The dedication took place at the Company's New York Headquarters, 590 Madison Avenue, in the presence of some 200 representatives of science and industry, to whom the existence of the machine came as a surprise; that an undertaking of such magnitude could be carried out for two years in complete secrecy is a tribute to the numerous members of the IBM organization who were involved in the plan.

Within its huge glass-and-steel room, the giant SSEC—containing 12,500 electronic tubes, 21,400 relays, and 40,000 pluggable connections—was computing positions of the moon from a lengthy formula, representing a function of time at 6-hour intervals. Each computation was completed and checked in 7 minutes, although it involved no less than 10,710 additions and subtractions, 8,680 multiplications, and 1,870 references to a table of sines. This impressive speed is due to several factors, of which the most important are

1. The ability of the machine to read, modify, and execute instructions stored in the same manner as numbers.
2. The 8 high-speed busses (each consisting of 78 channels) which connect the electronic memory with the arithmetic unit, allowing a speed of 3,500 additions, 50 multiplications and 30 divisions per second.
3. The 36 table look-up units which make it possible to carry out a complete search in a table containing a maximum of 100,000 decimal digits in, at most, 3 seconds.

Scientists interested in the services of the SSEC are advised to apply to Dr. W. J. Eckert, Director of the Watson Scientific Computing Laboratories at Columbia University or, preferably, to their local IBM office.

OTHER AIDS TO COMPUTATION

A New Class of Computing Aids

Students of computing aids have been accustomed to putting computers into one of two classes. One class includes "continuous" devices that represent mathematical quantities by measurements of some analogous, continuously variable physical quantity, like length, voltage, angle, etc. The accuracy of such computing aids is strictly limited by the errors in the physical measurements. The second class consists of digital computers that represent mathematical quantities first in a digital or radix notation, such as the decimal or the binary notation; they then represent each digit of this notation by setting up certain discrete physical situations, like 10 stable positions of a counter wheel or the off-or-on conditions of an electric switch. The "capacity" of a digital machine is the number of digit combinations it can handle, and is unrelated to errors of measurement or of construction provided these remain within certain broad limits.

I would like to suggest that a recently developed computing element makes a third category desirable, for it does not belong to either of those mentioned. The "function unit" computer represents numbers by a counting process instead of a measurement. Since counting is an exact process, the capacity of a function unit is limited only by the number of counts it can make and not by mechanical precision. On the other hand, the function unit does not represent quantities in a digital notation, so it is not a digital computer.

The name I suggest for the third class of computers is *counting computers*.

Some later remarks on the properties of counting computers will be more easily understood with a short description of a member of the class as background. The "function unit" is a mechanical representation of a function of one variable. Typically, the input or argument variable, x, is introduced into the function unit by rotating a shaft, and counting the revolutions of that shaft. Similarly, the output or function, y, is supplied by the function unit in the form of rotations of an output shaft.

The relation between y and x may be written

$$y = f(x).$$

It is stored in the form of a "tape" record, that is "read" mechanically by the function unit. Several modes of recording and reading have been devised, but the essential features are, first, that the tape is divided into recognizable steps that can be counted; second, that the record instructs the output shaft to rotate in countable steps; and third that the tape or record can be made to have as many steps as may be desired, by making it long enough. In brief, the function unit says that y is a function of x and any particular tape inserted in the unit specifies the particular values of the integers y that belong to the integral values of x over some extensive range of values of x.

Other elements adaptable to use in a counting computer are gear trains and the special combination of gears called "adders," or in engineering language, "differentials." Since gear teeth mesh positively in a one-to-one relation, and since the number of revolutions that gears can make is unlimited, these are suitable components for a counting computer. They supply us with means for adding, and for multiplying by a constant. Thus we can mechanize three kinds of mathematical relations in a counting computer:

$$\begin{aligned} y &= f(x) && \text{(function unit)} \\ w &= u + v && \text{(adder)} \\ w &= ku && \text{(gear train).} \end{aligned}$$

In these relations, x and y are essentially integers (some forms of function unit combine the counting principle with a continuous subdivision of the integers), u, v and w are real numbers and k is a fixed rational number that cannot be changed without changing the gears.

In contrast, the desk calculator—a digital machine—performs the operations

$$w = u + v, \qquad w = u - v, \qquad z = u/v,$$

where u, v and w are integers and z is rational. Continuous computers perform a multitude of operations which may include

$$z = \int xdy, \qquad z = dy/dx, \qquad z = \sin x, \text{ etc.}$$

While, at first glance, it may seem that the operations mechanized by the counting computer are too restricted to be useful, further consideration shows that it is possible to combine them in many ways. Many functions of two or more variables can be expressed as linear combinations of functions of single variables, where the latter are, in turn, linear combinations of the original variables. Thus a frequently used function of two variables is the product,

$$z = xy.$$

We can form new variables u and v where

$$u = x + y, \qquad v = x - y,$$

by the use of adders. The shafts representing u and v drive function units with square-law tapes, whose outputs are t and w, where

$$t = f(u) = u^2, \qquad w = f(v) = -v^2.$$

The outputs of these function units enter an adder which forms

$$t + w = (x + y)^2 - (x - y)^2 = 4xy.$$

Finally a gear with a ratio $k = 1/4$ leads to

$$z = 1/4(4xy) = xy.$$

Continuous functions, like $\sin u$, $\log u$, or arbitrary functions are easily approximated in the function unit, since the variable u can be multiplied by a very large constant, k, such as $k = 5000$, to give us a new variable, x. When u ranges from $-\pi$ to π, for example, x goes from $-15{,}708$ to $+15{,}708$. Similarly we can multiply $\sin u$ by a constant, say 5000 again, and let $f(x)$ be the integer nearest to

$$5000 \sin (x/5000),$$

where x is the integer nearest to 5000 u. Then the error in approximating u is no greater than half of 1/5000 or .0001, and the error in the function is no more than an additional .0001. The probable error is, of course, much less.

Similarly, the function $\log u$ can be approximated to about 1 part in 40,000 if we choose the variables

$$x = 10000\, u, \qquad y = 20000 \log u.$$

A small portion of the log function approximation looks like this

Log Approximation

x	u	$\log u$	$20000 \log u$	y	$\frac{y}{20000} - \log u$
.	.	.	.	.	.
.	.	.	.	.	.
12010	1.2010	.079543	1590.86	1591	+.000007
12011	1.2011	.079579	1591.58	1592	+.000021
12012	1.2012	.079615	1592.30	1592	−.000015
12013	1.2013	.079651	1593.02	1593	−.000001
12014	1.2014	.079687	1593.74	1594	+.000013
12015	1.2015	.079723	1594.46	1594	−.000023
.	.	.	.	.	.
.	.	.	.	.	.

There are a number of ways of mechanizing products. For positive numbers only, we can use logarithms, in the form

$$t = \ln u, \qquad w = \ln v, \qquad z = e^{t+w}.$$

Another possible relation is trigonometric. Let U and V be constants that are larger than the largest values of u and v to be multiplied. Let

$$t = \arcsin (u/U), \qquad w = \arcsin (v/V),$$

then

$$\begin{aligned} uv &= U \sin t \cdot V \sin w, \\ &= \tfrac{1}{2} UV [\cos (t - w) - \cos (t + w)]. \end{aligned}$$

This is in a form that can be represented by a counting computer. A slight modification would give us such expressions as $r \sin u$, $r \cos u$, etc.

Arbitrary functions of several variables can be approximated by trigonometric sums. If the number of terms in such sums is not too great, the functions can be mechanized by function units. As a simple case, consider a function of the two variables, u and v, defined for u and v between $-\pi$ and π. Under favorable conditions the function can be approximated by a few terms of the trigonometric series

$$\begin{aligned} F(u, v) &= \sum_j \sum_k F_{jk} e^{iju+ikv} \\ &= \sum_j \sum_k F'_{jk} \cos (ju + kv) + \sum_j \sum_k F''_{jk} \sin (ju + kv). \end{aligned}$$

Each argument, $ju + kv$, is obtainable by the use of gears and adders, each sine and cosine function can be represented by a function unit, the constants F'_{jk} and F''_{jk} become gear trains and $F(u, v)$ is, finally, obtained by adders that combine the outputs of the function units.

We return to the consideration of the properties of the class of counting computers as they are related to other computers.

In the point of accuracy, digital computers are practically unlimited. Capacities of 10^{10}, 10^{24} and even 10^{45} have been attained. The cost, bulk and slowness of operation of digital computers are roughly proportional to the number of digits, i.e. to the logarithms of the capacity. Counting computers are limited only by the bulk of the record and by the time required to pass from one part of the record to another. The bulk and slowness are roughly proportional to the capacity of the computer. Capacities of 10,000 to 50,000 distinct values are obtainable in practice with counting computers. Continuous computers are inherently relatively low-precision devices by comparison, being limited by the errors in measurement. Their cost and capacity are not related by any simple rule, but beyond a certain point, very great increase in cost produces only a slight increase in capacity. To hold errors below 1 part in 10,000 is almost impossible.

Digital computers are either restricted to simple operations, like multiplication and division, or else are extremely elaborate general-purpose machines. If we include the human operator as part of the system, it may be said that digital computing systems are complicated and flexible. Counting computers are less flexible, because they are characteristically capable of solving problems with special functional relations. Continuous computers are usually extremely special, although there are exceptions, like the slide rule. This specialization on the part of counting and continuous computers is not logically necessary; it comes about because these devices are relatively cheap, and can therefore be applied to solve special problems at a reasonable cost. To build a digital computer for a special problem would be prohibitively expensive. Furthermore both continuous and counting computers can be built with a great variety of functional relations.

It is generally recognized that the automatic control of machine tools and other devices is closely related to automatic computation, and in fact, that machine control is essentially a computing process that may, in some instances, be highly involved and demand great accuracy. Because counting computers are essentially more precise than continuous computers they are well adapted to such controls.

In view of the foregoing discussion, we conclude that counting computers form a new category, being neither digital nor continuous. This class is intermediate between digital and continuous ones in capacity and in the generality of problems handled by one computer. Finally, because counting computers are capable of higher precision than are continuous devices they are adapted to controlling the production of the latter, this control being a kind of computation in a broad sense.

G. R. STIBITZ

EDITORIAL NOTE: On May 16, 1948 Dr. Stibitz issued a somewhat elaborate Report no. 1309. *Function Unit Theory of Computing Counters.* 120 mimeographed p. 21 × 28 cm. This summarizes and extends the work done in recent reports. Copies of this Report may be obtained from manufacturers of the Function Unit Computers, Taller & Cooper Inc., 75 Front St., Brooklyn, N. Y.

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6. P. V. H. WEEMS, "Navigation computer series," *Navigation*, v. 1, June, 1947, p. 134–137.

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An instrument consisting of jointed links and slides.

8. A. LEĬBIN, "Polîârograf-Prilad dlîâ pobudovi vzaemno polîârnikh krivikh" [Polarigraph-apparatus for drawing reciprocally polar curves], *Geometrichniĭ Zbirnik*, v. 2, 1940, p. 109–114.

A description of an instrument invented by the author, consisting of jointed links and slides, for drawing, with respect to a given circle, the polar of any given curve.

9. D. C. DEPACKH, "A resistor network for the approximate solution of the Laplace equation" *Rev. Sci. Instrs.*, v. 18, 1947, p. 798–799. See *Math. Rev.*, v. 9, 1948, p. 160, G. KRON.

NOTES

93. A FILM OF PART OF KULIK'S MAGNUS CANON FOR SALE.—In *MTAC*, v. 2, p. 139–140, some details were given concerning this great 8-volume factor table which became the property of the Academy of Sciences, Vienna. Professor D. N. LEHMER secured a photostatic copy of the latter part of volume 1 of this table, that is, for the numbers 9 000 000 to 12 642 600 inclusive. Of this photostatic copy the Carnegie Institution of Washington (1530 P Street, N.W.) has made (in 1947) a negative microfilm. The Institution is prepared to supply positive microfilm copies at $1.00 per film.

94. LAMBERTIAN OR LAMBDA FUNCTION.—Let the circle (1) $x^2 + y^2 = 1$ and the hyperbola (2) $x^2 - y^2 = 1$ with common center C be tangent at Q, denoting the common tangent there by l. Let q be any point on (2), and u be the area of the hyperbolic sector qCQ. Project q on l at P and let the angle $PCQ = \omega$. In his memoir "Observations trigonométriques," *Histoire de l'Académie Royale des Sciences*, Berlin, for the year 1768, 1770, p. 327–354, J. H. LAMBERT gave the formula

$$u = \ln \tan (45° + \tfrac{1}{2}\omega)$$

and also (p. 353–354) a table which professedly gives the value of u, to 7D, for $\omega = 0(1°)90°$. What Lambert really gives, however, is the values of $\log \tan (45° + \tfrac{1}{2}\omega)$ so that in order to get the corresponding values for u all the approximate values of the table must be multiplied by 2.30258 509. Lambert's table is given also in his *Zusätze zu den logarithmischen und trigonometrischen Tabellen*, Berlin, 1770, p. 176–181; in the FELKEL edition of this, Lisbon, 1798, p. 164–168; and in Lambert, *Opera Mathematica*, v. 2, 1948 (see RMT **521**). [J.F.W. GRONAU, *Tafeln für die hyperbolischen Sectoren* (also as Neueste Schriften der naturf. Gesell. in Danzig, v. 6, Heft 4), Danzig, 1862, gives a table of $\log \tan (45° + \tfrac{1}{2}\omega)$, for $\omega = [10'(10')5°(1')\text{-}83°(10')90°$; 5 or more D].

$$u = \int_0^\omega \sec x dx = \ln \tan (\tfrac{1}{4}\pi + \tfrac{1}{2}\omega) = \ln (\sec \omega + \tan \omega) = gd^{-1}\omega = \text{lam}\, \omega,$$