The work forms given are essentially those in Ageton, plus about eight new ones. No warning is given of the difficulties which arise when the foot of the perpendicular lies near a pole; the smaller interval of the argument makes this less serious than in Ageton's tables.

The author's Case 7 (Latitude when the Sun or Star is near the Meridian) and Case 9 (Longitude when the Sun or Star is near the Prime Vertical) will probably appeal to many practical navigators. It is unfortunate that the author did not warn the reader of the troubles which arise in these cases if the altitude is near 90°.

The formula used in Case 9 is

$$2B(t/2) = B(s) + B(s - z) - B(d) - B(L)$$

where t and d are the meridian angle and declination respectively of the celestial body, L is the observer's latitude, z is $90° - H$ or the body's zenith distance, $s = (z + d + L)/2$. Table S, based upon this formula, gives t and z for Betelgeuse, each to the nearest minute of arc for L, 0(1°)60° and H, 20°(5°)35°. This table is quite similar to, and possesses many of the advantages of, *Tafeln zur astronomischen Ortsbestimmung*, by ARNOLD KOHLSCHÜTTER, Berlin, 1913. A set of similar star tables prepared today for the northern temperate zone would make a valuable addition to navigational literature. They would possess the great advantage of a direct approach, allowing the navigator to enter a table with the observed altitude as an argument, and with an assumed latitude or longitude, find the corresponding longitude or latitude.

A brief examination of the principal table (II) indicates that the tabular values are much more reliable than those in Ageton.

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518[V].—G. MORETTI, "Scie piane turbolente," *L'Aerotecnica*, v. 27, 15 June 1947, p. 210–221. 20.5 × 29 cm.

The tables, p. 219–220, are (1) of $M(x|\alpha, \xi)$, $N(x|\alpha, \xi)$, $P(x|\alpha, \xi)$, $\alpha = 1.069$, $\xi = .765$, for $x = [0(.1)2, 2.2, 2.5, 3; 3\text{–}4D]$, where

$$M = [\alpha^2 e^{-x^2}/C(\xi)]\begin{vmatrix} A(x)C(x) \\ A(\xi)C(\xi)\end{vmatrix}, \qquad N = [\alpha^2 e^{-x^2}/C(\xi)]\begin{vmatrix} B(x)C(x) \\ B(\xi)C(\xi)\end{vmatrix},$$

$$P = [2\alpha e^{-x^2}/C(\xi)]\begin{vmatrix} x^2-\frac{1}{2} & C(x) \\ \xi^2-\frac{1}{2} & C(\xi)\end{vmatrix}, \qquad A(x) = \tfrac{1}{2}e^{-x^2} + x\int_0^\infty e^{-t^2}dt,$$

$$B(x) = \tfrac{1}{2}e^{-x^2} - x\int_0^\infty e^{-t^2}dt, \qquad C(x) = \tfrac{1}{2}e^{x^2} - x\int_0^\infty e^{t^2}dt;$$

(2) of $y(x|\alpha, \xi, \mu) = M(x|\alpha, \xi) + \mu N(x|\alpha, \xi)$ and of $y(x|\alpha, \xi, \lambda) = M(x|\alpha, \xi) + \lambda P(x|\alpha, \xi)$, where $\mu = 1.5369, 1, 0, -2, -5, -10$, $\lambda = -1.2155, -1(1)2, 5, 10$ and in both cases $\alpha = 1.069$, $\xi = .765$, $x = [0(.1)2, 2.2, 2.5, 3; 3D]$.

Extracts from text

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **477** (Cazzola), **484** (NBSCL), **486** (Anan'ev), **495** (Chebyshev, Whittaker & Robinson), **503** (Magnus & Oberhettinger), **506** (U. S. Navy), **510** (Skinner); N **87** (German Tables), **92** (Fontoura & Penteado).

124. J. R. AIREY, "Tables of the Bessel functions $J_n(x)$," $n = 0(1)13$, $x = [6.5(.5)16; 10D]$, BAAS, *Report*, 1915, p. 30–32.

We note the following 51 cases, for $n = 2(1)13$, where Airey and Harvard (RMT **380**, **440**, **501**) differ:

n	x	Airey	Harvard
2	6.5	−0.30743 02368	−0.30743 03906 30...
3		−0.03534 65366	−0.03534 66312 85..
4		+0.27480 26645	+0.27480 27310
5		0.37356 52006	0.37356 53771
6		0.29991 30288	0.29991 32338
7		0.18012 03909	0.18012 05930
8		0.08803 85825	0.08803 88126
9		0.03658 99659	0.03659 03304
10		0.01328 74770	0.01328 82562
11		0.00429 45787	0.00429 66118
12		0.00124 80202	0.00125 41220
13		−0.00031 35057	+0.00033 39927
4	8.5	−0.20767 73541	−0.20770 08835
5		+0.06715 51647	+0.06713 30194
6		0.28668 34302	0.28668 09063
7		0.33757 43838	0.33759 29660
8		0.26932 14373	0.26935 45671
9		0.16938 36158	0.16942 73956
10		0.08937 32784	0.08943 28589
11		0.04090 64511	0.04100 28606
12		0.01650 22421	0.01669 21921
13		0.00568 81149	0.00612 80346
11	9.5	0.08969 64138	0.08969 64137
12		0.04269 16061	0.04269 16060
13		0.01815 60647	0.01815 60646
6	11.5	−0.24508 38970	−0.24508 14040
7		−0.08462 70668	−0.08462 44653
8		+0.14205 96418	+0.14206 03158
9		0.28227 52641	0.28227 36003
10		0.29976 25107	0.29975 92326
11		0.23905 08414	0.23904 68041
12		0.15755 21425	0.15754 76971
13		0.08975 36298	0.08974 83898
8	13.5	−0.20367 10209	−0.20367 08728
9		−0.02727 91962	−0.02727 93539
10		+0.16729 87593	+0.16729 84008
11		0.27512 92100	0.27512 88367
12		0.28105 99533	0.28105 97034
13		0.22453 29292	0.22453 28582
9	14	−0.11430 71982	−0.11430 71981
3	14.5	−0.21021 97923	−0.21021 97924 22...
4		−0.02612 20608	−0.02612 25583
5		+0.19580 76209	+0.19580 73465
6		0.16116 17993	0.16116 21076
7		−0.06243 23387	−0.06243 18091
8		−0.22144 12987	−0.22144 10957
9		−0.18191 66806	−0.18191 69861
10		−0.00438 63048	−0.00438 68871
11		+0.17586 66050	+0.17586 61074
12		0.27121 83952	0.27121 82225
13		0.27304 66008	0.27304 68125

R. C. A.

The discrepancies indicated above have all been tested by comparison with the original calculations, performed under the supervision of L. J. COMRIE, for the forthcoming BAAS, *Mathematical Tables, Bessel Functions*, part II. In every case the Harvard value is confirmed, although incompletely in 2 cases. These two cases are:

(i) $J_9(14)$ where Comrie's 12-decimal value ends ...81 50. The value given by Meissel (see GRAY, MATHEWS & MACROBERT *A Treatise on Bessel Functions*) has ...81 497..., so that Airey's error may presumably be explained as a rounding-off to 12 decimals, followed by another rounding-off to 10 decimals without reference back to Meissel's table, which Airey quotes as his source for integer x. This error is, however, quite trivial.

(ii) $J_9(14.5)$. The B.A. 12-decimal value ends ...61 51..., but is subject to a possible error of 2 units or so in the last figure. It has not seemed worthwhile to pursue the matter further.

It is disconcerting to find serious errors of this nature in any work of Airey's, even though it is comparatively early work. It is thus desirable to investigate more closely. A large pile of Airey's manuscripts were handed over to Dr. Comrie after Airey's death in 1937, but the calculations for this particular table do not seem to have been included.

The published values were therefore tested by formation of the values of

$$E = x\bar{J}_{n-1}(x) - 2n\bar{J}_n(x) + x\bar{J}_{n+1}(x)$$

in which $\bar{J}_n(x)$ is used to denote Airey's tabulated value.

The following values are seriously in error.

x	n	E in units of the 10th decimal
6.5	1	+ 10000
	12	+ 638
8.5	3	+20 00002
11.5	5	− 2 86690
13.5	7	− 19996
	8	+ 44998
14.5	3	+ 72132

Apart from these the greatest value of $|E|$ is 18 units for $x = 14.5$, $n = 2$, arising from the small error at $n = 3$.

Thus most of Airey's errors are readily explained. All of them, except the end figure ones, arise from the major computation errors indicated in the list. Four of these errors (the 1st, 3rd, 5th and 6th) are almost certainly accounted for as errors in addition, subtraction or transcription. The second seems to fall into the same class, since $638 = 650 - 12 = 100x - 12$, and 12 units of the 10th decimal is a residual that might reasonably arise from rounding-off errors. The cause of the other two is, however, obscure.

It therefore appears probable that the errors have arisen through the publication of an early work without applying any check, since any check must surely have brought to light at least one of these errors, and so have directed Airey's attention to the need for a thorough examination. It is unfortunate that he did not carry the values for $x = 6.5$ a little beyond $n = 13$, for the error is within 4 or 5 lines of taking complete charge, and swamping the true value of $J_n(x)$ entirely.

J. C. P. Miller

1 January 1948

Editorial Note: There are 49 titles (1911–1938) in the list of published mathematical tables by J. R. Airey (1868–1937). The title here in question is no. 11. Dr. Miller's report on Errata in the 6D portion of this table will appear in *MTAC* 23.

125. Albert Gloden, "Table de factorisation des Nombres $X^4 + 1$ dans l'intervalle $1000 < X \leq 3000$"; see RMT **348**, *MTAC*, v. 2, p. 211.

Corrections of five errors in this table are as follows:

P. 73 $1120^4 + 1 = 17 \times 89 \times 1\ 039\ 999\ 577$,
p. 82 $2310^4 + 1 = 17 \times 41 \times 40\ 852\ 171\ 033$,
p. 76 $\frac{1}{2}(1417^4 + 1) = 90\ 841 \times 22\ 190\ 521$,
p. 77 $\frac{1}{2}(1623^4 + 1) = 17 \times 204\ 077\ 517\ 313$,
p. 82 $\frac{1}{2}(2313^4 + 1) = 433 \times 593 \times 55\ 735\ 249$.

17 of the blank spaces in the table may now be filled in as follows:

P. 74 $1140^4 + 1 = 592\ 649 \times 2\ 849\ 849$,
p. 74 $1242^4 + 1 = 565\ 921 \times 4\ 204\ 657$,
p. 75 $1354^4 + 1 = 593\ 081 \times 5\ 667\ 097$,
p. 76 $1444^4 + 1 = 539\ 089 \times 8\ 065\ 073$,

p. 79 $1904^4 + 1 = 595\ 201 \times 22\ 080\ 257$,
p. 82 $2190^4 + 1 = 530\ 713 \times 43\ 342\ 777$,
p. 83 $2332^4 + 1 = 518\ 417 \times 57\ 047\ 281$,
p. 83 $2406^4 + 1 = 17 \times 511\ 457 \times 3\ 854\ 113$,
p. 74 $\frac{1}{2}(1229^4 + 1) = 597\ 769 \times 1\ 908\ 279$,
p. 75 $\frac{1}{2}(1299^4 + 1) = 527\ 377 \times 2\ 699\ 513$,
p. 79 $\frac{1}{2}(1867^4 + 1) = 563\ 081 \times 10\ 788\ 881$,
p. 79 $\frac{1}{2}(1869^4 + 1) = 522\ 553 \times 11\ 675\ 537$,
p. 80 $\frac{1}{2}(2005^4 + 1) = 2089 \times 3\ 868\ 023\ 217$,
p. 80 $\frac{1}{2}(2055^4 + 1) = 17 \times 572\ 233 \times 916\ 633$,
p. 82 $\frac{1}{2}(2189^4 + 1) = 526\ 249 \times 21\ 815\ 329$,
p. 87 $\frac{1}{2}(2895^4 + 1) = 570\ 001 \times 61\ 615\ 313$,
p. 87 $\frac{1}{2}(2969^4 + 1) = 520\ 529 \times 74\ 639\ 009$.

Furthermore, besides the factors given at the following 17 entries, the remaining factor in each case is a 12-figure prime of the form $8k + 1 < 600\ 000^2$: (a) $X^4 + 1$, p. 77, $X = 1562$; p. 79, $X = 1818$; p. 79, $X = 1848$; p. 82, $X = 2262$; p. 82, $X = 2302$; p. 84, $X = 2468$; p. 84, $X = 2476$; p. 86, $X = 2808$; p. 88, $X = 3000$. (b) $\frac{1}{2}(X^4 + 1)$, p. 78, $X = 1709$; p. 78, $X = 1715$; p. 82, $X = 2211$; p. 82, $X = 2299$; p. 84, $X = 2533$; p. 84, $X = 2577$; p. 85, $X = 2669$; p. 85, $X = 2683$.

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126. O. A. WALTHER, "Bemerkungen über das Tschebyscheffsche Verfahren zur numerischen Integration," *Skandinavisk Aktuarietidskrift*, v. 13, 1930, p. 168–192.

On p. 177–179 are given the roots of Chebyshev's polynomials of degree n (those employed in numerical integration with equal weight factors) for n = [1(1)7; 10D], n = [8(1)-10; 5D]. This fact was unknown to the writer when he also gave, among other quantities, the roots, to 10D, for those polynomials having only real roots (see RMT **495**). The writer had relied on FMR, *Index*, p. 360, where no mention was made of Walther's roots which supersede the calculations of most of the authors cited there; hence the writer's statement on p. 192–193 regarding other tables of roots must be slightly modified in order to take Walther's tables into account.

Comparison of Walther's roots with the writer's, where they overlapped, revealed one appreciable error in Walther's calculations which was greater than a unit in the last place. On p. 178, for Walther's $n = 4$ (corresponding to the writer's $n = 5$) a pair of roots is given as ±0.83249 74841, whereas it should have been ±0.83249 74870.

HERBERT E. SALZER

127. DOV YARDEN, (**a**) "Table of Fibonacci numbers"; (**b**) "Table of the ranks of apparition in Fibonacci's sequence," *Riveon Lematematika*, v. 1, 1946, p. 35–37; 54; v. 2, Sept. 1947, p. 22. The errata below supplement those listed in *MTAC*, v. 2, p. 343–344.

(**a**) In the last number (5) of *Riv. Lem.*, v. 1, June 1947, p. 99, are the following corrections in factorizations of U_n and V_n:

	U_n	n
5·28657·(*33720414042782577761*)	483162952612010163284885	115
353·*709*·*8969*·*336419*·2710260697	2046711111473984623691759	118

	V_n	n
$3 \cdot \mathit{347} \cdot \mathit{1270083883}$	1322157322203	58
$2^2 \cdot 19 \cdot 199 \cdot \mathit{991} \cdot \mathit{2179} \cdot 9901 \cdot \mathit{1513909}$	4895267005239686661124	99
$2 \cdot 3^2 \cdot \mathit{227} \cdot 29134601 \cdot (\mathit{5608975608563})$	66771477840504325965 1218	114
$2^2 \cdot 19 \cdot 79 \cdot 521 \cdot 859 \cdot (\mathit{1052645985555841})$	2828485190904971853895196	117

(**b**) On p. 22 of v. 2, line 13, the author noted the following corrections:

p = 1031, *for* 2·5·103 1030, *read* 2·5·103 206;

p = 1231, *for* 2·3·5·41 1230, *read* 2·3·5·41 410.

UNPUBLISHED MATHEMATICAL TABLES

Reference has also been made to Unpublished Tables in RMT **485** (Glaisher), **491** (Gloden); Q**24** (Wrench).

67[F].—P. Poulet, "Suites de totalics au depart de $n \leq 2000$." Hectographed copy on one side of each of 56 leaves, in possession of D. H. L. 20 × 24.8 cm.

By a "totalic series" or "aliquot series" is meant a sequence of positive integers, each term of which is the sum of the proper divisors of its predecessor. Two simple examples are

18, 21, 11, 1

1420, 1604, 1210, 1184, 1210, 1184, $\cdots$.

The first of these terminates with its fourth term; the second ultimately becomes periodic of period two. It has been conjectured[1] that aliquot series either terminate or become periodic. The present tables show this to be the case for all such sequences whose "leaders" (first terms) do not exceed 2000, with the possible exception of about 25 series which are left unfinished. For each such leader are given those terms of the sequence which are $\leq n$. When a term n_1 finally falls below n, the reader is referred to the previous series whose leader is n_1. When the leader is a term of a previous series, reference is made to the leader of this series. Prime leaders are of no interest and are omitted. Beyond $n = 200$ only abundant leaders n are listed. Other leaders would have given second terms not greater than the leaders.

Some leaders generate unusually long sequences. The longest completed series is

936, 1794, 2238, 2250, $\cdots$, 74, 40, 50, 43, 1

and runs to 189 terms, the largest term being

$$3328\ 91620\ 99526 = 2 \cdot 25943 \cdot 641582741.$$

Thus the three dots of this series represent a formidable calculation. It is due to B. H. Brown who (since 1940) also contributed many terms to several of the other still incomplete series. The incomplete series with the smallest leader is

276, 396, 696, 1104, $\cdots$, 5641400009252, $\cdots$ (58 terms).

Besides giving the terms in their decimal representation, the author gives their canonical factorization into primes. This table is an extension of a previous table of Dickson[2] for leaders ≤ 1000.

D. H. L.

[1] L. E. Dickson, *History of the Theory of Numbers*, v. 1, Washington, Carnegie Institution, 1919; offset print, New York, Stechert, 1934, p. 48–49.

[2] L. E. Dickson, "Theorems and tables on the sum of the divisors of a number," *Quart. Jn. Math.*, v. 44, 1913, p. 267–272. For additions and corrections see P. Poulet, *La Chasse aux nombres*, Brussels, v. 1, 1929, p. 69–72; v. 2, 1934, p. 187–8.

68[G].—Herbert E. Salzer, *Chebyshev Polynomials*, ms. in possession of the author at NBSCL.

C. Lanczos, in his "Trigonometric interpolation of empirical and analytical functions," *Jn. Math. Phys.*, v. 17, 1938, p. 140, gave the coefficients of the Chebyshev polynomials $C_n(x)$ adjusted to the range [0, 1], up to $n = 10$. Due to their importance, these coefficients