QUERIES

27. First Table of $\tan^{-1} x$.—When was the first table of $\tan^{-1} x$ published? In *MTAC*, v. 3, p. 42, it is suggested that this may have been in 1819 when the table of William Spence first appeared. The following copy of a letter addressed to J. W. L. Glaisher indicates that this date may be set back more than 50 years:

"20, Girdler's Road
Brook Green. W.
30 Sept. 1877

"My Dear Glaisher

My son has just brought me an old German schoolbook of 1764—Segner, Anfangsgründe der Arithmetik Geometrie u.s.w., Halle. It contains a curiously arranged table, the argument being the natural numbers from 1–1000, and the entries (1) the common logs; (2) $\sin^{-1}(x/1000)$; (3) $\tan^{-1}(x/1000)$; (4) $\cot^{-1}(x/1000)$. It is a very coarse expedient for contraction; but I never saw one before, and I thought you might like to hear of it in case it should be new to you also.

Yours ffly
C. W. Merrifield

The book is strictly an octavo of very small size, the page $7'' \times 4''$." [There are various references to Merrifield in *MTAC*, v. 1.]

The book here referred to was evidently written by Johann Andreas von Segner (1704–1777), professor of physics and mathematics at the University of Göttingen (1735–1755) and then professor in the same subjects at the University of Halle. A long list of his publications is given in "Poggendorff," v. 2, one entry being as follows: "Elementa arithmeticae et geometriae, 8°, Gotting. 1739, umgearbeitet unter d. Titel: Elementa arithmeticae geometriae et calculi geometrici, 8°, Halae 1756 et 1767 (deutsch von seinem Sohne Johann Wilhelm, 8°, Ib. 1764 u. 1773)." Thus the very book that Merrifield lists was a German translation, of one of Segner's Latin books, by his son. Where may a copy of this translation, or of the Latin original, be seen? The possibility of the $\tan^{-1} x$ table appearing in 1756, or even in 1739, is here definitely suggested. This Segner work of 1739 is mentioned in A. von Braunmühl, *Vorlesungen über Geschichte der Trigonometrie*, v. 2, Leipzig, 1903, p. 89, but tabular details are not mentioned.

In v. 1, 1900, p. 84–86, of Braunmühl's work is material about Abû'l Hasan Ali, of Morocco, a thirteenth century scholar whose notable Arabic compilation was translated into French and published by J. J. Sédillot as *Traité des Instruments Astronomiques des Arabes*, 2 v., Paris, 1834. In v. 1, p. 170, there is a table equivalent to $y = \tan^{-1} x$, where $y = 1°(1°)60°$, and $60x$ equals the number of parts, minutes and seconds, given in the table, from $1^p 2' 51''$ to $103^p 55' 22''$. The gnomon $(=60)$ is here horizontal and the shadow vertical. Thus, while this table is not the earliest published, it seems to be the most ancient table of $\tan^{-1} x$ now known. Compare *MTAC*, v. 2, p. 21.

There are, however, much older tables of $\cot^{-1} x$. On p. 168–169 of the same Sédillot volume is a table equivalent to $y = \cot^{-1} x$, where the horizontal shadow varies from $1(1)140$ and the gnomon $= 12$, the corresponding angles being given in degrees and minutes. This same table was given much earlier by AL-KHOWĀRIZMĪ (fl. about 825); see H. SUTER, *Die astronomischen Tafeln des Muhammed Ibn Mūsā Al-Khowārizmī*, Copenhagen, 1914, Table 60, p. 174 (Danske Vidensk. Selsk., *Skrifter*, 7s., *Hist. og Filos.*, v. 3, no. 1).

R. C. A.

QUERIES—REPLIES

35. TABLES OF TAN^{-1} (m/n) (Q **14**, v. 1, p. 431; QR **18**, v. 1, p. 460; **20**, v. 2, p. 62; **24**, v. 2, p. 147; **28**, v. 2, p. 287).—A table has been prepared which expresses $\tan^{-1}(m/n)$, for $0 < m < n \leq 50$, $0 < m < n = 100$, as a sum of multiples of $\tan^{-1} n_i$ where the n_i are fundamental in the sense of QR**28**. From this table a table of $\tan^{-1}(m/n)$ can be obtained by addition of values from such tables as that of the NBSCL. The manuscript is in the possession of the National Bureau of Standards.

JOHN TODD

NBSINA

CORRIGENDA

V. 1, p. 59, l, -21, *delete* correct.

V. 2, p. 137, $F(\theta, \phi)$, at $\theta = \phi = 86°$, substitute: *for* 3.17204 1744, *read* 3.17030 9981[1].

V. 3, p. 84, l. -6, -5, *for* It may be the first six-place table of the kind, but as long ago, *read* As long ago; p. 106, l. 19–20, *for* .49, *read* $-.49$; p. 129, l. 23, *for* A. H. BURKS, *read* A. W. BURKS. These errors in v. 3 were due to errata in the texts which were being reviewed.

Messrs. B. L. COLEMAN & SIDNEY MICHELSON of The British Electrical and Allied Industries Research Association, 5 Wadsworth Road, Greenford, Middx., England, reported the following, which Professor LEHMER accepts: v. 1, p. 379, l. 20–21, *for* $B_2^{(m)} = \alpha_1^s\alpha_2^{s-1}$ *read* $B_2^{(m)} = \alpha_1^s\alpha_2^{s-1} + \alpha_1^{s-1}\alpha_2^s$; *for* $B_3^{(m)} = -\alpha_1^s\alpha_2^s\alpha_3^{s-1}$, *read* $B_3^{(m)} = -[\alpha_1^s\alpha_2^s\alpha_3^{s-1} + \alpha_1^s\alpha_2^{s-1}\alpha_3^s + \alpha_1^{s-1}\alpha_2^s\alpha_3^s]$. Equation (11) should then be

$$\begin{aligned} 1/\alpha_1 &= B_1^{(m)}/A_1^{(m)}, \\ 1/\alpha_2 &= B_2^{(m)}/A_2^{(m)} - B_1^{(m)}/A_1^{(m)}, \\ 1/\alpha_3 &= B_3^{(m)}/A_3^{(m)} - B_2^{(m)}/A_2^{(m)}. \end{aligned}$$

In general, if the equation has real and complex roots, and α_k is simple, it is given by

$$1/\alpha_k = B_k^{(m)}/A_k^{(m)} - B_{k-1}^{(m)}/A_{k-1}^{(m)},$$

whilst complex α's are given by the expression (13) on page 380.