67371	13200	70932	87091	27443	74704	72306	96977	20931	01416
92836	81902	55151	08657	46377	21112	52389	78442	50569	53696
77078	54499	69967	94686	44549	05987	93163	68892	30098	79312
77361	78215	42499	92295	76351	48220	82698	95193	66803	31825
28869	39849	64651	05820	93923	98294	88793	32036	25094	43117
30123	81970	68416	14039	70198	37679	32068	32823	76464	80429
53118	02328	78250	98194	55815	30175	67173	61332	06981	12509
96181	88159	30416	90351	59888	85193	45807	27386	67385	89422
87922	84998	92086	80582	57492	79610	48419	84443	63463	24496
84875	60233	62482	70419	78623	20900	21609	90235	30436	99418
49146	31409	34317	38143	64054	62531	52096	18369	08887	07016
76839	64243	78140	59271	45635	49061	30310	72085	10383	75051
01157	47704	17189	86106	87396	96552	12671	54688	95703	50354
02123	40784	98193	34321	06817	01210	05627	88023	51930	33224
74501	58539	04730	41995	77770	93503	66041	69973	29725	08868
76966	40355	57071	62268	44716	25607	98826	51787	13419	51246
65201	03059	21236	67719	43252	78675	39855	89448	96970	96409
75459	18569	56380	23637	01621	12047	74272	28364	89613	42251
64450	78182	44235	29486	36372	14174	02388	93441	24796	35743
70263	75529	44483	37998	01612	54922	78509	25778	25620	92622
64832	62779	33386	56648	16277	25164	01910	59004	91644	99828
93150	56604	72580	27786	31864	15519	56532	44258	69829	46959
30801	91529	87211	72556	34754	63964	47910	14590	40905	86298
49679	12874	06870	50489	58586	71747	98546	67757	57320	56812
88459	20541	33405	39220	00113	78630	09455	60688	16674	00169
84205	58040	33637	95376	45203	04024	32256	61352	78369	51177
88386	38744	39662	53224	98506	54995	88623	42818	99707	73327
61717	83928	03494	65014	34558	89707	19425	86398	77275	47109
62953	74152	11151	36835	06275	26023	26484	72870	39207	64310
05958	41166	12054	52970	30236	47254	92966	69381	15137	32275
36450	98889	03136	02057	24817	65851	18063	03644	28123	14965
50704	75102	54465	01172	72115	55194	86685	08003	68532	28183
15219	60037	35625	27944	95158	28418	82947	87610	85263	98139
55990	06738								

Values of the auxiliary numbers arccot 5 and arccot 239 to 2035D are in the possession of the author and also have been deposited in the library of Brown University and the UMT FILE [1] of *MTAC*.

GEORGE W. REITWIESNER

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

[1] See *MTAC*, v. 4, p. 29.

RECENT MATHEMATICAL TABLES

691[A].—M. LOTKIN, "Table of the first 200 factorials to 20 places," Ballistic Research Laboratories, Aberdeen Proving Ground, *Technical Note* no. 106, 1949, 11 p. mimeograph, 21.7 × 27.8 cm.

The table gives the first 20 significant figures of $n!$ for $n = 1(1)\,200$ together with the exponent of the power of 10 by which the figure should be multiplied to give the approximate value of $n!$ The author was unaware of a previous table by UHLER[1] giving the exact values of these factorials.

Professor Uhler reports that a comparison shows that the present tables are quite without error. For references to other tables of large factorials see *MTAC* v. 1, p. 125, 163, 312, 452, v. 3, p. 205, 340, 355.

[1] H. S. UHLER, *Exact values of the first 200 factorials*, New Haven, 1944. [*MTAC*, v. 1, p. 312].

692[A].—H. S. UHLER, "The Arabian Nights' factorial and the weighted mean factorial," *Scripta Math.*, v. 15, 1949, p. 94–96.

This note gives the values of $450!\cdot 10^{-111}$ and $448!\cdot 10^{-109}$. The number 450! has 1001 digits, hence the title. The author gives the frequency of each digit 0–9 in the 890 digits of $450!\cdot 10^{-111}$ from which we deduce that the probability of obtaining such a distribution from a wholly random sequence of digits is a little less than 1/5. For other large factorials computed by the author and others see the preceding review.

693[C, E, K].—K. M. MATHER, "The analysis of extinction time data in bioassay," *Biometrics*, v. 5, 1949, p. 127–143.

This paper contains two tables. Table I (p. 136–137) gives $\ln \ln (1/x)$ for $x = 0(.001).999$ to 3D. Table II (p. 138–139) gives 4D values of $x + e^{-x}$, $-e^{-x} \exp (e^x)$, $e^{2x}/[\exp (e^x) - 1]$ for $x = -5(.1)-2(.05)+1(.1)1.9$. The values of $-e^{-x} \exp (e^x)$ are nearly all incorrect and appear to have been computed in a very casual manner. For example, $x = 0$, the value $-e$ is given as -2.7181. Other errors are by no means confined to the last decimal. For example for $x = 1.9$, the author has -114.9425 instead of -119.8085 and for $x = -4.6$ the author has -100.0000 in lieu of -99.4843. This table should not be trusted beyond 3 significant figures.

D. H. L.

694[C].—C. S. SMITH, "The intercommunication of atomic and weight percentages," p. 196–199 of *Metals Handbook, 1948 edition*, Cleveland, Ohio, 20.7 × 27.8 cm.

On p. 198 are two 4D tables of $10 + \log [x/(100 - x)]$ covering the range $x = 0(.01)5(.1)94.9$.

R. C. A.

695[F].—F. V. ATKINSON & LORD CHERWELL, "The mean-value of arithmetic functions," *Quart. Jn. Math.*, v. 20, 1949, p. 65–79.

On p. 76 there is a table of the number of k-th power free numbers $<250\,000$ of the form $n^k + h$ for $k = 3, 4, 5$ and $h = 1, 2, 3$ together with the corresponding values obtained from an approximate formula.

696[F].—J. LEHNER, "Further congruence properties of the Fourier coefficients of the modular invariant $j(\tau)$," *Amer. Jn. Math.*, v. 71, 1949, p. 373–386.

The function j may be defined by

$$j = x^{-1}\{1 + 240 \sum_{m=1}^{\infty} m^3 x^m (1 - x^m)^{-1}\}^3 \prod_{n=1}^{\infty} (1 - x^n)^{-24}$$
$$= x^{-1} + 744 + 196884x + 21493760x^2 + \cdots = \sum_{n=-1}^{\infty} c(n)x^n.$$

Although this fundamental function was first investigated by FELIX KLEIN half a century ago, it is only in recent years that some attention has been paid to the properties of the coefficients $c(n)$. A small table of $c(n)$ for $n = 0(1)24$ has been given by ZUCKERMAN.[1] From this table the author has derived a table (p. 384) showing the highest power of p dividing $c(n)$ for $p = 2, 3, 5, 7, 11$ and $n = 1(1)24$. The additional value

$$c(25) = 12\ 18832\ 84330\ 42251\ 04333\ 51500,$$

given by LEHMER,[2] produces

$$25 \mid 2 \ ,3 \ ,3 \ ,0 \ ,0$$

as the 25th line of the table, as noted by the author (p. 386).

D. H. L.

[1] H. S. ZUCKERMAN, "The computation of the smaller coefficients of $J(\tau)$," Amer. Math. Soc., *Bulletin*, v. 45, 1939, p. 917–919.
[2] D. H. LEHMER, "Properties of the coefficients of the modular invariant $J(\tau)$," *Amer. Jn. Math.*, v. 64, 1942, p. 488–502, (p. 491).

697[F].—A. V. PRASAD, "A non-homogeneous inequality for integers in a special cubic field, I." K. Acad. van Wetenschappen, Amsterdam, *Proc.*, v. 52, 1949, p. 240–250. *Indagationes Math.*, v. 11, 1949, p. 55–65.

On p. 247(62) there is a 6D table of n-th powers of $\theta = 1.32471795$, where θ is the real root of $\theta^3 - \theta - 1 = 0$ for $n = -7(1) - 2(\frac{1}{2})4, 5$.

698[F].—H. C. ROBERT, "Pythagorean triangles and their inscribed circles," *Duodecimal Bulletin*, v. 5, 1949, p. 41–46. 13.8 × 21.5 cm.

A table (p. 44–46) is given of right triangles with integral sides arranged according to the radius R of the inscribed circle for $R = 1(1)17$. In addition to the sides of the triangle, the perimeter and the pythagorean generators are given. Of the 74 triangles listed, 31 are primitive. The table is given in duodecimal notation.

699[F].—P. VARNAVIDES, "On the quadratic form $x^2 - 7y^2$," R. Soc. London, *Proc.*, v. 197A, 1949, p. 256–268.

On p. 259 there is a table of 10 integers in the field $k(\sqrt{7})$ which are particularly small in absolute value.

700[F].—PIET WIJDENES, *Beginselen van de Getallenleer*. Second ed., Groningen Noordhoff N.V., 1949, 260 p. 15.5 × 24.2 cm. Paper cover 8.25 florins; bound 10.50. The first edition, 236 p. appeared in 1937.

On p. 218–224 is a factor table of numbers less than 20000 not divisible by 2, 3, 5, 7, 11.

701[F].—D. YARDEN, "Table of the distribution of zeros in the period mod p of a recurring sequence of order 3," (Hebrew) *Riveon Lemat.*, v. 2, 1948, p. 65–66.

Four recurring series are involved in this note [*MTAC*, v. 3, p. 519]

$$U_n = U_{n-2} + U_{n-3} \qquad V_n = V_{n-2} + V_{n-3}$$
$$\bar{U}_n = -\bar{U}_{n-2} + \bar{U}_{n-3} \qquad \bar{V}_n = -\bar{V}_{n-2} + \bar{V}_{n-3}$$

with initial conditions

$$(U_0, U_1, U_2) = (\bar{U}_0, \bar{U}_1, \bar{U}_2) = (0, 0, 1)$$
$$(V_0, V_1, V_2) = (\bar{V}_0, \bar{V}_1, -\bar{V}_2) = (3, 0, 2).$$

The tables give for each series the values of $n(\text{mod } P)$ for which the n-th term of the series is divisible by p together with the number of such $n < P$. The primes p considered are those $\leqslant 31$.

702[G].—RAGY H. MAKAR, "The irreducible representation of the symmetric group of degrees 3, 4, and 5," Math. and Phys. Soc. of Egypt, *Proc.*, v. 3, 1948, p. 13–21.

The matrix representations of the elements of the symmetric groups of degrees 3, 4, and 5 are set forth in an abbreviated tabular form.

703[G].—JOHN RIORDAN. "Inversion formulas in normal variable mapping," *Annals Math. Stat.*, v. 20, 1949, p. 417–424.

If $G_1(g)$, $G_2(g)$, $\cdots$ are assigned polynomials, and if

$$x = g + \sum_{n=1}^{\infty} G_n(g)y^n/n!,$$

defines x in terms of g and a parameter y, then

$$g = x + \sum_{n=1}^{\infty} X_n(x)y^n/n!,$$

where

$$-X_n = Y_n(aG_1(x), aG_2(x), \cdots, aG_n(x)),$$

Y_n being the multivariate polynomial of the reviewer[1] in the variables $G_1(x)$ to $G_n(x)$ and the symbolic variable a which is such that

$$a^i \equiv a_i = (-d/dx)^{i-1},$$

with differentiations on all products $G_1(x)$ to $G_n(x)$ associated with it in the polynomial. This is the author's *first inversion formula*. Table 1 gives the explicit forms of Y_n $(fg_1, fg_2, \cdots, fg_n)$ for $n = 1(1)8$.

E. T. BELL

California Institute of Technology
Pasadena, California

[1] E. T. BELL, "Exponential polynomials," *Annals of Math.*, v. 35, 1934, p. 258–277.

704[I].—R. E. GREENWOOD & M. B. DANFORD, "Numerical integration with a weight function x," *Jn. Math. Phys.*, v. 28, 1949, p. 99–106.

The author considers two quadrature formulas

$$(1) \quad \int_0^1 xf(x)dx = \frac{1}{2n} \sum_{i=1}^{n} f(x_{i,n}) + R_n^{(1)}(f)$$

$$(2) \quad \int_{-1}^1 xf(x) = k_n \sum_{i=1}^{n} [f(y_{i,n}) - f(y_{i+n,n})] + R_n^{(2)}(f)$$

suggested by CHEBYSHEV[1] and of interest because the equal coefficients on the right minimize the probable error in the "observed" values of f.

As in the case of the unweighted formula

$$\int_{-1}^{1} f(x)dx = \frac{2}{n}\sum_{i=1}^{n} f(z_{i,n}) \tag{3}$$

of Chebyshev,[1] the optimal quantities $x_{i,n}$, $y_{i,n}$ are algebraic numbers which have, as n increases, the unpleasant tendency of leaving the interval of integration thus rendering the proposed quadrature useless [*MTAC*, v. 3, p. 97]. This phenomenon occurs in the case of (1) and (2) for $n \geqslant 4$, whereas in the case of (3) it occurs for $n = 8, 10, 11, \cdots$.

The present paper gives $x_{i,n}$ to 8D for $n = 1, 2, 3$, and k_n, $y_{i,n}$ for $n \leqslant 4$. For $n = 2, 3$ there are two possible values of k_n and two sets of y's; for $n = 4$, there are four values of k_n and four sets of y's. All results are to 8D, except for $n = 4$, when only 7D are given. The formula (2) for $n = 3$ is illustrated in two cases for $f(x) = e^x$ and compared with (3) for $n = 6$ and the NEWTON-COTES formula with 7 ordinates. The results speak well for (2).

D. H. L.

[1] P. L. CHEBYSHEV, "Sur les Quadratures," *Jn. de Math.*, s. 2, v. 19, 1874, p. 19–34. *Oeuvres*, St. Petersberg, v. 2, 1907, p. 165–180. See also R. RADAN, "Sur les formules de Quadrature a coefficients egaux," Inst. de France, Acad. Sci., *Comptes Rendus*, v. 90, 1890, p. 500–503, which contains data on (2) for $n = 2, 3$; a comparison with results of the present paper shows a number of minor errata.

705[I, M].—H. E. SALZER, "Coefficients for repeated integration with central differences," *Jn. Math. Phys.*, v. 28, 1949, p. 54–61.

In a previous paper[1] [*MTAC*, v. 3, p. 107] the author has given a table of coefficients for the repeated integration with forward and backward differences. In the present note the coefficients are based on central differences and were obtained by repeated integration of EVERETT's interpolation formula. The table extends from the case of 2-fold integration to 6-fold integration. In the important case of 2-fold integration the first 25 pairs of coefficients are given, the first 11 exactly and the others to 16D. For k-fold integration $k = 3(1)6$ the coefficients, which are all small, are given to 8 or 9S.

[1] H. E. SALZER, "Table of coefficients for repeated integration with differences," *Phil. Mag.*, s. 7, v. 38, 1947, p. 331–338.

706[K].—S. CHANDRASEKHAR, "On a class of probability distributions," Cambridge Phil. Soc., *Proc.*, v. 45, 1949, p. 219–224.

The function β_p under tabulation is the p-th moment of $W(\beta)$, where

$$W(\beta) = 2\beta\pi^{-1}\int_0^\infty e^{-u}y \sin \beta y dy, \quad u = y^{3/n}.$$

The function β_p is given explicitly by

$$\beta_p = 2\pi^{-1}(p+1)\Gamma(p)\Gamma(1 - np/3) \sin \tfrac{1}{2}\pi p$$

and is tabulated to 5S for

$n = 1.6$, $p = .25(.25)1.75, 1.80, 1.85$
$n = 2$, $p = .25(.25)1.25(.05)1.45, 1.475$
$n = 3$, $p = .2(.2).8, .9, .95, .975$
$n = 4$, $p = .1(.1).5, .55, .575$
$n = 6$, $p = .1(.1).4, .45, .475$
$n = 8$, $p = .1(.1).3, .325, .35, .36$
$n = 10$, $p = .1, .2, .25, .275, .280$

On p. 222, there is a 5S table of

$$[\tfrac{2}{3}\pi n(n+3)^{-1}\Gamma(3/n)\sin(\tfrac{3}{2}\pi n^{-1})]^{n/2}$$

for $n = 1.51, 1.52(.02)1.6(.1)2(.5)4(2)10(5)25$.

707[K].—H. J. GODWIN, "Some low moments of order statistics," *Annals of Math. Stat.*, v. 20, 1949, p. 279–285.

Let $\chi(i, n)$ be the i-th largest in a sample of n from the normal population with density function $(2\pi)^{-\frac{1}{2}}e^{-t^2/2}$. HASTINGS *et al.*[1] gave (a) the expectations and standard deviations of $\chi(i, n)$ to 5D, and (b) the covariances to 2D, for $n = 1(1)10$ and all i. JONES[2] obtained some of these values explicitly for $n = 4$. In the present paper more exact numerical integration is employed to improve the accuracy of tables in footnote 1, giving (a) to 7D and (b) to 5D. Correlations are given in a 4D table. The author also extends the results of Jones,[2] providing 26 new explicit values, for $4 \leqq n \leqq 6$.

J. L. HODGES, JR.

Univ. of California
Berkeley, California

[1] CECIL HASTINGS, FREDERICK MOSTELLER, JOHN W. TUKEY, & CHARLES P. WINSOR, "Low moments for small samples: a comparative study of order statistics," *Annals of Math. Stat.*, v. 18, 1947, p. 413–426.
[2] HOWARD L. JONES, "Exact lower moments of order statistics in small samples from a normal distribution," *Annals of Math. Stat.*, v. 19, 1948, p. 270–273.

708[K].—FRANK E. GRUBBS, "On designing single sampling inspection plans," *Annals of Math. Stat.*, v. 20, 1949, p. 242–256.

Let $P(c, n, p) = \sum_{k=0}^{c} \binom{n}{k} p^k(1-p)^{n-k}$, and define p_1 and p_2 by $P(c, n, p_1) = 0.95$ and $P(c, n, p_2) = 0.1$. Interpolating to about 3S in published tables[1] [*MTAC*, v. 1, p. 76–79] of the beta and F distributions, the author tables p_1 and p_2 for $c = 0(1)9$, $n = 1(1)150$. A corresponding single entry table based on the POISSON approximation is also given. The tables are intended to aid in selecting sample size and acceptance number in sampling inspection plans having 5 per cent producer's risk and 10 per cent consumer's risk.

J. L. HODGES, JR.

[1] CATHERINE M. THOMPSON, "Tables of percentage points of the incomplete beta-function," *Biometrika*, v. 32, 1941, p. 168–181.
M. MERRINGTON & C. M. THOMPSON, "Tables of the percentage points of the inverted beta (F) distribution," *Biometrika*, v. 33, 1943, p. 73–88.

709[K, L].—E. J. Gumbel, "Asymptotic distribution of range from that of reduced range," *Annals of Math. Stat.*, v. 18, 1947, p. 384–412.

The author considers the functions

$$\psi(x) = e^{-x} \int_{-\infty}^{\infty} \exp(-e^{t} - e^{-t-x})dt$$

and

$$\Psi(x) = \int_{-\infty}^{\infty} \exp(t - e^{t} - e^{-t-x})dt.$$

Table 1, p. 193–196, gives $\Psi(x)$, $\Delta\Psi$, $\psi(x)$ to 5D for $x = -3(.5)10.5$.

Table 1A, p. 397, gives the inverse function $\Psi^{-1}(x)$ to 2D for $\Psi(x) = .0002(.0001).001(.001).01(.01).1(.1).9(.01).99(.001).999(.0001).9997$.

710[L].—André Angot, *Compléments de Mathématiques à l'Usage des Ingénieurs de l'Électrotechnique et des Télécommunications.* Préface de Louis de Broglie. Paris, Éditions de la Revue d'Optique, 1949, viii, 660 p. 16.3 × 25.3 cm. Price 2500 francs, unbound.

This volume, by a professor at the École Supérieure d'Électricité and a "Lieutenant-Colonel des Transmissions," contains tables and rather rough graphs which may be noted even if the tables contain nothing new. In no case has the originator of any table been definitely indicated.

P. 333–334: Graphs of $\sinh x$, $\cosh x$, $\tanh x$ and 5 or 6S tables of e^{x}, e^{-x}, $\cosh x$, $\sinh x$ for $x = 0(.2)6$.

P. 336–339: $\mathrm{si}(x) = -\int_x^\infty t^{-1} \sin t\, dt$, $\mathrm{Si}(x) = \frac{1}{2}\pi + \mathrm{si}(x)$. There are tables of $\mathrm{Si}(x)$, $\mathrm{Ci}(x)$ for $x = 0(.01)1(.1)6(1)15(5)100(10)200(100)1000$, 10^4, 10^5, 10^6, 10^7, $4\ \infty$ D; 4–7D, mostly 4D. There are also 5–6D tables of the maxima and minima of $\mathrm{Ci}(x)$ for $x/\pi = .5(.5)15.5$ and of $\mathrm{Si}(x)$ for $x/\pi = 1(1)15$, as well as graphs of $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$.

P. 342: Graphs and 5D table of $\theta(x) = 2\pi^{-\frac{1}{2}}\int_0^x e^{-t^2}dt$ with Δ for $x = .05(.05)2$.

P. 349–350: Graph and table of $\Gamma(1+x)$, for $x = [0(.01)2; 4D]$, $x = [2(.01)3.99; 4\text{–}5S]$. Apparently reprinted from Jahnke & Emde.

P. 375–376: Graphs of $I_n(x)$, $n = 0(1)11$; $K_n(x)$, $n = 0, 1$; those of $I_n(x)$ apparently copied from Jahnke & Emde.

P. 380: Graphs of $\mathrm{ber}(x)$, $\mathrm{bei}(x)$, $M_0(x)$, $\theta_0(x)$.

P. 403–407: Tables of $J_0(x)$, $J_1(x)$, $Y_0(x)$, $Y_1(x)$ for $x = [0(.1)16; 4D]$.

P. 408–409: Tables to 4D of $J_n(x)$, $n = 2(1)9$, $x = 0(1)24$; $n = 10(1)17$, $x = 4(1)29$.

P. 410: First to ninth roots (4D) of $J_n'(x) = 0$, $n = 0(1)22$. Also first to eighth root (4–5D) of $J_n(x) = 0$, $n = 0(1)19$.

P. 411–412: Tables of $J_{n/2}(x)$, $\pm n = 1(2)13$, $x = [0(1)24; 4D]$.

P. 413–415: Tables of $\mathrm{ber}x$, $\mathrm{ber}'x$, $\mathrm{bei}x$, $\mathrm{bei}'x$, $\mathrm{ker}x$, $\mathrm{ker}'x$, $\mathrm{kei}x$, $\mathrm{kei}'x$ for $x = 0(.1)10$ mostly 4S.

P. 416–417. Tables of $M_0(x)$, $\theta_0(x)$, $M_1(x)$, $\theta_1(x)$, $x = 0(.05)1.7(.1)3(.2)5(.5)6(1)12(2)20(5)45$.

P. 442–444: Tables of Legendre polynomials $P_n(x)$, $n = 1(1)7$, $x = [0(.01)1; 4D]$, apparently reprinted from Jahnke & Emde.

P. 445–446. Graphs of the associated Legendre functions of the first kind. Apparently taken from Jahnke & Emde, figs. 60–63.

P. 497–505: Tables of LAPLACE transforms; graphs of discontinuous functions, p. 502–505.

R. C. A.

711[L].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 12: *Tables of the Bessel Functions of the First Kind of Orders fifty-two through sixty-three*. Cambridge, Mass., Harvard University Press, 1949, x, 544 p., 20 × 26.6 cm. Offset print. Price $8.00.

Sixteen previously published volumes of the *Annals* have been reviewed in *MTAC*, v. 2, p. 176–177, 185–187, 261–262, 344, 368, v. 3, p. 41, 102, 185–186, 311–314, 367, 432–440, 474–475, 517–518. These volumes on publication listed at $10.00 each, are now listed at $8.00 each, which is more reasonable for offset-printed volumes.

The volume under review is the tenth in the Harvard series of tables of Bessel functions of the first kind, which, after three more volumes have been published, will contain tables, to 10D at least, for $J_n(x)$, for $n = 0(1)100$, and $x = 0(.01)100$; for $n = 0(1)3$, $x = [0(.001)25(.01)99.99;\ 18D]$, and $n = 4(1)15$, $x = [0(.001)25(.01)99.99;\ 10D]$, but beginning with order 16 the argument interval of the tables is constantly .01. The values of $J_n(100)$, $n = 0(1)100$ are to be given in the final volume 15. In the first two volumes only two orders were tabulated, while there are twelve in the current volume in which the first significant values .00000 00001 occur in connection with $J_{52}(27.53)$ and finally $J_{63}(36.34)$.

The tables in this volume are wholly new. The Harvard Computation Laboratory is at present the outstanding center in the world for the computation and publication of mathematical tables. In five years not only have there been 15 volumes of this kind in the *Annals* series, but also other tables which have been reviewed in *MTAC*, v. 2, p. 218, 300, 307.

R. C. A.

712[L].—MARIETTE LAURENT, "Table de la fonction elliptique de Dixon pour l'intervalle 0–0.1030," Acad. r. de. Belgique, *classe des sciences*, *Bull.*, s. 5, v. 35, 1949, p. 439–450, 15.8 × 25.1 cm.

On p. 441–445 is a table of values of $sm\,u$ for $u = [0(.001).103;\ 10D]$, Δ^2, where $x = sm\,u$ and $u = \int_0^x(1 - t^3)^{-2/3}dt$, and on p. 446–450 is a table of u with argument $sm\,u = [0(.001).103;\ 10D]$, Δ^3. The author states that 11 decimals were used in the calculations, the 9-th decimal corresponding to the precision of a centimeter in geodesic applications, and that there may be unit errors in the tenth decimal place.

For previous tables see *MTAC*, v. 3, p. 249, and A. C. DIXON, *Quart. Jn. Math.*, v. 24, 1890, p. 167–233. The connection between the Dixon function $sm\,u$ and the equianharmonic WEIERSTRASS function is given by $sm\,u = [2\sqrt{3}\wp(u/\sqrt{3})]/[\sqrt{3} - \wp'(u/\sqrt{3})]$.

R. C. A.

713[L].—A. D. MacDonald, "Properties of the confluent hypergeometric function," *Jn. Math. Phys.*, v. 28, 1949, p. 183–191, 17.4 × 25.3 cm.

On p. 184–190 are 6S tables for $z = .5(.5)8$, and $\alpha = .001, .01, .05, .1, .2, .25, .3(.1)1$ of

$$M(\alpha; \gamma; z) = \sum_{n=0}^{\infty} \frac{\Gamma(\gamma)\Gamma(\alpha + n)z^n}{\Gamma(\alpha)\Gamma(\gamma+n)n!}$$

for $\gamma = \frac{1}{2}(\frac{1}{2})2$, and of the logarithmic solution for $\gamma = 1, 2, 3$.

R. C. A.

714[L].—Z. Mursi, "On the relation of Airy and allied integrals to the Bessel functions," Math. and Phys. Soc. of Egypt, *Proc.*, v. 3, 1948, p. 23–38.

If $f(x)$ stands for one of the functions $Ai(x)$, $Bi(x)$ or $aAi(x) + bBi(x)$, then the derivatives of $f(x)$ can be expressed as

$$f^{(2n)}(x) = P_n f(x) + Q_n f'(x)$$

$$f^{(2n+1)}(x) = R_n f(x) + S_n f'(x),$$

where P_n, Q_n, R_n, and S_n are polynomials in x. These polynomials are tabulated on p. 37–38 for $n = 1(1)15$.

R. C. A.

715[L].—Fritz Oberhettinger & Wilhelm Magnus, *Anwendung der elliptischen Functionen in Physik and Technik.* (*Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen*, v. 55.) Berlin-Göttingen-Heidelberg, Springer, 1949, viii, 126 p. 16 × 24.1 cm. Price 15.6 German marks in paper cover; bound 18.3.

Tables on p. 43–126 are as follows:

A. Complete normal elliptic integrals.

(i) 5D values of $k^2 = \sin^2\alpha$, K and E for $\alpha = 0(1°)70°(30')80°(12')$-$89°(6')90°$.

(ii) 5D values of K, K', K'/K, K/K', $\log q$, $\log q'$ for $k^2 = 0(.01).5$.

(iii) 5D values of K, K', K'/K, K/K' for $k^2 = .000001(.000001).00001$, $.0001(.0001).003$.

B. Tables of normal elliptic integrals of the first kind; values of $F(k, \phi)$, $k = \sin\alpha$ for $\alpha = 5°(5°)90°$ and $\phi = [1°(1°)90°; 5D]$ values for $\phi \leqslant 5°$, 4D values for $\phi > 5°$.

C. Tables of normal elliptic integrals of the second kind, 4D values of $E(k, \phi)$, $k = \sin\alpha$, for $\alpha = 5°(5°)90°$ and $\phi = 1°(1°)90°$.

R. C. A.

716[L].—W. Peremans & J. Kemperman, "Nummeringspribleem van S. Dockx, Mathematisch Centrum. Amsterdam," *Rapport* ZW; 1949–005,4 leaves, 19.8 × 34 cm.

On leaf 4 is a table of $a_k = \frac{2}{3}k(k + 1)(2k + 1) = \frac{4}{3}B_3(k + 1)$, where B_3 is the third Bernoulli polynomial, for $k = 1(1)100$.

R. C. A.

717[L].—R. A. Rankin, "The theory of the motion of rotated and unrotated rockets," R. Soc. London, *Phil. Trans.*, v. 241, 1949, p. 457–585, 23.4 × 29.9 cm.

This work contains two tables of "Fresnel functions." The main table gives

$$A(x) = \pi^{-1}2^{-\frac{1}{2}} \int_0^\infty t^{-\frac{1}{2}}(1 + t^2)^{-1} \exp(-\tfrac{1}{2}\pi x^2 t)dt$$
$$= \{\tfrac{1}{2} - S(x)\} \cos \tfrac{1}{2}\pi x^2 - \{\tfrac{1}{2} - C(x)\} \sin \tfrac{1}{2}\pi x^2$$

and

$$B(x) = \pi^{-1}2^{-\frac{1}{2}} \int_0^\infty t^{\frac{1}{2}}(1 + t^2)^{-1} \exp(-\tfrac{1}{2}\pi x^2 t)dt$$
$$= \{\tfrac{1}{2} - S(x)\} \sin \tfrac{1}{2}\pi x^2 + \{\tfrac{1}{2} - C(x)\} \cos \tfrac{1}{2}\pi x^2$$

and

$$Z(x) = \pi \int_0^x A(u)du,$$

where $S(x)$ and $C(x)$ are the Fresnel integrals

$$C(x) = \int_0^x \cos \tfrac{1}{2}\pi u^2 du, \quad S(x) = \int_0^x \sin \tfrac{1}{2}\pi u^2 du.$$

These are tabulated to 4D for $x = 0(.01)1(.05)1.5(.1)7.(.2)10(.5)15$ with first differences, and in the case of $Z(x)$, second differences. For $x \geq 1$ the table gives also

$$A_1(x) = (\pi x)^{-1} - A(x)$$
$$Z_1(x) = Z(x) - \ln x.$$

The second table gives 4D values of the integrals

$$A^*(x) = \int_x^\infty A(u)du/u$$
$$B^*(x) = \int_x^\infty B(u)du/u$$

for $x = 0(.1)5$ with first and second differences. For $x < 1$, $A^*(x) + \ln x$ and $B^*(x) + \ln x$ are also given.

D. H. L.

718[L].—Norbert Wiener, *Extrapolation, Interpolation, and Smoothing of Stationary Time Series with Engineering Applications.* Published jointly by Mass. Inst. Technology, Cambridge, Mass., and John Wiley & Sons, New York, 1949, x, 163 p. 14.6 × 22.6 cm. Price $4.00.

Appendix A of the work consists of a table of what the author calls Laguerre Functions. These are more explicitly

$$F_n(x) = (-1)^n 2^{\frac{1}{2}} e^{-x} L_n(2x)/n!$$

where $L_n(t)$ is the usual Laguerre polynomial given by

$$L_n(t) = e^t d^n(t^n e^{-t})/dt^n = n!M(-n, 1, t),$$

M being the confluent hypergeometric function. Thus

$$L_4(t) = t^4 - 16t^3 + 72t^2 - 96t + 24.$$

The range of x and n is

$$x = 0(.01).1(.1)18(.2)20(.5)21(1)26(2)30; \quad n = 0(1)5.$$

The table is given mostly to 5S, but in some cases only to 3S. A spot check reveals a number of last digit errors and the following gross error

$$x = 4.7, n = 4, \quad \textit{for } 88260 \quad \textit{read } 89267.$$

No statement is made as to the method of construction of the table. Apparently no really good tables of $L_n(t)$ have been published. See *MTAC*, v. 2, p. 267, FMR *Index*, p. 337.

D. H. L.

719[Q].—V. KRAT & S. PETROV, "Tablit͡sy vspomogatelnich funkt͡siĭ ψ i χ dli͡a opredeleni͡a elementov sistem zatmennykh peremenykh" (Tables of auxiliary functions ψ and χ for determination of the elements of systems of eclipsing variables) II, Central Astronomical Observatory, Pulkovo, *Izvestia*, v. 17, No. 5, 1947, p. 117.

This paper consists, in essence, of three tables. Tables 1 and 2 contain numerical values of RUSSELL's well-known $\psi(k, \alpha = n)$ function which is needed for the computation of the ratio k of the radii of components of an eclipsing binary system from an analysis of light curves due to total eclipses of a star which is completely darkened at the limb. A definition of this auxiliary function in terms of the basic p-functions was first given by Russell (*Astrophys. Jn.*, v. 35, 1912, p. 315); it is repeated in the introduction to the tables under review.

Of these, Table 1 contains 3D values of $\psi(k, n)$ appropriate for the partial phase of a total eclipse, while Table 2 gives values of the same function appropriate for the annular phase of a transit. The arguments of tabulation are $k = .1(.1)1$, $n = 0(.1)1$ for Table 1, and $k = 2(.1)1$, $n = 0(.1)1$ for Table 2. The intervals of tabulation in both arguments are too large to make the tables easy of interpolation. Both tables are not original, but are revised versions of earlier tables of the same functions published by RUSSELL & SHAPLEY (*Astrophys. Jn.*, v. 36, 1912, p. 239; Table IIx on p. 245 corresponds to Krat and Petrov's Table 1; while Table IIy on p. 391 of the same volume of the *Astrophys. Jn.* corresponds to Krat and Petrov's Table 2). A comparison of the corresponding entries of the new and old tables reveals discrepancies attaining the second significant place, and due no doubt to the inferior accuracy of the old tables which were based on inaccurate p-functions. The new Russian tables are based on the extensive and accurate 5D tables of $p(k, n)$ which were published in 1939 by T͡SESEVICH [*MTAC*, v. 3, p. 191–194].

Table 3—the main feature of the paper under review—contains a set of 4D tables of Krat's auxiliary functions $\psi(k, n\alpha_0)$ and $\chi(k, \alpha_0)$ appropriate for partial eclipses of stars exhibiting uniformly bright disks. The arguments of tabulation are $k = .1(.1)1$, $\alpha_0 = .1(.1)1$, and $n = 0(.1).9$. The reader is cautioned to notice that Krat's functions ψ and χ are *not* identical with

Russell's well-known functions denoted currently by the same symbols; for the definition of Krat's functions tabulated in the paper under review cf. *Russian Astronomical Jn.*, v. 11, 1934, p. 412 (Russian, with English summary).

ZDENĚK KOPAL

Massachusetts Institute of Technology
Cambridge, Massachusetts

720[R].—KARL REICHENEDER, "Fehlertheorie und Ausgleichung von Rautenketten in der Nadirtriangulation," Deutsche Akademie der Wissenschaften zu Berlin, *Veroffentlichungen des Geodätischen Institutes in Potsdam*, No. 1, 1949, viii, 98 p., 20.7 × 29.6 cm.

Contains tables of the probable errors in photogrammetric control extension by nadir point triangulation. The tables, p. 82–96, are based on a triangulation net of fifteen rhombuses (nadir numbers, N_0 to N_{15}), in which are located two fixed points. Fifteen error-tables are given, in which N_0 is the first fixed point, and the second fixed point is placed successively at $N_1, N_2, \cdots$ to N_{15}.

On p. 28 is a table giving the coefficients in the expansion of Lucas $U_n = (a^n - b^n)/(a - b)$ and $V_n = a^n + b^n$ as polynomials in $a + b$ and ab for $n = 0(1)16$ together with numerical values of U_n and V_n in the cases $ab = 1, a + b = 3, 4$. These are used to compute tables of weighting coefficients p. 34–46.

C. J. VAN TIL

University of California
Berkeley, California

721[V].—E. N. FOX, "The diffraction of two dimensional sound pulses incident on an infinite uniform slit in a perfectly reflecting screen," R. Soc. London, *Phil. Trans.*, v. 242, 1949, p. 1–32.

On p. 19 there are two tables of the function $G_0(Y, \tau) = \frac{2}{\pi} \tan^{-1} (\tau/Y)^{\frac{1}{2}}$ for $\tau = 0(.2)3.8$, $Y = 2.2(.2)3.8$ and for $\tau = 0(.2)1.8$, $Y = 2.2(.2)3.8$.

On p. 20, there are tables of

$$f_1(Y, \tau) = G_0(Y, \tau + 1) - G_0(Y, 1) - \frac{1}{\pi}\int_0^{\tau/2} \frac{Y^{\frac{1}{2}} G_0(x, \tau - 2x)}{(1 + x)^{\frac{1}{2}}(1 + x + Y)}\,dx$$

for $\tau = 0(.2)2.8$, $Y = .1(.1)1.0$ and for $\tau = 0(.2)1.8$, $Y = 1.2(.2)3.0$ and a table of

$$f_2(Y, \tau) = \frac{1}{\pi}\int_0^{\tau/2} \frac{Y^{\frac{1}{2}} f_1(x, \tau - 2x)}{(1 + x)^{\frac{1}{2}}(1 + x + Y)}\,dx$$

for $\tau = 0(.2)1.8$, $Y = .2(.2)2.0$. All tables are to 4D.

722[V].—VIRGINIA GRIFFING & FRANCIS E. FOX, "Theory of ultrasonic intensity gain due to concave reflectors," Acoustical Soc. of Amer., *Jn.*, v. 21, 1949, p. 348–359.

On p. 350 there is a table of $[Si(k) - (1 - \cos k)/k]/\pi$ for $k = .1(.1).4(.2)2(.5)5(1)16, 20(10)50$, and for $k = n\pi$ with $n = 1(1)5$ to 4S.