Table B

Complex Zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$

	$z_{0,s}$		$z_{1,s}$		$z_{1,s}'$	
s	Real	Imag.	Real	Imag.	Real	Imag.
1	− 2.40302	.53988	− .50274	.78624	+ .57679	.90398
2	− 5.51988	.54718	− 3.83354	.56236	− 1.94048	.72119
3	− 8.65367	.54841	− 7.01590	.55339	− 5.33348	.56722
4	−11.79151	.54882	−10.17358	.55127	− 8.53677	.55606
5	−14.93091	.54900	−13.32374	.55046	−11.70618	.55286
6	−18.07106	.54910	−16.47066	.55006	−14.86367	.55150
7	−21.21163	.54915	−19.61587	.54984	−18.01557	.55080
8	−24.35247	.54919	−22.76009	.54970	−21.16440	.55038
9	−27.49348	.54922	−25.90368	.54961	−24.31135	.55012
10	−30.63461	.54923	−29.04683	.54955	−27.45706	.54995
11	−33.77582	.54925	−32.18968	.54950	−30.60193	.54982
12	−36.91710	.54926	−35.33231	.54947	−33.74619	.54973
13	−40.05843	.54926	−38.47477	.54945	−36.88999	.54966
14	−43.19979	.54927	−41.61710	.54942	−40.03345	.54961
15	−46.34119	.54927	−44.75932	.54941	−43.17663	.54956

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[1] A. HILLMAN, "On the reality of zeros of Bessel functions," Amer. Math. Soc., *Bull.*, v. 55, 1949.

[2] NBSCL, *Tables of the Bessel Functions $Y_0(z)$ and $Y_1(z)$ for Complex Arguments*, New York, Columbia University Press, publication announced for 1949.

[3] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. Second ed. Cambridge and New York, 1944, p. 505–507.

A Method of Plotting on Standard IBM Equipment

The advent of automatic computing machinery in research in physics and chemistry has eliminated bulky arithmetic procedures, but in many cases this advantage is lost by the bottleneck of plotting the results; as for example in spectrum analysis where many computed curves have to be compared with the data in the form of a graph. If the results are computed on cards, or are in volume enough to be punched on cards, the following procedure simplifies the problem of plotting a large number of points.

The cards are sorted on abscissa and are fed into an IBM tabulator (preferably a 405) fitted with two digit selectors and preferably six class selectors. Each spacing of the platen (corresponding to each line in a typewritten roll of paper) is taken as unit increase in the abscissa. The range of the abscissa is then infinite—with obvious practical limitations. The unit spacing of the ordinate is that between type bars, of which there are 88 across the paper plus a space between the alphabetical and numerical sections. The range of ordinates is therefore limited to 0 to 88. This, however, is a suitable match for the abscissa scale for a reasonable length of paper. If the numbers to be plotted do not lie between 0 and 88 or lie in only a small fraction of this range, say 0 to 20, they can be machine-multiplied beforehand.

The problem then is, for a given value of the ordinate (punched on the card), say 35, to actuate the thirty-fifth type bar, which prints a symbol

(a digit 1–9) which is the "point" on the plot. This is done, in principle, as follows. The tens digit (3 in the example) is picked up by the upper control brushes as the card enters the machine. This signal is filtered through a digit selector (DSC No. 1). The "3" picks up a class selector No. 3. The units position is picked up by the lower brushes and goes through the other DSC No. 2. In the example the "5" goes to the fifth hub on every class selector. But, as we have seen, only selector No. 3 is picked up. The selectors are wired so that the impulse (in the example, "5") from the lower brush via DSC No. 2 goes through class selector No. 3 to type bar No. 35. A "5" is printed 35 spaces from the left edge of the paper (or suitable base line). The "points" on the plot then are the printed units-place digit. They can be connected by hand or ruler to form a graph. The accuracy is clearly about 0.3 in 88, and is quite adequate for plotting data with two significant figures.

In practice the above scheme is modified, to allow ordinates and abscissas to be printed for reference: to use counters as well as selectors, printing zeros, allowing for the "missing" type bar between alphabetical and numerical sections, etc.

Details: For operators of the 405 Tabulators the following detailed account of the method may be given in the standard IBM terminology. By the above scheme one could print numbers 0 to 88, but it is well worth while to sacrifice the first four alphabetical type bars for printing the abscissa. This also constructs the abscissa scale. It is also worth while to sacrifice the last three type bars to print the ordinate, as it is convenient to have the ordinate printed for later reference or as a check while drawing in the curve. It is also convenient to have three significant figures printed even though they are not plotted. The printing is then as follows:

Alphabetical type bars AT 1–4, abscissa.
AT 5–43, ordinates plotting points 0–38.
Gap between AT 43 and NT 1 corresponds to ordinate 39, which is printed
    as below.
Numerical type bars NT 1–42, correspond to ordinates 40–81.
NT 43–45, ordinate.

Type bars AT 5 to 43 are operated by the usual class selectors A to D picked up by impulse 0, 1, 2, 3, from DSC No. 1 to their D hubs. (Note that for numbers less than 10 the tens digit, 0, must be punched on the card.) Type bars NT 1 to 20 can be actuated through two other class selectors E and F when they are available, otherwise by counters, as below. The remaining numerical type bars are actuated through counters, e.g., 2D + 8D, 2C + 8C, and 2B. The counters have their plug to C supplied through X distributors No. 1, 2, 3, which in turn are picked up by impulses 6, 7, 8 from DSC No. 1 to their D hubs.

The units digit, picked up from a lower brush, goes to DSC No. 2. The hubs 1 to 9 of this DSC go to positions 2 to 10 in class selector and counter groups. Note that the DSC hubs are double. One set can go to a counter. Entry hubs are also double, so can be wired to all the other counters in succession. The last free set of hubs can then go to a class selector C hub. The other DSC hubs go to another selector. If only single points are plotted from each card, the other selector C hubs can be wired from No. X hubs of the two

already wired in. However, if multiple printing is anticipated, split wires will be needed.

To print zeros at 0, 10, 20, 30, etc., the units position picked up by upper control brushes goes to a comparing magnet. An unequal impulse there picks up X distributor (No. 4, say) which passes on No. X a "hot 9" to the first position on all selectors and counters. Thus a "9" is printed instead of a "0" in these cases. This is slightly confusing since all other plotted "points" are precisely the units-place digits, but this is a very trivial point. As a matter of fact the zero in 0, 10, 20, and 30 can be picked up on the control brush and wired to the zone magnets on the alphabetical type bars. All hammersplit levers are up except for AT 1 to 4 and NT 43 to 45.

The ordinate "39" needs special attention since there is no type bar corresponding to this ordinate. A simple way to take care of this is to print an asterisk in NT 1 instead of the "9" which would appear there, correctly, for ordinate 40. This can be done as follows. The tens digit is taken from the control brushes to one side of a comparing magnet. The other side of this magnet is connected to DSC No. 1 hub 3. Thus an impulse arises in this comparing magnet whenever the tens digit is not "3." The neighboring comparing magnet compares the units position as detected on the control brushes and compares it with a hot 9. Thus this gives a signal unless the units position is a "9." The "unequal" impulse leads of the two comparing magnets are bottle-plugged together, and the connecting hub wired to the pickup hub of a three pole X distributor (say No. 5). One of the common hubs of this distributor receives an "SUP" signal and its No. X hub is wired to NT 1, through No. X and C of another circuit in the distributor. The third circuit receives the "9" for normal printing of "40" in its C hub, which is passed to X, then to X of the second circuit—which, when the distributor is picked up, goes through C hub (of the second circuit) to NT 1. So far, the circuits do not discriminate between 09 and 39. Thus the SUP passes through an X distributor picked up by the "3."

One convenient feature of the method is that plots can be made from numbers punched anywhere on the cards. Only two wires for the upper and lower brushes need be shifted to change fields.

Double and triple spacing can be obtained, thus giving factors of 2 and 3 on the abscissa.

With certain modifications, the method can be used to plot accumulated totals, by card cycle transfer.

The method is also useful in plotting points not necessarily falling on a continuous curve. For example, in a correlation diagram, a number of values of x are plotted against y. If there is no correlation, the points lie at random, and if there is, they cluster in some region or around a 45° line, etc. A large number of such points can be plotted very quickly by this method. Some difficulty arises when there are more than one pair (x, y) to be plotted at the same place, which can however be easily resolved by coding.

The technique of plotting was tested by reproducing the original drawing of the experimental data. The drawing of the data was cut out from a reprint of the original article, projected through an enlarger on graph paper. The ordinate at each abscissa was read off the graph prepared in this way. These numbers were key-punched. The cards were put through the tabulator

with the plotting plug board. The points printed by the tabulator were connected. This forms our graph of the original data. To test the method this curve was followed by a pantograph which reduced it to the size of the original drawing. This small scale plot was compared directly with the original. The agreement was excellent in spite of the number of steps.

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EDITORIAL NOTE: Although the method here described appears to have been original with Mr. KING, W. J. E. informs us that it has been used in several places for several years, but does not seem to have been previously described in print.

RECENT MATHEMATICAL TABLES

578[A].—Schomann's, *(1 × 1 Tabelle) 1–99 × 99 und 1–999 × 9.* Hamburg, Germany, Verlag Br. Sachse, n.d., 16 p. 14.2 × 20.6 cm.

This little paper-covered multiplication table gives, p. 2–9, the results of multiplications of pairs of numbers 1(1)99 and 1(1)99; and, p. 10–15, of pairs 1(1)999 and 1(1)9. The use of the table to find 8379 × 5623 and 8967 × 456 is indicated.

579[A].—H. S. UHLER, "Twenty exact factorials between 304! and 401!," Nat. Acad. Sci., *Proc.*, v. 34, Aug. 1948, p. 407–412. 17.4 × 25.7 cm.

The text: "In the year 1944 the author published privately a little book entitled *Exact Values of the First 200 Factorials.* [See *MTAC*, v. 1, p. 312.] Subsequently he computed with great care the exact values of $n!$ from $n = 201$ to $n = 300$. The data of this third century have not appeared in print. One consultable copy has been deposited in the library of Brown University, Providence, Rhode Island, and another copy is in the possession of Doctor J. C. P. Miller, technical director of Scientific Computing Service Limited, 23 Bedford Square, London, W.C.1, England.

"Recently the author has computed a skeleton table of 42 exact factorials beginning with 303! and ending with 400!. This table was built up by first calculating the values of $n!$ for which $n + 1$ was one of the 17 primes from $n = 307$ to $n = 401$, so that Wilson's theorem could be applied as a more exacting check in addition to congruence testing with moduli such as $10^5 + 1$, $10^8 + 1$, etc. Incidentally the values of 350!, 372!, 375!, 378! and 400! as found by the author in February, 1945, were reproduced identically in the work performed three years later. In order to make a few of these arithmetical constants available to other investigators requiring exact values in the fourth century of $n!$ the following table of equally spaced but non-consecutive data is presented." [Then follows $n!$, $n = 305(5)400$; in the last there are 869 digits.]

EDITORIAL NOTE: Professor UHLER has reported a printers' error under $340!/10^{23}$, in the second line, 11th pentad, which reads 58229 erroneously instead of the correct order 85229. This correction was made in reprints.

580[C, D].—FRANCE, INSTITUT GÉOGRAPHIQUE NATIONAL, *Tables des Logarithmes à Huit Décimales.* Tome 1: *Logarithmes des Nombres entiers de 1 à 120 000;* Tome 2: *Logarithmes des Fonctions Circulaires de dix secondes en dix secondes d'arc dans le Système de la Division Centésimale du Quadrant.* Paris, 1944 [x, 216, x, 402], p. 20.7 × 27 cm. 2700 francs, unbound.

This is the second edition of the great work issued by the Service Géographique de l'Armée in 1891, to which we have had occasion more than once to make reference (*MTAC*, v. 1, p. 36, 85, 145; v. 2, p. 181). The first edition was a single-volume work with pages of