7. G. I. Gavrilko, "Noviĭ konikograf" [A new conicograph], *Geometrichniĭ Zbirnik*, Kharkov Univ., Ukrainskiĭ Institut Matematiki i Mekhaniki, Kharkov, v. 2, 1940, p. 107–108.

An instrument consisting of jointed links and slides.

8. A. Leĭbin, "Polîârograf-Prilad dlîâ pobudovi vzaemno polîârnikh krivikh" [Polarigraph-apparatus for drawing reciprocally polar curves], *Geometrichniĭ Zbirnik*, v. 2, 1940, p. 109–114.

A description of an instrument invented by the author, consisting of jointed links and slides, for drawing, with respect to a given circle, the polar of any given curve.

9. D. C. dePackh, "A resistor network for the approximate solution of the Laplace equation" *Rev. Sci. Instrs.*, v. 18, 1947, p. 798–799. See *Math. Rev.*, v. 9, 1948, p. 160, G. Kron.

NOTES

93. A Film of part of Kulik's Magnus Canon for sale.—In *MTAC*, v. 2, p. 139–140, some details were given concerning this great 8-volume factor table which became the property of the Academy of Sciences, Vienna. Professor D. N. Lehmer secured a photostatic copy of the latter part of volume 1 of this table, that is, for the numbers 9 000 000 to 12 642 600 inclusive. Of this photostatic copy the Carnegie Institution of Washington (1530 P Street, N.W.) has made (in 1947) a negative microfilm. The Institution is prepared to supply positive microfilm copies at $1.00 per film.

94. Lambertian or Lambda Function.—Let the circle (1) $x^2 + y^2 = 1$ and the hyperbola (2) $x^2 - y^2 = 1$ with common center C be tangent at Q, denoting the common tangent there by l. Let q be any point on (2), and u be the area of the hyperbolic sector qCQ. Project q on l at P and let the angle $PCQ = \omega$. In his memoir "Observations trigonométriques," *Histoire de l'Académie Royale des Sciences*, Berlin, for the year 1768, 1770, p. 327–354, J. H. Lambert gave the formula

$$u = \ln \tan (45° + \tfrac{1}{2}\omega)$$

and also (p. 353–354) a table which professedly gives the value of u, to 7D, for $\omega = 0(1°)90°$. What Lambert really gives, however, is the values of $\log \tan (45° + \frac{1}{2}\omega)$ so that in order to get the corresponding values for u all the approximate values of the table must be multiplied by 2.30258 509. Lambert's table is given also in his *Zusätze zu den logarithmischen und trigonometrischen Tabellen*, Berlin, 1770, p. 176–181; in the Felkel edition of this, Lisbon, 1798, p. 164–168; and in Lambert, *Opera Mathematica*, v. 2, 1948 (see RMT **521**). [J.F.W. Gronau, *Tafeln für die hyperbolischen Sectoren* (also as Neueste Schriften der naturf. Gesell. in Danzig, v. 6, Heft 4), Danzig, 1862, gives a table of $\log \tan (45° + \frac{1}{2}\omega)$, for $\omega = [10'(10')5°(1')$-$83°(10')90°$; 5 or more D].

$$u = \int_0^\omega \sec x dx = \ln \tan (\tfrac{1}{4}\pi + \tfrac{1}{2}\omega) = \ln (\sec \omega + \tan \omega) = gd^{-1}\omega = \text{lam}\, \omega,$$

according to a notation suggested by Professor E. V. HUNTINGTON,[1] to connect the function in name with Lambert. $\omega = \text{lam}^{-1} u = gd\, u =$ the gudermannian of u, a name introduced by CAYLEY (1862).

With a possible exception to be described later, the first tables of u were published by LEGENDRE (a) T.4, $\omega = [0(0^\circ.5)90^\circ;\ 12D]$, Δ^5, (b) T.9, $\omega = [0(1^\circ)90^\circ;\ 9D]$, both of them in *Exercices de Calcul Intégral*, Paris, 1816, p. 160–163, 414–416; and also in *Traité des Fonctions Elliptiques*, v. 2, Paris, 1826, p. 256–259, 361–363.

Other tables of lam ω with the argument in degrees or parts of a degree are those of SAKAMOTO,[2] $\omega = [0(1'' \text{ to } 30'')1^\circ 2'30'';\ 9D]$, Δ, $[1^\circ(1' \text{ to } 4')88^\circ;\ 6D]$, Δ, $[88^\circ(2'' \text{ to } 1')90^\circ;\ 6D]$, Δ^3 (radian arguments also given throughout); CAYLEY,[3] HOBSON,[4] and BOLL,[5] $\omega = [0(1^\circ)90^\circ;\ 7D]$ (radian arg. also given); POTIN,[6] and VASSAL,[7] $\omega = [0(1')90^\circ;\ 5D]$; FORTI,[8] $\omega = [0(2')90^\circ;\ 5D]$; HAYASHI,[9] $\omega = [0(10')90^\circ;\ 5D]$; DALE,[10] $\omega = [0(30')90^\circ;\ 5D]$, Δ; and GREENHILL,[11] $\omega = [0(1^\circ)90^\circ;\ 5D]$ (radian arg. also given).

Two other tables of lam ω with argument in radians are those of MILNE-THOMSON & COMRIE,[12] $\omega = [0(.01)1;\ 4D]$, Δ, $[1(.01)1.47(.001)1.57;\ 3D]$, Δ; and of HALL,[13] $\omega = [0(.01)1.57;\ 4D]$.

The second published table of lam ω was that of GUDERMANN [14] with main argument in grades, (a) $\omega = [0(0^g.01)100^g;\ 7D]$, Δ (sexagesimal arguments also given); (b) $[88^g(0^g.01)100^g;\ 11D]$; in the same argument were tables by POTIN,[15] $\omega = [0(0^g.01)100^g;\ 5D]$; and by HOÜEL,[16] $\omega = [0(0^g.1)95^g;\ 4D]$, $[95^g(0^g.1)100^g;\ 3D]$, Δ.

Gudermann, a favorite teacher of WEIERSTRASS and a notable contributor to hyperbolic functions, used k for ω and $\mathfrak{L}k$ for u, and his elaborate table to every centesimal minute is for $u = \mathfrak{L}k$. This table is entitled "Tabelle der Längezahlen . . . aller Kreisbogen für den Radius = 1 . . . behufs der Zurückführung der hyperbolischen Functionen auf die cyklischen und umgekehrt." In Gudermann's notation $\mathfrak{L}k$, the $\mathfrak{L}$ and k were chosen because "Länge" and "Kreisbogen" are involved.

In the United States, at least, one meets also with the term "lambda function," $u = \lambda(\omega) = \text{lam}\ \omega$; for example, by G. H. CHANDLER [17] in 1907. Was the choice made because "Länge" was regarded as fundamental? Or was "Legendre" or "Lambert" in the thought of the originator? The term is also used in H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, New York, 1934, and rev. ed., 1947.

There are also a number of tables of lam ω expressed in minutes of arc, namely: $(1/\pi)10800'$ lam ω, when ω is given in degrees and minutes. A list of some of these is given in FMR, *Index*, p. 185; for example, there is a table to the nearest $0'.01$, $\omega = 0(1')90^\circ$ in BECKER & VAN ORSTRAND, *Hyperbolic Functions*, fifth reprint, 1942, p. 309–318, which was taken from some edition of J. INMAN, *Nautical Tables*, say, London, 1858, p. 364–372. See MTE **129**. Such tables of "meridional parts" for a spherical earth give the distances of the parallel of latitude ω from the equator on a MERCATOR (1512–1594) chart, in terms of the chart distance representing $1'$ of longitude at the equator.

The first table of this kind was the very celebrated one given by EDWARD WRIGHT (1558?–1615) in his *Certaine Errors in Navigation, Arising either of the ordinarie erroneous making or vsing of the sea Chart, Compasse, Crosse-staffe, and Tables of declination of the Sunne, and fixed Starres detected and* *corrected. By E. W.* London, 1599, 279 p.; there were two different t.p. imprints in this year. "A Table for the true diuiding" occurs on p. [43–48]; this is a 3–6S table throughout the quadrant interval 10′. In the third edition, 1657, by Joseph Moxon, which I have also inspected in the John Carter Brown Library of Brown University, this table has been expanded, p. 14–36, at interval 1′. Through the gracious kindness of Mr. HENRY TAYLOR of New York, I have also inspected a copy of the second very rare edition, "with many additions," 1610, with this same expanded "table of latitudes." A few extracts from the third table page of the 1599 edition are given by F. CAJORI, on p. 97 of *Napier Tercentenary Memorial Volume*, ed. C. G. KNOTT, London, 1915.

Thus Wright finally computed his table by the continued addition of the secants of 1′, 2′, 3′, etc. (Cajori errs, p. 96, in stating 1″, 2″, 3″) and thereby secured an approximation sufficiently exact for the mariner's use. HENRY BOND, a seventeenth century teacher of navigation (see C. HUTTON, *Philos. and Math. Dict.*, v. 1, 1815), discovered by chance that Wright's table was analogous to a scale of logarithmic tangents of half the complement of latitude. He published his discovery in an *Addition* to RICHARD NORWOOD's *Epitome*, 1645. JAMES GREGORY (1638–1675) was the first to demonstrate that Bond's observation was correct, in showing that $\int \sec x dx = \ln(\sec x + \tan x)$, in his *Exercitationes Geometricae*, London, 1668. See *James Gregory Tercentenary Memorial Volume* . . ., ed. by H. W. TURNBULL, London, 1939, p. 18, 236, 459–464. See also J. Gregory, "Analogia inter lineam meridianam planispherii nautici, et tangentes artificiales, geometrice demonstrata, &c.," F. MASERES, *Scriptores Logarithmici*, v. 2, London, 1791, p. 6f.

About a century after Wright's table appeared, EDMOND HALLEY (1656–1742) wrote an interesting paper [18] summarizing the work of prominent mathematicians proving Bond's surmise, and offering his own method for constructing the table. In particular he notes that "The Last, or 89°59′ is 30374.9634311414228643, and not 32348.5279 as Mr. *Wright* has it, by the addition of the *Secants* of every whole Minute."

A biography of Edward Wright was one of the 904 such sketches written for the *Dict. Nat. Biog.* (v. 63, 1900) by J. K. LAUGHTON, whose output in this case is not without blemish. One of his errors would have been avoided if he had consulted W. W. R. BALL, *A History of the Study of Mathematics at Cambridge*, Cambridge, 1889, p. 25–27. A. DEMORGAN's excellent sketch in *The Penny Cyclopaedia*, v. 27, London, 1843, may also be noted. On Mercator's chart of 1556 the lengthening of the degrees of latitude had no foundation in scientific theory. In this connection Wright's contribution based on solid mathematical foundations, before the integral calculus had been developed, was a notable achievement.

In 1599 Wright published also *The Haven finding Art, or, the way to find any Haven or place at Sea, by the latitude and variation.* This was an adaptation and extension of SIMON STEVIN, *De Havenvinding*, Leyden, 1599, of which there were in 1599 also French and Latin editions. The 1657 reprint (20 p.) of Wright's edition, appended to the third edition of his *Certaine Errors*, is overlooked by E. J. DIJKSTERHUIS in his *Simon Stevin*, The Hague, 1943; and also by BIERENS DE HAAN, *Bibliographie Néerlandaise* . . . Rome, 1883. There was then no way of determining the longitude at sea. Wright effected a revolution in the science of navigation. Through Mr.

Taylor's courtesy Brown University filmed for its Library both the French edition of Stevin's *De Havenvinding* and Wright's *Certaine Errors*, 1610.

When the invention of logarithms became public Wright applied himself to the study of the new method and translated Napier's Description of his Canon. This translation was forwarded to Napier at Edinburgh, received his approbation and a few lines of addition, and was returned for publication. But Wright died soon after receiving it back in 1615. It was published in 1616 by his son SAMUEL WRIGHT. The so-called 1618 edition is simply that of 1616 with the title page cut out and a new one substituted.

R. C. A.

[1] A. E. KENNELLY, "Gudermannians and Lambertians with their respective addition theorems," Amer. Phil. Soc., *Proc.*, v. 68, 1929, p. 179. The well-known formula given here on p. 183, namely: lam $2\omega = 2\tanh^{-1}(\tan\omega)$, was intended in the 1909 edition of G. F. BECKER & C. E. VAN ORSTRAND, *Smithsonian Mathematical Tables. Hyperbolic Functions*, with $gd^{-1}\,2\omega$ for lam 2ω, p. xv (see *MTAC*, v. 2, p. 311).

[2] SAMATA SAKAMOTO, *Tables of Gudermannian Angles and Hyperbolic Functions*, Tokyo, 1934, p. 14–94.

[3] A. CAYLEY, "On the orthomorphosis of the circle into the parabola," *Quart. Jn. Math.*, v. 20, 1885, p. 220; also in *Coll. Math. Papers*, v. 12, 1897, p. 336.

[4] E. W. HOBSON, *A Treatise on Plane Trigonometry*. Cambridge, 1891, and second ed., 1897, p. 316; third ed., 1911, fourth ed., 1918, and fifth ed., 1921, p. 336.

[5] M. BOLL, *Tables Numériques Universelles*, Paris, 1947, p. 487.

[6] L. POTIN, *Formules et Tables Numériques* . . . Paris, 1925, p. 450–494.

[7] V. VASSAL, *Nouvelles Tables donnant avec cinq Décimales* . . . Paris, 1872, p. [67]–[111].

[8] A. FORTI & O. F. MOSSOTTI, "Tavole dei logaritmi delle funzioni circolari ed iperboliche," filling the whole of *Annali delle Università Toscane*, Pisa, v. 6, 1863, 4to; Forti's part of the work consists of the tables on [228] unnumbered pages, and the introduction, p.27–48; table of u, p. [183–228]. This is preceded by Mossotti's "Teoria ed applicazioni delle funzioni circolari ed iperboliche," p. 7–26 + plate. In 1863 this work seems to have been published separately at Pisa with the following title: *Tavole dei Logaritmi delle Funzioni Circolari ed Iperboliche, precedute dalla Storia e Teoria delle Funzioni stesse e da Applicazioni.* Second ed., *Tavole di Logaritmi dei Numeri e delle Funzioni Circolari ed Iperboliche, precedute dalla Storia e Teoria delle Iperboliche, da Applicazioni, e da altre Tavole di Uso Frequente.* Turin, Florence, Milan, Paravia & Co., 2 v., 1870; third ed., Turin and Rome, 1877, 584 p.

[9] K. HAYASHI, *Fünfstellige Funktionentafeln* . . . Berlin, 1930, p. 2–19.

[10] J. B. DALE, *Five-Figure Tables of Mathematical Functions*, London, 1903, p. 67.

[11] G. GREENHILL, *The Applications of Elliptic Functions*, London, 1892, p. 16. French ed., Paris, 1895, p. 569.

[12] L. M. MILNE-THOMSON & L. J. COMRIE, *Standard Four-Figure Mathematical Tables*, London, 1931, p. 208.

[13] W. HALL, *Tables and Constants to Four Figures*, Cambridge, 1905, p. 48–49.

[14] C. GUDERMANN, "Potenzial- oder cyklisch-hyperbolische Functionen," *Jn. f. d. reine u. angew. Math.*, v. 7, 1831, p. 72–96, 176–200; v. 8, 1832, p. 64–116; v. 9, 1832, p. 362–378. Reprinted in *Theorie der Potenzial- oder cyklisch-hyperbolischen Functionen*, Berlin, 1833, p. 159–260, 337–350.

[15] L. POTIN, *Formules et Tables Numériques*, Paris, 1925, p. 496–595.

[16] The Société d. Sciences Physiques et Naturelles de Bordeaux, *Mémoires*, v. 4; Cahier 2, 1866, contains the first edition complete, lxxi, [64], 2 p. of J. HOÜEL, *Recueil de Formules et de Tables Numériques*, printed at Paris by Gauthier-Villars. On the title page of this first edition appears also "Extrait des Mémoires de la Soc. d. Sci. phys. et nat. de Bordeaux." The table in which we are interested occurs on p. [36]–[55]. Second ed., a reprint, Paris, 1868; third ed., 1885; third ed. reprinted, 1901; third ed. rev. and corrected, 1927.

[17] G. H. CHANDLER, *Elements of the Infinitesimal Calculus*. Third ed. rewritten, New York, 1907, p. 298.

[18] E. HALLEY, "An easie demonstration of the analogy of the logarithmick tangents to the meridian line or sum of the secants: with various methods for computing the same to the utmost exactness," R. Soc. London, *Phil. Trans.*, v. 19, no. 219, Jan.–Feb. 1695/6, p. 202–214; the v. is dated 1698. Also in his *Miscellanea Curiosa*, second ed., v. 2, 1708, p. 20–36; and third ed., v. 2, 1723, p. 20–36.