The inequality (8) may be reduced to

$$\frac{a - a'}{a'' - a'} < \frac{V - V'}{V'' - V'}$$

or,

$$a(V'' - V') + a'(V - V'') + a''(V' - V) < 0,$$

or

$$(13) \qquad \frac{s}{A}\left(\frac{1}{A''} - \frac{1}{A'}\right) + \frac{s'}{A'}\left(\frac{1}{A} - \frac{1}{A''}\right) + \frac{s''}{A''}\left(\frac{1}{A'} - \frac{1}{A}\right) < 0.$$

Multiplying both members of (13) by $A'AA''$ gives

$$s(A' - A'') + s'(A'' - A) + s''(A - A') < 0$$

which is an expanded form of inequality (6) which was proved to be true.

By Theorem X *the maximum error given by* (*12*) *also applies to the* $a_{\overline{n}|}$ *table.*

Since the yield on a bond may be found approximately by interpolating in the $a_{\overline{n}|}$ *table, the maximum error is given by* (*12*).

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[1] It can be shown that the value of N obtained by interpolation is the exact value of n if simple interest is used for the fractional interest period involved.

[2] In W. L. HART, *Mathematics of Investment*, second ed. Boston, 1929, p. 244, a proof is given that the error is at most $\frac{1}{2}$ of the interest rate per period.

[3] The value of $n = n_1 + f$ ($f < 1$) obtained in the $a_{\overline{n}|}$ table has the following useful interpretation: f is the final payment due at the end of $n + 1$ interest periods.

[4] See THEODORE E. RAIFORD, *Mathematics of Finance.* Boston, 1945, p. 25, note, and W. L. Hart, *Mathematics of Investment*, third ed., Boston, 1946, p. 75 and p. 138. In these texts the following statement is made. Experience shows that it is safe to assume that a value of I found by interpolation is in error by not more than $\frac{1}{20}$ of the difference of the table rates used in the interpolation.

RECENT MATHEMATICAL TABLES

608[A, D, S].—G. H. GOLDSCHMIDT & G. J. PITT, "The correction of X-ray intensities for Lorentz-polarization and rotation factors," *Jn. Sci. Instrs.*, v. 25, Nov. 1948, p. 397–398. 20.2 × 27.2 cm.

There are two tables. **T. 1**, Inverse Lorentz-polarization factor as a function of $\rho = 2\sin\theta$; $(LP)^{-1} = \sin 2\theta/(1 + \cos^2 2\theta) = \frac{1}{2}\rho(4 - \rho^2)^{\frac{1}{2}}/(2 - \rho^2 + \frac{1}{4}\rho^4)$, for $\rho = [0(.01)2; 4D]$. **T. 2**, Rotation factor D as a function of ξ and ζ for equi-inclination conditions, $\xi = \zeta D/(1 - D^2)^{\frac{1}{2}}$; $D/(1 - D^2)^{\frac{1}{2}}$ is given, 2–3 decimal places, for $D = -1, .2(.1).9, .95, .975$.

Extracts from text

609[B].—LUDWIG ZIMMERMANN, *Vollständige Tafeln der Quadrate aller Zahlen bis 100 009 berechnet und herausgegeben.* Fourth edition, Berlin Grunewald, 1941, xix, 187 p. (*Sammlung Wichmann Fachbücherei für Vermessungswesen und Bodenwirtschaft*, v. 8.) 19.4 × 24.9 cm.

In the publisher's preface we are told that Zimmermann died 15 Aug. 1938. Compare *MTAC*, v. 2, p. 206–207; the errors of the third edition (1938), in T. III, there noted, here persist. The second edition was published at Liebenwerda in 1925; and the first in 1898.

Zimmermann was also the author of: (a) *Rechentafeln, grosse Ausgabe.* Liebenwerda, 1896, xvi, 205 p.; second ed., 1901; third ed., 1906; fourth ed., 1923, xxxix, 225 p. (b) *Rechentafeln, kleine Ausgabe.* Liebenwerda, 1895; fourth ed., 1926, xxv, 38 p. (c) *Tafeln für die*

Teilung der Dreiecke, Vierecke, und Polygone. Second enl. and impr. ed., Liebenwerda, 1896, 118 + 64 p. (d) *Die gemeinen oder briggschen Logarithmen der natürlichen Zahlen 1–10009 auf 4 Decimalstellen nebst einer Productentafel, einer Quadrattafel und einer Tafel zur Berechnung der Kathete und Hypotenuse und zur Bestimmung der Wurzeln aus quadratischen Gleichungen. Zum Gebrauch für Schule und Praxis.* Liebenwerda, 1896, 40 p. (e) *Mathematische Formelsammlung . . . zur Vorbereitung für das Einjährig-Freiwilligen-Examen. Essen,* Baedeker, 1910, iv, 55 p.

610[D, S].—J. D. H. Donnay & G. E. Hamburger, *Tables for Harmonic Synthesis, giving Terms of Fourier Series to one decimal at every millicycle tabulated for coefficients 1 to 100, and fiducial cosine values to eight Decimals.* Baltimore, The Johns Hopkins University, Crystallographic Laboratory, copyright 1948, [103] leaves. 21.6 × 28 cm. The leaves are enclosed in a strong ring binder. Purchasable from Professor Donnay at the Laboratory, $10.00.

The tables on 100 leaves give $10F\cos X$, for $F = 1(1)100$, $X = 1(1)1000$. The unit of angle, $2\pi/1000$, is called a millicycle (mC). Since $\sin X = \cos(X + 750)$, the table also gives $10F\sin X$. In all 200 000 values are thus available. They are represented by 25 000 entries arranged as follows: one page for every value of F and 250 entries on each page.

On leaf [103] is a table of $\cos X$, $X = [1(1)250; 8D]$; 1 mC = .36°. This is simply a table of extracts from Earle Buckingham, *Manual of Gear Design*, Section 1, 1935. [See *MTAC*, v. 1, p. 88–92]. The table on leaf [3] is equivalent to 1D from Buckingham's table and leaf [102] is equivalent to 3D from the same table.

A comparison of the 15D table of sines and cosines in F. Callet, *Tables Portatives de Logarithmes*, Paris, 1795, revealed two last-decimal unit errors in Buckingham, namely: cos 27 mC = .98564460 (not .98564459), and cos 232 mC = .11285638 (not .11285639).

The tables are intended to facilitate computations in problems of harmonic synthesis. In structural crystallography, for example, it may be used for the summation of Fourier Series representing either the electron intensity $\rho(xyz)$ at a point xyz, or the structure factor $F(hkl)$ of a reflection hkl.

Example: $$\rho(xyz) = \sum_h \sum_k \sum_l A(hkl)\cos 1000(hx + ky + lz)^{\mathrm{mC}} + \sum_h \sum_k \sum_l B(hkl)\sin 1000(hx + ky + lz)^{\mathrm{mC}}.$$

The required multiples of the trimetric coordinates x, y, z are tabulated once and for all. The angle $(hx + ky + lz)$ is calculated for every triplet hkl; in each case, the first three decimal places give the X of the table.

Alternately the three-dimensional triple sums may be replaced by one-dimensional single sums,

$$\sum_H A(H)\cos 1000HX^{\mathrm{mC}} + \sum_H B(H)\sin 1000HX^{\mathrm{mC}},$$

by letting $H = n^2h + nk + l$, where $n = 100$ (or 1000) according as xyz are given to two (or three) places, and

$$X = (n^3z + n^2y + nx)/n^3.$$

The basis for the second method is that evaluating the electron density along a (one-dimensional) row $[n^2nl]$ amounts, if n is sufficiently large, to evaluating it throughout the whole three-dimensional cell.

Extracts from text

Editorial Note: The millicycle angular unit here used is $\frac{2}{5}$ of a grade = .001 of a gone = .1 of a Cir. See *MTAC*, v. 1, p. 40–41.

611[E, H].—C. I. ROBBINS & R. C. T. SMITH, "A table of roots of $\sin z = -z$," *Phil. Mag.*, s. 7, v. 39, Dec. 1948, p. 1004–1005. 17.2 × 25.6 cm.

On writing $z = x + iy$ and separating into real and imaginary parts this equation is replaced by the pair of equations

$$f(x, y) = \cosh y + x/\sin x = 0, \qquad g(x, y) = \cos x + y/\sinh y = 0.$$

A 6D table of the first 10 non-zero roots in the first quadrant is given. These values were calculated by Newton's rule in the form

$$f(x + \delta x, y + \delta y) \approx f(x, y) + \delta x \frac{\partial f}{\partial x} + \delta y \frac{\partial f}{\partial y} = 0,$$

$$g(x + \delta x, y + \delta y) \approx g(x, y) + \delta x \frac{\partial g}{\partial x} + \delta y \frac{\partial g}{\partial y} = 0$$

Starting from values x, y these two equations determine δx, δy and thus improved approximations $x + \delta x$, $y + \delta y$ to the roots. This process was repeated until the corrections did not affect the eighth decimal. Consequently the roots are believed reliable to 6D. This table is similar to that of A. P. HILLMAN & H. E. SALZER, giving 6D values of the first 10 zeros of $\sin z - z$, *Phil. Mag.*, s. 7, v. 34, 1943, p. 575.

Extracts from text

EDITORIAL NOTE: In *MTAC*, v. 2, p. 60–61, Jan. 1946, MITTELMAN & HILLMAN published 7D values of the first four non-zero values of the zeros of $\sin z + z$. In 1940 J. FADLE published 5D values in *Ingenieur-Archiv*, v. 11, p. 129. See also *MTAC*, v. 1, p. 141, **50**.

612[G, K].—S. M. KERAWALA & A. R. HANAFI, "Table of monomial symmetric functions of weight 12 in terms of power-sums," *Sankhyā*, v. 8, June, 1948, p. 345–359. 23 × 29.5 cm.

This table is an extension of previous tables of symmetric functions of weights less than 12 described in RMT **463** (*MTAC*, v. 3, p. 24). Each line of the table gives the coefficients of the polynomial in $S_1, S_2, \cdots, S_{12}$ which represents a given monomial symmetric function $\Sigma\, \alpha_1^{p_1}\alpha_2^{p_2} \cdots$ where $p_1 + p_2 + \cdots = 12$. Here S_k is the sum of the kth powers of the α's. For some reason the last line of the table has been omitted; that is to say, there is no expression given for the 12-th elementary symmetric function $\Sigma\, \alpha_1\alpha_2 \cdots \alpha_{12}$ as a polynomial in the S's.

D. H. L.

613[I].—HERBERT E. SALZER, *Table of Coefficients for Obtaining the First Derivative without Differences.* (NBS, *Applied Mathematics Series*, no. 2.) Washington, D. C., 1948, [ii], 20 p. 19.8 × 26 cm. For sale by the Superintendent of Documents, Washington, D. C., 15 cents. See *MTAC*, v. 3, p. 187–188.

The present set of tables is designed to expedite the numerical estimation of the values of the derivative of a function, $f(x)$, which has been approximated by means of the interpolation formula of LAGRANGE. In 1944 the NBSCL under the direction of Dr. A. N. LOWAN provided a large volume of the coefficients in the Lagrange approximation formula

$$f(x_0 + ph) \sim \sum_{i=-[\frac{1}{2}(n-1)]}^{[\frac{1}{2}n]} A_i^{(n)}(p) f(x_0 + ih)$$

where $[m]$ means the largest integer in m. (For a review see *MTAC*, v. 1, p. 314–315.)

If the derivative of both sides of this approximation is taken with respect to p there results:

$$f'(x_0 + ph) \sim \frac{1}{hC(n)} \sum_{i=-[\frac{1}{2}(n-1)]}^{[\frac{1}{2}n]} C_i^{(n)}(p) f(x_0 + ih).$$

The present work provides tables for the coefficients in this sum. To quote the author:

"$C_i^{(n)}(p)$, or simply C_i, is a polynomial in p of the $(n-2)$th degree, and $C(n)$, or simply C, is the least integer chosen, so that $C_i^{(n)}(p)$ will have integral coefficients $\cdots$. The accompanying table gives the exact values of these polynomials $C_i^{(n)}(p)$, for p ranging from $-[(n-1)/2]$ to $[n/2]$. For $n = 4$, 5, and 6, the polynomials $C_i^{(n)}(p)$ are tabulated at intervals of 0.01; for $n = 7$, they are tabulated at intervals of 0.1.

For $n = 3$, no table is needed, because we have the quite simple formula

$$f'(x_0 + ph) \sim \frac{1}{h}[(p - \tfrac{1}{2})f_{-1} - 2pf_0 + (p + \tfrac{1}{2})f_1]."$$

H. T. D.

614[I].—P. M. Woodward, "Tables of interpolation coefficients for use in the complex plane," *Phil. Mag.*, s. 7, v. 39, Aug. 1948, p. 594–604. 16.9 × 25.1 cm.

T. I: $C_2(p, q) = \frac{1}{6}pq(p^2 - q^2)$, for p and $q = [0(.05)1; 7D]$, with coupled second differences—exact to 8D; **T. II**: $C_4(p, q) = -[pq/(360)][4 + (p^2 - 3q^2)(3p^2 - q^2)]$, for p and $q = [0(.05)1; 7D]$, also with coupled second differences. Fourth differences, which never exceed 60, are not tabulated.

Extracts from text

615[L].—P. K. Bose, "On recursion formulae, tables and Bessel function populations associated with the distribution of classical D^2-statistic," *Sankhyā*, v. 8, Oct. 1947, p. 235–248. 22.7 × 29.3 cm.

On p. 247–248 are 6D tables of $e^{-x}I_0(x)$, $e^{-x}I_1(x)$ for $x = 16.08$, 16.2, 16.68, 16.92, 17., 17.04, 17.16, 17.4, 18, 18.48, 16.16(.16)18.88 [except 17.6, 18.4], 19, [200 other values], 49.2, 49.28, 49.44, 49.6, 49.68, 49.92, 50.

In *MTAC*, v. 1, p. 226, we have given references to tables of these functions in (a) BAASMTC, *Math. Tables*, v. 6, 1937, for $x = [16(.1)20; 8D]$, δ^2; and (b) Badellino, 1939, for $x = [20(1)50; 9D]$.

As to the earlier pages, J. W. Tukey noted in *Math. Revs.*, v. 9, p. 620: "The distribution in question was found by R. C. Bose, *Sankhyā*, v. 2, 1936, p. 143–154. It can be put in the form

$$P(L) = \int_0^P x^{q+1}\lambda^{-q}e^{-\frac{1}{2}(x^2+\lambda^2)}I_q(x\lambda)dx,\ q = \tfrac{1}{2}p - 1,$$

where $2L^2 = \bar{n}pD_1^2 = \bar{n}pD^2 + 2p$, $2\lambda^2 = \bar{n}p\Delta^2$ and Δ^2 and D^2 are the population and estimated squared distances of two p-variate samples whose harmonic mean size is $\bar{n}$." Values of L are tabulated for $P = .99$, .95, .05 and .01, $p = 1(1)10$, that is, $q = -\frac{1}{2}(\frac{1}{2})4$, and $\lambda = 0(.5)3(1)6$, 8, 12(6)24, 36, 54, 72, 108, 216, 432 to $2D$.

R. C. A.

616[L].—C. H. Collie, J. B. Hasted, & D. M. Ritson, "The cavity resonator method of measuring the dielectric constants of polar liquids in the centimetre band," Phy. Soc., London, *Proc.*, v. 60, 1948, p. 71–82. 17.8 × 25.9 cm.

On p. 78–79 is a 4D or 4S table of the real and imaginary parts of $z^{-1}J_1(z)/J_0(z)$, $z = x - iy$, for $x = .5(.1)2$, $y = 0(.1)1$, $y \leqslant x$. On p. 77 is a graph of the imaginary part of the function.

617[L].—A. Ghizzetti, "Tavola della funzione euleriana $\Gamma(z)$ per valori complessi dell' argomento," Accad. Naz. Lincei, *Atti*, *Rend.*, s. 8, v. 3(2), 1947, p. 254–257. 18 × 26.5 cm.

Table of $\Gamma(x+iy)$, $x = 4(.1)5$, $y = 0(.1)1$, to 5S. For interpolation the marginal values $3.9 + .2ni$, $5.1 + .2ni$, $4 - .1i + .2n$, $4 + 1.1i + .2n$, $n = 0(1)5$. See further *Math. Revs.*, v. 8, p. 619 (S. C. VAN VEEN).

618[L].—S. GOLDMAN, *Frequency Analysis, Modulation and Noise.* New York, McGraw-Hill, 1948. "Appendix F. Table of Bessel Functions of the first kind of constant integral argument and variable integral order," p. 421–427.

This is a Table of $J_n(x)$ for $x = [1(1)33; 4\text{–}5S]$, $n = 0(1)N-1$, $15 \leq N \leq 37$. It is said to be of particular use in determining the sideband magnitudes in frequency and phase modulation. That part of the Table $n = 1(1)29$, $15 \leq N \leq 35$ is evidently abridged from JAHNKE & EMDE's expanded abridgment (1938) of MEISSEL's table of 1895, because J. & E.'s error in $J_9(21)$ is faithfully copied. The remaining part of the table, p. 427, was abridged from the HARVARD COMPUTATION LABORATORY'S Bessel Function calculations. See N99, no. 7.

R. C. A.

619[L].—R. E. GREENWOOD & J. J. MILLER, "Zeros of the Hermite polynomials and weights for Gauss' mechanical quadrature formula," Amer. Math. Soc., *Bull.*, v. 54, Aug. 1948, p. 765–769. 15.2 × 24.1 cm.

The Hermite polynomials are defined by the relation

$$H_n(x) = (-1)^n e^{x^2} d^n(e^{-x^2})/dx^n$$
$$= (2x)^n - n(n-1)(2x)^{n-2}/1! + n(n-1)(n-2)(n-3)(2x)^{n-4}/2! - \cdots.$$

Some writers, including many statisticians, prefer to use

$$h_n(x) = e^{\frac{1}{2}x^2} d^n(e^{-\frac{1}{2}x^2})/dx^n$$

as the defining relation for Hermite polynomials. The relation between these two sets of polynomials is given by

$$H_n(x) = (-2^{\frac{1}{2}})^n h_n(2^{\frac{1}{2}}x).$$

The approximate numerical integration formula for functions $f(x)$ on the infinite range $(-\infty, +\infty)$ with the weight function $\exp(-x^2)$ is

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx \backsimeq \sum_{i=1}^{n} \lambda_{i,n} f(x_{i,n}) \tag{1}$$

where the set $\{x_{i,n}\}$ is the set of roots defined by $H_n(x) = 0$, and where the set $\{\lambda_{i,n}\}$ is given by[1] $\lambda_{i,n} = \pi^{\frac{1}{2}} 2^{n+1} n!/[H_n'(x_{i,n})]^2$. If $f(x)$ is a polynomial of degree $(2n-1)$ or less, integration formula (1) is exact.[2]

The zeros $\{X_{i,n}\}$ for the polynomials $h_n(x)$, for $n = [1(1)27; 6D]$ have been tabulated by SMITH.[3] The corresponding zeros are given by $x_{i,n} = 2^{-\frac{1}{2}} X_{i,n}$.

Tables, 9–12D or S, are given of $\{x_{i,n}\}$ and of Christoffel numbers $\{\lambda_{i,n}\}$ for $n = 1(1)10$, $i = 1(1)5$.

Extracts from text

[1] G. Szegö, *Orthogonal Polynomials*, Amer. Math. Soc., *Colloquium Publs.*, v. 23, 1939, p. 344.

[2] SZEGÖ, *loc. cit.*, chap. XV; and C. WINSTON, "On mechanical quadratures formulae involving the classical orthogonal polynomials," *Annals Math.*, s. 2, v. 35, 1934, p. 658–677.

[3] E. R. SMITH, "Zeros of the Hermitian polynomials," *Amer. Math. Mo.*, v. 43, 1936, p. 354–358. [EDITORIAL NOTE: see *MTAC*, v. 1, p. 152–153; for varying definitions of $H_n(x)$, see here, and v. 1, p. 50; v. 2, p. 25, 30; v. 3, p. 26, 167. Greenwood & Miller give no reference to the table of REIZ, reviewed in *MTAC*, v. 3, p. 26.]

620[L].—U. S. Navy, Naval Research Laboratory, *Extended Tables of Fresnel Integrals*. Boston, Mass., 470 Atlantic Ave., 1948, 7 + [6] hektographed leaves, 26.6 × 20.3 cm.

The Tables involve $C(u)$, $S(u)$, $c_1(u) = \int_\infty^u \cos(\frac{1}{2}\pi t^2)dt = C(u) - \frac{1}{2}$, $s_1(u) = \int_\infty^u \sin(\frac{1}{2}\pi t^2)dt = S(u) - \frac{1}{2}$.

Table I is of $C(u)$, $S(u)$, $[C(u)]^2$, $[S(u)]^2$, $\{[C(u)]^2 + [S(u)]^2\}^{\frac{1}{2}}$, $[c_1(u)]^2$, $[s_1(u)]^2$, $\{[c_1(u)]^2 + [s_1(u)]^2\}^{\frac{1}{2}}$, for $u = [0(.1)20; 4\text{D or }4\text{S}]$.

Table II is of $C(u)$ and $S(u)$, for $u = [8(.02)15.98; 4\text{D}]$.

Among tables of this kind are those of $C(u)$ and $S(u)$ for $u = [0(.1)8.5; 4\text{D}]$ in JAHNKE and EMDE, 1945, American edition, which are in agreement with the tables under review except for the Navy error in $S(7.3)$, where *for* .3189, *read* .5189.

R. C. A.

621[L, M].—J. C. JAEGER, "Repeated integrals of Bessel functions and the theory of transients in filter circuits," *Jn. Math. Phys.*, v. 27, Oct. 1948, p. 210–219. 17.3 × 25.3 cm.

$Ji_{n,r}(t) = \int_0^t dt \int_0^t dt \cdots \int_0^t J_n(t)dt$, **T. I** is of $2^r Ji_{n,r}(t)$, for $r = 1(1)7$, $t = [0(1)24; 8\text{D}]$. $\Phi_n(t) = \int_0^\infty J_0[2v(ut)]J_n(u)du$, $J_n(t) = \int_0^\infty J_0[2v(ut)]\Phi_n(u)du$, **T. II–III** give 4D values of $\Phi_n(t)$, and $\Phi_n'(t)$, for $n = 1(1)7$, $t = 0(1)24$. $\psi_n(t) = \frac{1}{2}[\Phi_{2n}(t) + \Phi_{2n+2}(t)]$, **T. IV** gives 5D values of $\psi_n(t)$, for $n = 0(1)2$, $t = 0(1)24$.

Extracts from text

622[L, Z].—P. I. ZUBKOV, "Primenenie universal'nogo raschetnogo stola peremennogo toka dli͡a tabulirovanii͡a otnoshenii͡a modifit͡sirovannykh funkt͡siĭ Besseli͡a" [The application of a universal alternating current computer to the tabulation of ratios of modified Bessel functions], Akad. N., SSSR, *Izvestii͡a, Otdelenie tekhnich. n.*, 1948, p. 489–498.

The computer mentioned in the title is not described in the present paper. It appears to be a sort of AC network analyzer with adjustable resistances, inductances and capacities. The author mentions negative resistances from which one would infer the existence of two or more amplifiers probably with feedback facilities.

The functions referred to are the familiar $zI_{n-1}(z)/I_n(z)$ whose well known continued fraction development is exploited by the computer. Just how many terms of this development the machine can use is not revealed. The values of n considered are $n = \pm .1$. The complex number z is of the form $i^{\frac{1}{2}}x$ where x is real. Tables are given to 4S of the real and imaginary parts of the functions for $x = 0(.2)2$. These tables were computed by hand. Corresponding readings taken from the machine bear no superficial resemblance whatever. With a certain amount of careful guesswork, values of the functions can be derived from the readings. The agreement is said to be within 2 percent.

D. H. L.

623[M].—E. C. BULLARD & R. I. B. COOPER, "The determination of the masses necessary to produce a given gravitational field," R. Soc. London, *Proc.*, v. 194A, 1948, p. 332–347.

$\lambda(x) = 2(\sigma/\pi)^{\frac{1}{2}} \cos 2\sigma x \exp[\sigma(1 - x^2)] - (1/\pi)$, $\psi(x, \sigma^{-\frac{1}{2}})\psi(x, \eta) = \eta^{-1}\pi^{-\frac{1}{2}}\int_{-\infty}^{\infty}(1 + y^2)^{-1} \times \exp[-(x - y/\eta)^2]dy$; $\Lambda(r) = r\int_0^\infty e^p e^{-p^2/4\tau} J_0(pr)pdp$.

T. 1 gives values of $\lambda(x)$ for $\sigma = 1(4\text{D})$, $4(3\text{D})$ and $x = 0(.1)5$, with modified second differences; also for $\sigma = 1(4\text{D})$ the values of $\int_0^x \lambda(x_1)dx_1$; also 4D values of $\Lambda(r)$ and $\int_0^r \Lambda(r_1)dr_1$, for $\tau = 1$. Graphs of $\lambda(x)$ for $\sigma = 1$ and $\Lambda(r)$ for $\tau = 1$. The column giving $\lambda(x)$ for $\sigma = 4$ cannot be interpolated.

Beyond the limits of **T. 1**, the following expressions will give results accurate to four places of decimals:

$$\pi\lambda(x) \sim -1/x^2 - (3/2\sigma - 1)/x^4, \quad \pi\int_0^x \lambda(x_1)dx_1 \sim \tfrac{1}{2}\pi + 1/x + (3/2\sigma - 1)/3x^3;$$
$$\Lambda(r) \sim -1/r^2 - 3(3/2\tau - 1)/2r^4, \quad \int_0^r \Lambda(r_1)dr_1 \sim 1 + 1/r + (3/2\tau - 1)/2r^3.$$

T. 2, 3D values of $\int_{x_1}^{x_2}\lambda(x)dx$, for intervals 0 to .25; .25 to .5; .5 to 1; 1 to 1.5; 1.5 to 2; 2 to 3; 3 to 4; 4 to 5; 5 to 10; 10 to 20; 20 to ∞.

T. 3, 1D values of $\int_{r_1}^{r_2}\Lambda(r)dr$, for intervals 0 to .23; .23 to .34; .34 to .44; .44 to .54; .54 to .65; .65 to .83; .83 to 1.35; 1.35 to 1.59; 1.59 to 2.12; 2.12 to 2.54; 2.54 to 3.4; 3.4 to 5.05; 5.05 to 10.02; 10.02 to ∞.

Extracts from text

624[M].—H. G. Hay, & Miss N. Gamble, "Five-figure table of the function $\int_0^\infty e^{-zy}\cdot Ai^2(y - j_1)dy$ in the complex plane," *Phil. Mag.*, s. 7, v. 39, Dec. 1948, p. 928–946. 17.2 × 25.6 cm.

Except for two references to recent literature this paper is simply an edition for the general public of the report in Nov. 1946, which we have already reviewed in *MTAC*, v. 2, p. 344–345. In the reprint a small misprint has been introduced, p. 931, l. −1; *for* 3.3, *read* 3.2. We are told that a full description of the method used in the computation is due to G. G. Macfarlane, "The application of a variational method to the calculation of radio wave propagation curves for an arbitrary refractive index profile in the atmosphere," Phys. Soc. London, *Proc.*, v. 61, July 1948, p. 48–59. A reference is also given to P. M. Woodward, *Phil. Mag.*, s. 7, v. 39, Aug. 1948, p. 594–604 (RMT **614**), for his method of using coupled differences in bivariate interpolation for a function of a complex variable. A single-page sample of Woodward's tables appeared in the 1946 report of Hay & Gamble.

R. C. A.

625[Q].—Paul Herget, *The Computation of Orbits*. Published privately by the author, University of Cincinnati, and lithographed by Edwards Brothers, Inc., Ann Arbor, Michigan, 1948, ix, 177 p. including 24 p. of tables. Light cardboard binding, 22 × 28 cm. $6.25.

There are so few works on the computation of orbits and perturbations in English, while foreign works are so thoroughly out of print, that Herget's highly condensed but surprisingly comprehensive treatment of the subject must perforce be welcomed wherever advanced astronomy is studied. Some devotees of Leuschner's modification of Laplace's method will be stunned by its omission, but Herget has treated the various methods of computing preliminary orbits with his own independent and experienced approach. His inclusion of numerical differentiation and integration, special perturbations, and the application of Hansen's method of general perturbations as well as the computation of preliminary and corrected orbits in so small a volume is remarkable. Compactness is gained by the use of vector notation, unfortunately at times because of the difficulty in distinguishing vector from scalar font. The few, highly specialized, tables are also extremely compact and most of them require second difference interpolation. Most of these tables are available elsewhere in various forms.

The book should contribute appreciably to the preservation of orbit work in present-day astronomy.

Table **I** (3D) gives constants for correcting rectangular coordinates of the sun from an origin at the center of the earth to the position of the observer. Arguments: astronomical latitude and selected Observatories.

T. **II** is a critical table (4D) of Everett second differences interpolating coefficients. This interpolating formula is often useful in tables where the fourth differences can be neglected, and especially where also the second differences are tabulated.

If S is the chord between positions in a parabolic orbit at times t_i and t_j and solar distances r_i and r_j, then T. **III** (7D) gives $S/(r_i + r_j) = \eta\zeta$ as a function of $\eta = 2k(t_j - t_i)/(r_i + r_j)^{\frac{3}{2}}$, 0(.01).6; it gives also $\bar{y}$, the ratio of the sector area to triangle area, for the same argument. In adjacent columns T. **III** (7D) gives the solution y of the equation

$$y^3 - y^2 - hy - h/9 = 0$$

with argument $h = 0(.01).6$, and also $\Delta\bar{y}_0$ used in numerical approximating. These tabulated quantities are useful in the Gaussian method for calculating parabolic orbits.

T. **IV** (7D) gives, both for ellipse and hyperbola, two highly specialized quantities used in solving Lambert's equation and used in GAUSS's method mentioned above. Included also in T. **IV** is the function f, (6D), where

$$fq = 1 - (1 + 2q)^{-\frac{3}{2}}$$

with argument $q = - .03(.001) + .03$. This function occurs in ENCKE's method of special perturbations.

In relating true anomaly with time in a nearly parabolic orbit, either elliptic or hyperbolic, certain auxiliary quantities, A, B (8D), C (7D), and D (7D), given with argument $A = 0(.0001).3$, in T. **V** reduce the numerical work involved.

T. **VI** gives the interpolating coefficients to be used in tables of double and single numerical integration in an "Everett" form, where only even-order differences "on the line" need be tabulated. The argument is $n = 0(.001)1$, the fraction of the interval, for the coefficients (6D and 7D) of the second and lower order differences. Coefficients (5D) of the fourth and sixth differences are given at the end of the main table.

Herget states: T. **VII** "is an 'optimum-interval' table which gives $1/r^3$ with the argument r^2. The interpolating formula is

$$F(r^2) = F_0 - N(D_1 - ND_2)$$

where N consists of all of the portion of r^2 which is not printed in full-size type in the r^2 column." The quantities F_0, D_1, and D_2, to seven significant figures (8D or 9D), are given with argument $r^2 = 4(.01)40$, where the second or the first and second decimals are small-size type. The table covers only three pages!

[T. **I** is an abbreviation of a table by E. C. BOWER in Lick Observatory, *Bull.*, v. 16, 1932, p. 41; parts were given earlier in the British *Nautical Almanac*.—T. **II**, **VI**, see J. D. EVERETT, BAAS, *Report 1900*, p. 648–650; E. T. WHITTAKER & G. ROBINSON, (a) *A Short Course in Interpolation*. London, 1923, p. 40; (b) *The Calculus of Observations*. . . . 1924, p. 40.—T. **III**. For $\Delta\bar{y}$, see RAS, *Mon. Not.*, v. 90, 1930, p. 814; $\eta\zeta$ is condensed from T. 26 in J. BAUSCHINGER, *Tafeln zur theoretischen Astronomie*, second ed. by G. STRACKE. Leipzig, 1934.—T. **V**, Logarithmic equivalents are given by A. MARTH, *Astron. Nachrichten*, v. 43, 1856, cols. 121–134.—T. **VII**, see, for example, British *Nautical Almanac* 1933, Table X, "Planetary coordinates for the years 1800–1940."]

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EDITORIAL NOTE: On Jan. 6, 1949, we received from Professor Herget a "First List of Errata and Addenda" containing 34 entries.

626[U].—ISTITUTO IDROGRAFICO DELLA MARINA, *Tavole H per il calcolo delle Rette d'Altezza.* (Publication No. 3118.) Genoa, 1947. xxv, 30 p. and 2 charts. 17.1 × 24 cm. 600 lire.

This attractive small volume, bound in tan cloth, was prepared, according to its preface, to replace "Tavole F" which was out of print. Although no information on the point is included in this volume, one may reasonably suspect that "Tavole F" was an Italian reprint of the German "F-Tafeln" [*MTAC*, v. 2, p. 81–82]. In any case, the preface states specifically that this volume, save for a few additions and changes in the explanatory material and a few examples, is a direct copy of *Tavole A B per le rette di altezza*, a publication of the

Istituto Geografico Militare, which was in turn made up of the table of S. OGURA, and the azimuth diagram of A. RUST.

Thus this table may be compared to H.O. no. 208 (DREISONSTOK) [*MTAC*, v. 1, p. 79–80], with the inclusion of Rust's Azimuth Diagram permitting a reduction in the amount of material in Tables I and II. The astronomical triangle is divided into two right triangles by a perpendicular dropped from the zenith upon the hour circle of the celestial body; N is the length of this perpendicular and K is the declination of its foot. In the usual American notation, in which L, d, t, h, Z are the latitude of the observer, the declination, hour-angle, altitude and azimuth of the celestial body, the formulae used to compute the values in Table I are:

$$\tan K = \tan L \sec t, \qquad \text{and} \qquad \cos N = \sin L \csc K.$$

Table I is a double-entry table with vertical argument, $L = 0(1°)65°$, and horizontal argument, 12 values to a page, $t = 0(1°)180°$; for each argument pair are tabulated the values of $A = 10^5 \log \sec N$ to 0.1 for $t = 0(1°)20°$ and $160°(1°)180°$ and to the nearest integer for intermediate values of t, and of K to $0'.1$ for $t = 0(1°)180°$.

Table II is one of the values to the nearest 0.1 of $10^5 \log \sec (K - d)$ for $(K - d)$ $0(1')10°$ and to the nearest integer for $(K - d)$ $10°(1')80°$, and of $10^5 \log \csc h$ to the nearest integer for $h = 10°(1')80°$ and to the nearest 0.1 for $h = 80°(1')90°$.

h and Z may be obtained from these tables by the use of the formulae

$$10^5 \log \csc h = 10^5 \log \sec N + 10^5 \log \sec (K - d),$$
$$10^5 \log \csc Z = 10^5 \log \csc t + 10^5 \log \sec d - 10^5 \log \sec h,$$

or Z may be obtained from the Rust diagrams.

These tables have the common disadvantage of having the values corresponding to a single latitude scattered over all of the pages of Table I. Also they are useful only between 65°S and 65°N latitudes. They have the great advantages of simplicity and compactness and they can be used for all four of the basic problems of celestial navigation: the computation of altitude, of azimuth, the identification of stars, and the computation of great circle courses.

It might be added that the end papers in the front and back are very nice examples of a repetitive pattern involving a number of objects common in the navigator's world; these papers have no particular value in navigation but they do add to the attractiveness of the small volume. There are doubtless many navigators who would rather have some of the frequently used small tables, refractions, etc., on the inside covers and reserve the fine printing for the end papers of volumes on art and history.

To obtain an estimate of the accuracy of the tabular values, 1040 values of A in Table I were examined and 171 values were found to be in error. However, all of these errors were rounding-off errors of one unit in the last place given, except for a single error of two units. This would indicate that *Tavole H* is slightly more accurate than H.O. no. 208 (DREISONSTOK).

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627[U].—LAPUSHKIN, *Morekhodnye Tablitsy 1943g* [Nautical Tables for the year 1943], Leningrad and Moscow, Hydrographic Administration, Military and Maritime Fleet, USSR, 1944. lxiii, 245 p. 16.5 × 25.7 cm. + 13 cards 15.2 × 24.9 cm.

This collection of tables was designed for the use of the surface navigator and is based upon an earlier publication of the same name, edited by V. V. AKHMATOV and published in 1933, with second and third editions appearing in 1934 and 1939. It is somewhat similar to, but rather more extensive than, the tables in BOWDITCH, *American Practical Navigator*.

There are 57 pages of explanation followed by 59 tables, not counting those on the cards, found in a pocket on the inside of the back cover. Actually tables 44 to 59 inclusive are almost exclusively conversion tables, time to arc and conversely, inches to millimeters to millibars, degrees Fahrenheit to degrees Centigrade, etc.

Tables **1–6** are 4D: **1–2**, logarithms and antilogarithms; **3–4**, addition and subtraction logarithms; **5–6**, natural values for $0(10')90°$, and logarithmic values for $0(1')90°$, of the six trigonometric functions.

T. 7 is a collection of 3D tables corresponding to **T. 1–6**.

T. 8 gives the 4D values of log $\sin^2 \frac{1}{2}t$ and $\sin^2 \frac{1}{2}t$ for $t = 0(1')180°$. $\text{Sin}^2 \frac{1}{2}t$ will be recognized as the familiar haversine. **T. 9** and **11** give the corrections to $0'.1$ to observed altitudes 5° to 90° of the lower limb of the sun and of stars to take care of refraction, height of eye 0, 10(2)60 feet, and semi-diameter of the sun, the latter in **T. 9** only. **T. 9a** gives the additional correction necessary to take into account the variable semi-diameter of the sun and **T. 10** tabulates the values of the diameter of the sun on selected dates of the year.

T. 12 and **13** give the corrections to $0'.1$ to be applied to the altitudes of the lower and upper limbs of the moon for refraction, semi-diameter, parallax and height of eye 20 feet. **T. 12a–13a** are identical and provide the corrections necessary to take into account heights of eye other than 20 feet, 8(2)60 feet. **T. 14** gives the dip of the horizon to $0'.1$ for height of eye 0(0.5)3(1)18 meters; it is probable that the user will find the change of units, from feet in **T. 12a** and **13a** to meters in **T. 14**, confusing. One wonders also whether 20 feet will not be rather a small height of eye for the average sea-going vessel. **T. 15** provides the corrections to be applied to an altitude measured from a shore line rather than from a visible horizon.

T. 16 yields the corrections to $0'.1$ for refraction for low altitudes $-10'(2')1°30'(10')5°$, as well as for other altitudes $5°(20')10°(30')12°(1°)25°(2°)37°$, $40°(5°)80°$, 90° for a standard temperature of 10°C. and a standard barometric pressure of 760 mm. of mercury. **T. 16a–16b** give the corrections to be applied to the values taken from **T. 16** for temperatures $-20°(5°)40°$ and barometric pressures 720(5)780 mm of mercury respectively.

T. 17–19 are intended for use in determining the correction to the meridian of altitudes observed near the meridian. The formula used is

$$C = (200 \sin^2 \tfrac{1}{2}t)/K \text{ arc } 1' - \tfrac{1}{2}C^2 \tan H \text{ arc } 1'$$

where $K = 100 \tan L - 100 \tan d$, and H is the approximate meridian altitude. **T. 17** is a critical table of values of $100 \tan x$ for x, 0 to 90°, allowing one to evaluate K quite easily. **T. 18** is a double-entry table providing the first (and often the only significant) term to $0'.1$ with arguments K and t. **T. 19** yields the second term to $0'.1$ with arguments, the approximate value of the first term and the approximate meridian altitude.

T. 20 gives the range in hours and minutes for the use of near-meridian altitudes with arguments latitude $0(5°)40°(4°)60°(2°)80°$ and declination, same name and opposite name, $0(5°)20°$, 24°. **T. 21**, intended to be used in correcting the time of culmination, gives values of $15.28 \tan x$ to .01 for $x = 0(1°)79°$. **T. 22** gives the correction to .01 to be applied to an altitude measured near the meridian to obtain the corresponding meridian altitude.

T. 23–24 give the hour angle to the nearest minute of time and the altitude to $0°.1$ of a celestial body on the prime vertical with declination $1°(1°)24°(2°)52°(4°)60°$ at a point in latitude $1°(1°)40°(2°)80°$. **T. 25–26** give the change in altitude to $0'.1$ in one minute of time and the interval of time to $0^s.1$ corresponding to a change in altitude of 1′, for latitudes $0(10°)80°$ and azimuths $5°(5°)50°(10°)90°$.

T. 27, well hidden two thirds of the way through the volume shares with **T. 8**, one third of the way through, honors in importance as a navigating table. It occupies 23 pages and is intended to be used with the formulae given below in the usual American notation where t, d, L, Z and h are the hour angle and declination of the celestial body, the latitude of the observer and the azimuth and altitude of the celestial body:

$$T(K) = T(d) + S(t),\ T(Z) = T(t) - S(K) + C|K - L|,\ T(90° - h) = T|K - L| + S(Z).$$

The tabulated quantities, each given to the nearest integer, are:

$$C(x) = 2(10)^4 \log \csc x, \quad S(x) = 2(10)^4 \log \sec x, \quad T(x) = 2(10)^4 \log \tan x + 70725,$$

for $x = 0(1')90°$. These formulae correspond to a division of the astronomical triangle into two right spherical triangles by a perpendicular dropped from the celestial body upon the meridian. K is the declination of the foot of the perpendicular.

T. 28–29 give the apparent azimuth angle to 0°.1 of the rising and setting upper limb of the sun for latitudes 0(5°)20°(1°)75° and declination, same name as latitude and opposite name, 0(1°)24°.

T. 30–33 give respectively the change in longitude resulting from a 1′ change in latitude, the change in latitude corresponding to a 1^s change in time, and the changes in latitude and in time for a 1′ change in altitude. **T. 34** provides the values to 0′.1 of the difference of latitude and departure corresponding to a distance 0(1)100 nautical miles for course angles 1°(1°)90° [courses 1°(1°)360°]. **T. 35** gives the difference in longitude to .01 corresponding to a departure 1(1)9, 100 and a mid-latitude 0(1°)30°(0°.5)60°(0°.2)70°(0°.1)81°. **T. 36** is one of meridional parts based on BESSEL's formula; values are given to .1 for latitudes 0(1′)89°59′. **T. 37, 37a–37b** are for navigation along an arc of a great circle; they provide the latitude L of a point on the great circle path corresponding to a longitude λ as a solution of the equation:

$$\tan L = \sin (\lambda - \lambda_0) \tan C_0,$$

where λ_0 is the longitude of the nearer point of the path on the equator and C_0 is the course along the great circle at that point. **Table 38** gives the distances to .01 of an object from two points by two bearings measured with respect to the ship's course at these two points, and the distance to .01 from the point where the second bearing was taken to the point of closest approach to the object, each distance being given in terms of the distance run between the first and second bearings.

T. 39a is a double-entry table giving the speed of sound in sea water to the nearest integer in meters per second corresponding to a salinity of 0(5)40 per cent and a temperature of 0(5)30 degrees Centigrade. **T. 39b** is another double-entry table giving the correction to be applied to the depth 5, 10(10)500 meters found by an echo-sounding device when the speed of sound varies from the standard for the device by 5(5)100 meters per second.

Turning to the cards which are contained in a pocket on the inside of the back cover, Nomogram (N.) **I** gives the correction of an altitude to the meridian; it may be used instead of table 18. **N. II** is designed to permit the plotting of a position line from an azimuth observation; the example given to illustrate the use of the nomogram appears to be in error. If one uses $h = 19°$ instead of 10° as given, one obtains the answer given. The numerals on this particular nomogram are almost illegible, even in a good light. **N. III** provides azimuth angle from altitude and conversely, when local hour angle and declination are known. **N. IVa** yields the distance at which an object of known height above sea level can be seen by a person whose height of eye above the ocean is known. **T. IVa** and **IVb** give the distance to the visible horizon for heights of eye in feet, 0 to 23000, and in meters, 0 to 5100.

T. V gives the distance in miles to .01 with arguments, minutes of time elapsed 1(1)10 and speed in knots 1(1)60. **T. Va** gives minutes of time to .01 with arguments miles 1(1)10 and speed in knots 1(1)60. This latter table appears to be superfluous and impractical. All of the information in it likely to be of value is contained in **T. V**. One finds difficulty in imagining a circumstance where one will need to know the time to a hundredth of a minute required to travel an integral number of miles at an integral number of knots. However the criticism of superfluity can be levelled at a number of other tables in this volume.

T. VI–VIIb and **N. VIIa** are for computations made to allow for the ship's log and currents. **T. VIII** and **N. VIIIa** provide corrections to the ship's course. There are five tables, **IXa, IXb,** etc., yielding the distances of objects from observed vertical angles. **T. X** is provided for computation involved in manoeuvering.

The volume is well bound in fabrikoid and the paper is rather better than was formerly found in Russian publications.

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