with the plotting plug board. The points printed by the tabulator were connected. This forms our graph of the original data. To test the method this curve was followed by a pantograph which reduced it to the size of the original drawing. This small scale plot was compared directly with the original. The agreement was excellent in spite of the number of steps.

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EDITORIAL NOTE: Although the method here described appears to have been original with Mr. KING, W. J. E. informs us that it has been used in several places for several years, but does not seem to have been previously described in print.

RECENT MATHEMATICAL TABLES

578[A].—Schomann's, *(1 × 1 Tabelle) 1–99 × 99 und 1–999 × 9*. Hamburg, Germany, Verlag Br. Sachse, n.d., 16 p. 14.2 × 20.6 cm.

This little paper-covered multiplication table gives, p. 2–9, the results of multiplications of pairs of numbers 1(1)99 and 1(1)99; and, p. 10–15, of pairs 1(1)999 and 1(1)9. The use of the table to find 8379 × 5623 and 8967 × 456 is indicated.

579[A].—H. S. UHLER, "Twenty exact factorials between 304! and 401!," Nat. Acad. Sci., *Proc.*, v. 34, Aug. 1948, p. 407–412. 17.4 × 25.7 cm.

The text: "In the year 1944 the author published privately a little book entitled *Exact Values of the First 200 Factorials*. [See *MTAC*, v. 1, p. 312.] Subsequently he computed with great care the exact values of $n!$ from $n = 201$ to $n = 300$. The data of this third century have not appeared in print. One consultable copy has been deposited in the library of Brown University, Providence, Rhode Island, and another copy is in the possession of Doctor J. C. P. Miller, technical director of Scientific Computing Service Limited, 23 Bedford Square, London, W.C.1, England.

"Recently the author has computed a skeleton table of 42 exact factorials beginning with 303! and ending with 400!. This table was built up by first calculating the values of $n!$ for which $n + 1$ was one of the 17 primes from $n = 307$ to $n = 401$, so that Wilson's theorem could be applied as a more exacting check in addition to congruence testing with moduli such as $10^6 + 1$, $10^8 + 1$, etc. Incidentally the values of 350!, 372!, 375!, 378! and 400! as found by the author in February, 1945, were reproduced identically in the work performed three years later. In order to make a few of these arithmetical constants available to other investigators requiring exact values in the fourth century of $n!$ the following table of equally spaced but non-consecutive data is presented." [Then follows $n!$, $n = 305(5)400$; in the last there are 869 digits.]

EDITORIAL NOTE: Professor UHLER has reported a printers' error under $340!/10^{23}$, in the second line, 11th pentad, which reads 58229 erroneously instead of the correct order 85229. This correction was made in reprints.

580[C, D].—FRANCE, INSTITUT GÉOGRAPHIQUE NATIONAL, *Tables des Logarithmes à Huit Décimales*. Tome 1: *Logarithmes des Nombres entiers de 1 à 120 000;* Tome 2: *Logarithmes des Fonctions Circulaires de dix secondes en dix secondes d'arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1944 [x, 216, x, 402], p. 20.7 × 27 cm. 2700 francs, unbound.

This is the second edition of the great work issued by the Service Géographique de l'Armée in 1891, to which we have had occasion more than once to make reference (*MTAC*, v. 1, p. 36, 85, 145; v. 2, p. 181). The first edition was a single-volume work with pages of size 27.5 × 34.7 cm. In the offset reproduction the print page has been reduced in size in the ratio 27:22. A new preface is added. Log sin, log cos, log tan, log cot are tabulated for each $0^g.001$. All of the errors we listed on p. 85 have been corrected, but not the single known error of v. 1, noted on p. 181.

R. C. A.

581[C, E].—GEOFFREY BEALL, "The transformation of data from entomological field experiments so that the analysis of variance becomes applicable," *Biometrika*, v. 32, p. 243–262, 1942.

On p. 250–251 there is a table of $k^{-\frac{1}{2}} \sinh^{-1} (kx)^{\frac{1}{2}}$ for $x = [0(1)50(5)100(10)300; 2D]$, $k = 0(.02).1(.05).3(.1).6(.2)1$.

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582[D, P].—CONSIGLIO NAZIONALE DELLE RICERCHE, *Questioni di Matematica Applicata. Trattate nel 1° Convegno di Matematica Applicata (Roma, 1936), da M. Picone, G. Krall, C. Ferrari.* Bologna, Zanichelli, 1939, iv, 155 p. 15 × 23.5 cm. 200 lire. GIULIO KRALL, "Strutture in foglio (a scatola), volte-travi e volte secondo superfici di traslazione. Applicazioni alle costruzioni civili ed idrauliche," p. 37–131.

Tables, p. 57–58, of $\alpha(K, \psi)$ and $\beta(K, \psi)$, $K = 0(.1)1$, $\psi = [0, 7°.5, 15°(15°)90°; 3D]$, where $\alpha = (l + K^2 \cot^2 \psi)^{-\frac{1}{2}}$, $\beta = K(K^2 l + \tan^2 \psi)^{-\frac{1}{2}}$, $K = b/a$, the ratio of semi-axes of an ellipse, and $l = 1$.

Tables, p. 59–68, of $t_2, s, t_1, t_2', s', t_1'$, for $K = .1(.1)1$, $\psi = [0, 7°.5, 15°(15°)90°; 5\text{–}6S]$, where $t_2 = q^3K^{-1} \cos \psi$, $t_2' = q^3K^{-1} \cos^2 \psi$, $s = \sin \psi[2 + 3\epsilon^2q^2K^{-2} \cos^2 \psi]$, $t_1 = \cos \psi[2Kq^{-3} + 3\epsilon^2K^{-1}q^{-1} (\cos^2 \psi - 2q^2K^{-2} \sin^2 \psi)]$, $s' = \sin 2\psi\ (1 + \epsilon^2q^2K^{-2} \cos^2 \psi)$, $t_1' = 3[(Kq^{-3} + \epsilon^2 \cos^2 \psi K^{-1}q^{-1}) \cos 2\psi + \frac{1}{2}qs^2K^{-3} \sin 2\psi]$, $\epsilon^2 = 1 - K^2$, $q = K(\sin^2 \psi + K^2 \cos^2 \psi)^{-\frac{1}{2}}$.

Tables, p. 82–83, of $f(n, \psi) = (n^2 \cos^2 \psi + \sin^2 \psi)^{\frac{1}{2}}$, for $n = [1(.5)5; 4D]$, $\psi = 0, 7°.5, 15°(15°)90°$, and of $[f(n, \psi)]^{-1}$ to 5D.

Tables, p. 94–97, 99–101, 3D, of

$$s(\phi, \delta) = 2K^{-1} \cos^2 \phi_0 \cos^3 \delta_0 \tan \phi \tan \delta - 2 \cos^2 \delta_0 \sin \phi \tan \delta - 2K^{-1} \cos^2 \phi_0 \tan \phi \sin \delta,$$

and

$$\begin{aligned} t_\phi(\phi, \delta) = 2K^{-2} \cos^2 \phi_0 \cos^3 \delta_0 (\cos \phi - \cos \phi_0)(\cos \phi \cos^2 \delta)^{-1} \\ + \cos^2 \delta_0 K^{-1} (\sin^2 \phi - \sin^2 \phi_0)(\cos \phi \cos^2 \delta)^{-1} \\ -2K^{-2} (\cos \phi)^{-1} \cos^2 \phi_0 \cos \delta (\cos \phi - \cos \phi_0) \end{aligned}$$

for (p. 94–95) $\phi_0 = \delta_0 = \frac{1}{4}\pi$, $K = 1$, ϕ and $\delta = 0(.1).7, \frac{1}{4}\pi$; for (p. 96–97) $\phi_0 = .487^r = 27°53'6''$, $\delta_0 = \frac{1}{4}\pi$, $K = .8$, $\delta = 0(.1).7, .785$; $\phi = 0(.1).4, .487$; for $s(\phi, \delta)$ and $t_\phi(\phi, \delta)$ (p. 99–101) ϕ_0 and $\delta_0 = 30° = .524^r$, ϕ and $\delta = 0(.1).5, .524$, and ϕ_0 and $\delta_0 = 15° = .262^r$, ϕ and $\delta = 0$ (for t_ϕ only), .1, .2, .262.

Extracts from Text

583[D, P].—WILHELM JORDAN (1842–1899), *Hilfstafeln für Tachymetrie*, Twelfth ed. Stuttgart, Metzlersche Verlagsbuchhandlung, 1939. xvi, 246 p. 15.4 × 22.9 cm.

The first edition of this work was published in 1880, the eighth in 1924, and the ninth in 1928. The twelfth edition is an unchanged reprint of the ninth, which in turn was an unchanged reprint of the eighth.

The first 243 pages are filled with tables of $N \sin \alpha \cos \alpha$ and $N \cos^2 \alpha$, $N = 10(1)250$. Up to $N = 100$, $N \sin \alpha \cos \alpha$ is given for $\alpha = [0(3')29°57'; 2D]$, but $N \cos^2 \alpha$ for

$\alpha = [0(1°)10°(30')20°(20')30°; 1D]$. Then for $N = 100(1)175$, $N \sin\alpha\cos\alpha$ is given for $\alpha = [0(2')19°58'; 2D]$, but $N\cos^2\alpha$ for $\alpha = [0(30')10°(20')20°; 1D]$. For $N = 175(1)250$, $N \sin\alpha\cos\alpha$ is given for $\alpha = [0(1')9°59'; 2D]$, and $N\cos^2\alpha$ for $[0(30')5°(20')10°; 1D]$. On p. 244 is a table of $100 \sin\alpha\cos\alpha$ for $\alpha = [0(1')9°59'; 3D]$, and of $100 \cos^2\alpha$ for $\alpha = [0(10')10°; 2D]$. On p. 245 is $100 \sin^2\alpha$, $\alpha = [0(1')11°59'; 3D]$. On p. 246 is a correction table giving $l = -Ck^{-1} + Dk^{-1}$, for $C = 0$ and .5, $k = 99(.2)101$, $D = [10(10)300; 1D]$. Compare *MTAC*, v. 1, p. 38; v. 3, p. 88, 94.

R. C. A.

584[E, P].—N. I. KARIÂKIN, "Metod uzlovykh deplanatŝiĭ dlîa rascheta tonkostennykh mnogoproletnykh sterzhneĭ na kruchenie" [The method of nodal levelling for the solution of thin multiple-spanned rods under torsion.], *Vestnik Inzhenerov i Tekhnikov*, Moscow, v. 24, May 1948, p. 114–117.

T. 3, p. 116, is a 3D table of η, μ, $\nu = \eta + \mu$, for $u = 0(.1)6$ where

$$\mu = (u^2 - u\sinh u)/[2(\cosh u - 1) - u\sinh u],$$
$$\eta = (u\sinh u - u^2\cosh u)/[2(\cosh u - 1) - u\sinh u].$$

The values of ν for $u = 2.9$ and 3.1 are erroneous.

585[F].—ANON., "List of primes 12855 [=25409] and powers of primes 5954 [=10000] with periods not greater than 100 [=144]." *The Duodecimal Bulletin*, v. 4, Oct. 1948, p. 20–26. 14 × 21.5 cm.

This table is in duodecimal notation and gives the primes and powers of primes, as limited in the title, which divide $a^n - 1$ for $a = 2, 3, 5, 6, 7, 10, 11, 12$ and $n = 1(1)144$. Thus, when written to the base a, the number $1/p$ (where p is a prime or a power of a prime) will be periodic of period n. The table has been "translated" from A. J. C. CUNNINGHAM, H. J. WOODALL, & T. G. CREAK, *Haupt-Exponents, Residue-Indices, Primitive Roots and Standard Congruences*, London, 1922.

D. H. L.

586[F].—ALBERT GLODEN, "Solutions minima de la congruence $X^4 + 1 \equiv 0 \pmod{p^\alpha}$, $\alpha = 2, 3$, ou 4, pour $p < 10^3$," *Euclides*, Madrid, v. 8, 1948, p. 126. 16.6 × 24.1 cm.

This small table is a kind of supplement to the very extensive tables of the solutions of the congruence $X^4 + 1 \equiv 0 \pmod{p}$ reported in *MTAC*, v. 2, p. 71–72, 210–211, 300–301. The title is a little misleading. The actual table is for $\alpha = 2$ and all possible primes $p < 1000$, together with the 4 cases $p^\alpha = 17^3, 17^4, 41^3, 41^4$.

D. H. L.

587[F].—M. KRAITCHIK, "On the divisibility of factorials," *Scripta Math.*, v. 14, Mar. 1948, p. 24–26. 17 × 24.6 cm.

Tables are given of the factors of $n! \pm 1$ and of $P_n \pm 1$ where P_n denotes the product of all primes not exceeding n. The factorization of $n! - 1$ is complete through $n = 21$ while small factors, under 1000, are given up to $n = 40$. For $n! + 1$ the factorization is complete through $n = 22$. Small factors are also given through $n = 40$. $P_n - 1$ and $P_n + 1$ are completely factored through $n = 47$ and 53 respectively. Small factors are given through $n = 89$. The numbers $P_n + 1$, sometimes called Euclidean numbers, are of interest because of Euclid's proof of the infinity of primes according to which these numbers are either primes or products of primes greater than n. The first 5 Euclidean numbers are all primes. These are followed by 5 composite numbers. The 11th number $P_{31} + 1 = 200560490131$ was identified as a prime by D. N. LEHMER on Sept. 16, 1934. The present table shows that $P_n + 1$ is composite for at least the next seven cases.

D. H. L.

588[I].—HERBERT E. SALZER, "Tables of coefficients for interpolating in functions of two variables," *Jn. Math. Phys.*, v. 26, 1948, p. 294–305. 17.5 × 25.3 cm.

In a previous note[1] the author has shown that the multiple GREGORY-NEWTON interpolation formula of order n can be rewritten in the form of a LAGRANGE-type formula, thus

$$f(x + ph_1, y + qh_2) = \sum_{i+j=0}^{n} A_{i,j} f(x + ih_1, y + jh_2).$$

Here the summation ranges over all combinations i, j for which $i + j \leq n$ and

$$A_{ij} = \binom{n-p-q}{n-i-j}\binom{p}{i}\binom{p}{j}.$$

The present paper contains a new proof of this formula and its generalization to an arbitrary number of independent variables, as well as tables.

The tables are of exact values of the coefficients $A_{i,j}$ for $n = 2,3,4$ (in the paper the coefficients corresponding to $n = 2,3,4$ are denoted by $A_{i,j}$, $B_{i,j}$, and $C_{i,j}$, respectively). Each of the three sets contains 9 smaller tables corresponding to the values $q = .1(.1).9$. In these tables $p = .1(.1).9$ is the argument, and the $A_{i,j}$ the functions. When $n = 2,3,4$ there are, respectively, 6, 10, and 15 columns for the $A_{i,j}$.

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[1] H. E. SALZER, "Note on interpolation for a function of several variables," Amer. Math. Soc., *Bull.*, v. 51, 1945, p. 279–280.

589[J].—R. LIENARD, *Tables Fondamentales à 50 décimales des Sommes* S_n, u_n, Σ_n. Paris, Centre de Documentation Universitaire, 5 Place de la Sorbonne, 1948, ii, 54 p. 21 × 27.3 cm. Compare *MTAC*, v. 1, p. 456–457; v. 2, p. 17, 138–139; v. 3, p. 42.

$$S_n = 1 + 2^{-n} + 3^{-n} + 4^{-n} + \cdots,$$
$$s_n = 1 - 2^{-n} + 3^{-n} - 4^{-n} + 5^{-n} - 6^{-n} + \cdots,$$
$$u_n = 1 - 3^{-n} + 5^{-n} - 7^{-n} + \cdots,$$
$$\Sigma_n = 2^{-n} + 3^{-n} + 5^{-n} + 7^{-n} + \cdots.$$

T. I (p. 15–18) is of S_n, $n = 1(1)167$; **T. II** (p. 19–22): $2^{-n}S_n$, $n = 1(1)167$; **T. III** (p. 23–30): s_n and $1 - s_n$, $n = 1(1)167$; **T. IV** (p. 31–34): $2^{-n}s_n$, $n = 1(1)167$; **T. V** (p. 35–38): U_n, $n = 1(1)105$; **T. VI** (p. 39–46): u_n and $1 - u_n$, $n = 1(1)105$; **T. VII** (p. 47–50): ln S_n, $n = 1(1)167$; **T. VIII** (p. 51–54), Σ_n, $n = 1(1)167$. All of these functions have been tabulated before, but none of them to the extent given by Lienard.

Four of them were first tabulated by EULER: $2^{-n}S_n$ and Σ_n in 1748, S_n in 1755, u_n in 1785. PETERS & STEIN, in *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, p. 90–94, gives 32D values of S_n and $2^{-n}S_n$ for $n = 2(1)100$; $1 - u_n$, for $n = 1(1)53$, and $1 - s_n$, for $n = 2(1)100$. J. W. L. GLAISHER tabulated several of the functions as follows: 32D values of s_n and $1 - s_n$ for $n = 1(1)107$ in 1914, and S_n for $n = 2(1)107$ in 1914; 24D value of Σ_n and ln S_n, $n = 2(2)80$, in 1891; 18D value of u_n, $n = 1(1)38$, in 1912. J. P. GRAM tabulated (1884) ln S_n, to 15D, for $n = 2(1)34$. In H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 2 (1935), published tables of S_n (p. 244), to 32D, $n = 2(1)107$; ln S_n and Σ_n (p. 249–250), to 24D, $n = 2(1)80$; s_n (p. 247–248), to 32D, $n = 1(1)100$; and u_n (p. 304), to 18D, $n = 1(1)38$.

Pages 4–14 are devoted to comments on the methods for calculating the tables.

R. C. A.

590[K].—D. J. FINNEY, "The Fisher-Yates test of significance in 2 × 2 contingency tables," *Biometrika*, v. 35, 1948, p. 145–156. 19.1 × 27.2 cm.

Consider a double classification of a population in which each element belongs to one of the classes C_1, C_2 and, at the same time, either to C_1', C_2' (double dichotomy). The observed numbers in a sample can, in an obvious way, be arranged in the form of a 2 × 2 'contingency table'

$$\begin{matrix} a & A-a \\ b & B-b \end{matrix}$$

where A and B are the totals in the classes C_1 and C_2, respectively, and $a+b$, $A+B-a-b$ the totals in C_1', C_2'. If the two characteristics of classification are statistically independent, then the expected values of the two ratios $(A - a)/a$ and $(B - b)/b$ are the same, but chance fluctuations will produce deviations from this expectation. Various tests of significance of the observed deviations have been proposed, but the matter is still under discussion. The present paper is not intended as a contribution to this controversy, but is devoted mainly to tables facilitating the application of one particular test, proposed by R. A. FISHER. For reasons of symmetry the four entries can be so arranged that $A \geq B$ and $a/A \geq b/B$. Assuming statistical independence, and fixed values of a, A, B', the probability of finding in the left lower corner the particular value b is

$$P_b = \frac{A!B!(a+b)!(A+B-a-b)!}{(A+B)!a!b!(A-a)!(B-b)!}$$

The probability of a deviation as great as or greater than the deviation when the observed number is b is $P_b^* = P_b + P_{b-1} + P_{b-2} + \cdots + P_k$, the summation being continued until $k = a + b - A$ or 0 (whichever is greater). The statistician prescribes an arbitrary significance level p, say $p = .05$, and asks for the largest value b which will make the result significant, that is, the largest integer b satisfying $P_b^* \leq p$. In the tables under review this value b is tabulated for all permissible combinations a,A,B, with $A \leq 15$, $B \leq 15$, and for the significance levels .05, .025, .01, .005, thus permitting either one-sided or two-sided tests. In addition, the value P_b^* is given to three places.

These tables were constructed to solve the same class of problems as the corresponding tables of FISHER & YATES (*MTAC*, v. 1, p. 316–320); however, they possess the advantage of requiring no auxiliary computations as required in the latter tables. The Fisher-Yates tables are based on an approximation whereas these tables are exact to the accuracy recorded; however the range of applicability of these tables is considerably smaller.

WILL FELLER

591[K].—D. J. FINNEY & W. L. STEVENS, "A table for the calculation of working probits and weights in probit analysis," *Biometrika*, v. 35, 1948, p. 191–201. 19.1 × 27.2 cm.

A variety of statistical estimation problems (in particular in dose-mortality studies, detonation of explosives, etc.) require rather heavy algebra which can be considerably simplified by the use of a change of variables called the probit transformation. The main advantage of the latter is that it reduces the estimation problem to a problem of a more familiar kind, namely, linear regression theory. It is impossible to describe the statistical and computational problems in a short space. The mathematical definitions are as follows. Let P, $0 < P < 1$, be considered as the independent variable, and $Q = 1 - P$. Then the probit Y is a function of P defined by $P = \Phi(Y - 5)$, where $\Phi(x)$ is the normal distribution function

$$\Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt.$$

Four auxiliary functions are required for the estimation technique. The *range*, usually denoted by $1/Z$, is defined by dY/dP. The *maximum* and *minimum working probits* are defined by $Y_{max} = Y + Q/Z$, $Y_{min} = Y - P/Z$. Finally, the weighting coefficient is Z^2/PQ. The FISHER & YATES tables (*MTAC*, v. 1, p. 316–320) give the necessary quantities to carry out this estimation technique.

The tables under review give these same quantities correct to four places, which is the accuracy employed in the Fisher & Yates tables, with the exception of the five-place accuracy employed by them in the weighting coefficient; however, the argument Y is tabulated to hundredths rather than to tenths as in the Fisher & Yates tables. The tables also include the minimum working probit, $Y - P/Z$, which may be convenient at times but which is not essential to the technique.

WILL FELLER

592[K].—R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research.* Edinburgh, Oliver and Boyd. Third ed., rev. and enl., 1948, 112 p. 21.5 × 28 cm. 16 shillings. Compare *MTAC*, v. 1, p. 316–320.

Relatively few changes have been made from earlier editions. To the tables of the 20, 5, 1 and 0.1 percent significance levels of z and the variance ratio $F = e^{2z}$ that appear in previous editions, the authors have added tables of the 10 percent levels. Both 10 percent tables have been published previously (in a slightly different form), that of z by PANSE & AYACHIT (*Indian Jn. Agr. Sci.*, v. 14, p. 244–247, 1944) and that of e^{2z} by MERRINGTON & THOMPSON (*Biometrika*, v. 33, p. 73–88, 1943; see *MTAC*, v. 1, p. 78–79).

The table of the orthogonal polynomials $\xi_r'(x)$, where $x = 1(1)n$, $r = 1(1)5$, has been extended from $n = 3(1)52$ to $n = 3(1)75$. These functions are polynomials in x of degree r, and satisfy the relations

$$\sum_{x=1}^{n} \xi_i'(x)\xi_j'(x) = 0, \qquad i \neq j.$$

Further, each function is multiplied by the smallest factor that makes all values of $\xi_i'(x)$ integral. The functions greatly facilitate the work of fitting a polynomial by least squares to a set of data $y(x)$ recorded at equally-spaced intervals in x. Similar tables over the range $n = 1(1)104$ have been published by R. L. ANDERSON & E. E. HOUSEMAN, Iowa Agr. Exp. Sta., *Res. Bull.*, 297, 1942; see *MTAC*, v. 1, p. 148–150.

An unspectacular addition which is likely to be appreciated by statisticians is the inclusion of a table of natural logs (5D) of numbers in the range .100(.001).999. Many statistical operations necessitate taking the natural log of a probability and for this purpose the authors' previous tables, for arguments between 1 and 100, were unsuitable.

The two remaining new tables were constructed for rather specialized statistical computations and will be described only in general terms. The first, due to FINNEY, gives a series of weighting coefficients for dosage-mortality experiments where there is an appreciable natural mortality among the animals or insects that receive no toxic agent. This makes it necessary to adjust the observed mortalities of the groups that receive the toxic agent. The adjusted death rates are then transformed to a scale (probits) on which they may be expected to bear a linear relation to the log dose. The two constants that define the line are estimated by fitting a weighted linear regression, using the weights developed by Finney.

The second table (due to Fisher) gives a series of "scores" that are used very ingeniously in estimating the amount of linkage by maximum likelihood, from the progeny of crosses between double heterozygotes.

Progress continues to be made in the authors' catalogue of balanced incomplete block designs that require ten or fewer replications. The second edition listed 12 cases where the existence of a design had neither been proved nor disproved. The unsolved cases are now reduced to 5. Solutions have been found for $t = 16$, $k = 6$; $t = 21$, $k = 7$; $t = 25$, $k = 9$;

and $t = 31$, $k = 10$, while three cases, $t = 15$, $k = 5$; $t = 22$, $k = 7$; $t = 29$, $k = 8$, have been shown to be impossible. (t = number of treatments, k = number of units per block.)

Minor changes in the presentation of some of the other tables have been made, while parts of the introduction have been re-written.

W. G. COCHRAN

593[K].—JAPAN, HYDROGRAPHIC DEPARTMENT [*Interpolation Tables* 1 *and* 2], Tokyo, Dec. 1946 and Nov. 1947. 94 p. and 107 p., 18.0 × 25.5 cm. In Japanese; printed for home use only and not available for sale.

These tables have been prepared by the Japanese Hydrographic Department to facilitate the sub-tabulation which forms such a large part of the calculation of navigational ephemerides. They clearly owe much to the inspiration of the (British) *Nautical Almanac*, particularly to the section on interpolation (by L. J. COMRIE) in the 1937 edition, reprinted as *Interpolation and Allied Tables*. The notation used is the same and several tables have almost certainly been copied directly. It is, however, the new tables that possess the greatest interest.

All concerned with sub-tabulation on a large scale must, at some time, have made or have contemplated making tables to give directly the end-figures of the interpolates or their differences; the present volumes contain systematic tables for determining the first differences of the interpolates for a variety of intervals. The basis of the method is the splitting of the first and second difference contributions into exact and remainder parts; the various remainders, including where necessary the third difference contributions, are then combined by means of special double- (or triple-) entry tables. There is thus considerable similarity with E. W. BROWN'S (*Tables of the Motion of the Moon*, 1919) tables for interpolation to twelfths and some with Comrie's end-figure method of sub-tabulation (see the *Nautical Almanac* for 1931). With the availability of adding machines of large capacity (the Burroughs, National, punched card, relay and electronic machines) essentially simpler and more powerful methods of continuous sub-tabulation can efficiently be used; but these methods involve the use of extra figures which make them unduly laborious for hand calculation. The present tables will be of considerable value where such machines are not available.

The first volume of tables is concerned primarily with interpolation in which third differences (using the Besselian interpolation formula) can be ignored. Tables are given for sub-tabulation to halves, thirds, fourths, fifths, sixths, eighths, tenths and twelfths. Each table consists of two parts. The first gives, under the symbol A, the exact first differences of the second-difference contribution corresponding to chosen values of the double second difference; the range is to about 40,000 and the interval usually the minimum possible (e.g. 400 for tenths and 576 for twelfths), though it is deliberately increased for the larger sub-intervals (e.g. 464 for halves and 306 for thirds) to assist pagination. The second table is double-entry with arguments: the remainder, r, of the first difference divided by the number of intervals, n; the difference, at intervals of 5, between the actual double second difference and the tabular value used in the first table. It gives, under the symbol B, the first difference of the residual first and second difference corrections. The signs of the various contributions are given for positive values of the arguments; other combinations are easily obtained.

Sub-tabulations to $n$ths is thus performed by the following process: put the first difference equal to $qn + r$; take the series of values A from the first table with the nearest tabular value of the double-second difference; take the series of values B from the second table with the remainder r and the residual double-second difference; the series of first differences is the sum of q, A and B; the interpolates are then built up from this series to reproduce the pivotal value as a check on the arithmetic.

Other tables in the volume are straightforward tables giving (i) 4D critical tables of B^{ii} and B^{iii}; (ii) B^{ii}, B^{iii} (7D), B^{iv}, B^{v} (6D) and B^{vi} (5D) at interval .001. A short explanation gives illustrations of the use of both methods, formulae for the throw-back and limits for the neglect of various differences.

The second volume extends the method of the first to include third differences. This requires the use of a triple-entry table which severely limits the scope; the tables are, in fact, restricted to interpolation to fourths, fifths and sixths. In each section there are tables giving series of first differences: A, exact values from second differences; B, exact values from third differences; C, values corresponding to the remainders of the first, second and third differences. The intervals and ranges in the various tables are:

A, double-second difference: $n = 4$, 64, 32256; $n = 5$, 50, 30000; $n = 6$, 144, 20736

B, third difference: $n = 4$, 128, 16896; $n = 5$, 250, 30000; $n = 6$, 648, 23328

C, double-second difference 5 for all n, third difference 10 for $n = 4$, 20 for $n = 5$ and 6. The third table for $n = 6$ occupies 66 pages.

The use of the tables follows that of the first volume, with the exception that modified second and third differences can be used, in the usual way, to extend the method to cover fourth, fifth and sixth differences up to the usual limits. Here the first differences of the interpolates are the sums of q, A, B, C and these are built up to form the pivotal value as a check.

The errors of the method (apart from the use of the throw-back) are solely due to taking the nearest residual values of the second and third differences in forming the series C; they thus reach the maximum of .125 (due to the second difference) and .08 (due to the third difference). These are satisfactorily small and might be considered unduly so in view of the fact that the pivotal values can be in error by .5.

The only other tables in the volume are standard critical tables of the throw-back coefficients .184 and .108. There is, however, a short explanation with adequate examples.

D. H. SADLER

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Bath, England

594[K].—D. MAINLAND, "Statistical methods in medical research. I. Qualitative statistics (enumeration data)," *Canadian Jn. Research*, v. 26, 1948, p. 1–166. 17.1 × 25.5 cm.

The purpose of this monograph is to illustrate for workers in medical research the principal statistical techniques that are applicable to data expressed as fractions or percentages. The publication contains a number of tables which although not essentially new are more extensive than those already available and may be useful in many fields in which statistics is applied.

Table I gives confidence limits derived from a single binomial ratio. If a number A in a sample of N have a certain characteristic, the lower and upper confidence limits (f_L, f_u) for the fraction in the population that have this characteristic are defined by the following equations.

$$\sum_{r=A}^{N} \binom{N}{r} f_L{}^r(1 - f_L)^{N-r} = P : \sum_{r=0}^{A} \binom{N}{r} f_u{}^r(1 - f_u)^{N-r} = \mathrm{P},$$

where P is the confidence probability. The ranges are $A = 1(1)20$; $P = .10, .025, .005$. The range of N is irregular; it is approximately $N = 2A(1)2A + 20$, and thereafter by increasingly wide intervals up to $N = 1000$. The limits of f are given as percentages (1D), though strict accuracy to this extent is not claimed. Supplementary tables II and III cover the "large sample" case when A exceeds 20. These tables were computed by means of Table VIII$_1$ in FISHER & YATES' *Statistical Tables* (RMT 593): that is, by an approximate rather than an exact method. Over 100 entries were checked by exact computation.

Table IV is constructed for the comparison of binomial ratios from two independent samples of equal size N. The data are as follows:

	Sample		
Class	1	2	
C	a	c	$a+c$
not C	b	d	$b+d$
	N	N	$2N$

The expression

$$\frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{(2N)!\,a!\,b!\,c!\,d!}$$

is the probability that this table be obtained, given that (i) the probability of an observation falling in class C is the same in both samples and (ii) the marginal totals in the table are fixed. For $N = 1(1)20$, Table IV gives (4D) the significance probabilities for Fisher's "exact" test of significance in a 2×2 contingency table (*Statistical Methods for Research Workers*, eighth ed., 1941, §21.02), obtained by adding the probabilities given above. Thus if the first sample contains 3 successes and 9 failures, while the second has 8 successes and 4 failures, the probability tabulated (.0498) is the sum of the individual probabilities for the tables.

3	8
9	4

2	9
10	3

1	10
11	2

0	11
12	1

Table V lists all 2×2 contingency tables that reject the null hypothesis that the probability of a C is the same in both samples. The significance levels presented are the 2.5 and 0.5 percent levels, and the exact significance probability for each table is shown. The tables cover any pair of sample sizes (equal or unequal) up to 20. The tables were constructed by addition from the exact probabilities as given in the preceding paragraph.

The remaining tables are of a standard type and need not be discussed.

W. G. COCHRAN

595[K].—K. R. NAIR, "The Studentized form of the extreme mean square test in the analysis of variance," *Biometrika*, v. 35, 1948, p. 16–31. 19.3 × 27.3 cm.

Suppose $\chi_1^2, \chi_2^2, \cdots, \chi_k^2$ are k independent values drawn from a chi-square distribution with m degrees of freedom and arranged in ascending order of magnitude. Let χ_0^2 be a value drawn from a chi-square distribution with ν degrees of freedom and which is independent of the $\chi_1^2, \chi_2^2, \cdots, \chi_k^2$. Let ${}_\nu P_k(Q)$ be the probability that $(\nu\chi_k^2)/(m\chi_0^2) \le Q$.

(In statistical language ${}_\nu P_k(Q)$ is the probability that $(\nu\chi_k^2)/(m\chi_0^2)$, the largest of the k SNEDECOR F-ratios $(\nu\chi_1^2)/(m\chi_0^2)$, $(\nu\chi_2^2)/(m\chi_0^2)$, $\cdots$, $(\nu\chi_k^2)/(m\chi_0^2)$, will not exceed Q.)

The expression for ${}_\nu P_k(Q)$ is of the form

$${}_\nu P_k(Q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} P_k(Qt)\,dt$$

where

$$P_k(Qt) = \left[\frac{2(\frac{1}{2}m)^{\frac{1}{2}m}}{\Gamma(\frac{1}{2}m)} \int_0^{Qt} x^{m-1} e^{-\frac{1}{2}mx^2}\,dx\right]^k.$$

For $m = 1$,

$$(1) \qquad {}_\nu P_k(Q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} \left(\sqrt{\frac{2}{\pi}} \int_0^{Qt} e^{-\frac{1}{2}x^2} dx \right)^k dt.$$

For $m = 2$,

$${}_\nu P_k(Q) = \sum_{r=0}^{k} (-1)^r \binom{k}{r} \left(1 + \frac{2rQ^2}{\nu} \right)^{-\frac{1}{2}\nu}.$$

Finney (*Annals of Eugenics*, v. 11, 1941, p. 47) has tabulated, for $m = 2$, the values of Q for which ${}_\nu P_k(Q) = .95$ for $\nu = 1(1)10, 20, \infty$ and $k = 1(.1)3$.

Nair deals with tabulations only for $m = 1$. He develops an approximation to (1), making use of an expansion of ${}_\nu P_k(Q)$ in powers of $(1/\nu)$. Using terms up to and including terms of order $(1/\nu)^3$, Nair tabulates approximate values of Q for which ${}_\nu P_k(Q) = 0.95$ and 0.99 (i.e. 1% and 5% significance points of Q) for $k = 1(1)10$ and $\nu = 10,12,15,20,30, 60, \infty$. The author does not give much information on how close the approximations are.

Now let ${}_\nu P_k(q)$ be the probability that $(\nu\chi_1^2)/(m\chi_0^2) \geq q$. (In statistical language, ${}_\nu P_k(q)$ is the probability that $(\nu\chi_1^2)/(m\chi_0^2)$, the smallest of the k Snedecor F-ratios mentioned earlier, will exceed q.) The expression for ${}_\nu P_k(q)$ is of the form

$${}_\nu P_k(q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} P_k(qt) dt,$$

where

$$P_k(qt) = \left[\frac{2(\frac{1}{2}m)^{\frac{1}{2}m}}{\Gamma(\frac{1}{2}m)} \int_{qt}^\infty x^{m-1} e^{-\frac{1}{2}mx^2} dx \right]^k.$$

For $m = 1$,

$${}_\nu P_k(q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} \left(\sqrt{\frac{2}{\pi}} \int_{qt}^\infty e^{-\frac{1}{2}x^2} dx \right)^k dt.$$

For $m = 2$,

$${}_\nu P_k(q) = \left(1 + \frac{2kq^2}{\nu} \right)^{-\frac{1}{2}\nu},$$

from which 1% and 5% points can be readily computed, although no table is given by Nair or by Finney (*loc. cit.*).

For $m = 1$, Nair expands ${}_\nu P_k(q)$ into a power series in $(1/\nu)$ and obtains

$${}_\nu P_k(q) = 1 - \left\{ 1 - a_0 - \frac{a_1}{\nu} \left(1 - \frac{1}{8\nu} \right) \right\}$$

where

$$a_0 = \left((2/\pi)^{\frac{1}{2}} \int_q^\infty e^{-\frac{1}{2}x^2} dx \right)^k,$$

$$a_1 = k(a_0)^{(k-2)/k} \{ (k-1)qz + \tfrac{1}{4}(q^2+1) a_0^{1/k} \},$$

$$z = (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}x^2}.$$

Nair gives a table of values of a_0 and a_1 for $q = 0(.01).10$ and for $k = 2(1)10$. This table can then be used for determining for $k = 2(1)10$ and any value of ν the approximate value of q for which ${}_\nu P_k(q) = .90(.01).99$ (i.e. the 1(1)9 and 10% significance points of ν). The author actually gives a table of 1% and 10% significance points of q for $k = 1(1)10$ and $\nu = 10$.

S. S. W.

596[K, L, M].—Fritz Emde, *Jahnke-Emde Tables of Higher Functions, Treated by Fritz Emde. Fourth (revised) edition with 177 figures.* Leipzig, Teubner, 1948, xii, 300 p. There is also a German title page. 16 × 24 cm. See *MTAC*, v. 1, p. 106–109, 161, 198, 202, 204, 293–294, 386, 391–399; v. 2, p. 26, 47, 224, 350; v. 3, p. 254–255, 267–268, 314–315.

The first German edition of this famous work appeared in 1909. Of this first edition there were two corrected reprints in 1923 and 1928. Since Jahnke died in 1921 all later editions in

Germany were prepared by Professor EMDE. The second (revised) edition, 1933, contained xviii, 330 p. and was bilingual, English and German. The third (revised) edition appeared in 1938, xii, 305 p. and 181 figures; the 76 pages of the second edition devoted to elementary functions are here eliminated. Thus there have been nine editions or reprints of this work.

The present edition contains almost exactly the same number of pages as the third (fifth) since the deletions and additions almost counterbalance. Most of Professor Emde's preface, dated "Pretzfeld, January, 1948," is as follows:

"The Fourth Edition of the Tables of Higher Functions should have been issued in 1944. But after having been printed all copies were destroyed at the book-bindery by bombs and fire during the war. It is only now possible to reprint this edition from the same manuscript.

"As new matter this edition offers:

"1. an extension of the table of the error integral.
"2. the table of the functions of the parabolic cylinder, computed by J. B. Russel [*sic*] (*Journ. of math. and phys.* XII, 1932/3, p. 291–297), checked and corrected by S. Kerridge.
"3. the table of the Laguerre functions, computed by F. Tricomi (Atti R. Acc. Sc. Torino 76, 1941).
"4. the table of the spherical harmonics of the second kind, computed by F. Vandrey (Z. a. M. M. 20, 1940, S. 277–279).
"5. the tables of the incomplete Anger and Weber functions, computed by P. and E. Brauer (Z. a. M. M. 21, 1941, S. 180, 181).
"6. a table of the Bessel functions $J_{n/3}(iy)$ and $H^{(1)}_{n/3}(iy)$ $(n = 1,2)$, computed by S. Kerridge, instead of the earlier (incorrect) table of $J_{n/3}(iy)$ and $J_{-n/3}(iy)$, computed by Dinnik. If y is large these two last functions become nearly equal, thus do not represent two linearly independent solutions of the Bessel differential equation.
"7. formulae and figures for the use of Debye's convergently beginning series for the Bessel functions with complex argument and order.
"8. a list of 7 and more place logarithmic tables."

A number of the tabular changes have thus been suggested. Quite a few corrections in the tables have been made but many errors still remain.

The error integral table of $\Phi(x)$ has been extended from a 4D to a 5D table, for $x = 0(.01)3.09$, and a table of $e^{x^2}[1 - \Phi(x)]$, for $x = [3(.01)5; 5D]$ has been added. The former tables and graphs of the derivatives of the error integral have been dropped, and in their place Russell's tables of HERMITE (parabolic cylinder) functions, $\phi_n(x)$, $n = 1(1)11$, $x = [0(.04)1(.1)3.5; 5D]$ have been substituted; see *MTAC*, v. 1, p. 4, 152–153.

These are followed (p. 32–33) by TRICOMI's table of LAGUERRE functions $\mathrm{ln}\,(x) = e^{\frac{1}{2}x}(n!)^{-1}d^n(e^{-x}x^n)/dx^n$, $x = [.1(.1)1(.25)3(.5)6(1)14(2)34; 4D]$, $n = 1(1)10$; see *MTAC*, v. 2, p. 267. The errors which we noted in the $C(u)$ and $S(u)$ tables have been corrected.

In the section on elliptic functions the four pages of tables and graphs for the WEIERSTRASS functions in the equianharmonic case have been eliminated. There are 64 errors in the table of $\log q$ (p. 50); 16 errors in the K, E, θ table; and 7 in the $E(\theta, \phi)$, $F(\theta, \phi)$ tables (A. FLETCHER); also the right-hand member of the equation, p. 80, l. 5, is entirely wrong.

In the section on LEGENDRE functions VANDREY's table of spherical functions of the second kind, $Q_n(x)$, has been added, $x = [0(.01)1; 5D]$, $n = 1(1)7$. The four serious errors in this table which we noted, *MTAC*, v. 1, p. 446, are still in evidence. The quite erroneous figure for $P_\nu(x)$ in the neighborhood of $x = -1$ (which we noted *MTAC*, v. 1, p. 395, 398) has been replaced by an entirely new graph on p. 105 (108). The 53 errors corrected in the tables on p. 124–125 of the latest American ed. are all to be found on p. 120–121 of the work under review.

The greatest change in any section is in that devoted to Bessel functions, p. 125–264 (1945 Ed. p. 126–268). The sections on asymptotic representations and differential equations that give Bessel functions, have been elaborated and the section on integral representations deleted. The 6 pages of tables of the Struve functions S_0, S_1 were eliminated; the 3 pages devoted to S_0 were no great loss since they duplicated the table given for Ω_0 (with 40 unit errors not in S_0). The following two new BRAUER tables (which we listed *MTAC*, v. 1, p. 245, 282) are given (p. 218–219): Incomplete ANGER function, $\frac{1}{2}\int_0^x \cos \frac{1}{2}q[\frac{1}{2}\pi t - \sin(\frac{1}{2}\pi t)]dt$, $x = 0(.1)2$, $q = [.1(.1)1; 4\text{–}5D]$, and incomplete WEBER function, $\frac{1}{2}\int_0^x \sin \frac{1}{2}q[\frac{1}{2}\pi t - \sin(\frac{1}{2}\pi t)]dt$, for the same ranges, but mostly 5–6D.

The highly erroneous Dinnik tables involving $J_{\pm\frac{1}{3}n}(ix)$, $n = 1,2$, p. 235 have been replaced by Kerridge's full-page, 231, of tables involving $J_{\frac{1}{3}n}$ and $H^{(1)}_{\frac{1}{3}n}(ix)$, $n = 1,2$, $x = [0(.1)10; 4\text{–}5S]$.

Among the errors in this Bessel function section, already noted in *MTAC*, are the following (the first page given being that of the v. under review, the second that of the 1945 edition):

P. 125 (126).—Relief 66 (67), $J_p(x)$ is incorrect near the origin; see *MTAC*, v. 3, p. 315.
P. 164 (164).—$J_{\pm\frac{1}{4}n}(x)$, $n = 1,3$; only 6 of the 32 errors have been corrected.
P. 166 (166).—The error $J_1(x_{17})$ remains.
P. 168 (168).—$J_p(x_n)$, of the 27 errors in this table only the 7 most serious ones have been corrected.
P. 183 (183).—$\Lambda_3(6.4)$, diff. incorrect.
P. 204–205 (204–205).—Zeros of $J_p(x)N_p(kx) - J_p(kx)N_p(x)$, still three errors. Compare RMT 566.
P. 224–225 (228–229).—$I_0(x)$, $I_1(x)$, more serious errors have been eliminated but many last figure unit errors remain.
P. 230 (234).—Zeros of $J_n(ix)J_n'(x) - iJ_n(x)J_n'(ix)$, there are still 8 errors.
P. 232 (236).—$iH_0^{(1)}(i) = 3.006$ omitted.
P. 242–243 (246–247).—Re $J_0(x\sqrt{i})$, Im$J_0(x\sqrt{i})$, three errors.

The Section IX on RIEMANN zeta function, p. 265–270 (269–274), is practically unchanged, but under the table of the first 29 zeros of the function, $\sigma + it$, p. 270, references are given to papers by E. C. TITCHMARSH who shows, with tables, that between $t = 0$ and $t = 1468$, there are exactly 1041 zeros all on the line $\sigma = .5$; see R. Soc. London, *Proc.*, v. 151, 1935, p. 234–255, and v. 157, 1936, p. 261–263.

There is practically no change in the two final sections on confluent hypergeometric functions and MATHIEU functions.

In the concluding bibliography, two errors are made in an entry on p. 295, where it is stated that the second ed. of THOMPSON'S *Table of the Coefficients of Everett's Central-Difference Interpolation Formula*, was published in 1937 and contained xvi, 20 p. (see *MTAC*, v. 1, p. 185).

The board-binding and canvass back are only fairly substantial and many of the pages are apparently reproduced by the offset process from the corresponding pages of the third edition, or of a surviving volume of the 1944 fourth edition.

R. C. A.

597[L].—PIERO GIORGIO BORDONI, "Sulle funzioni di Stokes," Pontificia Academia Scientiarum, *Commentationes*, v. 9, no. 3, 1945, p.87–113. 17 × 24.5 cm.

The functions here tabulated (p. 104–112), and called respectively Stokes' functions of the first, second and third species, are as follows:

$$f_m(ix) = i^{-(m+1)}e^{ix}(\tfrac{1}{2}\pi x)^{\frac{1}{2}}[J_{m+\frac{1}{2}}(x) + i(-1)^m J_{-m-\frac{1}{2}}(x)]$$
$$= i^{-(m+1)}e^{ix}(\tfrac{1}{2}\pi x)^{\frac{1}{2}}H^1_{m+\frac{1}{2}}(x) = {}_2F_0(-m, m+1, \tfrac{1}{2}x^{-1});$$
$$F_m(ix) = ix(2m+1)^{-1}[mf_{m-1}(ix) + (m+1)f_{m+1}(ix)];$$

$\zeta_m(ix) = f_m(ix)/F_m(ix)$. The tabulation of real and imaginary parts is for $m = 1(1)5$, $x = .1(.05).2(.1)1(.25)4(.5)6(1)10(5)20, 30$. The tabulation is usually 4–5S. There are graphs illustrating the variations in connection with each of these functions.

The six functions $f_m(ix)$, $m = 0(1)5$, are as follows:

$f_0(ix) = 1$; $f_1(ix) = 1 - ix^{-1}$; $f_2(ix) = 1 - 3x^{-2} - 3ix^{-1}$;
$f_3(ix) = 1 - 15x^{-2} - i(6x^{-1} - 15x^{-3})$; $f_4(ix) = 1 - 45x^{-2} + 105x^{-4} - i(10x^{-1} - 105x^{-3})$;
$f_5(ix) = 1 - 105x^{-2} + 945x^{-4} - i(15x^{-1} - 420x^{-3} + 945x^{-5})$.

R. C. A.

598[L].—GREAT BRITAIN, Admiralty Computing Service, *Lateral Vibration of Beams of Conical Section.* No. SRE/ACS 92, August 1945, 5 p. (4 leaves with cover). 20.3 × 32.2 cm. This publication is available only to certain agencies and activities. Compare *MTAC*, v. 2, p. 289–297.

The differential equation $\dfrac{d^2}{dz^2}\left(z^4\dfrac{d^2y}{dz^2}\right) = z^2y$ has the solution, in terms of Bessel functions,

$$y = z^{-1}[AJ_2(2z^{\frac{1}{2}}) + BY_2(2z^{\frac{1}{2}}) + CI_2(2z^{\frac{1}{2}}) + DK_2(2z^{\frac{1}{2}})]$$

where

$$A/Y_3(\alpha) = -B/J_3(\alpha) = C/[rK_3(\alpha)] = D/[rI_3(\alpha)],$$

$$r = [Y_3(\alpha)J_3(\beta) - J_3(\alpha)Y_3(\beta)]/[I_3(\alpha)K_3(\beta) - K_3(\alpha)I_3(\beta)], \qquad \alpha = 2\sqrt{L}, \qquad \beta = 2\sqrt{aL}.$$

T.I gives, for $a = 0(.1).6$, 4–6S (one 7S), values of $L,A,B,10^5C,D$.

T.II gives 1–5S values of y, unit 10^{-4}, for $a = 0(.1).6$ and for 11 values of z from aL linear scale to L.

T.III gives 3–4S values of d^2y/dz^2, unit 10^{-5}, for the same values of a and z, as in **T.II**.

Reference: J. W. NICHOLSON, "The lateral vibration of bars of variable section," R. Soc. London, *Proc.*, v. 93, 1917, p. 506–519.

Extracts from text

599[L].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 10, *Tables of the Bessel Functions of the First Kind of Orders Twenty-Eight through Thirty-Nine.* By the Staff of the Computation Laboratory, Professor H. H. AIKEN, director. Cambridge, Mass., Harvard University Press, 1948, x, 694 p. 19.5 × 26.7 cm. $10.00. Offset print. Compare *MTAC*, v. 2, p. 176f, 261f; v. 3, p. 102, 117–118, 185–186.

This is the eighth of the thirteen planned volumes of the great edition of tables of Bessel Functions of the first kind prepared by Harvard's IBM Automatic Sequence Controlled Calculator. The offset print of the splendidly clear copy furnished from this machine is notable. The volume contains tables of $J_n(x)$, $n = 28(1)39$, $x = [0(.01)99.99; 10D]$. The first significant value .00000 00001 of the twelve functions are for $J_{28}(10)$, $J_{29}(10.65)$, $J_{30}(11.32)$, $J_{31}(11.99)$, $J_{32}(12.68)$, $J_{33}(13.37)$, $J_{34}(14.06)$, $J_{35}(14.77)$, $J_{36}(15.48)$, $J_{37}(16.2)$, $J_{38}(16.92)$, $J_{39}(17.65)$.

Practically all of the tens of thousands of entries in this table are new. The only previous duplicating values previously published, to at least 10D, are 50 values in CAMBI (1948, see RMT **535**) to 11D, and 24 given by HAYASHI (1930), for $n = 28(1)39$, $x = 20$, 30, and to at least 31D. In every case the rounded Cambi and Hayashi entries agree with those in the Harvard volume.

As in the case of the seventh volume the computation of the tables in the volume under review, and the preparation of the manuscript, were under the supervision of JOHN A. HARR.

R. C. A.

600[L].—K. A. KITOVER, "Tablifsy summ nekotorykh beskonechnykh trigonometricheskikh riadov" [Tables of the sums of certain infinite trigonometric series], Akad. Nauk SSSR *Prikl. Mat. Mekh.*, v. 12, 1948, p. 233–240. 16.2 × 26.1 cm.

The sums referred to in the title are

$$F_{2v}(x) = \sum_{n=1}^{\infty} n^{-2v} \sin nx, \qquad F_{2v+1}(x) = \sum_{n=1}^{\infty} n^{-2v-1} \cos nx$$

$$f_{2v}(x) = \sum_{n=1}^{\infty} (2n-1)^{-2v} \sin (2n-1)x$$

$$f_{2v+1}(x) = \sum_{n=1}^{\infty} (2n-1)^{-2v-1} \cos (2n-1)x.$$

F_k and f_k are tabulated to 4D for $k = 1(1)6$, and for $x = m\pi/180$, $m = 0(1)180$, that is for x in degrees. The corresponding radian values of x are also given. Because $f_k(\pi - x) = f_k(x)$ the table of f_k extends only as far as $x = \frac{1}{2}\pi$ with a pair of columns for $\pi - x$. The functions F_k and f_k are related by

$$f_k(x) = F_k(x) - 2^{-k}F_k(2x).$$

Seven other similar trigonometric sums are expressed in terms of F_k. Although the functions bear a strong superficial resemblance to the BERNOULLI functions which are merely polynomials in the unit interval, F_k and f_k are non-elementary functions, with the exception of

$$F_1(x) = -\ln (2 \sin \tfrac{1}{2}x), \qquad f_1(x) = \ln \cot \tfrac{1}{2}x.$$

The function $F_2(x)$ is known as CLAUSEN's integral and has been tabulated[1] by him to 16D for the same values of x. The other functions F_k and f_k appear to be new. They are said to be useful in the theory of elasticity. Graphs of these functions are also given.

D. H. L.

[1] T. CLAUSEN, *Jn. f. r. u. angew. Math.*, v. 8, 1832, p. 300, reprinted in F. W. NEWMAN, *The Higher Trigonometry*, Cambridge, 1892, p. 85; see *MTAC*, v. 1, p. 458.

601[L].—HERMANN KOBER, *Dictionary of Conformal Representations.* Admiralty, Department of Physical Research, Mathematical and Statistical Section. Part V: *Higher Transcendental Functions.* v. 30, 3 leaves. Number SRE/ACS 111 = ACSIL/ADM/48/329. London, 1948. 20.1 × 33.1 cm. This publication is not available for general distribution.

This is the final part of the *Dictionary* of which the four earlier parts were reported *MTAC*, v. 2, p. 296–297; v. 3, p. 103. Elliptic functions are dealt with, leaves 1–24; and other functions, leaves 25–30. "References" are listed in the final 3 leaves.

602[L].—WILHELM MAGNUS & FRITZ OBERHETTINGER, *Formeln und Sätze für die speziellen Funktionen der mathematischen Physik.* (*Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete*, v. 52.) Second ed. rev. and enl., Berlin, Göttingen, Heidelberg, Springer, 1948, viii, 230 p. 16 × 24.3 cm.

In RMT **503** (v. 3, p. 103–105) we gave the detailed contents and list of errata of the first edition of this work, published in 1943. In the present edition the errata have been corrected and 58 pages added. Every chapter has been enlarged (although the fifth has the same number of pages) as may be observed by comparing the following inclusive chapter page-numbers with those given earlier: **I** (1–9), Gamma function; **II** (10–24), Hypergeometric function; **III** (25–67), Cylinder functions; **IV** (68–101), Spherical functions; **V** (102–110) Orthogonal polynomials; **VI** (111–127), Confluent hypergeometric function and special cases; **VII** (128–158), Elliptic integrals, theta functions and elliptic functions; **VIII** (159–190), Integral transformations and inversions; **IX** (191–211), Coordinate transformations. Chapter VII has been entirely rewritten and doubled in size.

The first edition has been translated into English, and the New York publisher has informed me that he expects this translation to be available for distribution in January 1949.

R. C. A.

603[L].—SHEN YUAN, "The flow of a compressible fluid past quasielliptic cylinders at high subsonic speeds," Nat. Tsing Hua Univ., *Science Reports*, ser. A, *math. phys. Engin. Sci.*, v. 5, no. 1, Apr. 1948, p. 29–51.

On p. 44–45 are two tables of the Hypergeometric function $F(a, b, c; \tau)$ denoted for brevity as $F(m, \tau)$, where a, b, c, are functions of m as given by the equations $a + b = m - \frac{5}{2}$, $ab = -\frac{5}{4}m(m+1)$, $c = m + 1$. **T. 1**: m = [2(1)9(4)17; 4S], τ = .02(.02).16; also m = 30, τ = .02, .04. **T. 2**: $2m$ = [1(2)7; 4D], τ = .04(.02).3; $2m$ = [41; 4S], τ = .04(.02).2. Also $-2m$ = [1(2)15; 4D], τ = .04(.02)3; also $-2m$ = [17; 4–6S], τ = .22(.02).3; $-2m$ = [19; 4–6S], τ = .04(.02).3; $-2m$ = [25; 4–5S], τ = .18(.02).3; $-2m$ = [31; 1–4D], τ = .06(.02).3; $-2m$ = [35; 4–6S], τ = .06(.02).22(.04).3; $-2m$ = [41; 4–5S], τ = .06(.02).1, .18(.04).3.

S. A. J. & R. C. A.

604[M].—ROBERT LECOLAZET & PHILIPPE PLUVINAGE, "États de régimes permanents electrodynamiques dans l'atmosphère," *Annales de Géophysique*, v. 4, 1948, p. 96–108. 21.5 × 27.4 cm.

On p. 102 there is a table of the exact values of $A_{m,n} = \int_0^1 P_m(u)P_n(u)du$, $P_n(u)$ a Legendre polynomial, for $m = 0(1)5$, $n = 0(1)5$.

605[M].—V. A. UGAROV, "K teorii strat" [Concerning the theory of strata], *Zhurnal eksperimental'noĭ i teoreticheskoĭ Fiziki*, v. 18, no. 5, May 1948, p. 457–461.

Table, p. 458, of $J(\Omega) = -4\int_0^\infty xe^{-x^2}\cos \Omega x dx$,

Ω	0	1	1.57	1.85	2	2.5	3	4	5	6	$10 \to \infty$
$J(\Omega)$	-2	-1.05	$-.32$	0	.16	.48	.56	.4	.2	.12	$.05 \to 4/\Omega^2$

EDITORIAL NOTE: Corresponding to $\Omega = 10$, the table gave the incorrect value of $J(\Omega)$ as .5.

606[U].—H. A. GULDHAMMER, *Nautisk Tabelsamling*. Copenhagen, J. Jørgensen & Co., 1946, 174 p. 16.8 × 25.5 cm.

This volume contains a collection of forty numbered tables and nine pages of mathematical formulae and unnumbered tables of equivalent weights, measures, etc. No explanation of the tables and their use is included. There is not even a preface nor an index, just a one-page table of contents. All table headings are in Danish; only one English phrase was noted in the volume—"Duration of rise or fall" in parentheses below the heading of T. 33. However a person who can read German and is moderately well acquainted with navigation, should have no difficulty in using the tables.

T. 1 gives the course or bearing in degrees and tenths corresponding to the points of the compass. T. 2 is one giving difference of latitude and departure, each to 0.1 for course angle 1°(1°)89° and distance 1(1)300. T. 3 provides the change in longitude to 0'.01 corresponding to departures 1(1)10 nautical miles [actually to 0'.001 for 10 nautical miles] and for middle latitudes 1°(1°)60°(30′)70°(15′)80°. T. 4 is one of meridional parts to 0.1 for latitudes 0(1′)89°59′.

T. 5 gives the correction to 1′ to be applied to the middle latitude with arguments middle latitude 10°(10°)70° and change in latitude 1°(1°)15°. T. 6 yields the distance to the horizon to 0.1 nautical mile with argument height of eye 1(.5)10(1)30(2)100(5)170(10)200 meters. In T. 7 is tabulated the distance of an object to 0.1 nautical miles corresponding to height of the object 15(5)30(10)180, 200 meters and vertical angle subtended at the observer's eye 10′(5′)1°(10′)1°30′(15′)2°(30′)4°,5°. T. 8 gives the distance to 0.1 units of an object by two bearings with respect to the ship's course, with arguments first bearing 20°(5°)105° and second bearing 40°(5°)140°, the unit of distance being the distance run between the two bearings. There is also a brief section of this table with arguments given in points of the compass.

T. 9 gives the dip of the horizon to 0′.1 with argument height of eye .5(.5)18(1)42(2)66 meters. T. 10 gives corrections to 0′.1 to be applied to the dip of the horizon when the air is 1°(1°)10° Centigrade warmer or colder than the water. T. 11 provides corrections to 0′.1 to be applied to an observed altitude measured from a shore short of the horizon, the arguments being distance to the shore .5(.25)3(.5)5(1)7 nautical miles and height of eye 2(1)10(2)16 meters.

T. 12 is one of mean refraction to 1″ for 15° Centigrade and 760 mm. of mercury for measured altitudes of stars 0° to 90°. T. 13 gives the combined correction to 0′.1 for atmospheric refraction, semidiameter and height of eye 0.2(.5)8(1)18, 20 meters to be applied to observed altitudes of the lower limb of the sun 6° to 90°. T. 14–15 give the usual additional corrections to be applied for variable semidiameter of the sun and for upper limb of the sun for 15 intervals during the year. T. 16 is similar to T. 13, except that it includes no correction for semidiameter and hence is to be used with observed altitudes of stars and planets. T. 17–18 give the corrections to 1″ to be applied to the mean refraction 2′(2′)12′ given by T. 12 when the temperature is $-35°(5°)+35°$ Centigrade and the barometric pressure is 700(10)780 mm. T. 19 provides the correction to 0′.1 to the observed altitude of the moon's lower limb 3° to 90° for horizontal parallax 52′(1′)61′, and semidiameter and atmospheric refraction. T. 20–21 give the moon's diameter and semidiameter to 0′.1 corresponding to horizontal parallax 52′(1′)61′. T. 22 gives the day-numbers corresponding to 60 (or for leap years, 61) days in the year.

T. 23 provides the means of changing hours and minutes of time to decimal fractions to .01 of a day. T. 24 is for changing points and quarterpoints of a compass into degrees and tenths. T. 25–26 are for conversion of arc into time and conversely. T. 27 gives the amplitude to 0°.1 of celestial bodies which are rising or setting; the arguments are declination 0(30′)18°(15′)23°45′ and latitude 0(5°)20°(2°)50°(1°)66°, 66.5°.

T. 28 gives the change in altitude to 0″.01 during the last minute before and the first minute after culmination for latitudes 0(1°)46°(2°)70°, 80° and declination, same and opposite name, 0(1°)24°. T. 29 provides to 0.1 the square of the time interval from culmination in minutes, the argument being time $0(1^s)29^m59^s$. T. 30 provides the time interval from the meridian to the horizon in hours and minutes in latitudes 0(2°)10°(1°)66° and for declinations, same name and opposite, 0(1°)34°. T. 31 is intended for use with a radio direction finder; it provides the correction to a bearing due to the convergence of meridians for a change in longitude 1°(1°)14° and mid-latitude 4°(4°)20°(2°)60°(4°)72°.

T. 32 is a small table for interpolating the quantity required to change a solar time interval into a sidereal time interval. T. 33 is intended to be used in calculating the fractional height of tide at half hour intervals, knowing the duration of rise or fall $3\frac{1}{2}(\frac{1}{2})8$ hours. T. 34 provides a wind-scale; T. 35 is a distance-travelled table giving distance to 0.1 nautical miles for 1(1)60 minutes at 1(1)28 knots. T. 36 gives for $n = 1(1)240$ the values of n^2, n^3, $\sqrt{n}$ and $\sqrt[3]{n}$; the latter two are given to 4D. T. 37 is a 5D table of logarithms of numbers 10000(1)10999, 1100(1)9999, with P.P. T. 38 is a 3D table of the natural values of all six trigonometric functions for an angle 0(1°)90°. T. 39–40 provide 5D logarithms of trigonometric functions; T. 39 for sine, cosecant and cotangent for an angle 0(0′.1)2°12′ and T. 40 all six functions for an angle 0(1′)90°, Δ.

This volume has a neat blue cover with two gold lines around the edge of the front cover; it is well printed on a good grade of white paper. It seems likely that a person who uses the table frequently will grow to be very fond of the book.

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EDITORIAL NOTE: On p. 79, heading, *for* Tab. 39, *read* Tab. 30.

MATHEMATICAL TABLES—ERRATA

In this issue references have been made to Errata in RMT **579** (Uhler), **580** (France), **584** (Kariakin), **596** (Jahnke & Emde), **605** (Ugarov), **606** (Guldhammer); **N96** (Pitiscus).

145.—G. F. BECKER & C. E. VAN ORSTRAND, *Hyperbolic Functions*, Fifth reprint, 1942. See *MTAC*, v. 2, p. 311 and v. 3, p. 200.

Using the NBSCL, *Tables of the Exponential Function e^x*, second edition, 1947, I find the following values:

tanh 0.174 = 0.17226 50005 13, cosh 0.911 = 1.44446 49997 49,
tanh 0.932 = 0.73152 49994 56, tanh 1.381 = 0.88117 49957 43,
tanh 1.986 = 0.96302 50028 60.

The roundings of these values to five decimals had been left in doubt in MTE **129.** These results indicate the following three "errors in excess of 5 units in the next succeeding place of decimals" in *Hyperbolic Functions*:

page	u	function	*For*	*Read*
109	0.174	tanh u	.17226	.17227
124	0.932	tanh u	.73153	.73152
145	1.986	tanh u	.96302	.96303

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146.—R. A. FISHER, "On the 'probable error' of a coefficient of correlation deduced from a small sample," *Metron*, v. 1, no. 4, 1921, p. 3–32. On p. 26–27 is a table of $\tanh^{-1} x = \frac{1}{2}[\ln(1+x) - \ln(1-x)]$, $x = [0(.01).9(.001)1; 7D]$, δ^4.

On checking this table with a 9D table recently computed in this Laboratory we found only a single small error. In $\tanh^{-1}$ .918, *for* 1.576 159 6, *read* 1.576 159 5; our 9-place value is 1.5761 59504.

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EDITORIAL NOTE.—In this same paper of FISHER there is a table, p. 28, of $\tanh^{-1}\overline{(x - 2/x)} = \frac{1}{2}\ln(x-1)$, for $x = [1(1)100; 7D]$. This table is not listed in FMR, *Index*, although SPEIDELL's tables, 1622, of $\frac{1}{2}\ln x$ and of $\frac{1}{2}\ln(1/x)$, for $x = [1(1)1000; 6D]$, are noted.