features and characteristics discussed should be interpreted as performance specifications. Competent experts at the National Bureau of Standards are optimistic about the possibility of building machines to meet these specifications. However, a sound knowledge of the statistical applications of such machines must be based on actual use. Accurate and detailed descriptions of statistical applications of high-speed computing machines await three events. First, a machine must become available to a group engaged in large-scale statistical activity. Second, that group must become skillful in the use of the machine. Third, a comprehensive series of tests must be performed.

The words "high-speed" in the phrase "large-scale high-speed computing machine," when translated into operations per unit of time, are what interest most of us when we are introduced to these machines. It has become rather common in this field to use multiplication time as an over-all measure of the speed of operation of a machine. If we accept this convention, we cannot fail to agree that the machines are "high-speed" indeed when we are told that they will determine the product of two ten-digit decimal numbers in about two milli-seconds. These tremendously high speeds are possible because all calculations are accomplished by controlling the behavior of electrical pulses that can be generated and dispatched through the various circuits of the machine at rates of a million or more pulses per second.

The practice of expressing machine speed in terms of multiplication time originated with mathematicians, physicists, and engineers whose interest was in the development of equipment capable of providing numerical solutions according to complicated and involved formulae. From their experience with such formulae, they knew that in every 100 arithmetic operations there would be about 65 additions or subtractions, about 32 multiplications, and about three divisions. Since algebraic addition was most frequent but could be performed much faster than multiplication, while division, although slower than multiplication, occurred far less frequently, the selection of multiplication time as a measure of machine speed was a logical and intelligent choice.

Much of the statistician's work, however, differs significantly from the work of other scientists. Whereas the mathematician or physicist is confronted with a tremendous amount of arithmetic manipulation of a comparatively *small* volume of original data, frequently the statistician requires only a moderate amount of arithmetic manipulation of a tremendously *large* volume of original data.

This is a significant and important point. The largest of the high-speed machines conceived to date will have an internal memory of a few thousand words at most. The statistician will usually require several words for each message. It is, therefore, readily apparent that statistical studies involving thousands, or hundreds of thousands, or millions of cases will involve a number of "words" many times the capacity of the internal memory of one of these machines. The benefits of high-speed computation will be largely lost to the statistician unless there is associated with the machines some means whereby information can be delivered to the machine from the outside, and transmitted by the machine to the outside, at speeds comparable to, or at least in balance with, the internal speed of operation of the machine. In other words, the statistician demands high-speed input and output for a large and important area of his work. I believe the staff at the Bureau of the Census, in early discussions with designers and builders of these machines, were the first to call this extremely important requirement to their attention. It was pointed out that a convention that measured machine speeds in terms of the time required to perform a multiplication would be acceptable only if input and output speeds were in balance with the computing speeds of the machine.

The engineers were equal to this challenge. They have designed machines that will do a moderate amount of work on a very large amount of input data at speeds of the same order of magnitude as those that will be attained on problems involving a large amount of work on a small amount of input data. In other words, they have very nearly solved the problem of high-speed input and output.

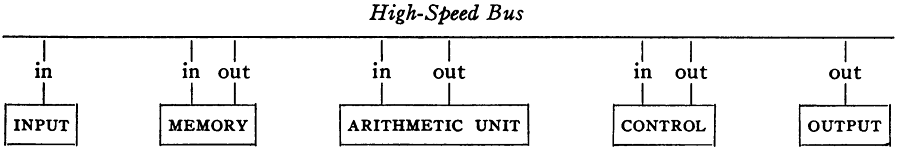
Metal wire or tape will be used on some, if not all, of the high-speed electronic machines as both an input and an output medium. Data will be recorded magnetically on these tapes and the machines will be able to read from, or write on, the tapes. To perform either of these operations, it will be necessary to move the tape physically. Since this mechanical movement of the tape will be involved, the speeds at which words can be read (or written) by the machine will be much less than the speeds at which the machine can transfer words internally. Here, however, the large internal memory of 1000 or more words comes to the rescue.

The machine will not read words from the tape one at a time as they are needed. Instead, it will read a block of words or several blocks of words from the tape, and store them in the internal high-speed memory to be examined and processed one word, or one message, at a time. Thus a portion of the internal memory will serve as a reservoir of unprocessed data. As the reservoir becomes depleted (but before it is exhausted), additional data can be read from the tape. This process of keeping a supply of data stored in the machine will be independent of the computing elements of the instrument. Thus, except for the time it takes to introduce data to the machine at the beginning of a problem, the input speed will be in balance with the internal speed of the machine. Since writing on a tape is simply the inverse of the reading process, the output speeds will also be in balance.

Now that we know the machines can accept vast quantities of data, perform certain arithmetic operations on those data, and deliver the results of those computations, all at very high rates of speed, we can explore the usefulness of the machines to statisticians.

Before investigating any specific types of statistical problems, however, I would like to discuss briefly a few of the properties and principles of operation of these machines.

The following diagram may help one to understand the logical organization of these machines.



It is important to note that the control has associated with it an "in gate" as well as an "out gate." That is, words can enter the control in the same manner that they can enter the memory or the arithmetic unit. *The instructions governing the routine of operations that the machine is to perform can be stored in the memory in exactly the same manner in which the numbers on which the machine is to operate are stored.* This means that the program for the solution of a problem can be recorded on magnetic tape and delivered to the machine via magnetic tape just as numbers are delivered to the machine. The length of the sequence of instructions can be flexible. If the problem involves only a short program of instructions, a comparatively short length of tape will be required. If a long routine of instructions is necessary, a longer length of tape will be required. Furthermore, the total number of instruction words may well exceed the capacity of the internal memory of the machine. The point is: not all of the instructions need be stored internally any more than all of the numbers need be stored internally. When additional instructions are required, they can be read into the machine from an instruction tape, just as additional numbers are read into the machine from a data tape when they are needed.

Another important characteristic of these machines is also clarified by this diagram. Since the instructions are stored in the memory along with numbers, the instruction words can be sent to the arithmetic unit just as number words can be sent to the arithmetic unit. *This means that the machine can generate or modify instructions to itself.*

It will not be necessary for the user of one of these machines to specify in his program a routine of instructions for each process each time that process occurs. All the programmer has to provide is instructions for a typical routine, and the machine will modify those instructions as the solution of the problem proceeds. For example, suppose we are obtaining a simple count of the number of whites and the number of nonwhites in a given population. Let us further suppose that each time we read from the data tape we provide the internal memory of the machine with the unclassified information for 50 persons. We, in effect, give the machine a sequence of instructions like this:

```
a Bring information for first 50 persons into the high-speed memory from the data tape
  and put it in memory positions 1 to 50.
b Bring information for the next 50 persons into the high-speed memory from the data
  tape and put it in memory positions 51 to 100.
c Examine race in memory position [1]. (The brackets indicate the number they
  bound will change.)
d Add "1" in the appropriate memory position (for example in memory position 201
  if white, 202 if nonwhite).
e Add "1" to the memory position specified in c.
f Compare memory position number in c with "51."
    1. If the memory position number in $c = 51$, bring information for the next 50
       persons into the high-speed memory from the data tape and put it in memory
       positions 1 to 50 (putting new data into memory positions 1 to 50 will erase the
       information previously stored in those positions). Then proceed to g.
    2. If the memory position number in $c \neq 51$, proceed to g.
g Compare memory position number in c with "101":
    1. If memory position number in $c = 101$,
        (a) bring information for next 50 persons into the high-speed memory from the
            data tape and put it in memory positions 51 to 100; then
        (b) subtract 100 from the memory position number in c; then
        (c) go back to c.
    2. If memory position number in $c \neq 101$, go back to c.
```

The instructions in *f* 1 and *g* 1 (a) are tape-reading instructions. As previously indicated, the machines will execute tape reading (or writing) instructions in parallel with the execution of other instructions. Thus the above sequence shows how a reserve of unprocessed data can be left in the internal memory as well as how the machine modifies its own instructions. There are many other ways in which the machine's ability to modify its own instructions is useful. This illustration is intended only to suggest the power of this feature.

The manner in which these machines read and write words should also be mentioned. In general, the process of "reading" a word does not erase that word. Only when a word is to be "written" in the space occupied by another word is the original word erased. This is true of the high-speed internal memory and usually of the magnetic tapes. (We can, if we wish, specify that after it is read a tape shall be erased.) Thus a tape containing the instructions for a recurring process (a monthly tabulation, for example) need be prepared only once. It can be read and reread as often as desired. On the other hand, in the course of rearranging data the machine may write intermediate results on tapes and leave them so written for only a short period, after which other results will be written on the same tapes.

Now let us examine some of the ways in which these machines may be useful to statisticians. First, and probably most obvious, is the reduction in the amount of preliminary rearrangement that will be required in the process of organizing and classifying data. Expressing this thought in the terminology of punch card tabulation, this means there will be a significant reduction of the volume of "sorting" necessary to prepare a tabulation. For example, assume a table with 20 columns and 25 lines—a 500-cell table. Such a table is not large, nor is it especially small. To prepare this table using punch cards, we would sort the cards into the groups representing the 25 lines and then run them through a tabulator to obtain the 20 columns on the other axis of the table. Now, suppose we have a high-speed computing machine with 1000 memory positions. We can make the very liberal allowance of 500 of these memory positions for the storage of instructions and unprocessed data and still have 500 memory positions in which to accumulate the 500 numbers we need for the table. In this case, it will be entirely unnecessary to rearrange the unprocessed data. We can leave it in whatever arrangement it appears on the data tape (random or otherwise), and in one passage of the tape through the machine we will obtain the desired table. Of course, for many tabulations some rearrangement of data will be necessary, but obviously the large number of memory positions in the proposed machines will effect substantial reductions in the number of "sort groups" that must be established to prepare a given tabulation.

Perhaps I can best illustrate other ways in which these machines will be useful by compounding the requirements associated with the 500-cell table. For the sake of illustration, assume that the 20-category classification refers to height and the 25-category classification refers to weight. We may also assume a machine with four tapes and their associated reading, writing, and driving mechanisms. Assume that the program is recorded on tape 1 and the data (say several thousand observations for each of which we have a height and a weight) are recorded on tape 2. Assume further that in addition to height and weight we also have the age of each individual recorded on tape 2. Age is not a variable in the table that we are preparing, but for future use it would be desirable to have the data arranged in order of age.

We can have on the first section of the program tape (i.e., tape 1) the program for arranging the data in order by age. This will involve transferring data between tapes 2, 3, and 4 until we have them in order of age, say, back again on tape 2.

Next, we can read from tape 1 (the instruction tape) the program for preparing the height-by-weight table. Let us suppose that the execution of these instructions terminates in recording, on tape 3, the 500 numbers corresponding to the 500 cells of the table.

We can now bring in some more instructions from tape 1. These might be a routine of orders for the computation of percent distributions. If so, they might direct the machine (a) to read the numbers from tape 3 (on which the 500 numbers in our table are recorded), (b) to accumulate the appropriate totals (for example, the total for each of the 20 columns), (c) to use those totals as bases and compute the desired percentages, and (d) to record the 500 percentages on tape 3.

Upon the completion of this routine, we can require still more instructions to be read from tape 1. These might be a program of orders directing the calculation of various statistical constants describing our distribution. These measures might include the mean height and mean weight, the variance of each variable, the covariance, the various correlation coefficients and the regression coefficients.

This example is representative of the type of work frequently done by statisticians. Facilities now available can provide all of the results described as obtainable by using a high-speed computing machine. How will the high-speed computing machines be superior to existing equipment? Obviously, we expect them to be much faster than anything now available to us. Suppose the work described in the illustration was for a study involving 100,000 observations. A high-speed computing machine should complete the job in a few hours at the most. Another and equally important contribution of these machines is their automatic nature. In the above illustration, the instructions on tape 1 directed the machine to perform a variety of operations. Data were rearranged, a table was prepared, percentages and other statistical measures were computed. All that was necessary to accomplish all of these objectives was the placing of tapes 1, 2, 3, and 4 on the machine and pressing a start button. Thereafter, the machine took over. This characterizes a major way in which these machines will help statisticians. Obviously, the incidence of human errors will be greatly reduced by a machine which eliminates human operators at so many stages of the tabulation and calculation processes.

BIBLIOGRAPHY Z–III

1. ANON., "Electronic computer assures solution of scientific problems," *Iron Age*, v. 157, 28 Feb. 1946, p. 132–134, 1 illustr. 22.8 × 29.2 cm.
2. ANON., "Electronic computer known as the ENIAC," *Mechanical Engineering*, v. 68, 1946, p. 560–561. 21.6 × 27.9 cm.
3. ANON., "Mathematics by robot," *Army Ordnance*, v. 30, 1946, p. 329–331, 4 illustrs. 21.6 × 27.9 cm.

A description of the ENIAC mentioning its purpose, size, phenomenal speed, operational characteristics, necessity for programming problems to be put on the machine, and future trends in electronic computing.

4. ANON., "War Department unveils 18,000-tube robot calculator," *Electronics*, v. 19, April 1946, p. 308–314, 3 illustrs. 20.3 × 29.8 cm.

An article dealing with ballistic applications, industrial uses, general details, operating procedure, arithmetic elements, memory elements, and control elements of the ENIAC.

5. O. CESAREO, "The relay interpolator," *Bell Laboratories' Record*, v. 24, 1946, p. 457–460, 1 illustration, 2 circuit diagrams. 25.4 × 17.8 cm.

Presented here is a brief description of the first all-relay digital computer, suggested by G. R. STIBITZ for use in gun director test equipment and developed by the Bell Telephone Laboratories for the National Defense Research Committee. The interpolator computations are based on the use of a bi-quinary adder and register circuits. These circuits are explained briefly. Block schematics and circuit diagrams showing inter-connection of contacts of the adder relays are included. The interpolator makes use of 493 U-type relays. Now in use at the Naval Research Laboratory, it was given to the Navy by the National Defense Research Committee.

6. JOSEPH JULEY, "The ballistic computer," *Bell Laboratories' Record*, v. 25, 1947, p. 5–9, 3 illustrations, 2 tables, 1 block layout. 25.4 × 17.8 cm.

The Ballistic Computer, like the Relay Interpolator (see above), was designed for the testing of gun directors. Director operators track a plan simulating a bombing run while director indicators and gun order indicators are photographed at regular intervals. With these data, transferred by key-punch operators onto teletype tapes, the computer calculates, for a successive series of instants, exactly where the plane and the shell would have been at the time of shell burst. The distance between the positions of shell and plane at each instant is the gun director error, given in errors in range, azimuth and angle of elevation.

The Ballistic Computer differs from the Relay Interpolator, its forerunner, in having more relays—1300 as compared to 493—and in having, in addition to register relays for adding, a set of multicontact relays used in multiplying and dividing. The computer can run continuously on one tape loading for 24 hours or longer. Useful checking circuits have been incorporated into the machine, making it quite feasible to run it unattended over long periods. The Ballistic Computer can accomplish, in five or six hours, work that five men, working steadily, could not complete in less than a week.

7. ANON., "The differential analyzer" [at Manchester University], *The Engineer*, London, v. 160, 1935, p. 56–58, 82–84, 12 illustrations. 24.3 × 36.8 cm.

This article describes the mechanical integrator, input table, torque amplifier, 2-stage torque amplifier, reversible torque amplifier, and frontlash unit.

8. D. R. HARTREE, "Differential analyzer," *Nature*, v. 135, 1935, p. 940–943, 2 illustrations. 18.4 × 27.3 cm.

This article discusses the construction, operation, and applications of the differential analyzer at Manchester University. It also treats the development of the torque amplifier by DR. VANNEVAR BUSH as then used at the Massachusetts Institute of Technology.

In earlier pages of *MTAC* there have been numerous references to Differential Analyzers: v. 1, p. 62–64, 96–97, 370, 430–431, 452–454 (Bibliography of 29 titles); v. 2, p. 55, 65, 89–91, 115–117, 150, 282, 293, 371. The first Differential Analyzer was described by the Russian A. KRYLOV in "Sur un intégrateur des équations différentielles ordinaires," 7 illustrations, Akad. N., S.S.S.R., *Bulletin*, s. 5, v. 20, Jan. 1904, p. 17–37.

9. Arthur W. Burks, Herman H. Goldstine, & John von Neumann, *Preliminary Discussion of the Logical Design of an Electronic Computing Instrument.* Second ed., part 1, v. 1, Princeton, N. J., Institute for Advanced Study, 1947, vi, 42 leaves. 21.6 × 27.9 cm. See MTAC, v. 3, p. 50–53, for a review of the first edition.

As the authors state in the preface, the arithmetic organ has been discussed in greater detail and the arithmetic processes treated more completely in this edition. In addition, certain sections of the report have been made somewhat more specific in the light of engineering advances in the authors' laboratory.

The treatment of the arithmetic operations on numbers expressed in binary form is more satisfying than the corresponding part of the first edition. A discussion of the choice of location of the binary point has been added. The arithmetic operations are carefully discussed in detail from the standpoint of adaptation to their execution by electronic organs. A few numerical examples provide useful illustrations for the reader. The treatment of the control organ is somewhat more closely integrated with the discussion of the arithmetic operations.

This second edition of the report is of interest in that it presents the most recent ideas of a competent group of designers of an automatically-sequenced, electronic digital computing machine. The format has been improved, affording more pleasant reading than the mimeographed first edition. Unfortunately, however, the report is marred by typographic errors which tend to confuse the reader. A list of errata would be a useful and welcome addition.

MDL

10. Moore School of Electrical Engineering, University of Pennsylvania, *Theory and Techniques for Design of Electronic Digital Computers.* Lectures delivered 8 July–31 August 1946. Two volumes, Philadelphia, The University of Pennsylvania, v. 1 (lectures 1–10) publ. 10 Sept. 1947, 161 leaves; v. 2 (lectures 11–21) publ. 1 Nov. 1947, 173 leaves. 21.5 × 27.7 cm. Offset printing of typescript on one side of each leaf.

These volumes were reviewed by Dr. C. H. Page & Mr. S. N. Alexander, both of the National Bureau of Standards, Washington, D. C. Mr. Alexander summarized and criticized those chapters of the report which are concerned with machine design. Dr. Page, on the other hand, has limited his review to those chapters of the report dealing with the mathematical phases of electronic digital computers.

Introductory Comment.—The first two reports contain twelve lectures on machine design considerations that range from the philosophy and economics of automatic computation to details about the organizational and technical features of the machines. In general, the reviewer has found most of the material to be considerably improved over the original form in which he heard the lectures delivered at the University of Pennsylvania. In several instances the subject matter appears either to have been augmented or brought up to date.

Lecture 1. *Introduction to the Course on Electronic Digital Computers* by G. R. Stibitz. This lecture provides an excellent background from which to view the present intensive activity being directed toward devising more powerful aids to computation. First, the motivations that have led men to devise such aids are reviewed. Out of this the predominantly economic justification that underlies the current programs is convincingly demonstrated. Some of the areas for future application of automatically sequenced digital computers are extrapolated from the limited existing experience in their use. However, the significant point is made that, as in other fields, we can expect a widening of the market in keeping with further reductions in the unit cost of computation. Certainly the salient feature of this lecture is the interesting manner in which the thread of economic justification is woven into the discussion of the computers.

Lecture 2. *The History of Computing Devices* by I. A. TRAVIS. This lecture gives a concise history of aids to computations and some of the circumstances that led to their invention and use. The present emphasis on the development of automatic digital machines is dated from 1938, with the prediction that this emphasis will continue during the next decade. An interesting chart is included, which enumerates what the lecturer considers to be the important dates in the growth of automatic computation.

Lecture 3. *Digital and Analogy Computing Machines* by J. W. MAUCHLY. The lecturer presents his thoughts regarding the appropriate fields of application for the newer digital machines as compared to the already well developed analogue devices. The analogue procedure for attaining numerical solutions is examined, and its characteristics and limitations are noted. Against this background the virtues of the digital procedure are pointed out. Paramount is the fact that the digital procedure offers a feasible attack on classes of problems not readily within the scope of analogue machines that have been devised thus far. The digital machines are further recommended for their inherently greater flexibility and their ability to attain greater accuracy whenever this accuracy is required. The lecturer then goes on to a discussion of the four steps into which he divides the over-all task of using a digital machine to obtain a solution: (1) coding, (2) set-up, (3) operations, (4) interpretation. The importance of balance among these steps is considered, particularly in relation to minimizing both the time and cost of obtaining a solution.

In closing, the lecturer covers in a very general manner the basic choice of the serial versus parallel organization of the machine itself, giving his views on the influence of this choice upon the speed, cost and reliability of the machine.

Lecture 8. *Digital Machine Functions* by A. H. BURKS. This lecture examines some of the factors that govern the advisability of having the digital computer always reduce the solution of each problem to first principles. The possibility of performing all the mathematical procedures in terms of four elementary logical operations is pointed out in a tantalizingly brief discussion. The editorial note on this topic, perhaps because of its excessive brevity, provides little clarification for a newcomer to such considerations. The lecturer then points out that practical considerations dictate the use of units at least capable of directly performing the arithmetic operations of addition and multiplication. The more elaborate operations such as finding roots, integration and sorting are better derived from the basic arithmetic operations through appropriate instructions to the machine. Most of the remaining text is devoted to methods of handling negative numbers and subtraction through the use of both the "nines" and "tens" complements. The special design considerations that are imposed on the adder and multiplier by each system of complements are also discussed. The concluding remarks point out how iterative procedures, based on the use of repeated addition and multiplication, can be employed to provide the operations of division and square root.

Lecture 9. *The Use of Function Tables with Computing Machines* by J. W. MAUCHLY. A rather detailed analysis of the general problem of referring to arbitrary functions is contained in this lecture. The complexity that arises for functions of several variables is brought out, and an elementary approach to this problem is cited. The relative merits of referring to the stored values of the functions in terms of a "selecting" or a "hunting" process are compared. Examples are given of arrangements where a combination of selecting and hunting is employed, together with a discussion showing the continuous transition that can occur between these two modes of reference to the function tables. Several examples are given of methods by which one can derive functional values between those tabulated within the machine. The discussion concludes with a few remarks on the importance of being able to provide extensive tabulations on the external memory of an EDVAC-type machine.

Lecture 10. *A Preview of a Digital Computing Machine* by J. P. ECKERT, JR. The design principles that are proposed for the EDVAC type of machine are discussed in the light of experience gained in the building and operation of the ENIAC. The major objections to the ENIAC are avoided in the EDVAC by (a) providing more memory, (b) separating the memory and arithmetic functions of the machine, (c) providing automatic setting up of the problem by the use of the internal control system, (d) use of a serial type of programming in order to simplify the planning of the problem, (e) the use of a more flexible type of input and output equipment such as the erasable magnetic recording device. The lecture concludes with a discussion of the functional arrangement of the EDVAC-type machine, some of the components that might possibly be used to accomplish the operations desired, and an example of a code by which the machine could be instructed to carry out the sequence of operations.

Lecture 11. *Elements of a Complete Computing System* by C. B. SHEPPARD. The summary that has been prepared from this lecture is a highly condensed, but nevertheless lucid, account of several devices for realizing the functions desired in the principal units of the computing machine. The machine is divided into four units, which are designated as the high-speed memory, the computer, the control, and the input-output mechanism. The operating principles for each device selected for discussion are briefly and clearly presented. The section devoted to the high-speed memory is expanded to provide a condensed, yet readable, technical description for several of these devices. Indeed, this summary contains much of the flavor of an unabridged dictionary of digital computing machine terms.

Lecture 13. *The Automatic Sequence Controlled Calculator*, and Lecture 14. *Electro-Mechanical Tables of the Elementary Functions* by H. H. AIKEN. The salient features of the electro-mechanical calculator in service at Harvard University are concisely described. The internal organization of the machine is covered, including the means by which the sequence mechanism and its punched control tape direct the solution of the problem. Reference is made to the *Manual of Operation* (1946, *MTAC*, v. 2, p. 185–187, 368), where this material is given in detail. This lecture closes with a discussion of the modifications in equipment and machine operation that appear advisable in the light of service experience.

The subject of elementary functions begins with the derivation of reciprocal powers through an iterative procedure using repeated addition and multiplication. Such operations as division and square root can then be derived without need for specialized equipment. Even though the Harvard machine does have a divider unit, the iterative procedure has been used to obtain extra quotient precision. The next topic covers the principles used in the elementary function units of the machine. These units provide values of the logarithm, exponential and sine functions. The lecturer not only considers these specialized units to be desirable in a machine of this type, but expresses the need for a $\tan^{-1}$ unit, which is being designed.

Even though the techniques used in this first automatically sequenced digital calculator have little in common with the electronic machines, familiarity with its internal organization is useful background material for the design of any digital machine. Indeed, the experience derived from planning problems for this computer has been a useful guide to several groups now developing electronic machines.

Lecture 15. *Types of Circuits—General* by J. P. ECKERT, JR. The general design considerations for electron tube circuits, as employed in digital computers, are introduced by comparison to the more familiar relay equivalents. The lecturer gives an appraisal of the relative merits of electron tubes versus relays on the basis of speed, actuating power, cost, reliability and life. This is followed by a discussion of those characteristics of relays and electron tubes that relate to operation in the synchronous and nonsynchronous type of machines. The remainder of the lecture discusses several basic circuits out of which the arithmetic and control functions of the computer can be compounded. The treatment of the amplitude-sensitive class of circuit is so sketchy that the unaided newcomer will sense little beyond their existence and the lecturer's disapproval of their use.

Lecture 16. *Switching and Coupling Circuits* by T. K. SHARPLESS. This lecture presents a few of the more conventional circuits and devices having two stable states that are of interest in digital computing. These are used to develop the design principles used in the ENIAC decade counters. The next main topic is a discussion of several techniques and devices for performing the distribution function needed for the control of the computing system. This includes the separation of a time sequence of pulses into individual pulses, each on a separate circuit, and the inverse operation of assembling a spatial distribution of pulses on separate circuits into a time sequence on a single circuit. The concluding topic is a qualitative discussion of coupling circuits for amplifiers that have been used in computer switching circuits. Emphasis is given to the departure from standard design permitted by the nonlinear character of the switching operations.

Lecture 20. *Reliability of Parts* by J. P. ECKERT, JR. This lecture gives an account of a number of the design choices made with respect to the ratings of parts that were used in the ENIAC. Much of it is then related to subsequent service experience. The surprisingly reliable service obtained from standard production types of electron tubes is of particular interest. (A recent summary of this experience is given in *Electronics*, v. 20, 1947, p. 116.) No doubt greater demands will be made on the electron tubes in future machines, but this initial experience is certainly encouraging. The quoted greater reliability of the electronic equipment in comparison to that of the standard punched card equipment associated with the ENIAC is a point of further significance to the future of this relatively new art.

Lecture 21. *Memory Devices* by C. B. SHEPPARD. After emphasizing the important role played by memory devices in the computer, the lecturer presents a table giving his appraisal of the relative merits of fourteen possible means of attaining useful memory. While the tabulation is an interesting one, it needs a fuller exposition of the sources and significance of the data than is provided. Next, some estimates are given regarding the minimum amounts of memory capacity that an electronic machine should contain. This is then divided into a high-speed main memory and a slow-speed auxiliary memory to conform to practical considerations imposed by available techniques. The case for magnetic wire and tape for the slow-speed memory is presented and is followed by a summary of the pertinent characteristics of these media. The remainder of the lecture is concerned with the high-speed memory problem. Although considerable attention is given to electrostatic types of memory tubes, the uninitiated reader may find the presentation somewhat confusing. Furthermore, he might obtain the erroneous impression that such tubes have emerged from the developmental laboratories. It is unfortunate that the subject of acoustic delay type of high-speed memory is disposed of by simply referring to Progress Reports on EDVAC of June 30, 1946. These reports have not been widely circulated, and the general reader is directed to a less complete account of this device given in *Electronics*, v. 20, Nov. 1947, p. 134.

The remaining nine lectures, on mathematical techniques involved in using digital machines, were as follows:

Lecture 4. *Computing Machines for Pure Mathematics* by D. H. LEHMER;
Lecture 5. *Some General Considerations in the Solution of Problems in Applied Mathematics*, by D. R. HARTREE;
Lectures 6, 7, 12, 17, 18. *Numerical Mathematical Methods*, 17 by A. W. BURKS, the other four by H. H. GOLDSTINE;
Lecture 10. *A Preview of a Digital Computing Machine* by J. P. ECKERT, JR.;
Lecture 19. *On the Accumulation of Errors in Numerical Integration of the ENIAC* by H. RADEMACHER.

Unfortunately, most of these were published in summary form only, reducing their utility to the mathematically minded engineer who is not familiar with the literature on computational methods.

The keynote is well expressed by Prof. HARTREE who made the point that developments in the design of mathematics for the machines must go hand in hand with the development of the machines for mathematics. Automatically sequenced machines, for example, are admirably adapted to iterative procedures, and the type of iterative algorithm most economically employed depends on the machine design, whether the machine employs a fixed time cycle or whether addition is performed more quickly than multiplication. Numerical procedures not useful on desk calculators, such as dividing by an iterative process involving only multiplication and addition, are often "natural" for machine operation. Variational and integral equation representations of problems may thus be more suitable than the customary procedures. Research in new computational methods is indicated, since most methods have been developed for use by human beings rather than by automatic machines.

Elementary processes of numerical quadrature and NEWTON-RAPHSON root evaluation are reviewed in several lectures. The iterative procedure for minimizing a function of several variables is derived in both algebraic and matrix notation, and the Steepest Descent and SOUTHWELL methods compared. The latter is simpler (and naturally more slowly convergent) for human use, but requires a machine to decide step-by-step which axial direction to follow in approaching the minimum of the function—a type of decision which may be costly to program into the machine. The minimization procedure is applied to a system of linear equations, and numerical examples given.

Ordinary differential equations with one-point boundary conditions can be integrated by an open-cycle process of successive low-order approximations applied separately to each subinterval of integration, or by single high-order approximations in each subinterval. The speed-accuracy compromise is different for human beings and machines, and may well differ among types of machines. The open-cycle method is discussed from the viewpoints of both the classical differential equation and the difference equation. Replacement of a single differential equation by a system of first order differential equations, and ultimately by a system of forward difference equations, is shown in an example to lead to an algorithm particularly adapted to machines.

The HEUN second order method of integrating differential equations, using an approximation corresponding to the trapezoidal rule in numerical quadrature, is analyzed for truncation error in the problem:

$$\frac{dx}{dt} = f(x, y), \qquad \frac{dy}{dt} = g(x, y).$$

The round-off errors in this problem are analyzed for the simpler first-order approximation method, and as in the case of numerical quadrature, are shown to depend on $(\Delta t)^{-1}$ in the maximal appraisal, and on $(\Delta t)^{-\frac{1}{2}}$ in the statistical appraisal. The statistical treatment assumes random individual round-off errors and so requires sufficiently large Δt for validity. This Δt is evaluated as

$$(\Delta t)^2 |x'''| > 10^{-k}, \qquad (\Delta t)^2 |y'''| > 10^{-k},$$

where k is the number of digits accommodated by the machine. The statement is then made that "This is a restriction on the fineness of the step of integration. Actual tests show that this is a conservative estimate. . . ."

The conclusion appears unwarranted, since the restriction on Δt was not derived as a restriction involving computational accuracy, but only as a restriction on the method used to analyze this accuracy. The reviewer feels that there is a "no-man's land" of interval sizes just below the quoted limit for random round-off, but that for intervals sufficiently smaller, a type of uniformity appears which again allows error analysis and perhaps a round-off procedure with reduced error.

NEWS

British Association for the Advancement of Science.—At the annual meeting held at Dundee from August 27 to September 3, 1947, the following papers were presented during the discussion of modern methods of computation:

"General survey of computational methods" by Dr. J. C. P. MILLER.

"Principles of programming on large-scale calculating machines" by J. H. WILKINSON.

"Relaxation methods" by L. FOX.

"An electronic differential analyzer" by J. B. JACKSON.

Prof. R. V. SOUTHWELL, Imperial College, London, announced that he expects soon to perfect a practical method of relaxation for functions of three variables.

Association for Computing Machinery (formerly Eastern Association for Computing Machinery).—The Executive Committee met on October 24, 1947, at Columbia University, New York (see *MTAC*, v. 3, p. 57). The Committee now consists of:

Dr. JOHN H. CURTISS (President, A.C.M.), National Bureau of Standards, Washington, D. C.

Dr. JOHN W. MAUCHLY (Vice-President, A.C.M.), Electronic Control Company, Philadelphia, Pa.

Mr. E. C. BERKELEY (Secretary, A.C.M.), Prudential Insurance Company of America, Newark, N. J.

Mr. ROBERT V. D. CAMPBELL (Treasurer, A.C.M.), Raytheon Manufacturing Co., Waltham, Mass.

Dr. FRANZ L. ALT, Ballistic Research Laboratories, Aberdeen, Md.

Mr. E. G. ANDREWS, Bell Telephone Laboratories, New York, N. Y.

Mr. PERRY CRAWFORD, Special Devices Division, Office of Naval Research, Sands Point, L. I., N. Y.

Dr. GEORGE B. DANTZIG, Office of the Air Comptroller, Air Force, Washington, D. C.

Dr. JAN A. RAJCHMAN, R.C.A. Laboratories, Princeton, N. J.

Dr. MINA REES, Office of Naval Research, Washington, D. C.

Prof. JOHN B. RUSSELL, Columbia University, New York, N. Y.

Prof. RICHARD TAYLOR, Massachusetts Institute of Technology, Cambridge, Mass.

Dr. CHARLES B. TOMPKINS, Engineering Research Associates, Washington, D. C.

The second meeting of the Association was held at the Ballistic Research Laboratories, Aberdeen Proving Grounds, Aberdeen, Md., on Thursday and Friday, December 11 and 12, at the invitation of Col. LESLIE E. SIMON.

The Ballistic Research Laboratories, probably the largest computing laboratory of the world, has a unique collection of large-scale computing equipment. This includes the ENIAC, the first, and so far the only, large-scale electronic digital calculator; the "Bell Relay Computer," a large automatic installation designed by the Bell Telephone Laboratories; a new sequence-controlled relay machine of medium size, built by the IBM Corporation; and a differential analyzer. Demonstrations of these machines were given Thursday afternoon and Saturday morning.

On Thursday the following papers were presented:

(1) "General principles of coding, with application to the ENIAC" by J. VON NEUMANN, Institute for Advanced Study, Princeton, N. J.

(2) "Adaptation of the ENIAC to von Neumann's coding technique" by R. F. CLIPPINGER, Aberdeen Proving Ground.

(3) "Optimum size of automatic computers," by G. R. STIBITZ, Burlington, Vt. see *MTAC*, v. 2, p. 362–364.

The program for the Friday meeting was as follows:

(4) "The UNIVAC" by J. W. MAUCHLY, Electronic Control Company, Philadelphia, Pa.

(5) "The machine of the Raytheon Company" by R. V. D. CAMPBELL, Raytheon Co., Waltham, Mass.

(6) "Operating characteristics of the Aberdeen machines" by F. L. ALT, Aberdeen Proving Ground.

(7) "Reduction of DOPPLER observations" by D. HOFFLEIT, Aberdeen Proving Ground.

(8) "Partial differential equations" by B. L. HICKS, J. H. LEVIN, M. LOTKIN, R. F. CLIPPINGER, J. V. HOLBERTON, Aberdeen Proving Ground.

(9) "Application of large-scale high-speed computing machines to statistical work" by J. L. MCPHERSON, Census Bureau, Washington, D. C.

The purpose of the Association is to advance the science, design, construction, and application of the new machinery for computing, reasoning, and other handling of information. The present membership comprises 246 individuals representing 80 organizations. Anyone interested in joining may become a member by sending his name, organization and address to the Secretary, Mr. EDMUND C. BERKELEY, Chief Research Consultant, the Prudential Insurance Company of America, Newark 1, N. J. Dues of $1.00 may be enclosed with this information or mailed directly to the Treasurer, Mr. ROBERT V. D. CAMPBELL, Raytheon Manufacturing Company, Waltham, Mass.

American Statistical Association.—"High-speed automatic computing machinery" was the discussion topic at a meeting of the local chapter of the American Statistical Association in Washington, D. C., on October 27, 1947, at 8:00 p.m. A survey of the national machine development program was presented by Dr. J. H. CURTISS, Chief of the National Applied Mathematics Laboratories, National Bureau of Standards. This was followed by a talk by Mr. J. L. MCPHERSON of the Census Bureau on "The application of high-speed automatic digital computing machinery to statistical tabulation" (see DISCUSSIONS).

On the evening of November 10, 1947, the chapter presented talks on "The solution of statistical problems for automatic computing machines." In keeping with this topic, Dr. E. W. CANNON, Chief of the Machine Development Laboratory of the National Bureau of Standards, discussed "Instruction codes for high-speed automatic computing machines," and Mrs. IDA RHODES, also of the Bureau, in a talk on "Programming problems for solution," presented the coding sequence to be used on the new machines for several problems of a statistical nature.

National Electronics Conference.—On Tuesday, November 4, 1947, a meeting on electronic computers was held at Chicago, Illinois. Dr. J. W. MAUCHLY of the Electronic Control Company, Philadelphia, Pa., discussed computers from the block diagram, or functional, standpoint. This was followed by a talk by Mr. J. M. COOMBS of Engineering Research Associates, St. Paul, Minnesota, on the development of magnetic disc memory devices. Mr. O. H. SCHUCK, of the Minneapolis-Honeywell Company, also spoke on analogue potentiometer-type computers for aeronautical navigation. Most of the discussion at the meeting centered around the computing ability, reliability, and time required for construction of the proposed machines.

OTHER AIDS TO COMPUTATION

BIBLIOGRAPHY Z-III

1. LEE JOHNSON, "How to speed up slide-rule work," *Engineering News-Record*, v. 139, July 24, 1947, p. 112–114. 21 × 28.6 cm.

2. "Kinks and short-cuts—Winder production aided by slide-rule calipers," *Textile World*, v. 97, no. 10, Oct. 1897, p. 154–156. 21 × 28.6 cm.

"Loss of production time can be avoided in the winder room by using this slide-rule caliper for estimating the amount of yarn on unfinished cones at shift-changing time. Instead of allowing the machine to remain idle while a part of a set could be run, the operator can run the part set with assurance that piece-rate pay for the number of pounds run will be equitably apportioned. The calipers indicate the number of pounds run according to package diameter and spindle assignment."

3. R. G. MANLEY, "Computer for principal stresses," *Engineering*, v. 164, Oct. 10, 1947, p. 340–341. 26.4 × 36.1 cm.

An instrument consisting essentially of two horizontal slides, one vertical slide and three cursors, one of which carries a radius arm and protractor.

4. L. E. WADDINGTON, "A slide rule for the study of music and musical acoustics," Acoustical Soc. Amer., *Jn.*, v. 19, Sept. 1947, p. 878–885. 20 × 26.6 cm.

"Musicians are seldom concerned with the mathematical background of their art, but an understanding of the underlying physical principles of music can be helpful in the study of music and in the considerations of problems related to musical instrument design. Musical