552[V].—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS, *Technical Note* no. 1428: *Notes and Tables for Use in the Analysis of Supersonic Flow.* By the staffs of the Ames Aeronautical Laboratory, Moffett Field, Calif. Washington, D. C., Dec. 1947, iv, 73 p. +7 plates. 20 × 26.2 cm.

This NACA *Technical Note* is a compilation of data found important for the analysis of compressible flow in connection with test work in a supersonic wind-tunnel. The text of the paper is devoted to a review of several fundamental aspects of the theory of supersonic flow. These include thermodynamics, equations of motion, nozzle theory, shock waves, expansion around a corner, airfoil theory, and flow about wedges and cones. The appendices give formulae for the calculation of the viscosity, Reynolds number, humidity relations, and atmospheric corrections for air.

The five tables included in the paper contain the following information: **Table I** gives various nozzle data for subsonic flow such as the ratios of the local pressure to rest pressure, the local density to the rest density, etc. for values of the Mach number. The Mach number $M = 0(.01)1$. **Table II** gives similar nozzle data for supersonic flow and in addition gives such data as the local Mach angle, the angle through which a supersonic stream is turned to expand from Mach number 1 to the local Mach number, etc. $M = 1(.01)3.5(.1)5(1)$-$10(5)20, 100, \infty$. **Table III** gives ratios of the local pressure to the rest pressure, the local density to the rest density, etc. on either side of a normal shock wave. **Table IV** gives the Mach number functions for use with small perturbation airfoil section theory, and **Table V** gives various properties of the standard atmosphere. All of the data given in the tables pertain to air where the ratio of the specific heats is 1.4. The functions are tabulated to at least 4S accuracy.

The graphs that are given include plots of maximum theoretical contraction ratio that permits start of supersonic flow in diffuser entrance against Mach number, and the variation of Reynolds number with Mach number. The well-known graphs for the characteristics of wedge and cone flow are also included.

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MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **521** (Lambert), **522** (Peters), **526** (Rybner), **529** (Kraïtchik), **531** (Pettit), **535** (Cambi), **541** (Lane & Sweeney), **548** (Ferrari, Russell & Shapley, T͡sesevich).

128. J. R. AIREY, "Tables of the Bessel functions $J_n(x)$," BAAS, *Report*, 1915, p. 29–30.

A list of errors was given in MTE 124 for the 10-decimal part of this table for $n = 0(1)13$' where $x > 6$. The whole table has now been compared with proofs of the forthcoming BAAS' *Bessel Functions*, part 2, and the following 23 further errors have been found for [$x = .2(.2)$-6; 6D]. Thus the total number of errors, large and small, in this table is 74.

x	*n*	*For*	*Read*	*x*	*n*	*For*	*Read*
1.8	9	—	+0.000 001	5.6	9	+0.012 893	+0.012 907
2.4	10	+0.000 002	+0.000 001		10	+0.003 870	+0.003 912
2.6	9	+0.000 024	+0.000 025		11	+0.000 930	+0.001 062
2.8	11	—	+0.000 001		13	+0.000 057	+0.000 059
3.8	11	+0.000 021	+0.000 022	5.8	4	+0.378 765	+0.378 766
	12	+0.000 004	+0.000 003		8	+0.046 382	+0.046 381
4.6	13	+0.000 005	+0.000 006		9	+0.016 641	+0.016 639
5.4	5	+0.310 074	+0.310 070		10	+0.005 261	+0.005 256
	13	+0.000 037	+0.000 038		11	+0.001 500	+0.001 486
5.6	6	+0.198 559	+0.198 560		12	+0.000 380	+0.000 381
	7	+0.094 452	+0.094 455		13	+0.000 088	+0.000 089
	8	+0.037 571	+0.037 577				

J. C. P. MILLER

6 Feb. 1948

129. G. F. BECKER & C. E. VAN ORSTRAND, *Hyperbolic Functions* (*Smithsonian Mathematical Tables*), Fifth reprint, 1942. See *MTAC*, v. 2, p. 311.

Comparison of this volume with recent publications [1] and the original sources of the tables [2] revealed the discrepancies listed in **A.** Errors and misprints; **B.** Errors in excess of 5 units in the next succeeding place of decimals or significant figure.

A.

Page	*u*	Function	*For*	*Read*
xvi		equation 86	+	−
18	0.0803	log tanh *u*	.90380	.90378
18	0.0803	log coth *u*	.09620	.09622
29	0.485	$\omega F_0'$	65.5	96.5
40	1.009	log cosh *u*	.18137	.19137
46	1.339	log tanh *u*	.04023	.94023
60	2.018	log sinh *u*	.56723	.56763
125	0.980	$\omega F_0'$	144	114
155	2.474	$\omega F_0'$	498	598
182	0.0414	*u*	.0411	.0414
212	1.027	*u*	3.3.96	33.96
214	1.117	*u*	5.7.79	57.79
238	1.217	e^{-u}	.296 1772	.296 1172
245	1.976	*u*	.970	.976
253	2.709	e^{-u}	.066 6039	.066 6034
261	0.096	*p*	2169	4169
270		*u*	2959	2939
270		*u*	3777	3877
271		*u*	5421	5431
272	7853	ln *u*	8.96765	8.96865
288	1.242	gd *u*	.008 4840	.008 4849
293	1.730	gd *u*	37.90	37.00
306	3.59	$\omega F_0'$	113.66	113.76
307	4.32	gd *u*	80 28 33.73	88 28 33.73
310	23	$\text{gd}^{-1} u$	505.53	565.53

B.

Page	*u*	Function	*For*	*Read*
32	0.631	log cosh *u*	.08126	.08127
34	0.703	log tanh *u*	.78266	.78267
34	0.703	log coth *u*	.21734	.21733
91	0.0157	coth *u*	63.699	63.700
91	0.0166	coth *u*	60.247	60.246
100	0.0635	tanh *u*	.06342	.06341
107	0.0972	tanh *u*	.09689	.09690
107	0.0982	tanh *u*	.09788	.09789
109	0.164	tanh *u*	.16254	.16255
111	0.254	tanh *u*	.24867	.24868
123	0.876	sinh *u*	.99241	.99242
130	1.242	tanh *u*	.84602	.84603
131	1.263	sinh *u*	.62661	.62660
132	1.319	tanh *u*	.86653	.86654
133	1.399	tanh *u*	.88513	.88514
134	1.449	tanh *u*	.89550	.89549
138	1.624	cosh *u*	.63522	.63523
141	1.775	tanh *u*	.94416	.94415
141	1.784	tanh *u*	.94513	.94512
143	1.876	tanh *u*	.95414	.95413
143	1.891	sinh *u*	.23753	.23754
144	1.932	tanh *u*	.95890	.95889
145	1.972	tanh *u*	.96199	.96200
171	5.60	cosh *u*	135.2150	135.2151
186	0.0627	cos *u*	.99804	.99803
212	1.015	$\omega F_0'$	85.0	84.9
216	1.229	*u*	59″.44	59″.45
224	2.6	*u*	08″.49624 24	08″.49624 25
224a	7.8	*u*	25″.48872 73	25″.48872 74
237	1.187	log e^u	.515 5075	.515 5076
251	2.521	e^u	12.441 032	12.441 031
252	2.626	log e^u	1.140 4572	1.140 4573
259	42	e^u	173 927 493	173 927 494

The values of tanh 0.174, cosh 0.911, tanh 0.932, tanh 1.381, tanh 1.986 cannot be rounded off correctly with existing tables. Following are three exceptional cases:

$$\sinh 0.876 = 0.99241\ 5002; \qquad \cos 0.0627 = 0.99803\ 4999;$$
$$\sin 0.394 = 0.38388\ 49999\ 93636.$$

The error of 5 units in the seventh decimal of $e^{-2.709}$ had its origin in a misprint in Newman's table; see *MTAC*, v. 1, p. 455.

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[1] F. E. PERNOT & B. M. WOODS, *Logarithms of Hyperbolic Functions to twelve Significant Figures*. Berkeley, Cal. 1918. (Univ. California, Publs. in Engineering, v. 1, no. 13.)
NBSCL, (a) *Tables of the Exponential Function* e^x. 1939;
(b) *Tables of Sines and Cosines for Radian Arguments*. 1940;
(c) *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*. 1940;
(d) *Tables of Natural Logarithms*, v. 1, 1941;
(e) *Tables of Circular and Hyperbolic Tangents and Cotangents for Radian Arguments*. 1943.

[2] C. GUDERMANN, *Theorie der Potenzial- oder cyklisch-hyperbolischen Functionen*. Berlin, 1833.

J. INMAN, *Nautical Tables designed for the Use of British Seamen*. Rev. by J. W. INMAN, London, 1867, p. 364–372.

F. W. NEWMAN, "Table of the descending exponential function to twelve or fourteen places of decimals," Cambridge Phil. Soc., *Trans.*, v. 13, part 3, 1883, p. 145–241.

J. W. L. GLAISHER, "Tables of the exponential function," Cambridge Phil. Soc., *Trans.*, v. 13, part 3, 1883, p. 243–272.

J. O. W. LIGOWSKI, *Tafeln der Hyperbelfunktionen und der Kreisfunktionen*. Berlin, 1889.

A. FORTI, *Nuove Tavole delle Funzioni Iperboliche*. Rome, 1892.

130. H. B. DWIGHT, *Mathematical Tables . . .*, 1944. See *MTAC*, v. 1, p. 180.

P. 45, sec 29°.62 to 29°.67 inclusive, *change* the third decimal figure, 1, in each of the six cases, *to* 0.

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131. H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, Revised edition. 1947. See *MTAC*, v. 2, p. 346.

P. 44, formula 195.01, first line, *for* $bg < 0$, *read* $b > 0$, $g < 0$.

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132. C. F. GAUSS, *Werke*, v. 3, 1866, zweiter Abdruck, 1876, "De curva lemniscata," p. 413–432. This item is in "Nachlass," first published in this v., 1866.

This second posthumous paper by Gauss on the subject of lemniscate functions is concerned principally with the expansion of these functions in trigonometric series.

Certain of the basic constants appearing in the coefficients of these series were evaluated by Gauss to as many as 57D. Recently I recalculated to a higher degree of approximation all the numerical data in this paper. A number of serious calculational errors, in addition to several minor terminal-digit inaccuracies, were thus discovered in the work of Gauss. Among these is a fundamental error in Gauss's approximation to the quarter-period $\bar{\omega}/2$ of the lemniscate functions.

For convenient reference, the errors and their corrections are presented in the following tabular array.

Page	Constant	*For*	*Read*
413	$\bar{\omega}/2$	680 320(7)	523 2420
	$(\bar{\omega}/2)^2$	11(7)	207
	$(\bar{\omega}/2)^4$	63(4)	942
	$(\bar{\omega}/2)^5$	70 0712625(4)	66 04823170
	$(\bar{\omega}/2)^6$	025(3)	4275
	$(\bar{\omega}/2)^8$	569(0)	7567
	$(\bar{\omega}/2)^9$	59	68
	$(\bar{\omega}/2)^{12}$	1749	8915
	$(\bar{\omega}/2)^{13}$	4025	7405
414	$\log (\bar{\omega}/2)$	26 9692(2)	45 79920
	$\ln (\bar{\omega}/2)$	0 6425	2 6427
	$\ln \bar{\omega}$	2 36577	4 36590
418	$\frac{\pi}{12} - \frac{1}{2}\ln 2$	372 7	024 8
	$\operatorname{csch} \pi$	1 57	5 30
	$\frac{1}{2}\operatorname{csch} 2\pi$	144 52	492 44
	$\frac{1}{3}\operatorname{csch} 3\pi$	86	79
	$\frac{1}{7}\operatorname{csch} 7\pi$	81	80
	$2/(e^{2\pi} - 1)$	98	97
	$1/(e^{4\pi} - 1)$	54	55
	$2e^{-9\pi/4}$	906	907
	$2e^{-25\pi/4}$	31 18	45 82
419	$2e^{-\pi}$	514	515
	$2e^{-4\pi}$	387 96535	532 53042
	$2e^{-9\pi}$	288	290
420	$A = \sqrt{\frac{\pi}{\bar{\omega}}}\Big/(2^{7/4}\cos \pi/8)$	6226 6118372314	8226 6118369169
	$2Be^{-\pi/2} = 2e^{-\pi/2}(A \cot \pi/8)$	19576 3585935635	21576 3585929459
	$2Ae^{-2\pi}$	9042	5000
	$2Be^{-9\pi/2}$	6398	5261
	$2Ae^{-8\pi}$	170	051
	$2Be^{-25\pi/2}$	94	99
421	$340-152\sqrt{5}$	01228	01221
	$\sqrt{340-152\sqrt{5}}$	7310507555 85731	6847092228 86965
	$(\sin \operatorname{lemn} \frac{1}{5}\bar{\omega})^4$	7731504831 80010	8194920158 78776
	$(\sin \operatorname{lemn} \frac{2}{5}\bar{\omega})^4$	2352519943 51472	1889104616 52706
	$(\sin \operatorname{lemn} \frac{1}{5}\bar{\omega})^2$	710	711
	$(\sin \operatorname{lemn} \frac{2}{5}\bar{\omega})^2$	029	052
423	$N = 2(\bar{\omega}/\pi)^2$	203	204
	$a = (2/\bar{\omega})^2$	803	802
	$(\ln 2)/\pi$	635	636
428	$e^{-\pi}$	636	633
429	$e^{-\pi/4}$	366	365
430	$e^{-\pi/4}$	820	785
431	$e^{-9\pi/4}$	4846487994 1872486024 8176915	6000857699 5032475616 1924591
431	$e^{\pi/2}$	44648993 1536	35666703 8331

The third calculation of $e^{-\pi/4}$, shown on p. 430, lacks one step for completion. The work to that point is correct to 48D.

Because of an error affecting the last four decimal places in the second line of work, the second calculation of $e^{\pi/2}$, on p. 432, when completed yields a result correct to only 30D.

An incorrect algebraic sign vitiates the last 20 decimal places in the first calculation of $e^{-\pi/2}$ outlined on p. 432. On the other hand, the second calculation of $e^{-\pi/2}$ when completed yields an approximation correct to 35D.

A few typographical errors appear in this paper. In the definition of A on p. 420 the exponent of 2 should read 7/4 instead of 7/2. On p. 421 the number $340-152\sqrt{5}$ should be equated to the square instead of the fourth power of $\frac{1}{2}(\sin \operatorname{lemn} \frac{2}{5}\bar{\omega})^4 - \frac{1}{2}(\sin \operatorname{lemn} \frac{1}{5}\bar{\omega})^4$

On p. 428 the larger prime factor of 32831087 should read 88019 in place of 8819.

It is believed that the preceding enumeration of numerical errors is exhaustive, so that all the data not explicitly mentioned as subject to error may be considered as entirely correct.

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EDITORIAL NOTE: As the result of this communication there will have to be certain revisions in FMR, *Index*, article 5.82.

133. NBSCL, *Tables of the Bessel Functions* $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leqq x \leqq 1$, 1948. See RMT **543**.

The following errata list was issued by the editors soon after publication:

P. vii, l. -6, *for* $\left(\frac{x}{2}\right)^{\nu+2k}$, *read* $\left(\frac{x}{2}\right)^{\nu+2k}$

p. 4, $x = .0096$, $Y_1(x)$, Δ *should read* .683520, *instead of* .652083
p. 11, $x = .0434$, $Y_1(x)$ *should read* -14.720493
p. 13, argument $x = .100$, *not* 100
p. 28, $x = .828$, $Y_1(x)$, Δ^2 *should be* -173
p. 35, $x = .0000$, $K_0(x)$, and $K_1(x)$ *should both read* ∞
p. 42, $x = .099$, $K_1(x)$, Δ *should read* -10198
$x = .100$, $K_1(x)$, Δ *should read* -9997, and Δ^2 *should read* 195
p. 54, $x = .700$, $K_1(x)$, Δ, *for* -2185, *read* -2158.

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134. TOKIHARU OKAYA, "Note sur le mouvement du fluide parallèle à l'axe d'un cylindre solide et sur le phénomène de Liesegang," *Japanese Jn. Physics*, v. 12, 1937, p. 9–25. See *MTAC*, v. 1, p. 227, 301.

In the table, p. 13, of $\phi_n(x) = (2\pi x)^{\frac{1}{2}}e^{-x}I_n(x)$ there are the following two errors: *for* $\phi_0(0.1^{-1}) = 1.0132970$, *read* 1.0132907; and *for* $\phi_1(0.08^{-1}) = 0.9691305$, *read* 0.9691905. The last decimal in each of the values $\phi_0(0.03^{-1})$ and $\phi_0(0.08^{-1})$ is one unit too high. In equation (15) on p. 12 *for* $\phi_n(x) = \sum_{k=0}^{p}$ *read* $\phi_n(x) = 1 + \sum_{k=1}^{p}$.

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135. J. T. PETERS, *Sechsstellige Tafel der trigonometrischen Funktionen . . . von zehn zu zehn Bogensekunden . . .*, 1929; second ed. 1939; third ed. 1946. See RMT **522**, and *MTAC*, v. 1, p. 121 (MTE **14**), 162 (MTE **27**); v. 2, p. 279 (MTE **103**).

A second edition, reproduced photographically from the first after correction of 22 errors, is reviewed in *Astronomische Nachrichten*, v. 269, p. 287–288 (December, 1939) by "No," presumably Frl. H. Nowacki of the Astronomisches Rechen-Institut, who gives a list of 23 errors in the first edition, including those in MTE **14** and **103**. She refers (see MTE **27**) to the inadequate reading of the leading figures of the first edition, and the subsequent reading of a fresh proof, but does not make it clear whether the list of errors she gives is printed in the second edition or not. She calls attention to the fact that one error out of the 23 (p. 190) has not been corrected in the second edition.

Through Dr. HANNA PETERS, daughter of the author, I have recently received a copy of the third edition (ausgegeben November, 1946); at the request of R.C.A. I have examined this to see if the known errors in the first edition had been corrected. The edition is a photographic reprint of the first; the cover and list of contents appear to have been reset, but the title page, the "Vorwort" and the remainder of the book appear to have been photographed from the first edition with (a) the addition of "Dritte Auflage" and change of publisher's imprint, (b) the omission of the original date of September, 1928 and (c) the correction of at least 22 errors in the tables. Generally, the reproduction is of a high standard. but occasionally there are minor variations of inking and focus which distinguish it from letter-press printing.

Examination of the Nautical Almanac Office copy of the first edition (in which the 23 errors mentioned above were corrected in early 1940) has revealed further errors which still persist in the third edition; those on p. 49, 269 are in the hand of L.J.C., the others appear to have been found by various members of the staff in the course of use. There is given below a complete list of all known errors in the first edition; those marked with an asterisk * are in the third edition, and presumably in the second edition as well. The error on p. 190 is the uncorrected error in the second edition referred to above. In the first edition a decimal point is omitted from the entry for cos 21°13′00″ on p. 151 and the argument for 66°59′10″ is out of alignment on p. 162; both of these are corrected in the third edition.

Close investigation of a suspected "wrong fount" on p. 55 reveals an amazing mixture of types in this apparently well printed volume. In the main table the figures in front of the decimal point are often in a different type from that generally used. The leading 0 in sin and tan on p. 24 is correct, but most (not all) of the other leading figures are incorrect up to page 81, at which point the mistake was apparently discovered and thereafter corrected; even so the incorrect fount is used in several cases on p. 85. The occurrence is by no means uniform; for instance, it actually spreads to the argument column on pages 32 and 33, while it is correct in the sin and tan columns on p. 43–59. Although the difference between the figures is quite noticeable when compared closely, it is too slight to be seen in normal use and it does not detract from the general appearance of the volume. This mixture occurs in both the first and third editions.

Page	Location	*For*	*Read*
8[1,2]	cotg 0°27′03″, 04″	127 and 126 to be lowered one line.	
12[1]	diff. cotg 0°40′57″/58″	24	34
48[1]	heading, last column	sec	cos
49[1,2]	sec 4°10′30″	2261	2661
*49	p.p. table for 86 × 8	68.7	68.8
55[1,2]	argument 5°14′	50″	30″
61[1,2]	sec 6°17′00″	0.	1.
*65	diff. cos 6°55′40″/50″	5	6
84[1]	argument 79°52′	49″	40″
90[1]	sec 11°03′20″	9909	8909
110[1]	argument 75°30′	20″	30″
118[1]	diff. sec 15°45′40″/50″	44	14
123[1]	diff. sin 16°38′50″/60″	57	47
127[1]	diff. tang 17°18′40″/50″	33	53
129[1,2]	sec 17°39′40″	8464	9464
156[1]	tang 22°06′00″	4.	0.
157[1]	diff. sin 22°10′00″/10″		45
169[1]	diff. sec 24°17′00″/10″	4	24
*170	p.p. table for 58 × 9	52.	52.2
*190[1]	tang 27°44′00″	5.	0.
233[1]	diff. tang 34°56′00″/10″	22	72
234[1,2]	heading, fourth, fifth cols.	cotg cosec	cosec cotg
261[1]	diff. sin 39°37′10″/20″	28	38
266[1]	diff. cotg 40°24′00″/10″	155	115
*269	heading, sin	the s is upside down	
278[1]	cotg 42°23′00″	0.	1.
291[1]	diff. sin 44°30′30″/40″	55	35

D. H. Sadler

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[1] These 23 errata were reported in *Astr. Nach.* (1939).—EDITOR.
[2] These 6 errata were reported in *MTAC* (1943 and 1947) by L. J. C.—EDITOR.

136. J. T. PETERS, *Siebenstellige Logarithmen der trigonometrischen Funktionen von 0° bis 90° für jedes Tausendstel des Grades.* Berlin, Preussische Landesaufnahme, 1921.

The following two errors were found during the comparison of this table with proofs of a new CHAMBERS' six-figure table. The comparison covered log sines and log tangents for 0°(0.°001)5°.

P. 74, log tan 3°.619, *for* 8.800 0440, *read* 8.801 0440;
P. 80, log sin 3°.933 *for* 8.836 2102, *read* 8.836 2602.

Neither of these errors occurs in the six-figure or ten-figure tables by Peters of the same functions, published in 1922 and 1919 respectively, and having the same argument.

L. J. C.

137. J. T. PETERS, *Zehnstellige Logarithmentafel*, v. 2, Berlin, 1919.

On p. 762, 38°.000–38°.050, the third digit from the left in the difference column for log tang and log cotg should be 6 instead of 7. Thus for the first difference read 156237, not 157237. This error persists for the entire page.

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UNPUBLISHED MATHEMATICAL TABLES

Reference has been made to an unpublished mathematical table by John Todd in QR35.

69[A].—H. E. SALZER, *Table of Factorials.* Manuscript in possession of the author, NBSCL.

The manuscript is a table of $N!$ for $N = [1(1)1000; 16S]$. Although only 16S are guaranteed, the entries are almost certainly correct to 17S, and there is a high probability that they are good to even 18S, with an error no more than several units in the eighteenth significant figure.

H. E. SALZER

70[E].—G. W. SPENCELEY, *Tables of Hyperbolic Functions.* Manuscript in possession of the author, Miami University, Oxford, Ohio.

I have computed tables of $\sinh x$ and $\cosh x$ for $x = [1°(1°)1080°; 28S]$.

G. W. SPENCELEY

71[I].—H. E. SALZER, *Tables of Coefficients for Checking and Interpolation of Functions Tabulated at Certain Irregular Logarithmic Intervals.* Tables in possession of the author.

Many small tables exist of functions that behave as polynomials in $\log x$. Such functions are usually tabulated for arguments proportional to

1, 2, 5, 10, 20, 50, 100, 200, 500, 1000.