

Mathematical Tables *and other* Aids to Computation

A Quarterly Journal edited on behalf of the
Committee on Mathematical Tables
and Other Aids to Computation

by

RAYMOND CLARE ARCHIBALD
DERRICK HENRY LEHMER

WITH THE COÖPERATION OF

EDWARD WHITNEY CANNON
LESLIE JOHN COMRIE
SOLOMON ACHILLOVICH JOFFE

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