To check such a table for smoothness or to interpolate one uses divided differences. The first of the ms. tables gives the requisite coefficients A for expressing the $(n-1)$-th divided difference of the function $F(x)$ as

$$A_1F(1) + A_2F(2) + A_3F(5) + A_4F(10) + \cdots.$$

The coefficients are given to 8S for $n = 3(1)10$. Two other tables are based on the points

1, 5, 10, 50, 100, 500, 1000 & 1, 2, 10, 20, 100, 200, 1000.

D. H. L.

AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. Cannon, 418 South Building, National Bureau of Standards, Washington 25, D. C.

Technical Developments

The leading article of this issue of *MTAC*, "The IBM pluggable sequence relay calculator," by Dr. W. J. Eckert is our current contribution under this heading.

Discussions

The second article of this issue of *MTAC*, "Inversion of a matrix of order 38" by Mr. H. F. Mitchell, Jr., is the first of our contributions under this heading. The following five papers are revised summaries of talks delivered at the meeting of the Association for Computing Machinery by members of the staff of the Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, on December 11–12, 1947; see *MTAC*, v. 3, p. 132–133.

Airflow Problem Planned for the ENIAC

Given a body of revolution with sharp nose immersed in a steady uniform flow at sufficiently high Mach number, it is permissible to neglect viscosity and body forces.

If the independent variables α and β, constant on characteristics, are introduced, it can be shown that the differential equations take the form

$$\begin{aligned}
&Hy_\alpha - (K+R)x_\alpha = 0\\
&Hy_\beta - (K-R)x_\beta = 0\\
&Hu_\alpha + (K-R)v_\alpha + (P+Q)x_\alpha = 0\\
&Hu_\beta + (K+R)v_\beta + (P-Q)x_\beta = 0\\
&dz = yA(-vdx + udy)
\end{aligned}$$

where x and y are cylindrical coordinates of a point P, u and v are the corresponding velocity components, and H, K, R, P, Q, A are known functions of y, u, v, z.

The boundary conditions which complete the formulation of the problem are of the form

$$dy/dx = F(x, y) \qquad v/u = F(x, y)$$

on the contour of the body, and

$$G(u, v) = 0 \qquad dy/dx = H(u, v)$$

on the shock-wave.

Approximating the solutions locally by polynomials of first, second, or third degree, one obtains procedures of differing orders of accuracy.

The Airflow Branch of the Ballistic Research Laboratories, with the aid of a group of mathematicians at the Moore School of Electrical Engineering, have programmed for computation by the ENIAC the solution of a 9-parameter family of problems of the above type by a second-order method. The parameters are the Mach number, the coefficients of two tangent, third-degree curves which make up the head of the projectile, and the coefficients of a third-degree curve defining the boat tail.

The differential equations are nonlinear, hyperbolic, and partial, with boundary conditions on a shock surface not specified in advance. The ENIAC will be able to compute the solution for one set of parameters at 1000 grid points in 20 minutes.

R. F. CLIPPINGER

Nonlinear Parabolic Equations

Numerical solutions of two nonlinear heat-flow problems have been obtained with desk calculators and with large computing machines using finite difference approximations. For both problems the net has been defined by $v_{ij} = v(i\Delta x,\ j\Delta t)$ and $\Delta t = \frac{1}{2}\Delta x^2$.

In the first problem the system of equations is of the form

$$\begin{aligned} &\partial v/\partial t = \partial^2 v/\partial x^2 + \exp(-1/v), && x \geqslant 0,\ t > 0 \\ &-\partial v/\partial x = H[v_g(t) - v], && x = 0,\ t > 0 \\ &\partial v/\partial x \to 0, && x \to \infty,\ t \geqslant 0 \\ &v = v^{(0)}, && x \geqslant 0,\ t = 0. \end{aligned}$$

The approximately equivalent difference equations

$$\begin{aligned} &v_{i,j+1} = \tfrac{1}{2}(v_{i-1,j} + v_{i+1,j}) + \Delta t \exp(-1/v_{i,j}), \qquad i > 0 \\ &v_{0,j+1} = v_{1,j} + H\Delta x(v_{gj} - v_{0,j}) + \Delta t \exp(-1/v_{0,j}) \end{aligned}$$

were solved by hand calculation and with the Bell Relay Computer for three values of H, four values of $v^{(0)}$, and various simple functions $v_g(t)$. It appears that the local truncation error committed at the boundary is greater than in the interior and that attainment of significantly smaller error may entail excessively laborious computation. The integrated error is difficult to estimate because of the nonlinearity. However, completed runs with several mesh sizes on the Bell machine may yield, at some later date, a better than qualitative estimate. An approximate analytic solution has been obtained which permits estimation of the t for which $v(0, t) = 0.05$ to an apparent accuracy of 20 percent in t over a range of 10^8. The performance of the Bell machine on this problem has been quite satisfactory. No errors have been detected in its 1000 hours of operation, about 50 percent of which were unattended.

The second problem requires the solution of the system

$$\begin{aligned} &\partial v/\partial t = \partial^2 v/\partial x^2 + m\mu(t)(\partial v/\partial x), && x > 0,\ t > 0 \\ &v = f(x), && x \geqslant 0,\ t = 0 \\ &v = f(0), && x = 0,\ t \geqslant 0 \\ &v \to 0, && x \to \infty,\ t \geqslant 0 \\ &\mu(t) = 1 + 2\partial v/\partial x, && x = 0,\ t \geqslant 0 \end{aligned}$$

where $f(x)$ is a given function, m is a parameter, and $\mu(t)$ is to be determined.

Numerical integration has been performed on both the IBM Relay Multipliers and the Bell Relay Computer, using the simplest finite difference approximation for the differential equation

$$v_{i,j} = \tfrac{1}{2}(1 - \tfrac{1}{2}m\Delta x\mu_j)v_{i-1,j} + \tfrac{1}{2}(1 + \tfrac{1}{2}m\Delta x\mu_j)v_{i+1,j}$$

and a third order forward difference formula for

$$\mu_j = 1 + \Delta x^{-1}[-\tfrac{11}{6}f(0) + 6v_{1,j} - 3v_{2,j} + \tfrac{2}{3}v_{3,j}].$$

Computation proceeds along "lines," the values for fixed j and all i being computed before going on to $j + 1$. It is necessary to start with a small Δx and to double it several times to cover the desired range in t.

The computation on the Bell Computer was relatively slow, about 50 seconds being required for each point. However, the operation was completely automatic; the machine ran unattended overnight several times. No machine errors were found.

The IBM Relay Multiplier calculates more rapidly, requiring only 1.2 seconds for each point, but the operation is not completely automatic. After each line the cards must be transferred by hand and the coefficients in (1) set on switches. Also, to check against machine errors, it is necessary to make a run of each line on a second Multiplier and compare $\sum v_{i,j}$.

The integral of $\mu(t)$ is also required and is computed automatically on the Bell Computer but must be done separately by hand from IBM results. For these reasons, the machine operating time alone does not determine the total time required for solutions.

Accuracy on the IBM Multiplier is severely limited because the machine operates with only 6D digits. When the interval size was reduced to cut down truncation errors, the rounding off errors became serious.

In conclusion, it should be mentioned that experience with solution of these nonlinear parabolic equations is in accordance with the experience of others, namely: that fast and long calculations on automatic or semi-automatic computing machines require a proportionately large amount of mathematical analysis, often complex in nature.

BRUCE L. HICKS & H. G. LANDAU

Laminar Boundary Layer Flow in a Compressible Fluid

A problem recently placed on the ENIAC involves the solution of the partial differential equations representing laminar boundary layer flow in a compressible fluid. The particular problem is for the case of a flat plate with the flow at zero incidence, assuming a constant but unspecified pressure gradient. The solution is obtained by series expansions in one of the independent variables with coefficients being functions of the other independent variable. The coefficients are obtained by the solution of a set of ordinary differential equations. A set of ordinary differential equations for each order of coefficients is obtained by substituting the series expansions in the original partial differential equations and equating coefficients of the independent variable of the series expansions.

In this application the great advantage of using a high-speed machine such as the ENIAC arises from the fact that the solution is obtained by meeting two boundary conditions at $Y = \infty$. This involves guessing two boundary conditions at $Y = 0$ and integrating the equations to find out if the boundary conditions at infinity are met. (In this problem ∞ is approximately 250 intervals of the independent variable Y.) It usually requires several runs to obtain an agreement with the boundary conditions specified at $Y = \infty$, and each integration is a sizable computing problem by hand. A machine with a larger memory would be more efficient than the present ENIAC since this would eliminate the printing of many intermediate quantities, which is necessary with the present machine.

This method of solution was proposed by Professor D. R. HARTREE of Cambridge University, England.

JOHN V. HOLBERTON

On the Approximate Solution of a Partial Differential Equation on the Differential Analyzer

An attempt was made to obtain an approximate solution on the Differential Analyzer for the partial differential equation $v_t = v_{xx}$, subject to boundary conditions $v = v_0(x)$ at $t = 0$ where $v_0 \to 0$ as $x \to \infty$, and $v =$ constant at $x = 0$.

The scheme tried consisted of replacing the above equation by the second-order ordinary differential equation

$$d^2[v_{j+1}(x) + v_j(x)]/dx^2 = 2[v_{j+1}(x) - v_j(x)]/\Delta t$$

where $v_j(x) \equiv v(x, j\Delta t)$, $j = 0, 1, 2, \cdots$. Fixing Δt and beginning with $j = 0$, an operator manually feeds in the function $v_0(x)$ from a graphical input table, obtaining $v_1(x)$ as a graphical solution. Then, feeding in $v_1(x)$ on the second run, the solution obtained is $v_2(x)$, etc. The solution desired on each run is that solution which approaches zero asymptotically.

It was found that the solution of the equation on the Analyzer is highly sensitive to errors in feeding the input function, and at the slightest provocation the output curve proceeds to diverge at a wild rate from the solution sought. An explanation of this behavior is found by rewriting the foregoing ordinary differential equation in the form

$$d^2[v_{j+1}(x) - v_j(x)]/dx^2 - 2[v_{j+1}(x) - v_j(x)]/\Delta t = - 2d^2v_j(x)/dx^2.$$

Since v = constant at $x = 0$, the general solution of this differential equation is $V_{j+1} = v_{j+1} + c \sinh (2/\Delta t)^{\frac{1}{2}} x$, where v_{j+1} is the particular solution satisfying the boundary condition at infinity, and c is an arbitrary constant. The second term on the right, which is the solution of the homogeneous equation corresponding to the differential equation, will, for nonzero values of c, cause V_{j+1} to diverge to $\pm \infty$. A nonzero value of c sufficiently large to produce very rapid divergence can be introduced by a very minute error in manual feeding of the input function. In the case $j = 0$, for example, it was found that even with the most painstaking following of the input v_0 not only is it impossible to obtain a solution V_1 which approaches zero, but in successive runs with the same initial and boundary conditions it is impossible to obtain the same V_1 twice. The worst divergence from the desired V_1 which occurred was caused by an error in manual following of 0.017 in. in the vertical scale over an interval of 0.5 in. in the horizontal scale. Since it is virtually impossible on the Aberdeen Analyzer manually to follow a graph with an accuracy of more than .02 inch, it is easy to see in a problem of this sort, and with the setup used, that only the remotest chance could yield a solution which satisfies the boundary condition at infinity. For the above reasons, the Analyzer approach to the solution of the partial differential equation given had to be abandoned.

JOSEPH H. LEVIN

Computation of the Airflow about a Cone Cylinder

In the past, airflow computations had to be performed with the aid of standard desk machines only. They will in the future be carried out by high-speed machines such as the ENIAC. In order to convey some idea of the complexity of this type of computation, a brief description is given here of a method now used for the determination of the supersonic flow of a compressible gas about a nonyawing cone cylinder placed in a uniform stream. Assuming steady and irrotational flow, the equations governing the flow are found to be

$$Hu_x + K(u_r + v_x) + Lv_r + P = 0, \qquad v_x - u_r = 0.$$

In these equations, (x, r) are Cartesian coordinates of a point in a fixed meridian plane, in the direction of the axis of the body and normal to it, respectively; u, v are the components of the velocity vector in the x, r directions; u_x is the partial derivative of u with respect to x, etc.; $a^2 = \frac{1}{2}(g - 1)(1 - u^2 - v^2)$ is the square of the local speed of sound, measured in units of the speed of efflux into a vacuum; $g = c_p/c_v$ is the ratio of the specific heats of the gas; and $H = a^2 - u^2$, $K = - uv$, $L = a^2 - v^2$, $P = a^2v/r$. The above system of partial differential equations is hyperbolic if the flow is supersonic. In this case it is convenient to introduce another pair of variables, called characteristic variables, and reduce the system to an equivalent but simpler system of differential equations.

The computation then proceeds as follows: Given the free stream velocity $\bar{q}_1$ and the semi-cone angle h, the location (x, r) of the characteristic SW off the shoulder S and the velocities u, v along SW may be obtained by integration of ordinary differential equations.

Integration of another ordinary differential equation is necessary to rotate $\bar{q}_1$ at S through the angle h until it becomes parallel to the cylindrical surface. Now the flow in the next region may be computed; here the characteristic differential equations are advantageously replaced by their equivalent difference equations; these permit computing x, r, u, v at P, once these quantities are known at the adjacent points below and to the left of P. The computation then proceeds to fill out subsequent regions.

From the results thus far obtained it would seem that even relatively coarse characteristic grids—which may be completed fairly quickly—lead to physically meaningful numbers.

Thus, good results have been obtained in many cases where characteristic grids consisting of approximately 120 points were used. Since it takes an experienced computer working with a standard desk machine up to 45 minutes to do one point of such a grid, a job of this nature requires roughly 90 hours.

When higher accuracy is desired, however, it frequently becomes necessary to work with much finer grids and, consequently, to perform an amount of work often larger by a factor of ten than that mentioned above. It is in cases such as these that high-speed computing machines, like the ENIAC, become most essential, especially if a large number of such large-scale calculations of the same type have to be carried out. But even the ENIAC, which was originally designed to do mainly firing-table work, has been found to lack the capacity needed for such extensive flow problems. Once this defect has been remedied, it is hoped that this machine, working at its best about 5000 times faster than man, will also be well suited for flow calculations.

M. Lotkin

Bibliography Z–IV

1. Howard H. Aiken & Grace M. Hopper, "The automatic sequence controlled calculator—I," *Electrical Engineering*, v. 65, 1946, p. 384–391, 6 figures. 21.6 × 27.9 cm.

Beginning with a short resumé of the major stages of computing machine development from the abacus through Charles Babbage's Analytical Engine, this article continues with a short discussion of the increased need for mechanical computation and goes on to describe the sequence control unit of the automatic sequence controlled calculator, the storage counter, and the "ganged counters." A bibliography of 21 items is appended.

With the increasing need for accurate computation occasioned by the use of numerical analysis in many scientific developments, as well as by greater accuracy of physical measurement, more and more time and effort must be devoted to computational labor, which is, moreover, always susceptible to human fallibility. However, the situation has improved somewhat with the development of the automatic sequence controlled calculator, which will carry out any selected sequence of the 5 fundamental operations of arithmetic (addition, subtraction, multiplication, division, and reference to tables of previously computed results) under completely automatic control. This article, Part I of 3 parts (see *MTAC*, v. 2, p. 316), describes the mechanism of the machine and explains its function in addition and subtraction.

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2. Anon., "Electronic digital computers," *Mechanical Engineering*, v. 69, May, 1947, p. 413–414. 28.6 × 20.9 cm.

3. Anon., "Mark II Calculator," *Rev. Sci. Instrs.*, v. 18, March, 1947, p. 202. 26.7 × 19.7 cm.

4. E. C. Berkeley, "Electronic machinery for handling information, and its uses in insurance," Actuarial Soc. Amer., *Trans.*, v. 48, part 1, 1947, p. 36–52. 22.9 × 15.2 cm.

This paper is divided into 4 sections. The first gives a concise description of 4 high-speed machines already in use: the Differential Analyzer at MIT, the Harvard Sequence Controlled Calculator, the ENIAC at Aberdeen, and the Bell Telephone Laboratories' Relay Calculator. In each case the author supplies information regarding memory capacity, storage medium, rapidity of access, types and speeds of operations performed, and reliability of results. He points out an especially attractive feature of the BTL Machine—its built-in checking circuit which stops the machine when a discrepancy is indicated. The performance of these machines proves, according to the author, that high-speed calculators are entirely successful, that great complexity of computational routine does not affect their efficiency, and that they are economically feasible. For, even at their present high cost—between one-fourth and one-half million dollars—the savings to an owner would be about 6 or 7 times the price if one assumes only a 10-year life span for a machine. Obviously, when a considerably cheaper standard model is manufactured, the savings will be far greater.

The second section deals with experiments in the field of electronic machine construction. The problem of devising an efficient and inexpensive medium for storage of information seems to have been solved by the use of magnetic wire or tape, which can store about 2×10^9 binary digits per cu. ft., retain the information indefinitely, and accept new information in place of old, the latter being erased as it is "written" over.

Experiments also show that extraordinarily high speeds may be achieved on the proposed electronic machines for the mathematical operations of addition, subtraction, multiplication, and division, as well as for the logical operations of comparing, selecting, sequencing, etc. Thus, electronic circuits are being developed that will perform up to 100,000 additions per second, and multiplication may reach a speed of 10,000 per second. Mr. Berkeley points out, however, that in the insurance business the need for such enormous speeds is at present not pressing.

The paper continues with a discussion regarding the programming of a problem for an electronic machine. All the instructions, as well as the data, will be stored in the memory, so that the entire problem can be handled by the machine without human intervention. The instructions will be such as to enable the machine to modify the sequence of operations automatically, depending on intermediate results obtained during the course of the computation.

Much thought has, of course, been given to making these machines highly reliable as regards accuracy of final results, infrequency of machine failure, rapidity of localizing the seat of failure when it does occur, ease of repair, and prolonged operation without the presence of human attendants. As might be expected, the new machines will not only be speedier and more trouble-proof than the first experimental models, but cheaper as well, their cost being estimated as between $100,000 and $125,000.

In the third section, the author lists the features of an electronic machine which would be ideal for performing the chores of a large insurance company. This machine would have a large memory capable of rapid reference, as well as ready access to about 200 tables aggregating some 100,000 ten-digit numbers. It would maintain accurate up-to-date files and take proper action based upon these data according to insurance company rules. As regards the input apparatus, Mr. Berkeley would like to have data converted for machine use from punch cards, punched tape, or a typewriter with equal ease. Since manual operations are exceedingly slow, it is obvious that multiple-input devices will be needed to keep up with the great speed of the electromatic machines. He also expects highly reliable performance, as well as a host of other things, hoping at the same time that the cost can be kept within reasonable limits. (It is the pleasant task of the MDL to report to the readers of MTAC that all these expectations are not far from realization.)

The last section gives a fairly complete listing of the various insurance problems which the electronic machine is expected to solve with great efficiency. As the paper under review is but a chapter of a forthcoming book, there is no need to go into greater detail here. The author points out one great advantage that these machines will bestow upon both clerks and actuaries of an insurance company: it will relieve them of much drudgery and enable them to use their time and abilities in fascinating investigation and research which lack of time has hitherto prevented.

MDL

5. ARTHUR W. BURKS, "Super electronic computing machine," *Electronic Industries*, v. 5, no. 7, p. 62–67, 96, 9 figures, 4 illustrs. 21.6 × 29.2 cm.

6. L. J. COMRIE, "Calculations and electronics, automatic computer designed by the National Physical Laboratory," *Electrician*, v. 137, Nov. 8, 1946, p. 1279–1280. 14 × 22.2 cm.

7. E. U. CONDON, "Electronics and the future," *Electrical Engineering*, v. 66, 1947, p. 355–356. 29.2 × 22.2 cm.

8. HASKELL B. CURRY & WILLA A. WYATT, *A Study of Inverse Interpolation on the ENIAC*, Ballistic Research Laboratories, Report No. 615, August, 1946, 58 pages, 50 figures. 27.9 × 21.6 cm.

Given $x = x(t, \phi)$ and $y = y(t, \phi)$ in tabulated form for equally spaced values of t and ϕ, the problem is to express t and ϕ in tabular form for equally spaced values of x and y. The purpose of the report is to provide a basic program for solving this problem on the ENIAC in connection with firing table computation.

Although the problem is one of interpolation in two variables, it is treated here as successive inverse interpolation on one variable at a time. The inverse interpolation is done by an iteration procedure since, the authors say, "it is eminently suitable for the ENIAC," and "the process is in principle independent of the choice of interpolatory approximation formula."

The chapter headings are Introduction, General Outline of the Calculation, Theory of the Iteration, Methods of Discriminating, Theoretical Considerations Related to Formulation of Linear Expressions, Schedule of Stages for the Basic Scheme, Detailed Program for the Basic Scheme, Modifications Near a Maximum, A Second Method of Successive Approximations, Composite Interpolation (interpolating for several functions simultaneously), and Concluding Remarks. A number of diagrams illustrate the interpolation process for various cases; others give the explicit ENIAC coding for the basic scheme and for various modifications.

MDL

9. J. S. KOEHLER, "An electronic differential analyzer," *Jn. Appl. Physics*, v. 19, 1948, p. 148–155. 26.7 × 19 cm.

"An electrical device is designed which will solve ordinary, nonlinear, nonhomogeneous, differential equations or the boundary value problems based on such differential equations. The device is based on the fact that the charge on the condenser in a series resonant circuit varies with the time in a manner described by a linear second-order differential equation. The desired variations of the coefficients with time are fed into the apparatus by using a variable voltage generator. This generator provides a small voltage which varies with time in accordance with any given curve. The variable voltage generator has been constructed and operates satisfactorily. The resulting solution is drawn by an oscillograph. The instrument is designed to furnish a solution which does not deviate from the true solution by more than 4%."

10. F. RUSSO, "Les grandes machines mathématiques," *Revue des Questions Scientifiques*, s. 5, v. 8, 1947, p. 611–616. 25.4 × 16.5 cm.

Summaries of talks delivered by Professor Léon Brillouin of Harvard University, and Louis Couffignal, in charge of mechanical computer studies at the Centre National de la Recherche Scientifique, at a meeting at the École des Telecommunications in Paris on June 12 and 13, 1947. The talks are specifically concerned with recent developments in computing and the need in France for large-scale computers. A project for the development of a large-scale electronic computer will be under way at the beginning of 1948, and it is hoped that the machine will be operating in 1949.

MDL

11. R. R. SEEBER, "Value of super-calculators," *National Underwriter* (Life Edition), no. 4, April 4, 1947, p. 6. 25.4 × 33 cm.

In an address to the junior branch of the Actuaries' Club of New York, Mr. Seeber stresses the possibilities for use of the "super-calculators" in certain types of actuarial calculations.

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12. S. Lilley, "ENIAC, ASCC and ACE, machines that solve complex mathematical problems," *Discovery*, v. 8, January, 1947, p. 23–27, 32. 25.4 × 18.4 cm.

13. Tjänste, Chalmers Tekniska Högskola, Göteborg, *Betankände med förslag till närmast erforderliga åtgärder för tillgodoseende av Sveriges behov av matematikmaskiner*, April 30, 1947, mimeographed. 36 leaves, 7 illustrs. 21 × 29.8 cm.

This report contains a proposal presented to the Secretary and Chief of the Royal Ecclesiastic Department by the Mathematical Machine Committee of Sweden for the acquisition of one or more of the new high-speed computers for Swedish use. It discusses the various developments in high-speed computation, a field in which Sweden has, up to the present time, done very little. In view of the urgent need of the Swedish Armed Forces and industries, it was concluded that an electronic computer should be immediately acquired from abroad, if possible, after a thorough study by Swedish experts of the projected foreign machines, and that these experts should familiarize themselves with computing developments and techniques in foreign countries. The price of the machine is not to exceed 2,000,000 kronor, and 100,000 kronor is to be allocated as a reservation estimate for the budget year 1947–1948. The Committee has also been instrumental in the establishment of an organization to undertake the concrete planning and action necessary for the purchase of a large-scale computer.

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14. Robert C. Tumbleson, "Calculating machines," *Federal Science Progress*, v. 1, June, 1947, p. 3–7, 12 tables. 29.2 × 22.9 cm.

Special mention is made of the growth, commercial applications, and projected developments in the field of automatic computing machinery. The author describes the essential features of the Automatic Sequence Controlled Calculator and the ENIAC and continues with a discussion of some of the big computer projects now under way, namely, the EDVAC, the computer under design at the Institute for Advanced Study, and the computers being designed for the Bureau of Census and the Office of Naval Research under the direction of the National Bureau of Standards. The far reaching significance of developments in this field is stressed. "For the first time, technical men are freed from the enormous drudgery of calculations. At last they are free to think."

MDL

News

Electronic Digital Computing in England.—There are four digital computing projects under way in England. One of these is located at the National Physical Laboratories at Teddington, Middlesex, another at the Mathematics Laboratories of Cambridge University, and a third at Manchester University. The fourth is under the sponsorship of the British Rubber Industries.

The work at the National Physical Laboratories (NPL) was started in the Mathematics Division under the direction of Dr. A. M. Turing. Until January of last year the efforts of his staff, consisting of one full-time and one half-time assistant, were concentrated on coding and logical design while the development work was let as a contract to the Post

Office Research Station at Dollis Hill. However, since the responsibilities of the Post Office group primarily involved communications, the work for NPL did not proceed very fast. This led part of the group at Teddington to begin the design and development work necessary to construct a pilot model. At this stage 6 people were doing this design and development as well as carrying on logical and coding investigations relating to the design of a large Automatic Computing Engine,[1] or ACE, as they called it.

In July 1947, Sir Charles Darwin, Director of the NPL, decided to expand the program for developing and constructing the computing engine. An immediate result was the formation of a group of about 15 persons in the Radio Division. Some of the personnel working on the pilot model were assimilated by this new group, and their supplies and equipment were taken over. However, there remained in the Mathematics Division about 5 persons who were to work on coding. At this time Dr. Turing, having requested and received a year's leave of absence, returned to Cambridge to resume a fellowship there.

The group in the Radio Division was officially established on August 15 with personnel mainly borrowed from other divisions of the laboratory. None of these people had any previous experience with computing machines, and most of them had little experience working at the pulse frequencies of such machines. As they had very little supplies and equipment, most of their time up to January 1948 was spent in obtaining supplies and equipment and in converting a room for use as a laboratory.

The development and construction of a mercury delay line computing machine was begun at Cambridge University by Dr. M. V. Wilkes. By means of a grant from the Department of Scientific and Industrial Research,[2] Dr. Wilkes was able to spend some time in the United States; in particular, he was able to attend some of the lectures on computing machines, given at the Moore School of Electrical Engineering at the University of Pennsylvania in Philadelphia during the summer of 1946. During 1947 he produced a circulation system in a mercury delay line operating at a 500 kilocycle pulse frequency. With the help of at first one and later two men, he then proceeded to design, construct, and test other components of his computing machine.

Professor F. C. Williams studied the problem of storing pulses on the face of a standard cathode-ray tube while still at Telecommunications Research Establishment at Malvern. In 1947 he moved to Manchester University to teach in the electrotechnics department and there continued his research on the storage problem. Professor M. H. A. Newman of the mathematics department of Manchester University is strongly interested in the development of a high-speed computing machine.

At the end of 1947, Professor Williams was so successful in storing 2048 pulses on the face of a twelve-inch tube that he hoped to be able to store 4096 pulses by essentially the same technique. In order to do this, he thought the tube might need a more spherical face to simplify the focus problem, a smaller gun aperture to produce smaller spots on the tube face, and possibly a metallic collector ring to avoid carbon particles in the fluorescent material on the face.

He hopes to build what he refers to as a baby machine by about July 1948. This machine will contain only a few storage tubes, rudimentary computing circuits, and probably a manual (keyboard) input and a visual output.

Mr. A. D. Booth has been working on automatic computing machines for some time under the sponsorship of the British Rubber Products Research Association. This work is being done at Welwyn Garden City, Hertfordshire. Although his first machines used relays, more recently he has used magnetic paper discs for the memory. Initially, this magnetic-disc machine used relays in the auxiliary circuits, but it is understood that the relays were to be replaced by "valves." Mr. Booth has been handicapped by the fact that there has been practically no development of magnetic tape or wire equipment in Britain. He obtained his paper discs in the United States and has had to wind his own recording and reading heads. The latest memory device used in this machine is a rotating drum on which magnetic impulses representing the data are impressed.

Returning our attention to the logical designs developed at the Mathematics Division of the NPL, we find these designs influenced by Dr. Turing. The machine has many logical facilities,[3] and as Mr. Todd[4] says, arithmetic facilities were added only as a concession to those interested in computing. The approach to the design has been from the viewpoint of a *universal* computing machine.[5] This is illustrated by the general principle of having as simple circuits as possible outside of the memory and doing the more complicated activities by relatively elaborate programs (sequences of instructions in the memory).

Emphasis on a simple machine led to more complicated systems of coding than were used in any of the computers being designed in this country. Thus, one instruction tells from what delay lines two numbers are to be taken, what operation is to be performed upon them, into which delay line the result is to be placed, and from which delay line the next instruction is to come. The actual words taken from the respective delay lines are determined by a timing number. The next instruction is taken from the memory as soon as the present instruction is obeyed. Thus, instructions to be obeyed successively must be scattered along in the particular delay line; for example, successive instructions may be in positions 1, 7, 29, 6, 13, 3, ... This appears to be a considerable complication, but those at NPL who are familiar with the system do not think so. They find it relatively easy to fill about 80 percent of a delay line with instructions before it is necessary to shift to a new one. Furthermore, it is not difficult, while shifting to new lines, to refer back and fill in any remaining gaps in the delay lines.

The following is a comparison of the pilot model and the ACE being designed at the NPL, the machine at Cambridge University, and the small machine being planned at Manchester University.

The memory of the pilot test assembly planned at the NPL was about 250 10-digit numbers and was to be realized in mercury delay lines about 5 feet long, with a pulse repetition frequency of one megacycle and using a carrier in the delay line of fifteen megacycles. The ACE has always been planned with a memory of 4000 to 6000 10-digit numbers, consisting either of delay lines similar to those in the test assembly or storage tubes of the type developed at Manchester if they were definitely successful. The memory of the Cambridge machine is about 512 10-digit numbers and is realized in mercury delay lines working at a pulse repetition frequency of 500 kilocycles. The memory of the Manchester machine is to be a "few" cathode-ray tubes. The effective pulse frequency in this machine is about 125 kilocycles.

At the NPL, Hollerith (IBM) equipment was to be used for input and output, and the possibility was held open for using magnetic wire or tape on the ACE when it was built. The Cambridge machine is to use teletype tape. At Manchester no development work has been done on input-output mechanisms; it is hoped that magnetic wire or tape equipment may be obtained from someone in the United States.

The word length ranges from 32 pulses, for the test assembly (at NPL) and the Manchester machine, to 36 pulses for the Cambridge machine, and 40 for the ACE. All machines are purely binary in character with conversion to be done by programming. The Cambridge and Manchester machines make use of a single address code very similar to that used at Princeton. As explained above, the NPL machines use a coding system quite distinct from any other.

Expected completion dates of the various machines may be of some interest. The test assembly at NPL may be finished this year, and it is hoped that the ACE will be completed in about three years. Dr. Wilkes expects to have the Cambridge machine in operation by the end of this year. The "baby" machine at Manchester should be ready for use sometime this summer, but no dates are quoted there concerning any full-sized machines.

HARRY D. HUSKEY

NBSMDL

[1] In England "engine" has been a favored term for automatic computing machines since the days of CHARLES BABBAGE and his "analytical engine."

[2] The NPL come under this department, although, due to their early history, they are not administered in the same manner as other subdivisions of D.S.I.R.

[3] For example, an "and" facility which gives a result only if the numbers stored in two locations in the memory agree in some of their digits.

[4] Mr. JOHN TODD of the University of London is spending some time with the Institute for Numerical Analysis of the National Applied Mathematics Laboratory.

[5] See A. M. TURING, "On computable numbers, with an application to the entscheidungsproblem," London Math. Soc., *Proc.*, s. 2, v. 42, p. 230–265, 1936.

Automatic Relay Computer (ARC)—Supplementary Notes.—The ARC, part relay, part electronic, calculating machine, referred to above, is now nearing completion (at the laboratories of the British Rubber Producers' Research Association), under the direction of Dr. A. D. BOOTH and Miss K. H. V. BRITTEN, both of King's College, London.

The logical design of the ARC, which was developed by Booth and Britten, is described in a report prepared by them at the Institute for Advanced Study, Princeton, N. J., in September, 1947. The coding is similar in principle to that proposed by VON NEUMANN and GOLDSTINE.

This machine is a completely automatic computer, capable of dealing with any problem which can be expressed in terms of the processes of ordinary arithmetic (addition, subtraction, multiplication, and division). It is designed to handle numbers of 20 dyadic digits.

The ARC comprises an arithmetic unit and control, both constructed of standard telephone relays, and a memory consisting of a rotating drum on which data are impressed in the form of magnetic impulses. The binary scale is used, and the machine is constructed primarily to work to 6D accuracy though more places can be retained if desired.

Problems are prepared for the machine by translating them in terms of its "code." This is simply the list of orders which it is possible for the machine to obey and consists of 26 orders of the type "Add number from position (x) of memory into accumulator" and "Multiply number in position (x) of memory by number in shifting register." When a problem has been "coded," the sequence of orders and any necessary initial data are punched onto teletype tape and transferred from there to the memory. Results are also recorded on tape.

The main bulk of construction is now complete, and the machine has been in action as a multiplier, multiplying two 20-binary digit numbers in 1/3 sec. A dividing unit which is expected to work at the same speed is nearly complete.

A note on size may be of interest. The arithmetic unit and control consist of about 600 relays, occupying two racks 5 ft. by 2 ft. The actual memorizing part of the memory will be 2 in. in radius and 14 in. long, and the accompanying electronic circuits will not contain more than 120 tubes. Thus, the whole machine, including input and output, will occupy a floor space of approximately 6 ft. by 3 ft.

The American Academy of Arts and Sciences.—On Wednesday, February 11, 1948, at Boston, Massachusetts, a meeting was held at which Professor JOHN VON NEUMANN of the Institute for Advanced Study, Princeton, N. J., spoke on "Electronic methods of computation." Professor von Neumann showed that fast computing, at electronic speeds, requires complete automatization of a large area in logic and extensive foresight concerning its possible ramifications, functions, and malfunctions. It promises, therefore, to initiate a new, quantitative branch of logic. It also provides a new approach to nonlinear problems, which now block many important avenues in mathematics and mathematical physics. Subsequent applications will be still broader.

Association for Computing Machinery.—Since the Association now contains over 350 paid-up members in all parts of the United States and in some foreign countries, the Executive Committee, at a meeting in New York on January 16, dropped the word "Eastern" and changed the name of the Association to the Association for Computing Machinery. A proposed constitution and bylaws have been drafted, submitted to the Executive Council for discussion, and revised. This new draft was sent to the members by mail for balloting.

IBM Selective Sequence Electronic Calculator.—The new IBM Selective Sequence Electronic Calculator was dedicated to the use of science by THOMAS J. WATSON, President of the Company, in a series of elaborate ceremonies lasting through January 27 and 28.

The dedication took place at the Company's New York Headquarters, 590 Madison Avenue, in the presence of some 200 representatives of science and industry, to whom the existence of the machine came as a surprise; that an undertaking of such magnitude could be carried out for two years in complete secrecy is a tribute to the numerous members of the IBM organization who were involved in the plan.

Within its huge glass-and-steel room, the giant SSEC—containing 12,500 electronic tubes, 21,400 relays, and 40,000 pluggable connections—was computing positions of the moon from a lengthy formula, representing a function of time at 6-hour intervals. Each computation was completed and checked in 7 minutes, although it involved no less than 10,710 additions and subtractions, 8,680 multiplications, and 1,870 references to a table of sines. This impressive speed is due to several factors, of which the most important are

1. The ability of the machine to read, modify, and execute instructions stored in the same manner as numbers.
2. The 8 high-speed busses (each consisting of 78 channels) which connect the electronic memory with the arithmetic unit, allowing a speed of 3,500 additions, 50 multiplications and 30 divisions per second.
3. The 36 table look-up units which make it possible to carry out a complete search in a table containing a maximum of 100,000 decimal digits in, at most, 3 seconds.

Scientists interested in the services of the SSEC are advised to apply to Dr. W. J. Eckert, Director of the Watson Scientific Computing Laboratories at Columbia University or, preferably, to their local IBM office.

OTHER AIDS TO COMPUTATION

A New Class of Computing Aids

Students of computing aids have been accustomed to putting computers into one of two classes. One class includes "continuous" devices that represent mathematical quantities by measurements of some analogous, continuously variable physical quantity, like length, voltage, angle, etc. The accuracy of such computing aids is strictly limited by the errors in the physical measurements. The second class consists of digital computers that represent mathematical quantities first in a digital or radix notation, such as the decimal or the binary notation; they then represent each digit of this notation by setting up certain discrete physical situations, like 10 stable positions of a counter wheel or the off-or-on conditions of an electric switch. The "capacity" of a digital machine is the number of digit combinations it can handle, and is unrelated to errors of measurement or of construction provided these remain within certain broad limits.

I would like to suggest that a recently developed computing element makes a third category desirable, for it does not belong to either of those mentioned. The "function unit" computer represents numbers by a counting process instead of a measurement. Since counting is an exact process, the capacity of a function unit is limited only by the number of counts it can make and not by mechanical precision. On the other hand, the function unit does not represent quantities in a digital notation, so it is not a digital computer.

The name I suggest for the third class of computers is *counting computers*.

Some later remarks on the properties of counting computers will be more easily understood with a short description of a member of the class as back-