$$335\ 54432\, x^{25} = H_{25} + 600\, H_{23} + 1\ 51800\, H_{21} + 212\ 52000\, H_{19}$$
$$+ 18170\ 46000\, H_{17} + 9\ 88473\ 02400\, H_{15} + 345\ 96555\ 84000\, H_{13}$$
$$+ 7710\ 08958\ 72000\, H_{11} + 1\ 06013\ 73182\ 40000\, H_{9}$$
$$+ 8\ 48109\ 85459\ 20000\, H_{7} + 35\ 62061\ 38928\ 64000\, H_{5}$$
$$+ 64\ 76475\ 25324\ 80000\, H_{3} + 32\ 38237\ 62662\ 40000\, H_{1}$$

$$671\ 08864\, x^{26} = H_{26} + 650\, H_{24} + 1\ 79400\, H_{22} + 276\ 27600\, H_{20}$$
$$+ 26246\ 22000\, H_{18} + 16\ 06268\ 66400\, H_{16} + 642\ 50746\ 56000\, H_{14}$$
$$+ 16705\ 19410\ 56000\, H_{12} + 2\ 75635\ 70274\ 24000\, H_{10}$$
$$+ 27\ 56357\ 02742\ 40000\, H_{8} + 154\ 35599\ 35357\ 44000\, H_{6}$$
$$+ 420\ 97089\ 14611\ 20000\, H_{4} + 420\ 97089\ 14611\ 20000\, H_{2}$$
$$+ 64\ 76475\ 25324\ 80000\, H_{0}$$

$$1342\ 17728\, x^{27} = H_{27} + 702\, H_{25} + 2\ 10600\, H_{23} + 355\ 21200\, H_{21}$$
$$+ 37297\ 26000\, H_{19} + 25\ 51132\ 58400\, H_{17} + 1156\ 51343\ 80800\, H_{15}$$
$$+ 34695\ 40314\ 24000\, H_{13} + 6\ 76560\ 36127\ 68000\, H_{11}$$
$$+ 82\ 69071\ 08227\ 20000\, H_{9} + 595\ 37311\ 79235\ 84000\, H_{7}$$
$$+ 2273\ 24281\ 38900\ 48000\, H_{5} + 3788\ 73802\ 31500\ 80000\, H_{3}$$
$$+ 1748\ 64831\ 83769\ 60000\, H_{1}$$

$$2684\ 35456\, x^{28} = H_{28} + 756\, H_{26} + 2\ 45700\, H_{24} + 452\ 08800\, H_{22}$$
$$+ 52216\ 16400\, H_{20} + 39\ 68428\ 46400\, H_{18} + 2023\ 89851\ 66400\, H_{16}$$
$$+ 69390\ 80628\ 48000\, H_{14} + 15\ 78640\ 84297\ 92000\, H_{12}$$
$$+ 231\ 53399\ 03036\ 16000\, H_{10} + 2083\ 80591\ 27325\ 44000\, H_{8}$$
$$+ 10608\ 46646\ 48202\ 24000\, H_{6} + 26521\ 16616\ 20505\ 60000\, H_{4}$$
$$+ 24481\ 07645\ 72774\ 40000\, H_{2} + 3497\ 29663\ 67539\ 20000\, H_{0}$$

$$5368\ 70912\, x^{29} = H_{29} + 812\, H_{27} + 2\ 85012\, H_{25} + 570\ 02400\, H_{23}$$
$$+ 72108\ 03600\, H_{21} + 60\ 57075\ 02400\, H_{19} + 3452\ 53276\ 36800\, H_{17}$$
$$+ 1\ 34155\ 55881\ 72800\, H_{15} + 35\ 21583\ 41895\ 36000\, H_{13}$$
$$+ 610\ 40779\ 26186\ 24000\, H_{11} + 6714\ 48571\ 88048\ 64000\, H_{9}$$
$$+ 43949\ 36106\ 85409\ 28000\, H_{7} + 1\ 53822\ 76373\ 98932\ 48000\, H_{5}$$
$$+ 2\ 36650\ 40575\ 36819\ 20000\, H_{3} + 1\ 01421\ 60246\ 58636\ 80000\, H_{1}$$

$$10737\ 41824\, x^{30} = H_{30} + 870\, H_{28} + 3\ 28860\, H_{26} + 712\ 53000\, H_{24}$$
$$+ 98329\ 14000\, H_{22} + 90\ 85612\ 53600\, H_{20} + 5754\ 22127\ 28000\, H_{18}$$
$$+ 2\ 51541\ 67278\ 24000\, H_{16} + 75\ 46250\ 18347\ 20000\, H_{14}$$
$$+ 1526\ 01948\ 15465\ 60000\, H_{12} + 20143\ 45715\ 64145\ 92000\, H_{10}$$
$$+ 1\ 64810\ 10400\ 70284\ 80000\, H_{8} + 7\ 69113\ 81869\ 94662\ 40000\, H_{6}$$
$$+ 17\ 74878\ 04315\ 26144\ 00000\, H_{4} + 15\ 21324\ 03698\ 79552\ 00000\, H_{2}$$
$$+ 2\ 02843\ 20493\ 17273\ 60000\, H_{0}.$$

HERBERT E. SALZER

NBSCL

RECENT MATHEMATICAL TABLES

519[A].—A. J. SACHS, "Babylonian mathematical texts, I. Reciprocals of regular sexagesimal numbers," *Jn. Cuneiform Studies*, v. 1, 1947, p. 219–240. 21.5 × 27.9 cm.

Regular sexagesimal numbers are those whose reciprocals may be expressed in a finite number of terms. In Old-Babylonian table texts (say, 1700 B.C.) the object of a set of multiplication tables was not only to yield the results of any multiplication, but also to give the multiples of reciprocals commonly used in division. "The existence of a multiplication table for the three-place number 44, 26, 40, for example, makes sense only in the light of the fact that 44, 26, 40 is the reciprocal of 1, 21."

Dr. Sachs has established the standard technique employed in the Old-Babylonian period to find the reciprocal of any regular number which is not contained in the standard reciprocal table (say, 2, 5 or 23, 43, 49, 41, 15).

Suppose c denotes a regular number whose reciprocal one wishes to find. Then choose two numbers a and b such that $c = a + b$ and a is a number which is found in the standard table of reciprocals. Using the ordinary notation $\bar{n} = 1/n$, it is evident that $\bar{c} = \bar{a}(\overline{1 + b\bar{a}})$.

In illustration of this analysis Dr. Sachs gives a transcription (p. 237) of the 21 paragraphs of tablet CBS 1215 of the Philadelphia collection. This is a table of the reciprocals of $2^n(2, 5)$ for $n = 0(1)20$, [that is 125 to 131 072 000], with the intermediate steps of the work; also for the reciprocals of the final results, showing that the number started with is found. All this material is rearranged as a table (p. 238–240), 7 columns on each page, headed $c, b, a, \bar{a}, 1 + \bar{a}b, \overline{1 + \bar{a}b}, \bar{c}$. For example, in no. 1, $c = 2; 5$, $\bar{c}$ is found to be 0; 28, 48. In no. 19, second part, $c = 0; 0, 0, 0, 23, 43, 49, 41, 15$, and $\bar{c}$ is found to be 2, 31, 42, 13; 20. In each part of the latter example three applications of the formula are necessary, after reaching the column $1 + \bar{a}b$.

This paper is the first of a series of articles dealing with unpublished Babylonian mathematical texts, chiefly at the University Museum in Philadelphia.

R. C. A.

520[A, J].—John Q. Stewart, "Empirical mathematical rules concerning the distribution and equilibrium of population." *Geographical Review*, v. 37, 1947, p. 461–485. 17 × 25.5 cm.

On p. 465 is a 2D table, 15 of the 29 different values of which were given in more extended form by J. W. L. Glaisher, "On the constants that occur in certain summations by Bernoulli's series," London Math. Soc., *Proc.*, v. 4, 1872, p. 55. These 15 values are of $\Phi(-1/m)$ for $m = 1(1)5$, 10, and $\Phi(-1 - 1/m)$, for $m = 1(1)9$, each to 6D. The remaining 14 values may be calculated from $\Phi(-1 - 1/m)$ for $m = 3/4$ and 2; and from $\Phi(-1/m)$, for $1/m = .3, .4, .6, .7, .9, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{5}{7}, \frac{10}{13}$. $\Phi(-1 - 1/m) = 1^{-1-1/m} + 2^{-1-1/m} + 3^{-1-1/m} + \cdots$ and $\Phi(-1/m) = 1^{-1/m} + 2^{-1/m} + \cdots + x^{-1/m} - mx^{1-1/m}/(m - 1) - \frac{1}{2}x^{-1/m} + x^{-1-1/m}/12m - (m + 1)(2m + 1)x^{-3-1/m}/720m^3 + \cdots, x = 10$.

R. C. A.

521[B, C, D, E, F, H].—J. H. Lambert, *Opera Mathematica. Volumen secundum: Commentationes Arithmeticae Algebraicae et Analyticae, pars altera.* Zürich, Füssli, 1948, xxx, ii, 324 p. 15.5 × 22.9 cm.

This second volume of the works of Lambert contains the conclusion of his work in the fields of Arithmetic, Algebra and Analysis. In reviewing the first volume (*MTAC*, v. 2, p. 339–341) we inadvertently quoted the German title page on the dust wrapper instead of the Latin title page of the volume. In the volume under review there are only three parts of interest to us because of tabular material.

I. We shall first consider Lambert's memoir on hyperbolic functions, "Observations trigonométriques," p. 245–269, presented to the Berlin Academy of Sciences in 1768; the table is on page 268–269. The notation we are going to use is explained elsewhere in this issue, in N94, on Lambertian Functions. The hyperbolic sector $u = \ln \tan (45° + \frac{1}{2}\omega)$, $x^2 - y^2 = 1$ or $x = \cosh u$, $y = \sinh u$ being equations of the hyperbola. For the circle the corresponding equations are $x^2 + y^2 = 1$ or $x = \cos \phi$, $y = \sin \phi$; $\tan \phi = \sin \omega$, $\tan \omega = \sinh u$, $\sec \omega = \cosh u$. In the table are 9 columns; the first is $\omega = 0(1°)90°$; the second column contains the values of the corresponding hyperbolic sector u (according to Lambert). Lambert really gives the values of $\log \tan (45° + \frac{1}{2}\omega)$, so that his values must be multiplied by 2.30258509 in order to give the true approximate values of the hyperbolic sectors u corresponding to successive values of ω. It is curious that Professor Speiser overlooked this Lambert error which makes the values in the following four succeeding columns of the table impossible to check: $\sinh u$ and $\log \sinh u$ (cols. 3, 5), $\cosh u$ and $\log \cosh u$ (cols. 4, 6). Column 9 gives the values of ϕ to the nearest tenth of a second corresponding to each value of ω; columns 7 and 8 give respectively the values of tan ϕ and log tan ϕ. The values in cols. 2–8 are all to 7D. The erroneous col. 2 appears also in Lambert's interesting volume of tables,

II. *Zusätze zu den logarithmischen und trigonometrischen Tabellen zur Erleichterung und Abkürzung der bey Anwendung der Mathematik vorfallenden Berechnungen.* Berlin, 1770, 4, 98, 210 p. We have noted some facts concerning the origin of this volume in our previous review (*l.c.*, p. 340). In the volume before us it occupies p. 1–69, 75–111 + a title-page leaf. It contains 44 tables which are reproduced in full except for 8, namely: T.I, the smallest divisors of numbers not divisible by 2, 3 or 5, from 1 to 102000, filling p. 2–69 of the original, only p. 2–3 are reprinted; T.VI, prime numbers 1–102000 (p. 73–117), only p. 73; T.XV, hyperbolic logarithms of 1.01(.01)10 to 7D (p. 125–133), only p. 125; T.XXV, $n \sin A$, $n = 1(1)9, A = [1°(1°)90°; 5D]$ (p. 152–157), only p. 152–153; T.XXXII, hyperbolic functions (p. 176–181), only p. 176–177; T.XXXV, N^2, $N = 1(1)1000$ (p. 184–189), only 184–185; T.XXXVI, N^3, $N = 1(1)1000$ (p. 190–195), only p. 190–191; T.XL, N^p, $p = 1(1)11$, $N = .01(.01)1$ (p. 202–207), only p. 202–203. All other tabular pages are printed in facsimile. Since the volume is so rare it is too bad that it was not completely reproduced. Some of the tables not previously mentioned are the following: T.VII–IX, of 2^n, $n = 1(1)70$, 3^n and 5^n, each for $n = 1(1)50$. T.XIX gives the exact values of $\sin 3n°$, $n = 1(1)30$. T.XXIII, lengths of arcs of a unit circle to 27D, subtended by central angles 1°(1°)100°, 120°(30°)270°, 330°, 360°. T.XXIX, for solving the equations $\pm x \mp x^3 = a$ (see *MTAC*, v. 2, p. 28–29). T.XXXVII, 30 terms of the first 12 figurate numbers: x, $\frac{1}{2}x(x+1)$, $\frac{1}{6}x(x+1)(x+2)$, etc. T.XLII, various rational approximations to $N^{\frac{1}{2}}$, $N = 2(1)12$; T.XLIV, decimal representation of the coefficients of the first 15 terms in $(1+x)^{+\frac{1}{2}}$, and $(1+x)^{-\frac{1}{2}}$.

A posthumous Latin edition of Lambert's Tables was published in 1798 under the direction of Anton Felkel, by the Royal Academy of Sciences at Lisbon: *Supplementa Tabularum Logarithmicarum et Trigonometricarum auspiciis Almae Academiae Regiae Scientiarum Olisiponensis cum versione introductionis germanicae in latinum sermonem secundum ultima auctoris consilia amplificata.* lxxvi, 203 p. + plate. In T.I, where Lambert gave only the least factor, Felkel provides all prime factors except the greatest, the least figures being printed in figures, and the others denoted by letters. The erroneous column in T.XXXII is perpetuated here.

III. We come now to p. 70–73, taken from the preface of Lambert's *Beyträge zum Gebrauche der Mathematik und deren Anwendung*, Berlin, 1772. There is here a table of 70 corrections, found by Wolfram, for T.I of Lambert's *Zusätze.* These have all been corrected in the *Supplementa*. It is also noted that in T.VI the prime number 91183 is missing; this is also wanting in the *Supplementa*. Various other corrigenda for other parts of the *Zusätze* are also listed.

Lambert's appeals to others, for calculating large factor tables, resulted in activity on the part of five persons, Oberreit, von Stamford, Rosenthal, Felkel, and Hindenburg. The correspondence in this connection is printed in the fifth volume of *Joh. Heinrich Lamberts . . . deutsche gelehrter Briefwechsel. Herausgegeben von Joh. Bernoulli*, Berlin, 1785. Felkel alone has left any record of his work; see *Scripta Mathematica*, v. 4, 1936, p. 336–337.

Lambert refers briefly to a number of tables, concerning which we may give further information in a few notes.

Büchner's table of squares and cubes from 1 to 12000 (Lambert, v.2, p. 4, 7, 8, 58, 70). J. P. Büchner, *Tabula Radicum*, Nuremberg, 1701.

Joncourt published a table of *Trigonalzahlen* (Lambert, v. 2, p. 4). This is É. de Joncourt, *Traité sur la Nature et sur les Principaux Usages de la Plus Simple Espèce de Nombres Trigonaux.* The Hague, 1762. Also Latin edition *De Natura et Praeclaro Usu simplicissimae speciei Numerorum Trigonalium*, The Hague, 1862, viii, 28, 224, 7 p. (This is the book which Prof. Speiser identifies as "Joncourt: *de la nature des nombres trigonaux*, 1742.")

Krüger-Jäger list of primes (Lambert, v. 1, p. 117; v. 2, p. 6, 7, 8, 14). J. G. Krüger, *Gedanken von der Algebra nebst den Primzahlen von 1 bis 1000000 . . .*, Halle, 1746. Algebra, 124 p.; list of primes, 47 p. The primes are from 1 to 100999, the title being incorrect. Lambert tells us (v. 2, p. 6) that Krüger received the table from Peter Jäger.

Anjema factor table from 1 to 10000 (Lambert, v. 1, p. 117–118; v. 2, p. 5, 7, 23). See *MTAC*, v. 2, p. 340, 374.

Poetius (Lambert, v. 1, p. 117, 118; v. 2, p. 5, 7, 14). J. M. POETIUS, *Rekenkunst mit den parallelen Algebra, Arithmetica*. Appendix, "anatomica numerorum." Leipzig, 1728. This appendix of primes and divisors up to 10000 is in [G. F. RICHTER] *Vollständigen mathematischen Lexici, zweiter Theil*. Leipzig, 1742, p. 530–543.

"Pell's" factor table (Lambert, v. 1, p. 118; v. 2, p. 5, 6, 7, 11, 14). Lambert erroneously attributes to Pell a factor table up to 100 000 (multiples of 2 and 5 omitted), in the compilation of which Pell had no part. It is interesting that in v. 2, p. 70, Lambert refers to his own factor table as a "Pell table." The facts are as follows: A Swiss, J. H. RAHN, wrote a work, *Teutsche Algebra oder Algebraische Rechenkunst*, Zürich, 1659, of which there was an English translation: *An Introduction to Algebra. Translated out of the High-Dutch into English, by Thomas Brancker . . . Much Altered and Augmented by D. P.* [Dr. John Pell]. *Also A Table of Odd Numbers less than One Hundred Thousand, showing Those that are Incomposit, and Resolving the rest into their Factors . . . Supputated by the same Tho. Brancker*. London, 1668, viii, 198, 50 p. + 1 plate. Thus we see by the title page itself that the original table of Rahn up to 24000 was extended by THOMAS BRANCKER up to 100 000. Also, the table is headed: "Tho. Branker's (*sic*) Table of Incomposit numbers, less than 100 000." Brancker's table is reprinted in F. MASERES, *The Doctrine of Permutations and Combinations . . .*, London, 1795, p. 366–416. From 20000 to 100000 it is also reprinted in E. HINKLEY, *Tables of the Prime Numbers, and Prime Factors of the Composite Numbers, from 1 to 100 000*. Baltimore, 1853, p. 165–205.

The reader interested in fuller information on early factor tables should turn to JAMES GLAISHER, *Factor Table for the Fourth Million*, London, 1879, p. 17–28; and J. W. L. GLAISHER, "On factor tables, with an account of the mode of formation of the factor table for the fourth million," Camb. Phil. Soc., *Proc.*, v. 3, 1878, p. 99–138.

In our previous review we referred (*MTAC*, v. 2, p. 341) to results of WALLIS (1685) giving the values of convergents of 17 terms of the continued fraction for $1/\pi = \frac{1}{3}+\frac{1}{7}+\frac{1}{15}+\frac{1}{1}+\frac{1}{292}+\frac{1}{1}+\frac{1}{84}+\frac{1}{2}+\frac{1}{1}+\frac{1}{1}+\frac{1}{15}+\frac{1}{3}+\frac{1}{13}+\frac{1}{1}+\frac{1}{4}+\frac{1}{2}+\frac{1}{6}+\frac{1}{6}+\frac{1}{99}+\frac{1}{1}+\frac{1}{2}+\frac{1}{2}+\frac{1}{6} \cdots$, that is, through the second $\frac{1}{6}$ before $\frac{1}{99}$. I have since noticed that WOLFRAM in a letter to Lambert on 3 Aug. 1772 gave (87 years after Wallis) this same value (*Joh. Heinrich Lambert deutscher gelehrter Briefwechsel* hrsg. JOH. BERNOULLI, v. 4, 1784, p. 445–450) and also the next convergent, for the first $\frac{1}{6}$ before $\frac{1}{99}$, namely:

18 50401 87797 33719 17511:5 89001 21117 19765 29866
= 3.14159 26535 89793 23846 26433 83279 50288 41971 6897, which is correct to 41D.

R. C. A.

522[D].—J. PETERS, *Sechsstellige Tafel der trigonometrischen Funktionen enthaltend die Werte der sechs trigonometrischen Funktionen von zehn zu zehn Bogensekunden des in 90° geteilten Quadranten und die Werte der Kotangente und Kosekante für jede Bogensekunde von 0°0′ bis 1°20′*. Third ed. Bonn, Dümmler, 1946, vi, 293 p. 19 × 26 cm.

This is a photographic reprint of a corrected first edition, Berlin and Bonn, Dümmler, but some errors still remain. A corrected second edition appeared in 1939. The first Russian edition came out at Moscow, in 1937, and the second Russian edition in 1938. For other details see MTE **135**, elsewhere in this issue.

523[D, R].—RUDOLF BOSSHARDT, *Tafeln zum Abstecken von Kreisbogen nach Polarkoordinaten*, Stuttgart, Konrad Wittwer, 1940, iv, 106 p. 12.5 × 17.7 cm.

Consider a circular arc ABC with center O. Let the tangents at A and C meet in T, and let OT meet the arc in B, and the chord of the arc CA in E; also let $OA = r$, $\angle COA = \alpha$.

Then (1) $AT = CT = r \tan \frac{1}{2}\alpha$; (2) arc $ABC = r\pi\alpha$ (sexagesimal)/180; (3) $= r\pi\alpha$ (centesimal)/200; (4) $BT = r[\sec(\frac{1}{2}\alpha) - 1]$; (5) $AE = CE = r \sin \frac{1}{2}\alpha$; (6) $EB = r(1 - \cos\frac{1}{2}\alpha)$. For $r = 100$, Table I gives 3D values of (1), (2), (4), (5), (6), each with Δ, for α(cent. degrees) $= 1^g(1^g)120^g(0^g.5)180^g$; and (2), (5), (6), each with Δ, for $\alpha = 180^g(1^g)200^g$.

Let $AC = s$, $\angle BAT = \psi$, and arc $ABC = b$; then with varying parameter range b, and $r = 12(1)50(2)100(5)200(10)500(20)1000(50)1500(100)5000$, Table II gives ψ(cent.) and ψ(sexag.), each to the nearest second; also $b - s$ (2D) with Δ(3D). For $r = 12 - 17$, $b = 1$, 2(2)24(1)45; for $r = 105$–155, $b = 1(5)70(2)100(5)140$; and for $r = 200$–5000, $b = 1$, 10(10)160.

Table III is for changing from centesimal to sexagesimal division, and conversely. The remaining pages are devoted to uses of the tables with surveying instruments and to a 23-item Literature-List.

The author expresses the view that he has made a new contribution to surveying tabular material.

R. C. A.

524[E].—NBSCL, *Tables of the Exponential Function e^x*. Second edition (211 copies). Washington, D. C., U. S. Govt. Printing Office, 1947. [xviii], 549 p. 20.7 × 27 cm. For sale by the Superintendent of Documents, Washington, D. C. $3.00. On the back of the volume MT2 is printed; presumably MT1 was *Table of the First Ten Powers of the Integers from 1 to 1,000.*

This volume of which the first edition appeared in 1939 (see *MTAC*, v. 1, p. 438), was the first of the extensive volumes which this computing group prepared. We have already noted that it has long been out of print (*MTAC*, v. 3, p. 65), and that six slips in it had been found (*MTAC*, v. 1, p. 161, 198; v. 2, p. 314, 352). All of these have been corrected in the new edition which is appreciably thinner than the old edition, on account of thinner, but good, paper having been used in its composition. The binding is very substantial. On the whole the printed pages of the table are not quite as black in the review copy as in the copy of the first edition now before me.

The number of pages in the volume is the same as formerly. The new title-page has been considerably changed. The old material of p. [vii–viii] has been eliminated, and a brief new "Preface to the second edition" by Dr. Lowan, chief of the Computation Laboratory, appears on p. [vii] of the new edition. Our total of 549 pages differs from the 535 on the last table page of the volume through the fact that title-pages preceding Tables II–VIII are not numbered. The title-page for Table I was counted in [xviii]. The increase of price of this excellent volume from $2.00 to $3.00 still leaves it an extraordinarily cheap product for these days, and will probably do little to retard its further rapid sale.

R. C. A.

525[E].—Jørgen Rybner, *Tabeller til Brug ved Løsning af Ligningen $tgh(b + ja) = r\underline{/\phi}$*. Copenhagen, Jul. Gjellerups Forlag, 1943, 8 p. 17 × 25 cm. Offset print.

These tables are of

$$f(r) = 2r/(1 + r^2), \qquad g(r) = 2r/(1 - r^2),$$

for $r = [0(.001).999; 6D]$, Δ.

If $r > 1$, take $1/r$ and $f(1/r) = f(r)$, $g(1/r) = -g(r)$.

If $\tanh(b + ia) = r(\cos\phi + i \sin\phi)$, $\tanh 2b = f(r)\cos\phi$, $\tan 2a = g(r)\sin\phi$. Compare the 16 nomograms for $\tanh(b + ia)$ in J. Rybner, *Nomogrammer over komplekse hyperbolske Funktioner*, 1947 (RMT 526).

526[E, H].—JØRGEN RYBNER, *Nomogrammer over komplekse hyperbolske Funktioner. Nomograms of Complex Hyperbolic Functions.* Copenhagen, Jul. Gjellerups Forlag, 1947, 36 p. text + 54 leaves printed on one side with nomogram material + plastic rule (3 × 31.1 cm.) with hair line. 21.3 × 31 cm.

The individual investigator with limited facilities at his disposal has always found the evaluation of a hyperbolic function of a complex variable to be a tedious and time consuming chore. Professor Rybner has taken great care to produce an accurate set of steel engraved nomograms covering the sinh, cosh and tanh (designated tgh) which should be very useful to such a worker who has a large number of cases to consider.

There are 13 separate sheets to cover the cosh with an accuracy approaching three figures. The cosh is expressed in Cartesian form by the equation

$$\cosh (b + ia) = p + iq.$$

The values of b, which in electrical work corresponds to attenuation, is expressed in both nepers and decibels, while a, which corresponds to phase shift, is given in degrees and radians, each divided on a decimal basis. Each nomogram carries the formulae for computation and the rules concerning sign and periodicity.

The sinh function also covers 13 sheets and is treated similarly.

The tanh function covers 16 sheets and is given in the polar form

$$\tanh (b + ia) = r\underline{/\theta}.$$

Again b is expressed in both nepers and decibels but a is given in radians only; θ is expressed in degrees and minutes or degrees and decimals. Suitable formulae for computation, for sign and periodicity are included on each sheet. The reason for expressing tanh in polar form rather than in the rectangular form of the sinh and cosh is not given. The polar form however is convenient for the transmission engineer who frequently wants to compute short circuit or open circuit impedances.

The text, which is given in both Danish and English, includes a number of formulae for circular and hyperbolic functions and also some formulae for four-terminal networks and transmission lines. Approximation formulae which appear to be new are given for very small values of b.

In addition to the above, certain nomograms useful to the engineer are also included. These are concerned with

1. Conversion from rectangular to polar form i.e., $x + iy = r\underline{/\theta}$.
2. Conversion from $R\underline{/a}$ to $(1 + r/\theta)$.
3. Reflection loss and reflection phase shift.
4. Resonance circuits and filters.

The ring binding at the top of the nomograms is somewhat less convenient than conventional binding at the side of a sheet.

The general usefulness of the nomograms would be enhanced by typical examples.

A number of errors in notation were observed in a somewhat cursory turning of the pages, as follows:

p. 32, l. −5, *for* B_2, B_2 *read* B_1, B_2.

sheet 10 for tanh, at the lower right, *for* $b = 0.75 - 0.5$, *read* $b = 0.25 - 0.5$.

sheets 1, 3, 6, 9, 10, 11, 12, 13, 14, 15, 16 at the top:

$$\textit{for } \operatorname{arctg} \frac{\sin 2a}{\sinh 2a}, \qquad \textit{read } \operatorname{arctg} \frac{\sin 2a}{\sinh 2b}.$$

The need for the tanh nomograms was realized many years ago by transmission engineers. Aside from Professor KENNELLY's extensive and well-known *Atlas*,[1] which uses r and θ as the basic coordinate system, the only other published set of charts known to the writer is that of Professor HENRY E. HARTIG, "Charts for transmission line problems," *Physics*, v. 1, 1931, p. 380–387, which interchanges the coordinates and contours of the *Atlas*. Professor Hartig's charts use a neper-quadrant coordinate system with $R/\underline{\theta}$ contours for sinh and cosh. For the tanh the contours are $R/\underline{\theta}$ in the 1st, 4th, 5th, etc. octants and $R^{-1}/\underline{\theta}$ contours in the 2nd, 3rd, 6th, 7th, etc. octants. With these charts the answer, as a function of length, always lies on a straight line. They also have the virtue of clarifying the periodic nature of the functions.

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New York

[1] A. E. Kennelly, *Chart Atlas of Complex Hyperbolic and Circular Functions.* Third ed. rev. and enl., Cambridge, Mass., 1924, 66 p. Kennelly published also *Tables of Complex Hyperbolic and Circular Functions.* Second ed. rev. and enl., Cambridge, 1921. vi, 240 p.

527[E, S].—Parry Moon, **A.** "A table of Planck's function from 3500 to 8000°K," *Jn. Math. Phys.*, v. 16, 1937, p. 133–157. 17.5 × 25.4 cm. Also M. I. T., E. E. Dept., *Contribution* no. 131, 1938. **B.** *A Table of Planck's Function 2000 to 3500°K*, Cambridge, Mass., Mass. Inst. Techn., Dept. Electr. Engineering, 1947, 80 p. 15.2 × 22.8 cm. **C.** "A table of Planckian radiation," Opt. Soc. Amer., *Jn.*, v. 38, 1948, p. 291–294.

The growing use, in science and engineering, of Planck's equation for the radiation from a blackbody emphasizes the need for more satisfactory tabulated values of this function. In photometry and radiometry, stress is being laid to an increasing extent upon calculations based on blackbody radiation. In colorimetry a similar movement is in progress—a movement that has been particularly marked since 1931, when the Commission Internationale de l'Éclairage standard distribution coefficients were introduced. A number of tables of Planck's function are available, but all have disadvantages when values are to be obtained with the minimum of calculation.

Planck's equation is

$$J(\lambda) = C_1\lambda^{-5}[e^{C_2/\lambda T} - 1]^{-1}, \tag{1}$$

where λ = wavelength (micron), T = absolute temperature (°K), $C_1 = 36970$, $C_2 = 14320$; C_1 and C_2 are in accordance with the international temperature scale;[9] $J(\lambda)$ = spectral radiosity of a blackbody (watts cm.$^{-2}$ micron^{-1}) at wavelength λ.

If $x = \lambda T/C_2$, $yx^5 = J(\lambda)\lambda^5/C_1$, equation (1) becomes

$$yx^5 = [e^{1/x} - 1]^{-1}. \tag{2}$$

If $C'_2 = .4342944819 \cdot C_2 = 6219.096981$,

$$J(\lambda) = 36970\lambda^{-5}[10^{6219.096981/\lambda T} - 1]^{-1}. \tag{3}$$

Actual computations were made from this equation. T.I. in **A** is for C'_2/T, for T = 2000°K-(10°)3500°(100°)10000°(1000°)20000°K, T.2 is of C_1/λ^5, for λ = .26(.01)1(.1)3(1)8; T.3 is a double entry table for λ and T, of $J(\lambda)$, for

$$\lambda = [.26(.01).75(.05)1(.1)3;\ 5S],\ \Delta, \text{ and } T = 3500(100°)8000°\text{K}.$$

It is believed that in general the values of this table are correct to the last figure, though because of rounding process there may be an error in a few instances of as much as 5 in the sixth figure.

In **B** the table is of $J(\lambda)$ for λ = [.38(.01).76; 8S], Δ, and T = 2000(10°)3500°K. The values were computed to 10 digits by means of electrically-driven computing machines. The values were checked by use of fifth differences, and were then rounded off to the 8 figures given in the table.

This new table should be particularly useful, since it embraces the region of filament temperatures of incandescent lamps, and the two tables extend from spectral distributions approximating the radiation from a carbon-filament lamp to those approximating the radiation from an overcast sky.

The tables in **A** and **B** were calculated under Mr. Moon's direction by more than a score of computers. In using (3) the powers of 10 were read directly from Peters' *Zehnstellige Logarithmen*, v. 1.

Tables based on the relation (2) have been given by JAHNKE & EMDE,[1] FREHAFER & SNOW,[2] FABRY,[3] YAMAUTI & OKAMATU.[4] Since there is only one independent variable, the table can be condensed into comparatively small space while still covering the entire range of wavelength and temperature. These tables, however, are anything but satisfactory. Interpolations are nearly always necessary; and the labor entailed in the interpolations, as well as in the computation of x from the interpolated values of λ and T and the computation of $J(\lambda)$ from the interpolated value of y, is considerable. In fact, comparative tests showed that $J(\lambda)$ could be computed directly from (1) more easily than it could be obtained from Yamauti & Okamatu tables,[4] a logarithm table and a modern calculating machine being used in both cases.

Present needs require a table of double entry, with the two independent variables λ and T. Tables of this kind have been computed by FORSYTHE,[5] FOWLE,[6] FREHAFER & SNOW,[7] and SKOGLAND.[8] These tables are based on the values of either 14330 or 14350 for C_2, instead of the international standard value[9] of 14320. The change to 14320 is easily made with existing tables, but requires additional computation which tends to reduce the serviceability of the tables. The Skogland results are being used extensively, both in this country and abroad, but they cover only a limited range of temperature and are confined to the visible spectrum. The Fowle table and Frehafer & Snow table are not subject to these restrictions in temperature and wavelength but the values are in general calculated to only 2 or 3 significant figures.

Extracts from text

EDITORIAL NOTE: The radiation formula of MAX K. E. L. PLANCK (1858–1947) was first published in "Ueber eine Verbesserung der Wienschen Spektralgleichung," Deutsche phys. Gesell., *Verhandlungen*, v. 2, 1900, p. 202. This note is reprinted in the *Ostwald's Klassiker d. exakten Wissenschaften*, v. 206, 1923, p. 53–55. In the bibliography of the function a reference may be given also to NBSCL, *Miscellaneous Physical Tables. Planck's Radiation Functions* . . ., 1941.

[1] E. JAHNKE & F. EMDE, *Tables of Functions*, second ed., Leipzig, 1933, p. 45–47 [Also in later editions.—EDITOR]

[2] MABEL K. FREHAFER & C. SNOW, *Tables and Graphs for Facilitating Computations of Spectral Energy Distribution*. Nat. Bureau of Standards, *Misc. Publs.*, no. 56, 1925, T. I.

[3] C. FABRY, *Introduction générale à la Photométrie*, Paris, 1927.

[4] Z. YAMAUTI & M. OKAMATU, "Tables for Planck's radiation formula," Tokyo, Electrotechn. Lab., *Researches*, no. 395, 1936, part I, 39 p. (English abstract); no. 402, 1937, part II, 51 p. in English.

[5] W. E. FORSYTHE, "1919 report of standards committee on pyrometry," Optical Soc. Amer., *Jn.*, v. 4, 1920, p. 331.

[6] F. E. FOWLE, "Radiation from a perfect (blackbody) radiator," in Nat. Res. Council, *Intern. Critical Tables*, v. 5, 1929, p. 238–243.

[7] MABEL K. FREHAFER & C. SNOW, *idem*, T.II.

[8] J. F. SKOGLAND, *Tables of Spectral Energy Distribution*. Nat. Bureau of Standards, *Misc. Publs.*, no. 86, 1929.

[9] G. K. BURGESS, "The international temperature scale," Nat. Bureau of Standards, *Jn. Research*, v. 1, 1928, p. 637.

528[F].—N. G. W. H. BEEGER, "Second extension of the table of least exponents ξ for which $2^\xi \equiv 1(\text{mod } p)$," *Nieuw Archief voor Wiskunde*, s. 2, v. 22, p. 310–311, 1948. 15.7 × 23.6 cm.

This is an extension for the range $309672 < p < 320000$ of the exactly similar table reviewed in RMT **279**, v. 2, p. 71. The existence of the manuscript of this table is there announced. The author calls attention to the fact that $p = 318781$ has the small exponent 828 so that p divides $2^{414} + 1$. Incidentally so does $p = 853669$.

D. H. L.

529[F].—M. KRAÏTCHIK, *Théorie des Nombres*, v. 3; *Analyse Diophantine et Applications aux cuboïdes rationnels.* Paris, Gauthier-Villars, 1947, xi, 135 p. 15.8 × 24.5 cm.

This work contains the following four tables:

T.I (p. 36–55). This table is to facilitate the solution of the diophantine equation

$$x^4 + \alpha x^2y^2 + y^4 = u^2$$

by giving solutions of the associated congruence

$$y^4 + \alpha y^2 + 1 \equiv u^2 \pmod{p}.$$

The trivial cases of $\alpha = \pm 2$ are omitted. The moduli considered are the 24 odd primes <100.

T.II (p. 83–85). This is a list of rational integral cuboids, that is, a list of triples (x, y, z) of positive integers such that the sum of the squares of any two of them is a perfect square. Thus x, y, and z are the edges of a rectangular parallelopiped whose faces have integral diagonals. The author considers only those cases in which x, y, z are not all even. This implies that exactly one of them is odd and this odd one is taken as z. The list is arranged according to z and extends to $z \leq 10^4$. There are 284 of these cuboids of which only 39 are primitive, that is, x, y, z have no common factor.

T.III (p. 112–113). This table is wrongly labeled. The title should read $x^2 + 16y^2 \equiv z^2 \pmod{p}$. The second column should be headed z^2, not x^2. The moduli considered are the primes ≤ 43 except 2, 3, 7. The table gives the values of x^2 or $y^2 \pmod{p}$ for each possible given value of $z^2 \not\equiv 0 \pmod{p}$.

T.IV (p. 122–131). This is a list of the 241 primitive cuboids with an odd value of $z < 10^6$. For each such z the values of x, y, z are given with three pairs of generators. See also *MTAC*, v. 2, p. 167.

D. H. L.

530[F].—D. B. LAHIRI, "On Ramanujan's function $\tau(n)$ and the divisor function $\sigma_k(n)$. Part II." Calcutta Math. Soc., *Bull.*, v. 39, 1947, p. 33–52. 18.7 × 24.2 cm. For a review of Part I see *MTAC*, v. 3, p. 23, RMT **459**.

This Part contains a large table of 171 congruence relations between Ramanujan's function $\tau(n)$, defined by

$$\Delta(x) = \sum_{n=1}^{\infty} \tau(n)x^n = x\{(1-x)(1-x^2)(1-x^3)\cdots\}^{24}$$

and the sums of powers of the divisors of n, the moduli being certain divisors of $2^{13}3^65^3\cdot 7\cdot 11\cdot 13\cdot 691$. The following is a typical example.

$$9504\tau(n) \equiv 13650\sigma_{11}(n) - 691[11(6n-5)\sigma_9(n) - 5\sigma_1(n)] \pmod{2^3\cdot 3\cdot 5\cdot 11\cdot 691}.$$

The general form is

$$A\tau(n) \equiv \sum_{k=0}^{6} P_k(n)\sigma_{2k+1}(n) \pmod{M}$$

where A and the polynomial P depend on M. The large number of apparently different relations is due in great measure to the numerous congruence relations existing between the σ's. In many cases, and in the above example, the constant A is not prime to the modulus M so that one obtains a congruence for $\tau(n)$ itself only for a certain divisor of M. This happens for instance whenever M is divisible by 2^{11}, 3^6, 11, or 13. The highest powers of the various primes for which there is a congruence relation for $\tau(n)$ itself are 2^{10}, 3^5, 5^3, 7, and 691. There is also an interesting table giving the 41 products of the functions $\Phi_{r,s}(x)$, defined in RMT 459, which are expressible linearly in terms of other Φ's and the generator Δ. It is from this table that the main table was produced.

D. H. L.

531[H].—John T. Pettit, "A speedy solution of the cubic," *Mathematics Mag.*, v. 21, Nov.–Dec. 1947, p. 94–98. 17.1 × 25.4 cm.

In solving $y^3 + py + q = 0$, if p and q have the same signs, change the signs of the roots and consider $y^3 \pm |p|y \mp |q| = 0$. With the substitution $y = |q|z/|p|$, transform the equation to $z^3/(1 - z) = \pm |p|^3/|q|^2 = K$. For the general cubic, $x^3 + bx^2 + cx + d = 0$, reduce to the previous case by taking $x = y - \frac{1}{3}b$, $p = c - \frac{1}{3}b^2$, $q = d - \frac{1}{3}bc + 2b^3/27$.

The relation $z^3/(1 - z) = K$ is represented by a graph on p. 96, and tables are given on p. 97–98 for $z = -3(.01) + 1.5$ and K to 3S, $-\infty < K < \infty$. Over this range z is a single-valued function of K, and one real root is thus obtained. On reducing the general equation to a quadratic the other roots are found to be $x = [q/2p][z \pm (-4K - 3z^2)^{\frac{1}{2}}] - \frac{1}{3}b$.

The procedure and the table printed in this article are essentially those given in more detail by H. A. Nogrady, *A New Method for the Solution of Cubic Equations*, 1936; see *MTAC*, v. 1, p. 441f. Other references on the numerical solution of the cubic are given in *MTAC*, v. 2, p. 28f.

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Note by S. A. J.: The Pettit table is obviously wholly unreliable, since there are from 2 to 78 units of error in the last-figure values of K, for the following 14 values of z: -2.22, -1.29, -1.22, -1.19, -1.04, -1.02, -0.53, -0.52, -0.51, -0.23, -0.22, $+0.34$, 0.52, 1.18.

532[H, I, L].—H. S. Carslaw & J. C. Jaeger, *Conduction of Heat in Solids.* Oxford, Clarendon Press, 1947, vi, 386 p. 15.5 × 24 cm. Miss Martha E. Clarke is credited with assistance in computation of the tables.

Appendix II: "The error function and related functions," p. 370–373. There are 4D tables of e^{x^2} erfc x, $4\pi^{-\frac{1}{2}}xe^{-x^2}$, $2\pi^{-\frac{1}{2}}e^{-x^2}$, erf x, erfc x, $2i$ erfc x, $4i^2$ erfc x, $6i^3$ erfc x, $8i^4$ erfc x, $10i^5$ erfc x, $12i^6$ erfc x, for $x = 0(.05)1(.1)3$.

$$\operatorname{erf} x = 2\pi^{-\frac{1}{2}} \int_0^x e^{-t^2}dt;\ \tfrac{1}{2}\pi^{\frac{1}{2}} \operatorname{erfc} x = \int_x^\infty e^{-t^2}dt;\ i^n \operatorname{erfc} x = \int_x^\infty i^{n-1} \operatorname{erfc} t dt,$$

$n = 1, 2, \cdots, i^0 \operatorname{erfc} x = \operatorname{erfc} x$.

Appendix IV: "The roots of certain transcendental equations," p. 377–379. T.1, the first six roots, α_n, to 4D, of $\alpha \tan \alpha = C$, for $C = .001, .002(.002).01, .02(.02).1(.1)1(.5)2(1)$-$10(5)20(10)60(20)100, \infty$. The roots of this equation are all real if $C > 0$. T.2, the first six roots, α_n, to 4D, of $\alpha \cot \alpha + C = 0$, for $C = -1(.005) - .99(.01) - .9(.05) - .8(.1) + 1(.5)2(1)10(5)20(10)60(20)100, \infty$. The roots of this equation are all real if $C > -1$. **T.3**, the first six roots, α_n, to 4D, of $\alpha J_1(\alpha) - CJ_0(\alpha) = 0$, for $C = 0(.01).02(.02).1(.05)$-$.2(.1)1(.5)2(1)10(5)20(10)60(20)100, \infty$. **T.4**, the first five roots, α_n, to 4D, of $J_0(\alpha)Y_0(k\alpha) - Y_0(\alpha)J_0(k\alpha) = 0$, for $k = 1.2, 1.5(.5)4$.

Appendix V: "Table of Laplace transforms $\bar{v}(p) = \int_0^\infty e^{-pt}v(t)dt$," p. 380–381. There are twenty-eight entries under $\bar{v}(p)$, $v(t)$.

Extracts from text

Editorial Note: A table of the first seven roots of $\alpha \tan \alpha = C$ was given by A. B. Newman & L. Green, "The temperature history and rate of heat loss of an electrically heated slab," Electrochem. Soc., *Trans.*, v. 66, 1934, p. 355, for $C = [0, .1, .5, 1(1)10, \infty$; 4D]. For various solutions of $\alpha \cot \alpha + C = 0$, or $\tan \alpha = -\alpha/C$, see *MTAC*, v. 1, p. 203, 336, 459; v. 2, p. 95. In this connection reference may also be given to J. C. Jaeger & Martha Clarke, "Numerical results for some problems on conduction of heat in slabs with various surface conditions," *Phil. Mag.*, s. 7, v. 38, 1947, p. 507.

533[K].—Bell Telephone Laboratories, *Table of Individual and Cumulative Terms of the Point Binomial* $(q + p)^n$. (Inspection Engineering Memoranda, no. IEM-4; Quality Assurance Dept. 2530.) June 4, 1947, 398 leaves. Hectographed on one side only. 21.5 × 27.7 cm. This edition is not available for general distribution but the volume is to be published by the Van Nostrand Company in 1949.

Consider N independent trials with a constant probability P of success. The probability of a total of exactly X successes is given by

$$b(X, N; P) = \binom{N}{X} P^X(1 - P)^{N-X}, \qquad 0 < P < 1.$$

This is the binomial distribution or, as it is sometimes called, the point binomial. The summatory function

$$B(X, N; P) = b(0, N; P) + b(1, N; P) + \cdots + b(X, N; P)$$

gives the probability of at most X successes. Both functions are of fundamental importance in all applications of probability theory. For a numerical evaluation of $B(X, N; P)$ tables of the beta function can be used, but this procedure is rather cumbersome. It is therefore customary to use the two classical approximations to $b(X, N; P)$ and $B(X, N; P)$. Putting

$$z = \{X + \tfrac{1}{2} - NP\}\{NP(1-P)\}^{-\frac{1}{2}}$$

the Laplace limit theorem states that as $N \rightarrow \infty$

$$B(X, N; P) \rightarrow 2\pi^{-\frac{1}{2}} \int_0^z e^{-\frac{1}{2}t^2} dt.$$

This approximation is accurate only if NP is very large. If N is large but $NP = a$ is of moderate magnitude, the Poisson approximation

$$b(X, N; P) \approx e^{-a}a^X/X!$$

is much better. For moderate values of N neither approximation is satisfactory. More accurate asymptotic expansions are available, but are rather cumbersome for routine use. In modern sampling, in industrial quality control, and many other applications we frequently deal with comparatively small values of N. In such cases one has to resort to tedious computations or to unsafe approximations. Therefore the present tables will be greeted with a feeling of relief by many practical statisticians: they solve the problem completely at least within the range $50 \leqslant N \leqslant 100$.

It should be noticed that $b(X, N; P) = b(N - X, N; 1 - P)$ and $B(X, N; P) = 1 - B(N - X - 1, N; 1 - P)$. Accordingly there is no need to tabulate the values for $P > \frac{1}{2}$. The present tables contain 11 independent sections, each covering one value of N in the range 50(5)100. In every case $b(X, N; P)$ and $B(X, N; P)$ are tabulated in adjacent columns for $P = [0(.01).50; 6D]$. The number of X-entries is usually considerably smaller than N since only values of X near NP correspond to $b(X, N; P)$ values significantly different from zero. The last digit is not absolutely reliable. It appears to the reviewer that the accuracy is sufficient for most practical applications, and that the tabular interval in P is sufficiently small. It is to be hoped that these useful tables[1] will be extended beyond $N = 100$.

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[1] The following acknowledgment is in the introduction: Thanks are extended to Mr. E. G. Andrews for his cooperation and assistance in computing the table; to Dr. H. Nyquist, Dr. John Riordan, and Dr. H. W. Bode for their aid in arranging for the project; to Dr. F. L. Alt and his able assistant, Miss Bettie Boyd, for setting up the project on the Computing System; to Miss Alice G. Loe for work on the printer, for checking all phases of the work, and for computing some 5% of the values appearing in the table which could not be completed by the Computing System because of limited time; and to other members of the Quality Assurance Department for guidance and cooperation.

Harry J. Romig

534[K].—C. V. L. Charlier, *Elements of Mathematical Statistics, also L. v. Bortkiewicz, Table of Poisson's Frequency Function edited and translated by* J. Arthur Greenwood. Cambridge, Mass., 1947, ii, 120 p. Orders to be placed with J. A. Greenwood, 25 Winthrop St., Brooklyn 25, N. Y. $3.00. Plastic binding. 17.5 × 22.7 cm.

The first edition of this little work of Carl Vilhelm Ludvig Charlier (1862–1934) appeared in 1910 as *Grunddragen af den matematiska Statistiken*, Lund. The next edition was by Charlier in German, *Vorlesungen über die Grundzüge der mathematischen Statistik*, Lund, 1920. 127 p. 17.5 × 22 cm. This differed from the Swedish edition by the addition of a fifteenth chapter and an appendix of the following five tables: **T.I**, $Q(x) = (2\pi)^{-\frac{1}{2}}\int_{-\infty}^{x} e^{-\frac{1}{2}x^2}dx$, for $x = [-2.9(.1) - 2(.01) + 2(.1)2.9; 4D]$; **T.II**, $x = R(u)$, inverse of Q for $u = [0(.01).99; 4D]$; **T.III**, $\phi_0 = (2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$, for $x = [0(.01)3(.1)4.9; 4D]$; **T.IV**, $\phi_3(x) = d^3\phi_0/dx^3 = (-x^3 + 3x)\phi_0$; **T.V**, $\phi_4(x) = d^4\phi_0/dx^4 = (x^4 - 6x^2 + 3)\phi_0$, Hermite polynomials ϕ_3 and ϕ_4 for $x = [0(.01)3(.1)4.9; 4D]$.

There are various small tables scattered throughout the work. In the English edition these are numbered 1–40 and the tables of the Appendix, p. 101–116, are numbered 41–44. **T.41** of $-x = -R(u)$, is **T.II**, above, rearranged. We are told that eight last-figure errors were corrected by comparison with the 10D table of T. Kondo & E. M. Elderton, *Biometrika*, v. 22, p. 368–370; also K. Pearson, *Tables for Statisticians and Biometricians*, v. 2, 1932, p. 2–10.

T.42 is of $\phi_{-1} = Q(x)$, ϕ_0, $-\phi_2$, ϕ_3, $-\phi_5$, for $x = [0(.01)4; 7D]$; ϕ_4 and $-\phi_6$ are also given, for the same range, to 4D. ϕ_{-1} and ϕ_0 are taken from those by W. F. Sheppard in *Biometrika*, v. 2, 1903, p. 182–188; also in Pearson, *Tables for Statisticians and Biometricians*, v. 1, Cambridge, 1914, p. 2–7. Mr. Greenwood states that ϕ_2, ϕ_3 and ϕ_5 were computed, partly to 13D, partly to 10D, using the NBSCL, *Tables of Probability Functions*, v. 2, 1942, and the tables of N. R. Jørgensen, *Undersøgelser over Frequensflader og Korrelation*, Copenhagen, 1916, and then rounded to 7D. The Greenwood values of ϕ_2, ϕ_3, and ϕ_5 are identical with those of Jørgensen, whose 7D table of ϕ_6 is rounded off to a 4D table. ϕ_4 is the same as Charlier's 4D **T.V**.

T.43 is the table referred to in the title of the book under review, L. v. Bortkiewicz, *Das Gesetz der kleinen Zahlen*, Leipzig, 1898, p. 49–52, giving the 4D values of $m^xe^{-m}/x!$ for $m = .1(.1)10$, $x = 0(1)24$. (Compare *MTAC*, v. 1, p. 19.)

"**T.44** was newly computed by the translator," it is stated. It gives "the limits within which the Skewness and Excess must lie in order that a frequency curve of type A may yield positive frequencies." There are only 18 entries $E = 0(.03).48, .50$, and for these the corresponding 3D values of $|S| <$ are given.

Such are the principal tables in the latest form of an elementary work first published nearly 40 years ago.

R. C. A.

535[L].—Enzo Cambi, *Eleven- and Fifteen-Place Tables of Bessel Functions of the First Kind, to All Significant Orders.* New York, Dover Publications, 1948, vi, 154 p. 22 × 29 cm. $3.95.

This new addition to the ever expanding list of tables of the Bessel functions consists essentially of two tables of the functions and two additional tables of the coefficients used in their computation. The author explains his purpose in this new work by the following statement in the preface: "A great scientific need was met just ten years ago, when the British Association for the Advancement of Science published its volume of ten place tables of *Bessel Functions . . . of Orders Zero and Unity*, for x varying from 0 to 16 at interval 0.001 and from 16 to 25 at interval 0.01. In many war time problems, however, there was a constant demand for tables of the values of such functions to higher integral orders, especially for Bessel functions of the first kind. The present volume offers a contribution in this field."

In Part I of the work, 143 pages, the essential tables are as follows:

Table I gives the values of $J_n(x)$ for $x = [0(.01)10.5; 11D]$ and the order n is extended to values for which J_{n+1} becomes smaller than $.5 \times 10^{-11}$ for the entire range of x. Actually a few values of J_{30} are included.

Table II includes the same functions as those in Table I except now the range is $x = [.001(.001).5; 15D]$ and is extended to include values of J_n which are greater than 10^{-15}. The table thus terminates with a few values of J_{11}.

In Part II, p. 144–148, we find the values of the coefficients of the Taylor's series in h for $x = 2(1)6$ and $n = 2(2)16$ to 12D. Through $n = 6$ the expansions include terms to h^{11} and through $n = 16$ they terminate with h^{10}.

In Part III, p. 149–153, there are recorded the values of A_m in the expansion,

$$J_n(x + h) = A_0J_0(h) + A_1J_1(h) + A_2J_2(h) + A_3J_3(h) + \cdots,$$

for $x = 7(1)10$ and $n = 2(2)30$. Through $n = 16$ these are to 12D, but "for the higher values of the order $n(n > 16)$, the coefficients are limited to the figures required for the computation of J_n to 12–13 places."

In a fourth part, p. 154, the author gives a brief bibliography of Bessel functions.

In computing the tables described above, the author began with the well-known tables of MEISSEL for J_0 and J_1. The values of $J_n(x)$, $n > 1$ were computed up to $x = 1.5$, by the power series, from 1.5 to 6.5 by means of Taylors' series, and from 6.5 to 10.5 through the addition formula, for even n. The recurrence formula for Bessel functions was used to compute the values for odd n, except in the cases $n = 3$ and $n = 5$, where the formula might have introduced errors beyond the accuracy limit of the table.

In checking the accuracy of the 11th place error in Table I, the following formulae were used:

$$1 = J_0 + 2J_2 + 2J_4 + \cdots,$$
$$\sin x = 2J_1 - 2J_3 + 2J_5 - 2J_7 + \cdots.$$

In computing Table II, J_0 was interpolated from the 16D table of HAYASHI and the values of the other functions of even order were computed from the power series and the use of interpolation formulae. The values of the functions of odd order were found by means of the recurrence formula: $J_n(x) = (x/2n)[J_{n+1}(x) + J_{n-1}(x)]$, which could be used safely since x was small.

The present table seems to fill a useful place in spite of the fact that the Computation Laboratory of Harvard University is providing more extensive tables of some of the same functions. This project is providing the values of $J_n(x)$ for $x = 0(.001)25(.01)99.99$, $n = 0(1)3$ to 18D and $n = 4(1)14$ to 10D; then for $n = 15(1)100$, $x = [0(.01)99.99; 10D]$. The first six volumes for $n = 0(1)15$ appeared in 1947. But this basic work because of its size and cost will not be available readily to every computer who well may wish to use Bessel functions. But the compactness and low cost of the 11D and 15D work under review will recommend it to everyone who may desire to include an extensive set of values of the Bessel functions in his library.

H. T. D.

EDITORIAL NOTE: Dr. CAMBI requests that the following corrections be made in his volume of tables reviewed above: p. v, l. −3, and p. 149, l. 1, *for* Taylor Series, *read* Additional Series; p. 47, $J_5(6.93)$, *for* .35385 88467 2, *read* .35385 88367 2.

536[L].—GEORGE A. CAMPBELL & RONALD M. FOSTER, *Fourier Integrals for Practical Applications*, New York, Van Nostrand, 1948. 178 p. 15.8 × 23.7 cm. $3.50.

The response of a linear oscillating system to a harmonic signal, $a \cos pt + b \sin pt$, is in general harmonic, $A \cos pt + B \sin pt$; and the behavior of such a system can be regarded as known if its response to a harmonic signal of arbitrary frequency and phase has been determined. The response to an arbitrary signal can then be obtained in three steps: (i) decomposition, or analysis, of the signal into its harmonic components, (ii) finding the response for each of the harmonic components, and (iii) superposition, or synthesis, of the component responses. If the incoming signal is periodic, of period T, its decomposition takes the form of a Fourier series,

$$\sum_{n=-\infty}^{\infty} c_n e^{2\pi i n t/T},$$

and the synthesis of the component responses amounts to the summation of a Fourier series. For non-periodic signals, for instance for so-called transients, the frequency spectrum is continuous in general, and the decomposition takes the form of a *Fourier integral*

$$G(t) = \int_{-\infty}^{\infty} e^{2\pi i f t} F(f) df. \tag{1}$$

Given the signal, $G(t)$, its analysis into harmonic components involves finding $F(f)$ which is given by

$$F(f) = \int_{-\infty}^{\infty} e^{-2\pi i f g} G(g) dg. \tag{2}$$

Again, if the harmonic components, with amplitudes $F(f)$, of the response are known, their synthesis is based on (1). Thus in the case of Fourier integrals harmonic analysis and synthesis use the same type of integral, and the practical importance of an extensive table of integrals of this type is apparent.

Fourier integrals have many other applications, for instance to the solution of certain types of differential and integral equations (Cf. E. C. TITCHMARSH, *Introduction to the Theory of Fourier Integrals*, Oxford, 1937). Again, the so-called Heaviside, or operational, calculus is based on Laplace integrals which are essentially one-sided Fourier integrals, that is Fourier integrals corresponding to functions $G(t)$ which vanish for negative t.

The book under review contains what is generally acknowledged to be the most extensive published list of Fourier integrals. It was originally published, under the same title, in 1931 as Monograph B-584 of the *Bell Telephone System Technical Publications*, reprinted in the *Collected papers of George A. Campbell*, New York, American Telephone and Telegraph Company, 1937; the original edition of 1931 was reproduced photographically, with minor corrections, in 1942. Though the present book does not mention any of the previous editions, it appears to be a photographic reproduction of the 1942 edition. No attempt seems to have been made at bringing the material up to date, even though the authors are known to have continued collecting valuable material after 1931.

There are 33 pages of explanatory text. An introduction explains the importance of Fourier integrals, the relationship between $F(f)$ and $G(g)$, the terminology, the role of the unit impulse and its relationship to the harmonic function cis $(2\pi ft) = e^{2\pi i f t}$, the use of the tables for obtaining Fourier integrals. Next the general processes for deriving pairs of functions F and G ("mates"), and elementary transformations as well as various properties of such pairs are explained. There follow brief sections on mates based on the normal error law, essentially singular pairs, such as $F = (2\pi i f)^n$ and its mate, pairs for which F and G are in a given relation, for instance $F(x) = G(x)$, on Fourier series, on the use of contour integrals, and on practical applications of the tables. There is also a summary of the descriptive part, and a list of notations and abbreviations used throughout the book.

Table I contains 763 pairs of mates F, G under the following 13 headings: General processes for deriving the mate; Elementary combinations and transformations; Key pairs; Rational algebraic functions of f; Irrational algebraic functions of f; Exponential and trigonometric functions of f or f^{-1}; Exponential and trigonometric functions of f^2; Other elementary transcendental functions of f; Other transcendental functions of f; Fourier series given as pairs; Contour integrals. Paths parallel real axis; Contour integrals. Closed paths; Contour integrals. Paths with arbitrary end-points.

Table II contains admittances of and transients in physical systems, illustrating the application of Fourier integrals to transient problems. There are 39 physical systems listed under 3 headings: Time variable; Space variable; Two space variables. In each case a brief description of the physical system is given together with its admittance (whose knowledge is equivalent to the knowledge of the response to an arbitrary harmonic signal), its response to an instantaneous unit impulse, to a unit step (a signal of constant intensity which starts at $t = 0$) and to a harmonic signal which arrives at $t = 0$.

A few valuable features of this carefully compiled book deserve special praise. They are: A carefully devised and scrupulously maintained system of notation which enables the authors to present the results very concisely and yet quite precisely (for instance v, w always stand for integers, m, n for non-negative, and j, k, l for positive integers). "Key pairs," formulae containing numerous parameters and thereby parent formulae for many special and limiting cases listed in the various special sections; such key pairs are collected separately. Numerous cross references. Very helpful special explanations, given in specially devised symbols, referring to alternative expressions, related transform pairs, special cases in which the restrictions may be relaxed, and similar matter. The only criticism which this reviewer has to level against this excellent work is—that there is not enough of it, i.e. that it has not been brought up to date.

A. Erdélyi

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537[L].—Gustav Doetsch (1892–), *Tabellen zur Laplace-Transformation und Anleitung zum Gebrauch.* (*Die Grundlehren der math. Wissen. in Einzeldarstellungen,* v. 54.) Berlin and Göttingen, Springer, 1947, x, 186 p. Offset print. 16 × 24.2 cm.

The first part of the book (p. 1–71) contains a condensed but clear account, without proofs, of the important properties of the Laplace transform and a description of how it can be used to solve differential equations. Specific illustrations of the various possibilities are given, including a heat conduction problem where a blind application of the complex inversion formula leads to an incorrect result. The author's notation is

$$f(s) = \int_0^\infty e^{-st}F(t)dt \equiv \mathfrak{L}\{F(t)\}.$$

The second part consists of a table of inverse transforms: each entry gives $f(s)$, the corresponding $F(t)$, and the abscissa of convergence. This is the arrangement adopted in the shorter tables of M. F. Gardner & J. L. Barnes, *Transients in Linear Systems studied by the Laplace Transformation*, New York, Wiley, 1942, v. 1, p. 334–356, and R. V. Churchill, *Modern Operational Mathematics in Engineering*, New York, McGraw-Hill, 1944, p. 295–302 (122 entries in the table), and appears to be more convenient for applications than the opposite arrangement as used, for example, by N. W. McLachlan & P. Humbert, *Formulaire pour le Calcul Symbolique* (*Mémorial des Sciences Mathématiques*, no. 100), Paris, Gauthier-Villars, 1941. In addition, pages 164–185 of Doetsch contain an index of functions, arranged alphabetically by the symbols used to denote the functions, with the definition of each function and references to all the formulae where it appears. This index enables the table to be used with some facility as a table of direct transforms.

This is the most extensive published table of Laplace transforms. Its scope can be seen from the following summary: Operations (56 formulae); Rational functions (65); Irrational functions (105); Logarithmic, inverse trigonometric functions (90); Exponential functions (102); Trigonometric and hyperbolic functions (74); Gamma and related functions (37); Integral functions (sine, cosine, exponential integrals, etc.) (42); Confluent hypergeometric functions (45); Bessel functions (108); Spherical harmonics (14); Elliptic integrals (28); Theta functions (15); Other special functions (12). The tables of J. Cossar & A. Erdélyi, *Dictionary of Laplace Transforms*, 1944–1946 (*MTAC*, v. 1, p. 424–425, v. 2, p. 76, 215–216), not yet made available to the general public, contain more formulae for transcendental functions, but fewer for elementary functions.

The book is reproduced from typescript, with the formulae very clearly written by hand.

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EDITORIAL NOTE: In effect, LAPLACE introduced in 1779 the transformation which has received his name by showing the correspondence between the two functional domains which it relates [P. S. LAPLACE, "Sur les suites," *Mém. Acad Sci.*, 1779; *Oeuvres Complètes*, v. 10, p. 1–89]. The Laplace transformation may be viewed as a limiting form of a transformation used by Euler [E. G. C. POOLE, *Introd. to the Theory of Linear Differential Eqns.*, Oxford, 1936, p. 137]. In this sense the transformation is related to work which preceded Laplace's. The definite integral form of the direct Laplace transformation was applied by Laplace to the solution of differential and difference equations in 1782 [*Oeuvres*, v. 10, p. 209–338]. Laplace's book on the theory of probability, 1812, illustrates the many uses to which he puts the transformation. This Note is adapted from GARDNER & BARNES, *l.c.*, p. 360.

There is also a table (112 entries) in ERNST HAMEISTER, *Laplace Transformation. Eine Einführung für Physiker, Elektro-Machinen und Bauingenieure.* Munich and Berlin, 1943; also Ann Arbor, Mich., Edwards Bros., 1946, 147 p.

538[L].—HARRY J. GRAY, RICHARD MERWIN, & J. G. BRAINERD, *Solutions of the Mathieu Equation*, Amer. Inst. Electr. Engineers, *Technical Paper* 48–70. 1948, 17 p. 20.5 × 27.8 cm. Offset print. To nonmembers by mail, 55 cents.

The equation whose solutions are tabulated is

$$d^2y/dt^2 + \epsilon(1 + k\cos t)y = 0. \tag{1}$$

If $2z$ be written for t, a for 4ϵ, and $-q$ for $2\epsilon k$, the resulting equation is

$$d^2y/dz^2 + (a - 2q\cos 2z)y = 0, \tag{2}$$

this form being preferable in analytical work. The equation has four types of solution depending upon the values of (a, q):

1°. The first solution is bounded and periodic in π or 2π, the second being non-periodic and tending to $\pm\infty$ as $z \to +\infty$.
2°. Both solutions are bounded and periodic with period $2s\pi$, s integral $\geqslant 2$.
3°. Both solutions are bounded, but non-periodic.
4°. Both solutions are non-periodic, but one tends to $\pm\infty$ while the other tends to zero as $z \to +\infty$.

The values of (a, q) pertaining to 1° lie on curves symmetrical about the a axis, and these divide the (a, q)-plane into regions[1] where the solutions have the forms in 2°–4°. For 2°, 3°, the region is called stable, and for 4° unstable, by virtue of the type of solution. The paper under review is concerned with solutions of the type 2°–4°, which occur in the following physical problems:

i. Frequency modulation in radio telephony, and the acoustical warble tone;
ii. Pendulum whose support executes harmonic motion in a vertical plane;
iii. Oscillations of side-rod electric locomotives;
iv. Sub-harmonics in mechanical vibrating systems, and in conical loud-speaker diaphragms;
v. Pseudo-rectilinear motion of loud-speaker coil in non-uniform radial magnetic field;
vi. Stability of structural columns subject to a longitudinal harmonic force;
vii. Unsymmetrical short-circuits on water-wheel generators under capacitive loading.

From this incomplete list, it is evident that solutions of Mathieu's equation for certain ranges of a, q are needed, and the question arises as to the most suitable way in which the data should be presented.

The two solutions of (1) are taken in the form

$$g(t) = u_1\cos\mu t - u_2\sin\mu t,$$

and

$$H(t) = u_1\sin\mu t + u_2\cos\mu t = Mh(t);$$

u_1 (even), u_2 (odd), being functions with period π or 2π,

$$\cos 2\pi\mu = 2g(\pi)h'(\pi) - 1,$$

and

$$M = [-g(\pi)g'(\pi)/h(\pi)h'(\pi)]^{\frac{1}{2}}.$$

The initial conditions are

$$g(0) = h'(0) = 1, \qquad g'(0) = h(0) = 0.$$

The quantities tabulated are: (a) $g(t)$, $h(t)$, for $t = 0(.1)3.1, \pi$; $\epsilon = 1(1)10$; and $k = 0(.1)1$; the total number of entries for *each* function is 3300; (b) μ, M for $\epsilon = 1(1)10$, and $k = 0(.1)1$, the total number of entries for *each* parameter[2] being 100. M is real, imaginary or complex, but it is preferable to have real solutions. In all cases the data are given to 5D.[3] Interpolation will usually not yield data comparable in accuracy with that tabulated. The interval 1 for ϵ precludes data being obtained for intermediate points. In the reviewer's opinion it is preferable to have the solution in functional rather than in numerical form, especially in cases where it is non-periodic. A clearer picture of the physical behavior of a system may then be visualized. Such solutions were discussed by Dr. GERTRUDE BLANCH, in *MTAC*, v. 2, p. 263–266, and examples have been given by the writer.[1,4] By having either a comprehensive table of a, $a^{\frac{1}{2}}$, q, μ, or a large scale chart (two or three feet square), and using a form of solution such that μ is *never* complex, the coefficients in the series solution may be found quickly. The functional form of the solution then follows immediately. For instance if in (2) we take $a = 1$, $q = .16$, then $\mu = \simeq .08$. Apart from a constant multiplier, using t for z, the two solutions are

$$y_1(t) = e^{.08t}[\cos t - .021 \cos 3t + \cdots + .94 \sin t - .0175 \sin 3t + \cdots], \tag{3}$$

$$y_2(t) = y_1(-t). \tag{4}$$

If these refer to a physical system, its behavior with increase in t can be visualized immediately, whereas from purely numerical solutions this would not be so. (3), (4) are neither odd nor even. This is advantageous, since odd and even solutions (like those tabulated for (a)) would *both* tend to $\pm\infty$ as $t \to +\infty$, whereas $y_1(t) \to \pm\infty$, while $y_2(t) \to 0$.

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[1] See N. W. McLACHLAN, *Theory and Applications of Mathieu Functions*. Oxford, 1947, figures 8, 11.

[2] The values of μ are half those corresponding to equation (2).

[3] The authors make the following statement: "The tables are given to five decimal places, but because they are extracted from larger ones and no rounding has been done, the digit in the fifth decimal place may be in error, and it seems best to state the accuracy of the tables as four decimal places. In this connection, it may be pointed out that the tables were obtained on the ENIAC, using 10-digit numbers and an interval $\Delta t = .0004$ in the corresponding difference equation. Rounding and truncation errors cut the accuracy so that 5- or 6-figure accuracy might be expected."

[4] N. W. McLACHLAN, "Computation of the solutions of $(1 + 2\epsilon \cos 2z)y'' + \theta y = 0$; frequency modulation functions," *Jn. Appl. Physics*, v. 18, 1947, p. 723–731.

539[L].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 9, *Tables of the Bessel Functions of the First Kind of Orders Sixteen through Twenty-Seven.* By the Staff of the Laboratory, Professor H. H. AIKEN, technical director. Cambridge, Mass., Harvard University Press, 1948, x, 764 p. This is the first v. of the Bessel function series to be paged. 19.5 × 26.7 cm. $10.00. Offset print. Compare *MTAC*, v. 2, p. 176f, 261f, 344; v. 3, p. 102, 117–118.

This is the seventh, and largest so far published, of the thirteen planned volumes in the monumental edition of tables of Bessel functions prepared at Harvard University by means of the IBM Automatic Sequence Controlled Calculator. It contains tables of $J_n(x)$, $n = 16(1)27$, $x = [0(.01)99.99; 10D]$. For $n = 0(1)15$, the interval up to 25 had been .001. Since many of the early values of the 12 functions are zero, the first cases of those with the significant value .00000 00001 are respectively as follows: $J_{16}(3.12)$, $J_{17}(3.60)$, $J_{18}(4.10)$, $J_{19}(4.62)$, $J_{20}(5.16)$, $J_{21}(5.72)$, $J_{22}(6.29)$, $J_{23}(6.88)$, $J_{24}(7.48)$, $J_{25}(8.09)$, $J_{26}(8.71)$, $J_{27}(9.35)$.

Among all entries in the table, 112 725, there are only 296 previously published to at least 10D; there are 288 given by MEISSEL (1895) for $x = [0(1)24; 18D]$, 1 given by MEISSEL (1891), $J_{20}(20)$ to 20D, and 7 given by HAYASHI (1930) for $x = 1, 2, 10(10)50$, to at least 15D. Since the Harvard volume was published, however, the tables by CAMBI have appeared (reviewed elsewhere in this issue, RMT **537**). The ranges of values here partially duplicating those of the Harvard volume are $x = [0(.01)10.5; 11D]$, $n = 16(1)27$.

The tapes used in computing the tables of the volume under review were coded by JOHN A. HARR, who also supervised the operation of the ASCC and the preparation of the manuscript.

R. C. A.

540[L].—P. I. KUZNETSEV, "Funktsii Lommeliâ ot Dvukh Mnimykh Argumentov" [Lommel functions with two imaginary arguments], *Prikladnaia Matem. i Mekhanika*, v. 11, Oct. 1947, p. 555–560.

$$Y_n(w, z) = i^{-n} U_n(iw, iz),\ \Theta_n(w, z) = i^{-n} V_n(iw, iz),$$

$$U_n(w, z) = \sum_{m=0}^{\infty} (-1)^m (w/z)^{n+2m} J_{n+2m}(z),\ V_n(w, z) = \sum_{m=0}^{\infty} (-1)^m (z/w)^{n+2m} J_{n+2m}(z),$$

$$Y_n(w, z) = \sum_{m=0}^{\infty} (w/z)^{n+2m} I_{n+2m}(z),\ \Theta_n(w, z) = \sum_{m=0}^{\infty} (z/w)^{n+2m} I_{n+2m}(z).$$

There are 6D tables of $Y_n(w, z)$, $\Theta_n(w, z)$, for $n = 1, 2$, $w = 1(1)6$, $z = 1(1)6$; with graphs. There are also 4D tables, and graphs, of $V(x, t)$, $10^3 J(x, t)$, for $t = 6.56$ and ∞, $x = 0(100)$-400; and for $x = 400$, $t = 6$, 10(5)25.

541[L].—W. LANE & D. SWEENEY, *Table of Legendre Polynomials P_n (cos θ) for $n = 0(1)20$, and $\theta = 0(1°)180°$ to Six Decimals.* United States Atomic Commission, Oak Ridge, Tennessee, Los Alamos Scientific Laboratory MDDC–780, LADC–361. Date of Manuscript Feb. 12, 1947; document declassified March 18, 1947. 10 p. 20.4 × 26.8 cm. Printed.

The contents of this publication are indicated by its title. The first sentence of the text is "The accompanying table of Legendre Polynomials was computed for use because no tables of the functions of the desired range were available from other sources." A correct statement would have been the following: "What we have tried to do is to compute again a small section of a 6D table readily available in many libraries, namely: $P_n(\cos \theta)$, $n = 1(1)32$, $\theta = 0(10')90°$, by A. H. H. TALLQVIST, Finska Vetenskaps-Societeten, *Acta*, s. 2, v. 2A, no. 11, 1938. We did not look at this table."

At the end of the first paragraph of the text is the following statement: "The table is accurate to six decimal places for values of $n < 15$, and five decimal places for values of $n \geqslant 15$." Here are suggestions as to how much dependence can be placed on this statement: A comparison of Tallqvist's table, mentioned above, was made with the 220 entries of Lane & Sweeney's table for $n = 1(1)20$, $\theta = 0(1°)10°$. Tallqvist's 10D table for $n = 1(1)8$, $\theta = 0(1°)90°$, Finska Vetenskaps-Societeten, *Acta*, v. 33, no. 4, 1905, and A. LODGE's 7D table for $n = 1(1)20$, $\theta = 0(5°)90°$, R. Soc. London, *Trans.*, v. 203A, 1904, p. 100–101, were used for further checking. For the 154 entries $n < 15$ said by Lane & Sweeney to be "accurate to six decimal places" there are 104 errors of from 1 to 11 units in the sixth decimal place; there are 34 unit errors and 35 cases of errors of more than 3 units. Next, let us consider the statement of the authors regarding the 66 entries for $n \geqslant 15$. Here there are 35 errors, 5 of them being errors of 2 units in the fifth decimal place.

Such are illustrations of ignorance and incompetence in a Government Laboratory at an important center.

R. C. A.

542[L].—NBSCL, *Tables of Bessel Functions of Fractional Order*, Volume 1. New York, Columbia University Press, 1948, xlii, 413 p. 19.7 × 26.5 cm. $7.50.

This is the first of two volumes dealing respectively with the tabulation of the functions $J_\nu(x)$ and $I_\nu(x)$, prepared under the able direction of Dr. LOWAN. The main part of this volume is occupied (p. 1–271) with the following tables of $J_\nu(x)$: $\nu = -\frac{3}{4}$ and $-\frac{2}{3}$, $x = [0(.001).9(.01)25;\ 10D]$; $\nu = -\frac{1}{3}$ and $-\frac{1}{4}$, $x = [0(.001).8(.01)25;\ 10D]$; $\nu = \frac{1}{4}$ and $\frac{1}{3}$, $x = [0(.001).6(.01)25;\ 10D]$; $\nu = \frac{2}{3}$ and $\frac{3}{4}$, $x = [0(.001).5(.01)25;\ 10D]$. δ^2, or modified δ^2, are provided throughout; and also δ^4, or modified δ^4, for $x = .05(.001).15$. Furthermore, for $x < .05$, tables for $x^{-\nu}J_\nu(x)$, with their second central differences, are also provided, where interpolation close to the origin is not feasible.

The interval in argument has been so chosen that interpolation may yield the maximum attainable accuracy over most of the range covered. Everett's interpolation coefficients E_2, F_2, E_4 and F_4 are tabulated at interval .001. In the range for fourth central differences indicated above, it is always possible to obtain an accuracy of at least 7S, by means of δ^2.

When x is large, use is made of the auxiliary tables (p. 273–383) of $A_\nu(x)$ and $B_\nu(x)$, for $x = [25(.1)50(1)500(10)5000(100)10000(200)30000;\ 10D]$; $J_\nu(x) = A_\nu(x) \cos (x - \frac{1}{2}\pi\nu - \frac{1}{4}\pi) - B_\nu(x) \sin (x - \frac{1}{2}\pi\nu - \frac{1}{4}\pi)$. There are 10D values of $\sin x$ and $\cos x$ (p. 386–387) for $x = 0(.01)1.6, 2(1)40$, and on p. xlii are 15D values of various constants.

The values of the first 30 zeros of $J_\nu(x)$, to 10D, and for $\pm\nu = \frac{3}{4}, \frac{2}{3}, \frac{1}{3}, \frac{1}{4}$, are given on p. 384–385. These appeared earlier (for the first 8 in more extended form) in *MTAC*, v. 1, p. 353–354, 1945; and v. 2, p. 118–119, 1946.

In the long Introduction is a note by Mr. H. E. Salzer on "Bessel functions considered as functions of their order." There are also (p. xxxiii–xli) "Note on modified second differences for use with Everett's Interpolation formula" and "Bibliography" (30 selected titles).

Bessel functions of fractional orders greater than unity may be obtained from the present tables by employing recurrence relations.

The preliminary manuscript of $J_\nu(x)$ to 13D or 13S, first prepared and subjected to a sixth difference test, has been previously referred to in *MTAC*, v. 1, p. 93, 300. No earlier published table gave more than 7D values of $J_\nu(x)$. Hence, even in this respect the volume under review marks a very notable contribution to tabular material in varied fields where such functions are of use, for example: wave propagation in stratified media with a constant gradient in the index of refraction,[1] asymptotic solutions of ordinary linear differential equations, quantum mechanics, and engineering problems.

R. C. A.

[1] R. E. LANGER, "On the connection formulas and the solutions of the wave equation," *Physical Rev.*, v. 51, 1937, p. 669–676.

543[L].—NBSCL, *Tables of the Bessel Functions* $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$. (*Applied Mathematics Series*, no. 1.) Washington, D. C., 1948, x, 60 p. 20 × 26 cm. For sale by the Superintendent of Documents, Washington. 35 cents.

These tables inaugurate the Applied Mathematics Series of the Computation Laboratory of the Bureau of Standards; see *MTAC*, v. 3, p. 65. In a foreword A. M. WEINBERG & E. P. WIGNER state that the tables were required in connection with the construction of huge nuclear chain reacting piles at Hanford, Washington.

The table gives (p. 3–31, 35–60) the values of $Y_0(x)$, $Y_1(x)$, for $x = 0(.0001).05(.001)1$, and the values of $K_0(x)$, $K_1(x)$, for $x = 0(.0001).033(.001)1$. A. N. Lowan states in the Introduction that "With the exception of a few entries close to the origin, the entries of the present table were obtained by interpolation in the BAAS tables. . . ." In general the functions are given to 8S or 8D except near the origin where a few values are given to 9S. First and second differences are provided.

Further to simplify the problem of interpolation in $Y_0(x)$ and $Y_1(x)$ for x small, the auxiliary functions $C_0(x)$, $C_1(x)$, $D_0(x)$, and $D_1(x)$ as defined in the British Association volume are given (p. 2) for $x = [0(.0001).005; 8D]$, Δ. Similar auxiliary functions of $K_0(x)$ and $K_1(x)$ are given (p. 34) for $x = 0(.001).03$. The Introduction states that the eight-figure values may be in error by as much as two units, and the nine-figure values may be in error by as much as four units of the last place retained. Known errors occur in $\Delta Y_1(0.0096)$ and $\Delta K_1(0.700)$, see MTE **133**. The tables have been spot checked by J. A. HARR, and several pages have been differenced by the reviewer. No other errors were found.

H. H. AIKEN

EDITORIAL NOTE: The tables of $K_0(x)$, $K_1(x)$ here printed were earlier listed as manuscripts in *MTAC*, v. 1, p. 165, 300.

544[L, S, T].—G. W. KING, "The asymmetric rotor. VI. Calculation of higher energy levels by means of the correspondence principle," *Jn. Chem. Phys.*, v. 15, Nov. 1947, p. 820–830. 19.7 × 26.8 cm.

This is the final printed record of an investigation, of which a duplicated account, embodied in two bimonthly progress reports, has been reviewed at some length in RMT **467** (*MTAC*, v. 3, p. 27–29). Table I (p. 825) is identical with Table I of the August report; Table II (p. 827) with Table II (June); and Table III (p. 829) with Table III (August).

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545[M].—G. R. MACLANE, "Table of integrals," p. 369–370 of H. M. JAMES, N. B. NICHOLS & R. S. PHILLIPS, *Theory of Servomechanisms*. (M.I.T. *Radiation Laboratory Series*, v. 25.) New York, McGraw-Hill, 1947.

The table is of integrals of the type

$$I_n = (2\pi i)^{-1}\int_{-\infty}^{\infty} g_n(x)dx/[h_n(x)h_n(-x)]$$

where

$$h_n(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n,$$
$$g_n(x) = b_0x^{2n-2} + b_1x^{2n-4} + \cdots + b_{n-1},$$

and the roots of $h_n(x)$ all lie in the upper half plane. The table lists the integrals $I_n(x)$, for $n = 1(1)7$. $I_1 = b_0/2a_0a_1$; $I_2 = (-a_2b_0 + a_0b_1)/2a_0a_1a_2$; etc.

546[M. S].—W. G. POLLARD & R. D. PRESENT, *On Gaseous Self-Diffusion in Long Capillary Tubes*. United States Atomic Energy Commission, Columbia University, MDDC–1521. Document declassified Dec. 11, 1947. 29 p. 20.4 × 26.8 cm. Offset print.

Appendix IV, "The evaluation of the integral Q," p. 28–29,

$$Q = \tfrac{1}{2}\int_0^{\frac{1}{2}\pi} \sin^2 t \cos^2 t \, dt \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos x \, e^{-2a \csc t \cos(x/\lambda)}dx = \pi/16 - \tfrac{1}{2}\pi K,$$

where

$$K = \int_0^{\frac{1}{2}\pi} \sin^2 t \cos^2 t \, [\mathbf{H}_1(2ai/\lambda \sin t) - iJ_1(2ai/\lambda \sin t)]dt.$$

The following five values of K: .0560, .0834, .1066, .1186, and .1250, are given as corresponding respectively to $a/\lambda = .25, .5, 1, 2,$ and ∞.

547[Q].—JOHN B. IRWIN, "Tables facilitating the least-squares solution of an eclipsing binary light-curve." *Astrophysical Jn.*, v. 106, 1947, p. 380–426. The numbering of equations in this review continues the numbering in RMT **549**.

As is well known, the fractional light l of an eclipsing binary system can, during eclipse, be written as

$$l = 1 - fL_1 \tag{6}$$

where L_1 denotes the fractional luminosity of the eclipsed component and $f \equiv f^x(k, p) \equiv f(\delta, r_1, r_2, x)$ is a function defined by equations (1)–(5). Moreover, the apparent separation δ of the centers of both components can be expressed as $\delta^2 = \sin^2\theta \sin^2 i + \cos^2 i$, where θ denotes the true anomaly of the eclipsing component in its relative orbit and i, the inclination of the orbital plane to the celestial sphere. Hence, in general, $f \equiv f(\theta, r_1, r_2, i, x)$, where θ is a known function of the time and r_1, r_2, i, x are constants to be determined by orbital analysis. Suppose that this has been done by some approximate method and that Δl represents the residual difference between light observed at any particular moment and that computed on the basis of the approximate elements. These residuals are due partly to the observational errors, and partly to possible inaccuracies in the adopted system of the elements. The corrections ΔL_1, Δr_1, Δr_2, Δi, and Δx required to minimize the sum of squares of the residuals can, however, be obtained as follows. Differentiating equation (6) above we find that

$$\Delta l = -f\Delta L_1 - L_1\left\{\frac{\partial f}{\partial r_1}\Delta r_1 + \frac{\partial f}{\partial r_2}\Delta r_2 + \frac{\partial f}{\partial i}\Delta i + \frac{\partial f}{\partial x}\Delta x\right\}. \tag{7}$$

The residual Δl of each observation of light during eclipse supplies us with an equation of condition of this form for determining the requisite corrections. Provided that these residuals are small and that their number exceeds sufficiently the number of the unknowns, a least-squares solution of such a set of equations then yields the most probable values of the required corrections as well as their errors.

The whole difficulty in application of this process centers around the evaluation of the four partial derivatives of f with respect to r_1, r_2, i, and x for every observed point. The derivative with respect to x presents the least difficulty; for a differentiation of equation (1) yields readily

$$\partial f/\partial x = 6(f^D - f^0)/(3 - x)^2. \tag{8}$$

The remaining three partial derivatives can be obtained by differentiating the right-hand sides of equations (2)–(5) behind the integral sign. In doing so we find that

$$\left\{\begin{aligned} &\frac{\partial f^0}{\partial r_2} = 2\frac{r_2}{r_1^2}I^0_{-1,0} \qquad &\frac{\partial f^D}{\partial r_2} = 3\frac{r_2^2}{r_1^3}\sqrt{\frac{\delta}{r_2}}\,I^0_{-1,1} \\ &\frac{\partial f^0}{\partial \delta} = -2\frac{r_2}{r_1^2}I^1_{-1,0} \qquad &\frac{\partial f^D}{\partial \delta} = -3\frac{r_2^2}{r_1^3}\sqrt{\frac{\delta}{r_2}}\,I^1_{-1,1} \end{aligned}\right. \tag{9}$$

where

$$\pi r_2^{\beta+\gamma+m+1}I^m_{\beta,\gamma} = \int_{\delta-r_2}^{c}[r_2^2 - (\delta - x)^2]^{\frac{1}{2}\beta}[2(s - x)]^{\frac{1}{2}\gamma}[\delta - x]^m dx$$

and $c = s$ or $\delta + r_2$ depending on whether the eclipse is partial or annular. Since, moreover, f is a homogeneous function of r_1, r_2, and δ(i.e., it depends, not on r_1, r_2, δ directly, but only on their ratios), Euler's theorem on homogeneous functions can be invoked to prove that

$$\frac{\partial f}{\partial r_1} = -\frac{r_2}{r_1}\frac{\partial f}{\partial r_2} - \frac{\delta}{r_1}\frac{\partial f}{\partial \delta} \tag{10}$$

while ultimately

$$\frac{\partial f}{\partial i} = -\sin 2i\frac{\partial f}{\partial \cos^2 i} = -\frac{\sin i \cos i \cos^2\theta}{\delta}\frac{\partial f}{\partial \delta}. \tag{11}$$

Five-decimal tables of the $I^m_{\beta,\gamma}$-integrals recently constructed by the reviewer for another purpose (see RMT **515**), can be used to facilitate the computation of the required derivatives by means of equations (8)–(11) above. This approach using closed formulae is, however, apt to be fairly laborious—chiefly because the I-integrals have not been tabulated in terms of the customarily known parameters k and p, but rather in terms of a certain auxiliary angle α which has nothing to do with the present problem. An alternative way for evaluating the requisite partial derivatives for any pair of k and p suggests itself: namely a numerical differentiation of T͡SESEVICH's tables.[1] The necessary formulae by which this can be done were outlined by the reviewer some time ago[2] and need not be reproduced here. The accuracy which can be reached in this way is naturally limited by the accuracy of T͡sesevich's tables; whereas a five-digit accuracy is easily attainable by means of the literal formulae (8)–(11) and Kopal's tables of the I-integrals, a numerical differentiation of T͡sesevich's tables cannot yield more than three correct figures in partial derivatives. This appears, however, to be all that is needed for the improvement of preliminary elements of eclipsing binary systems by way of differential corrections at present and probably for some time to come.

Computers of photometric orbits of eclipsing variables have recently been put under a deep debt of obligation to Professor Irwin for having completed the extensive set of bivariate tables of all partial derivatives of equation (7) in terms of k and p (or q for annular eclipses), which are the subject of the present review. They were evaluated, not by means of the literal formulae (8)–(11), but by a numerical differentiation of T͡sesevich's tables, although—Irwin states—the literal formulae and Kopal's tables were used to compute all derivatives for the $k = .975$ columns, the $p = -.975$ rows, and many values of the $p = -1$ row as well as numerous spot-checks. The extent of T͡sesevich's tables limited the accuracy of Irwin's tables to three significant figures. The arguments of tabulation are $k = 0(.1).8(.05)1$, $p = -1(.05) - .8(.1).8(.05)1$ if the eclipse is an occultation ($r_1 = r_a$), and $k = .2(.1).8(.05)1$, $p = -1(.05) - .8(.1).8(.05)1$ during a partial transit while, during annular phase, $q = .1(.1)1$. Except for tables 4, 8, 12, 16, 20, and 24, additional columns are inserted for $k = .975$ and $p = -.975$ to facilitate interpolation.

Irwin's paper contains altogether 32 bivariate tables which can be divided broadly in the following two groups:

I. Occultation 3D Tables (smaller star eclipsed; $r_1 = r_a$)

	$x = 0$	.4	.6	1
$r_a \dfrac{\partial f^x}{\partial r_b}$:	1A	9	17	1B
$r_a \dfrac{\partial f^x}{\partial r_a}$:	2A	10	18	2B
$\dfrac{-r_a r_b}{\cos^2\theta} \dfrac{\partial f^x}{\partial \cos^2 i}$	3A	11	19	3B

II. Transit 3D Tables (larger star eclipsed; $r_1 = r_b$)

	$x = 0$	.4	.6	1
$\dfrac{r_b}{k} \dfrac{\partial f^x}{\partial r_b}$:	5A	13	21	5B
$\dfrac{r_b}{k} \dfrac{\partial f^x}{\partial r_a}$:	6A	14	22	6B
$\dfrac{-r_b}{k \cos^2\theta} \dfrac{\partial f^x}{\partial \cos^2 i}$	3A	15	23	7

In addition, Tables 4 and 8 list 4D values of $\frac{1}{6}(3 - x)^2\partial f^x/\partial x$ for the case of an occultation and a transit, respectively. Tables 12 and 20 contain the occultation values of $\partial f^x/\partial x$ for $x = .4$ and .6, respectively, whereas tables 16 and 24 contain the corresponding transit values.

In the past several months the present reviewer has made extensive use of Irwin's tables (communicated to him in advance of publication) and had also a frequent opportunity for their spot-checking on the basis of the exact formulae. The tables were found to be correct to one unit of the last place in every case under examination. No tabular differences are given, but in most tables the first differences turn out to be small enough to permit a linear interpolation in both variables. There is no doubt that the publication of Irwin's tables will greatly facilitate the computation of accurate photometric orbits of eclipsing binary systems and, in this way, contribute significantly to further development of this rapidly growing subject. The only criticism which can be raised concerns some of Irwin's notations. Although these are adequately explained in the introduction preceding the tables, some of them are needlessly complicated (too many superscripts), and many depart rather drastically from common usage without sufficient reason. This fact, the reviewer feels, is bound to make the reading of Irwin's memoir inadvertently more difficult to a general reader than it need have been.

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[1] Leningrad, Astr. Inst., *Bi͡ull.*, nos. 45 (1939) and 50 (1940), see RMT **548**.
[2] Amer. Philos. Soc., *Proc.*, v. 86, 1943, p. 342.

548[Q].—V. P. T͡SESEVICH, "Tabli͡tsy dli͡a opredeleni͡ia elementov orbit zatmennykh zvezd tipa Algoli͡a" [Tables for the determination of the orbital elements of eclipsing stars of the Algol type], no. **45**, 1939, p. 115–152. "Tabli͡tsy fotometricheskikh faz zatmeniĭ peremennykh zvezd tipa Algoli͡a" [Tables of the photometric phases of eclipses of variable stars of the Algol type], no. **50**, 1940, p. 283–366. Leningrad, Astron. Institut, *Bi͡ull.*

The determination of orbital elements of eclipsing binary systems represents an important task of theoretical astronomy, calling for an extensive use of functions that can be practically dealt with only in tabular form. Some tables which have been constructed for this purpose represent rather remarkable feats of table construction and are, moreover, of a sufficiently general nature to warrant a comprehensive review.

The basic functions of double star astronomy whose numerical values are needed in tabular forms can be described as follows. Let $r_a, r_b (r_a \leqslant r_b)$ denote the radii of two luminous circular discs and let δ be the distance between their centers. If, at any moment, $\delta < r_a + r_b$, one disc will eclipse the other and a loss of light will result, which depends on the geometrical circumstances as well as on the distribution of brightness $J(r)$ over the disc undergoing eclipse. This latter function is usually approximated by a cosine law of the form $J(r) = J_0(1 - x + x \cos \gamma)$, where J_0 denotes the central brightness of the eclipsed disc, x is the "coefficient of darkening," and γ is such that $\sin \gamma = r$, the fractional distance from the center.

Analytical formulae expressing the resultant loss of light can be set up and evaluated in a closed form for any kind of eclipse.[1] It turns out that, if the disc undergoing eclipse appears uniformly bright ($x = 0$), the respective loss of light (proportional to the eclipsed area) can be expressed in terms of circular and algebraic functions; but if $x \neq 0$, we encounter elliptic integrals, not only of the first and second kinds, but also of the third kind which belong, fortunately, to the "circular" class and are therefore expressible in terms of incomplete elliptic integrals of the first and second kinds with complementary moduli. Even so, however, their explicit forms are so complicated as to render them of little practical use unless their numerical values are available in tabular forms.

The function f which represents the loss of light arising from an eclipse can, in general, be made to depend on two parameters formed by any ratio of r_a, r_b, and δ. For the purpose of table construction it is customary to adopt, as such parameters, the ratio of the radii $k \equiv r_a/r_b$ and the geometrical depth of the eclipse $p \equiv (\delta - r_b)/r_a$ chosen so that, by definition, $0 \leqslant k \leqslant 1$, while p is constrained to vary from 1 at the moment of first contact to -1 at the moment of internal tangency—regardless of whether the latter marks the beginning of totality or of annular phase. Furthermore, the loss of light f is, for practical reasons, customarily normalized to vary between 0 and 1 through dividing it by the respective loss of light at the moment of internal tangency, and so normalized values of the fractional loss of light are denoted as $\alpha^x(k, p)$. For any given pair of k and p the numerical value of $\alpha^x(k, p)$ depends, naturally, also on the degree of darkening x of the disc undergoing eclipse. In practice, however, it is sufficient to evaluate the α's for "uniform" ($x = 0$) and "completely darkened" ($x = 1$) discs only, because the α's appropriate for any intermediate value of x follow from the interpolation formula

$$(1) \qquad f^x(k, p) = [(3 - 3x)/(3 - x)]f^0(k, p) + [2x/(3 - x)]f^D(k, p).$$

It should be added that, because of the way in which the f's have been normalized, their numerical values depend also on whether the disc undergoing eclipse is the smaller or the larger of the two (i.e. whether the eclipse is an occultation or a transit). If the eclipsed disc appears uniformly bright, the ratio of the fractional losses of light during an occultation and transit for any given pair of k and p is equal to k^2; in the presence of a limb-darkening this ratio turns out, however, to be less simple and therefore separate tables of $\alpha^D(k, p)$ must be constructed depending on whether the eclipse is an occultation (α') or a transit (α'').

An inversion of $\alpha(k, p)$ yields the geometrical depth of the eclipse $p(k, \alpha)$ as a function of the ratio of the radii k and the fractional loss of light α. Unlike $\alpha(k, p)$, the inverse function $p(k, \alpha)$ defies, however, any explicit analytical formulation so that its values are obtainable only by numerical inverse interpolation. The values of appropriate to intermediate degrees of darkening cannot, moreover, be obtained by linear interpolation between the two extreme cases, but for each x the inversion has to be carried out anew.

Owing to their importance in the computation of orbits of eclipsing binary stars, several tabulations of the fractional loss of light $\alpha(k, p)$ and of the geometrical depth of the eclipse $p(k, \alpha)$ have been attempted in the past decades. The first of such tabulations, to 3D, of $p(k, \alpha)$ by H. N. RUSSELL[2] (*Astroph. Jn.*, v. 35, 1912, p. 315; T.I on p. 333) and of $p(k, \alpha')$ and $p(k, \alpha'')$ by H. N. RUSSELL & H. SHAPLEY (Tables Ix and Iy in *Astroph. Jn.*, v. 36, 1912, p. 243 and 390), are mentioned here for historical reasons only; for, these tables were both too small and insufficiently accurate to be of permanent value.[3]

In 1931, new tables of the α- and p-functions appropriate for eclipses of uniformly bright discs were completed by WEND [4] and HETZER [5] as dissertations under the guidance of the late Professor JULIUS BAUSCHINGER. Wend constructed a 4S table of α as a function of $k = .3(.01)1$ and δ/r_b (not p). Since there is no common upper (or lower) bound on δ/r_b during the eclipses, the range of his second parameter varies from table to table, though the increment is always equal to .01. While Wend completed a table essentially equivalent to $\alpha(k, p)$, Hetzer tabulated a quantity $W^{\frac{1}{2}} = 1 + kp(k, \alpha)$, to 4D, for $k = 0(.01)1$ and $\alpha = 0(.01)1$.

In 1938 and 1939 FERRARI published [6] 4D tables of the fractional loss of light, relevant to completely darkened discs, for both occultation and transit and covering partial as well as annular eclipses. His paper (i) contains a table of $\alpha'(k, p)$ for $k = 0(.05)1$ and $p = 1(-.01)-1$, while paper (ii) contains a table of $\alpha''(k, p)$, the range of the arguments being $k = .4(.05)1$ and $p = 1(-.01)-1$ for the partial (transit) eclipse, and $-1(-.01)-2.5$ for the annular eclipse; first tabular differences (k-wise) are also printed. An additional table gives $\alpha''(k, p)$, during annular eclipse, for $k = .3(.05).4$ and $p = -1(-.05)-3.25$.

The most extensive and accurate existing set of published α- and p-tables so far, however, are those (**45**, **50**) of T͡sesevich under review. No. **45** is devoted to tables appropriate to the case of complete darkening ($x = 1$). After a brief introduction, defining the function to be tabulated and describing the methods of their evaluation, we find T.I containing 5D values of $\alpha'(k, p)$ tabulated for $k = 0(.05)1$ and $p = 1(-.01)-1$, followed by a 4S table of its inverse function $p(k, \alpha')$ tabulated at the same intervals in k and for $\alpha' = 0(.01)1$ (T.II). T.III then gives 5D values of $\alpha''(k, p)$ for $k = .2(.05)1$ and $p = 1(-.01)-1$, followed again by its inverse $p(k, \alpha'')$ tabulated to 4D for $k = .2(.05)1$ and $\alpha'' = 0(.01)1$ (T.IV).

No. **50** opens (apart from certain auxiliary tables) with a 5S table of $\alpha(k, p)$ appropriate for uniformly bright discs and tabulated for $k = 0(.05)1$ and $p = 1(-.01)-1$ (T.V), followed by its inverse function $p(k, \alpha)$ tabulated for $k = 0(.05)1$ and $\alpha = 0(.01)1$ to 4D (T.VI). The main and most valuable feature of this number is, however, a group of 4D-tables of $p(k, \alpha'^x)$ (T.VII–X) and $p(k, \alpha''^x)$ (T.XI–XIV) relevant respectively to occultation and transit eclipses and each evaluated for $x = .2(.2).8$, for the same range and subdivision of the arguments as T.VI.

The concluding part of No. **50** is then devoted to the tables pertaining to annular eclipses. If the eclipse is annular, T͡sesevich replaces, for reasons of convenience, the geometrical depth p of the eclipse by $q = [k(1 + p)]/(k - 1)$, defines

$$\alpha''^x(k, q) = 1 + A^x(k)X(k, q),$$

and tabulates $X(k, q)$, to 5D, as a function of $k = .2(.05)1$ and $q = 0(.01)1$ (T.XV) as well as its invese $q(k, X)$, likewise to 5D, as a function of k and X for the same range and subdivision of the argument (T.XVI). T.XVIa, an expanded portion of T.XVI, gives the tabular values of $q(X, k)$ for $k = .2(.1)1$ and $X = .85(.005).95(.001).995(.0005).998(.0002).999(.0001)1$. No. **50** concludes with five univariate tables of $A^x(k)$ evaluated to 5D for $k = .2(.01)1$ and $x = 1(-.2).2$ (T.XVII–XXI). T.XVII giving $A(k)$ contains also first and second differences.

T͡sesevich's work leaves at present little to be desired and should probably be regarded as the standard set of fundamental tables that are likely to meet all astronomical requirements for a long time to come. The author did not give the reader as much insight into the processes by which his tables were constructed as one might wish to have, but they appear to have been essentially sound and their results impressive. Skeleton tables of all α-functions were apparently first computed to 6D; the tables enumerated above were obtained from them subsequently by interpolation and rounding off. Numerous and careful spot-checks of T͡sesevich's tables by Professor JOHN E. MERRILL of Hunter College, and by the present reviewer, disclosed that in general T͡sesevich's tables can be trusted to the number of digits given in his tables; rarely do departures from true values amount to more than one unit of the last place. In *Bulletin of the Panel on Orbits of Eclipsing Variables*, no. 2 (Harvard College Observatory, June 1946, p. 8–11), Merrill lists 42 errors exceeding 2 units of the last place found in Tsesevich's tables, and his list should be consulted by anyone planning to make extensive use of them. Misprints in T͡sesevich's tables are, unfortunately, many; most of them can, however, be easily detected from the behavior of local tabular differences. A systematic survey of tabular differences of T͡sesevich's tables would undoubtedly disclose a great many more misprints than have so far been noticed, and is much to be desired. Apart from misprints, however, the printing of the tables is generally good and clear, though the paper, especially that of no. **45**, is of poor quality. Both issues are provided with brief English summaries of their contents and also all headings are in Russian and English; but the wealth of information regarding the mathematical character of the tabulated functions, contained in the introduction to each of the issues, is accessible only to those who read Russian. It may be mentioned that no. **50** was published in Leningrad shortly before the outbreak of Russo-German war and apparently few copies were distributed before Leningrad became a theatre of war. The reviewer learns, however, through Professor SHAPLEY, that the stock of this valuable publication survived intact and is now available for distribution.

A comparison of T͡sesevich's 5S tables with the 4S tables of $\alpha'(k, p)$ and $\alpha''(k, p)$, published earlier by Ferrari,[6] affords a convincing check of the accuracy of the later numerical work. It should be pointed out that Ferrari did not evaluate the integrals expressing the α's in terms of the elliptic functions, but computed them with the aid of GAUSS's formula for approximate quadratures. Ferrari himself found that his series converged rather slowly, and the correction factor he obtained, by a comparison of the few entries directly computed for the transit, did not bring the systematic errors within satisfactory limits in many regions.

Entry-by-entry check of the 4S Ferrari against 5S T͡sesevich values rounded off to 4D, performed by Merrill (*l.c.*), disclosed that about three-fourths of Ferrari's values of $\alpha'(k, p)$ and one-half of his $\alpha''(k, p)$'s are a unit or more in error. Ferrari's α'''s are systematically too high (sometimes as much as 3 units) for positive p's and too low (frequently by 3 units) for negative p's. His α''''s are, in general, about one unit too low for large values of k and large positive p's one unit too high for k about 0.55 and large positive p's, as much as 4 units too low around $k = 0.55$ and $p = -0.85$, and too high by as much as 6 units around $k = 0.6$ and $p = -0.65$. Inasmuch as the appearance of more extensive and accurate tables by T͡sesevich has rendered all of Ferrari's work rather out of date, it does not seem worth while to publish any detailed correction table to his results.

The mathematical definitions of the tables discussed above may now be set forth. The loss of light f arising from the eclipse of an arbitrarily darkened disc obeys the equation (1) above, where

$$(2) \qquad \pi r_1^2 f^0 = \left\{ \int_s^{r_1} \int_{-(r_1^2-x^2)^{\frac{1}{2}}}^{(r_1^2-x^2)^{\frac{1}{2}}} + \int_{\delta-r_2}^{s} \int_{-[r_2^2-(\delta-x)^2]^{\frac{1}{2}}}^{[(r_2^2-(\delta-x)^2]^{\frac{1}{2}}} \right\} dxdy$$
$$= r_1^2 \cos^{-1} s/r_1 + r_2^2 \cos^{-1} (\delta - s)/r_2 - \delta(r_1^2 - s^2)^{\frac{1}{2}}$$

if the eclipse is partial, and

$$(3) \qquad \pi r_1^2 f^0 = \int_{\delta-r_2}^{\delta+r_2} \int_{-[r_2^2-(\delta-x)^2]^{\frac{1}{2}}}^{[r_2^2-(\delta-x)^2]^{\frac{1}{2}}} dxdy = \pi r_2^2$$

if it is annular; whereas

$$(4) \quad \pi r_1^3 f^D = \frac{3}{2} \left\{ \int_s^{r_1} \int_{-(r_1^2-x^2)^{\frac{1}{2}}}^{(r_1^2-x^2)^{\frac{1}{2}}} + \int_{\delta-r_2}^{s} \int_{-[r_2^2-(\delta-x)^2]^{\frac{1}{2}}}^{[r_2^2-(\delta-x)^2]^{\frac{1}{2}}} \right\} (r_1^2 - x^2 - y^2)^{\frac{1}{2}} dxdy,$$
$$= \pi r_1^3 - 2r_1^3\{(E - F)F' + E'F\} + [r_2(\delta/r_2)^{\frac{1}{2}}/3]\{7r_2^2 - 4r_1^2 + \delta^2\}(2E - F)$$
$$- [(\delta/r_2)^{\frac{1}{2}}/3\delta]\{5\delta^2 r_2^2 + 3(r_1^2 - r_2^2)^2 - 3\delta r_1^3\} F$$

if the eclipse is partial, and

$$(5) \qquad \pi r_1^3 f^D = \frac{3}{2} \int_{\delta-r_2}^{\delta+r_2} \int_{-[r_2^2-(\delta-x)^2]^{\frac{1}{2}}}^{[r_2^2-(\delta-x)^2]^{\frac{1}{2}}} (r_1^2 - x^2 - y^2)^{\frac{1}{2}} dxdy,$$
$$= 2r_1^3\{(E - F)F' + E'F\} + \tfrac{1}{3}\{7r_2^2 - 4r_1^2 + \delta^2\}[2\delta(r_2 - \delta + s)]^{\frac{1}{2}} E$$
$$+ \tfrac{1}{3}\{(\delta^2 - r_2^2)^2 - r_1^2(2r_1^2 - r_2^2) + 6\delta r_1^3\} F/[2\delta(r_2 - \delta + s)]^{\frac{1}{2}}$$

if it is annular. In these latter equations, $F \equiv F\left(\frac{\pi}{2}, k\right)$ and $E \equiv E\left(\frac{\pi}{2}, k\right)$ denote the Legendre complete integrals of the first and second kinds, with the modulus $k^2 = \frac{1}{2}[1 - (\delta - s)/r_2]$ for a partial eclipse and $= 2[1 - (\delta - s)/r_2]^{-1}$ for an annular eclipse. $F' \equiv F(\phi, k')$ and $E' \equiv E(\phi, k')$ stand for the incomplete integrals of the same kind with a complementary modulus $k' = (1 - k^2)^{\frac{1}{2}}$ and an amplitude $\phi = \sin^{-1}[2\delta/(r_1 + r_2 + \delta)]^{\frac{1}{2}}$ if the eclipse is partial, and $\phi = \sin^{-1}[(r_1 + r_2 - \delta)/(r_1 + r_2 + \delta)]^{\frac{1}{2}}$ if it is annular.

For the purposes of tabulation, we prefer to normalize the f-functions so as to make them vary between 0 at the moment of the beginning of the eclipse ($\delta = r_1 + r_2 = r_a + r_b$) and 1 at the moment of internal tangency of both discs ($\delta = r_a - r_b$). This can be achieved by putting, by definition, $f^0 = \alpha(k, p)$, if $r_1 = r_a$; or $f^0 = k^2\alpha(k, p)$ if $r_1 = r_b$. $f^D = \alpha'(k, p)$ if $r_1 = r_a$; or $f^D = \frac{3}{2}k^2\Phi(k)\alpha''(k, p)$, if $r_1 = r_b$, where

$$(6) \qquad \Phi(k) = 4\{\sin^{-1}\sqrt{k} + \tfrac{1}{3}(4k - 3)(2k + 1)\sqrt{k}(1 - k)^{\frac{1}{2}}\}/3\pi k^2.$$

$\Phi(k)$ is equal to $\frac{3}{2}k^2$ times the value of f^D at the moment when $\delta = r_1 - r_2$. T͡sesevich tabulates $\Phi(k)$ to 6D for $k = .2(.01)1$ (no. **50**, p. 307, auxiliary T.5). The function $\frac{3}{2}k^2\Phi(k)$ was also previously tabulated by Ferrari [6] to 4D for $k = .2(.05)1$. (v. 148, p. 221).

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[1] For their details see, for instance, Z. KOPAL, *Introduction to the Study of Eclipsing Variables* (*Harvard Observatory Monographs*, no. 6), Cambridge, Mass., 1946, chapter 2.

[2] This table was later recomputed by B. W. SITTERLY, Princeton Univ. Observatory, *Contributions*, no. 11, 1930, 41 p. 23 × 32 cm.

[3] According to T͡SESEVICH, no. **45**, p. 124, "T.I*y* [p. 390] is especially deficient in accuracy and contains systematic departures from accurate figures by twenty and more units of the third decimal."

[4] A. G. M. WEND, *Eine Tafel zur Theorie der Bedeckungsveränderlichen*. Diss. Leipzig. Weida i. Thür, 1931, 44 p. 17.7 × 26.3 cm.

[5] E. M. R. HETZER, *Beitrag zu H. N. Russell's Methode der Berechnung der Elemente von Verfinsterungsvariablen unter Voraussetzung von Kreisbahnen und gleichmässig hellen Sternscheiben*. Diss. Leipzig. Weida i. Thür, 1931, 56 p. 23.7 × 29 cm.

[6] K. G. FERRARI, "Zur Theorie des Bedeckungslichtwechsels bei vollständig randverdunkelten Sternscheiben." Akad. d. Wissen., Vienna, *Abt.* IIa, (i) v. 147, 1938, p. 497–511; (ii) v. 148, 1939, p. 217–235.

At the time of publication of Ferrari's memoirs, only a few skeletons of T͡sesevich's tables of $\alpha'(k, p)$ and $\alpha''(k, p)$ were available for comparison; see Pulkovo, Astron. Observatoriia, T͡sirkuliary [Circulars], no. 24, 1938, p. 41–45.

549[U].—JAPAN, HYDROGRAPHIC OFFICE, *Publication* no. 601. *Celestial Navigation Computation Tables.* Tokyo, Hydrographic Office, 1942. 138, xxix p. 18 × 25.3 cm.

These tables were intended for surface navigation; the volume contains practically everything that will be needed for celestial navigation beyond the material in a Nautical Almanac.

The reader will find the title page of this volume facing page 138! It is followed by a table of contents and twenty-seven pages of explanation including one page giving English equivalents of a number of Japanese characters appearing frequently in the text.

The first fourteen pages of the volume are given over to the usual auxiliary tables of navigation, conversion of time to arc, and corrections to be applied to observed altitudes. The latter are unusual in that they include corrections to be applied to altitudes less than 6°; special corrections for temperature, barometric pressure, and difference between air and water temperatures are given.

The basic table provided for the computation of the altitude and azimuth of a celestial body (no. 6, p. 15–79) is similar to that of YONEMURA [*Tables for calculating Altitudes and Azimuths of Celestial Bodies*, Tokyo, 1920]. In the usual American notation (used throughout this review) where t, d are the LHA and declination of the celestial body, L the latitude of the observer, and h, Z the altitude and azimuth angle of the celestial body, the formulae are (θ being an auxiliary angle)

$$\log(1/\text{hav}\,\theta) = \log(1/\text{hav}\,t) + \log\sec d + \log\sec L,$$

$\text{hav}(90° - h) = \text{hav}(L \pm d) + \text{hav}\,d$, $\log\csc Z = \log\csc t + \log\sec d - \log\sec h$. The table gives the values, to the nearest integer, of $10^5\log(1/\text{hav}\,x)$ and $10^5\log\csc x$ for $x = 0(1')360°$; and of $10^5\,\text{hav}(90° - x)$, $10^5\,\text{hav}\,x$, and $10^5\log\sec x$ for $x = 0(1')90°$.

Table 7 gives the altitude to the nearest 0°.1 of a celestial body of declination $d = 0(1°)$-$10°(2°)62°$ when it is on the prime-vertical at latitude $L = 0(1°)50°(2°)70°$.

Table 8 is designed for the computation of azimuth angle by means of the formulae:

$$\tan\tfrac{1}{2}(Z + q) = \cos\tfrac{1}{2}(L + d)\csc\tfrac{1}{2}|L - d|\tan\tfrac{1}{2}t,$$
$$\tan\tfrac{1}{2}(Z - q) = \sin\tfrac{1}{2}(L + d)\sec\tfrac{1}{2}(L - d)\tan\tfrac{1}{2}t.$$

with argument $S = L + d = 0(5')6°(10')90°$, the values of $X_1 = 3200 + 10^3\log\cos\tfrac{1}{2}S$ and $Y_1 = 3200 + 10^3\log\sin\tfrac{1}{2}S$ are given to the nearest integer. With argument $D = |L - d|$ $= 0(5')6°(10')174°(5')180°$, the values of $X_2 = 10^3\log\csc\tfrac{1}{2}D$; $Y_2 = 10^3\log\sec\tfrac{1}{2}D$ are given to the nearest integer. With argument $t = 0(5')6°(10')174°(5')188°(10')354°(5')360°$, the values of $X_3 = Y_3 = 3200 + 10^3\log\tan\tfrac{1}{2}t$ are given to the nearest integer. With argument x (or y) $= 0(0°.05)3°(0°.1)87°(0°.05)90°$, the values of $X_4 = 6400 + 10^3\log\tan x$ (or $Y_4 = 6400 + 10^3\log\tan y$) are given to the nearest integer. This table is especially convenient for the calculation of a series of azimuths of a celestial body for a fixed position of the observer and constant declination of the celestial body.

Table 9 is intended for star identification, that is, one can start with values of h, Z and L, and quickly obtain values of d and t, usually, enough to identify the celestial object. The formulae used are:

$$\cot K = \sec Z \tan h, \sin d = \sin h \sec K \sin (L + K), \tan t = \tan Z \sin K \sec (L + K).$$

With arguments $Z = 0(1°)180°$, $h = 0(1°)90°$, $K = 0(1°)90°$, $L + K = 0(1°)180°$, $d = 0(1°)90°$, $t = 0(1°)90°$, the values of $K_1 = 10^3 \log \sec Z$, $H_1 = 3000 + 10^3 \log \tan Z$, $K_2 = 3000 + 10^3 \log \tan h$, $D_1 = 3000 + 10^3 \log \sin h$, $K_3 = 3000 + 10^3 \log \cot K$, $D_2 = 10^3 \log \sec K$, $H_2 = 3000 + 10^3 \log \sin K$, $D_3 = 3000 + 10^3 \log \sin (L + K)$, $H_3 = 10^3 \log \sec (L + K)$, $D_4 = 6000 + 10^3 \log \sin d$, $H_4 = 6000 + 10^3 \log \tan t$ are given to the nearest integer. This description makes the tables sound cumbersome; actually they occupy only 3 pages and the formulae are written $K_1 + K_2 = K_3$; $D_1 + D_2 + D_3 = D_4$; $H_1 + H_2 + H_3 = H_4$.

Table 11A is essentially a "distance-travelled" table for use with hourly rate 6(.1)34 knots for time intervals ranging from 1^m to 10^d, with tabulated value in nautical miles. Table 11B yields distance travelled in meters in 1 to 60 seconds at speed ranging from 1 to 34 knots. Table 12 is a table of Difference of Longitude and Departure for courses 1°(1°)89° corresponding to distances of 0(1′)100′(100′)900′. Table 13 gives difference of longitude to 0.1 with arguments departure of 0(1′)10′(10′)100′(100′)900′ and middle latitude 0°, 4°(2°)-10°(1°)65°. Table 14 is one of meridional parts for latitudes 0(10′)90° with proportional parts for interpolation; it is based on Bessel's figures and hence now out of date. Tables 15–22 are respectively 4-place tables of $\log N$, $\log \sin x$ for $x = 0(1')10°$; $\log \tan x$ for $x = 10'(1')10°$; $\log \sin x$ for $x = 0(10')90°$; $\log \tan x$ for $x = 10'(10')89°50'$; $\sin x$ for $x = 0(10')90°$; $\tan x$ for $x = 0(10')89°50'$; $\sec x$ for $x = 0(10')89°50'$. Tables 23–25 include mathematical formulae and numerical values likely to be of use to mariners. Table 26 is one of equivalents —weights, measures, areas, etc.

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550[U].—JAPAN, HYDROGRAPHIC OFFICE, *Publication* no. 602. *Brief Celestial Navigation Table* (*Dead Reckoning Position Method*). Tokyo, Hydrographic Office, October 1942. 4, 50, xxiv p. 18.2 × 25.7 cm.

This volume presents a dead-reckoning position method based on a division of the astronomical triangle into two right triangles by a perpendicular dropped from the celestial body upon the meridian. K is defined in this book as the polar distance of the foot of the perpendicular. Using this, and the usual American notation (as indicated in the preceding review), the equations are:

$$\log \tan K = \log \cot d + \log \cos t$$
$$\log \cot Z = \log \cot t + \log \csc K + \log \cos (K + L)$$
$$\log \cot h = \log \cot (K + L) + \log \sec Z.$$

These are written $K_3 = K_1 + K_2$, $Z_4 = Z_1 + Z_2 + Z_3$, $A_3 = A_1 + A_2$. This notation makes the computing form simple, but it involves a certain amount of duplication in the tables provided. There are three tables, each with values given to four decimals. In the first, $\log \cot x$ is given for $x = 0(1')90°$. The second table gives $\log \cot x$ and $\log \cos x$ for $x = 0(1')360°$. The third table gives $\log \tan x$, $\log \csc x$, $\log \cot x$, $\log \sec x$ for $x = 0(1')90°$. These tables remind one of the four holes the hired man cut in the barn door for the mother cat and her three kittens.

The title page follows page 50 and is in turn followed by twenty pages of explanation in Japanese. The usual auxiliary tables giving corrections to observed altitudes are given in graphical form on the first four pages of the volume; they are also given in the usual tabular form (but to a low order of accuracy) on page 50. A multiplication and interpolation table for hourly differences 2(1)20 allows the determination of values corresponding to time intervals ranging from one second to eight days. Following this, there is a table giving the north–south and east–west components of distances 1′ to 200′ along courses 0 to 90°. Another table gives the longitude differences corresponding to distances (measured in an east–west direction) 1′(1′)14′ at latitudes 0°, 4°, 8°, 10°, 12°, 14°(1°)70°.

The design of this volume would indicate it was planned for men with a very limited background in celestial navigation. For this reason, it might have a strong appeal for many of the persons who now use Martelli's tables. A simple trimming of the top and bottom corners of pages helps one to locate the desired tables quickly.

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551[V].—MASSACHUSETTS INSTITUTE OF TECHNOLOGY, Department of Electrical Engineering, Center of Analysis, Technical Report no. 3, work performed under the direction of ZDENĚK KOPAL, under NOrd Contract No. 9169: *Tables of Supersonic Flow Around Yawing Cones.* Cambridge, Mass., 1947, xviii, 324 p. +2 folding plates, 20.2 × 26.8 cm. For sale by Superintendent of Documents, Washington, D. C., $2.50.

The supersonic flow of air around a non-yawing cone has been numerically solved and presented in rather complete tables entitled *Tables of Supersonic Flow around Cones* (RMT **475**, *MTAC*, v. 3, p. 37–40). Such information is very valuable in considering the performance of sharp-nosed projectiles or rockets when flying directly along the axis. Any device, however, which moves in free flight through the atmosphere can be maintained only approximately in a non-yawed position. Since yawing motion is inevitable, additional questions are raised which cannot be answered from the previously published tables. The first question of importance is the sensitivity of the drag to yaw; that is, will the forces on the nose of the projectile change appreciably with small angles of yaw? The second and somewhat more important question which is raised is that concerned with the normal force. Just as an airplane wing flying at an angle of attack will experience a lift force normal to its direction of motion, so a cone flying yawed at supersonic speed will experience a force normal to its direction of motion. Such normal forces, if large, would make the problem of control of the flight of a body difficult, since these forces must be properly counterbalanced if the motion is to be stable.

In the present work, the first-order improvement to the former tables is given. The angle of yaw ϵ is considered so small that its square and all higher powers can be ignored. With this approximation, the general equations of motion in polar coordinates are reduced to the following non-homogeneous, second-order, ordinary differential equation with non-constant coefficients:

$$\frac{d^2(x/d_1)}{d\theta^2} + A\frac{d(x/d_1)}{d\theta} + B(x/d_1) + C = 0.$$

In this equation x gives the correction to the radial velocity as a function of the polar angle θ. The coefficients A, B, C are rather intricate functions of θ, as well as of various quantities occurring in the solution for a non-yawed cone. Specifically, the two-velocity components, the speed of sound, the pressure, density, and ratio of specific heats all enter. As is noted in the text, these expressions are obtained after "requisite and rather troublesome algebra." The constant d_1 in this equation is evaluated from the boundary conditions at the shock wave.

In this work, while approximations are made appropriate to a slight yawing of the cone, there are no approximations made relative to the shock wave. The flow behind the shock wave is treated as rotational and of non-constant entropy as is appropriate to this problem. In this respect the present work, which follows the theory of STONE,[1] is an improvement on the earlier work of TSIEN [2] and SAUER.[3]

To obtain a solution an initial Mach number and shock wave angle θ_w were chosen, and the integration was carried out numerically toward decreasing θ until the appropriate boundary condition at the surface of the cone was obtained. From the resulting values of x all other information could then be simply computed. In particular, the yaw of the shock wave which is in general not the same as that of the cone, was found. In all of this work it was assumed that the Mach number of the free stream was high enough for the shock wave to be attached to the vertex of the cone. The results of the computation are given in sufficient detail to compute the variation in the three-velocity components and the pressure and density around a slightly yawing cone. In addition, normal force coefficients and drag coefficients are given. It is noted, however, that to the present approximation the drag coefficients are identical with those for a non-yawed cone as reported previously.

In three tables, included in the introduction, $\theta_s = 10°(5°)20°$, $\bar{u}_s = .4(.1).9$, a comparison is made of calculations of the normal force coefficients by use of Tsien's formula and the present tables. The error ranges from -12.2% to $+44.2\%$ (ignoring the fact that Tsien's formula for sufficiently high Mach number gives imaginary results). The tables presented are as follows:

I, p. 1–306: *Tables of Supersonic Flow of Air Around Slightly Yawing Cones.*

These tables give, successively, the contributions x, y, z to the velocity components u, v, w taken in the direction of increasing spherical polar coordinates r, θ, ϕ, due to a small yaw, as functions of the angular variable θ; and the corresponding proportional changes in pressure $(\eta/\bar{p})$ and density $(\xi/\bar{\rho})$ due to the same cause.

The velocities are expressed in terms of the velocity which the air in front of the shock wave would attain if allowed to expand adiabatically into a vacuum.

These 4D tables are for $\theta_s = 5°(2°.5)15°(5°)50°$, $\theta = 5°$ to $89°.612$ in steps ranging from $.125°$ to $1°$. These cover the entire range of interesting cone angles and speeds including some cases of the "second" solution, i.e., strong shock (see previous review).

II, p. 307–312: *Survey of the Results.* The individual columns indicate:

$\bar{u}_s$ = radial velocity-component of the axial flow along the solid surface;
θ_w = semi-apex angle of the shock wave;
M = Mach number (i.e., stream velocity divided by the velocity of sound in the undisturbed air in front of the shock wave);
d_1 = constant specifying the change of entropy across streamlines;
δ/ϵ = ratio of the yaw of the shock wave to the yaw of the solid cone;
K_D = coefficient of head drag on the cone;
K_N = coefficient of normal drag on the cone.

III, p. 315–317: *Table Giving the Shock-to-Cone Yaw Ratio,* δ/ϵ, in Terms of the Radial Velocity Component, u_s, along the Solid Surface, and the Semi-Apex Angle of the Cone, θ_s.

The values in this table are primarily 4D results interpolated for more convenient use from the previous computations.

IV, p. 319–321: *Table Giving the Coefficient of Normal Drag,* K_N, in Terms of the Radial Velocity Component, $\bar{u}_s$, along the Solid Surface, and the Semi-Apex Angle of the Cone θ_s.

Here again 4D interpolated values are given for convenience.

V: *Two folding charts* are given which show the yaw of the shock wave and the normal force coefficients as functions of the cone angle and velocity along the cone. These give a good general picture of the nature of the flow and make for ready practical use where high accuracy is not required.

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[1] ARTHUR H. STONE, *The Aerodynamics of a Slightly Yawing Supersonic Cone,* NDRC Report, Div. 1, 1944.

[2] HSUE-SHEN TSIEN, "Supersonic flow over an inclined body of revolution," *Jn. Aero. Sci.*, v. 5, 1938, p. 480.

[3] R. SAUER, "Überschallströmung um beliebig geformte Geschoszspitzen unter kleinem Anstellwinkel," *Z. f. Luftfahrtforschung*, v. 19, 1942, p. 148–152.

552[V].—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS, *Technical Note* no. 1428: *Notes and Tables for Use in the Analysis of Supersonic Flow*. By the staffs of the Ames Aeronautical Laboratory, Moffett Field, Calif. Washington, D. C., Dec. 1947, iv, 73 p. +7 plates. 20 × 26.2 cm.

This NACA *Technical Note* is a compilation of data found important for the analysis of compressible flow in connection with test work in a supersonic wind-tunnel. The text of the paper is devoted to a review of several fundamental aspects of the theory of supersonic flow. These include thermodynamics, equations of motion, nozzle theory, shock waves, expansion around a corner, airfoil theory, and flow about wedges and cones. The appendices give formulae for the calculation of the viscosity, Reynolds number, humidity relations, and atmospheric corrections for air.

The five tables included in the paper contain the following information: **Table I** gives various nozzle data for subsonic flow such as the ratios of the local pressure to rest pressure, the local density to the rest density, etc. for values of the Mach number. The Mach number $M = 0(.01)1$. **Table II** gives similar nozzle data for supersonic flow and in addition gives such data as the local Mach angle, the angle through which a supersonic stream is turned to expand from Mach number 1 to the local Mach number, etc. $M = 1(.01)3.5(.1)5(1)$-$10(5)20, 100, \infty$. **Table III** gives ratios of the local pressure to the rest pressure, the local density to the rest density, etc. on either side of a normal shock wave. **Table IV** gives the Mach number functions for use with small perturbation airfoil section theory, and **Table V** gives various properties of the standard atmosphere. All of the data given in the tables pertain to air where the ratio of the specific heats is 1.4. The functions are tabulated to at least 4S accuracy.

The graphs that are given include plots of maximum theoretical contraction ratio that permits start of supersonic flow in diffuser entrance against Mach number, and the variation of Reynolds number with Mach number. The well-known graphs for the characteristics of wedge and cone flow are also included.

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MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT **521** (Lambert), **522** (Peters), **526** (Rybner), **529** (Kraïtchik), **531** (Pettit), **535** (Cambi), **541** (Lane & Sweeney), **548** (Ferrari, Russell & Shapley, T͡sesevich).

128. J. R. AIREY, "Tables of the Bessel functions $J_n(x)$," BAAS, *Report*, 1915, p. 29–30.

A list of errors was given in MTE **124** for the 10-decimal part of this table for $n = 0(1)13$' where $x > 6$. The whole table has now been compared with proofs of the forthcoming BAAS' *Bessel Functions*, part 2, and the following 23 further errors have been found for [$x = .2(.2)$-6; 6D]. Thus the total number of errors, large and small, in this table is 74.

x	*n*	*For*	*Read*	*x*	*n*	*For*	*Read*
1.8	9	—	+0.000 001	5.6	9	+0.012 893	+0.012 907
2.4	10	+0.000 002	+0.000 001		10	+0.003 870	+0.003 912
2.6	9	+0.000 024	+0.000 025		11	+0.000 930	+0.001 062
2.8	11	—	+0.000 001		13	+0.000 057	+0.000 059
3.8	11	+0.000 021	+0.000 022	5.8	4	+0.378 765	+0.378 766
	12	+0.000 004	+0.000 003		8	+0.046 382	+0.046 381
4.6	13	+0.000 005	+0.000 006		9	+0.016 641	+0.016 639
5.4	5	+0.310 074	+0.310 070		10	+0.005 261	+0.005 256
	13	+0.000 037	+0.000 038		11	+0.001 500	+0.001 486
5.6	6	+0.198 559	+0.198 560		12	+0.000 380	+0.000 381
	7	+0.094 452	+0.094 455		13	+0.000 088	+0.000 089
	8	+0.037 571	+0.037 577				

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6 Feb. 1948