22. ROBERT M. WALKER, "An analogue computer for the solution of linear simultaneous equations," I.R.E., *Proc.*, v. 37, 1949, p. 1467–1473, figs.

Linear simultaneous equations occur frequently in science and in engineering. Their solution by numerical methods is straightforward, but the amount of work required increases rapidly with the number of unknowns. A device is described for the solution of systems of linear simultaneous equations with not more than twelve unknowns. It is an electrical analogue computer which accepts the problem information in digital form from a set of punched cards. This facilitates the preparation, checking, and insertion of the input data and greatly reduces some of the usual liabilities of an analogue device. No special preparation of the problem is required, other than a simple one of scaling the coefficients. Solutions of well-determined problems are easily and rapidly attained and may be refined to any desired accuracy by a simple iteration procedure.

Author's summary

NOTES

112. A COMMITTEE ON FACTOR TABLES.—In September, 1946, the Association Française pour l'avancement des Sciences established a committee consisting of A. GÉRARDIN (France), who to our regret could not participate in the work because of ill health, L. POLETTI (Italy) and the author, for the purpose of extending the factor table. The committee was joined later by Dr. A. GLODEN (Luxemburg). We agreed that only a table practically free from error may have any significant value. Hence, the necessity to check the existing manuscript tables against one another. These tables are: KULIK's famous manuscript,[1] Poletti's table of the 11-th million, GOLUBEV's table of the 11-th and 12-th millions, R. J. PORTER's two tables[2] of the 11-th million. The Carnegie Institution of Washington presented us with two microfilms of the 11-th and 12-th millions of Kulik's manuscript. Poletti's table is in our hands. A request of a photostat of Golubev's table in the Steklov Institute at Moscow finally was denied. Porter's tables are unfortunately in symbols quite different from Kulik's. The committee decided to extend the existing printed table to the 11-th and probably the 12-th million. The necessity of checking Kulik's table against Poletti's made it necessary to get large photos of the relevant part of the microfilm. The Lord Mayor of Luxemburg presented us with large photos of the second half of the 11-th million of the microfilm and the author ordered large photos of the other half. Since Kulik used letters instead of numbers, the author is to transcribe the photos in the interval from 10^7 up to $10^7 + 5 \cdot 10^5$, Dr. Gloden in the interval $10^7 + 5 \cdot 10^5$ up to $10^7 + 10^6$. Every page of the new manuscript will be checked against Poletti's table. The checking of the manuscript against a third table, which seems necessary is still a problem to the committee.

N. G. W. H. BEEGER

Nicolaas Witsenkade 10
Amsterdam Z, Holland

[1] See *MTAC*, v. 2, p. 139–140, v. 3, p. 222.
[2] *MTAC*, v. 1, p. 451.

113. SUPERSONIC FLOW CALCULATIONS.—The following is an account of work done on the ENIAC on supersonic flow past cone cylinders.

Numerical solutions for equations for the flow of a compressible gas at supersonic speeds past a cone cylinder with attached shock wave at various combinations of Mach number and cone semiangle have been computed on the ENIAC [*MTAC*, v. 3, p. 206–207]. The general method of obtaining these solutions will be described first. Using characteristic variables, the equations of irrotational motion for the supersonic flow past a body of revolution may be written as follows:

$$Hy_\alpha - (K + R)x_\alpha = 0$$
$$Hy_\beta - (K - R)x_\beta = 0$$
$$Hu_\alpha + (K - R)v_\alpha + (a^2v/y)x_\alpha = 0$$
$$(K - R)u_\beta + Lv_\beta + (a^2v/y)y_\beta = 0,$$

where x, y are cylindrical coordinates with the x axis along the axis of the projectile, u, v are reduced velocity components in the direction of the x, y axes, where $u = \bar{u}/c$, $v = \bar{v}/c$, and c is the limiting velocity as the velocity of sound approaches zero, $\bar{u}$, $\bar{v}$ are the original velocity components, $a^2 = \bar{a}^2/c^2$ is the similarly reduced velocity of sound:

$$H = a^2 - u^2, \qquad K = -uv$$
$$L = a^2 - v^2, \qquad R = a(q^2 - a^2)^{\frac{1}{2}}.$$

The surface of the body is assumed to be a stream surface, and with the RANKINE-HUGONIOT shock wave conditions give us the following boundary conditions, in which the subscripts 1 refer to conditions ahead of the shock and the subscripts 2 refer to conditions immediately behind the shock:

$$y = F(x) \text{ is the contour of the given body, } v = uF'(x),$$
$$v_2^2[(\gamma - 1)/(\gamma + 1)q_1 + 2q_1/(\gamma + 1) - u_2]$$
$$= (q_1 - u_2)^2[u_2 - (\gamma - 1)/(\gamma + 1)q_1]$$

$dy/dx = (q_1 - u_2)/v_2$, where dy/dx is the slope of the tangent.

$\gamma = 1.4$ is the ratio of the specific heats.

The values of x, y, u, v were computed at the intersections of all characteristics $\alpha = $ const., $\beta = $ const. in the following region. The characteristic $\alpha = 0$ was chosen as the downstream characteristic emanating from the shoulder in TAYLOR-MACCOLL flow about the cone. Along this characteristic the values of x, y, u, v were computed at k intervals $\Delta\beta$ terminating at the shock wave.

The characteristic $\beta = k$ emanates from the intersection of $\alpha = 0$ and the shock wave, and the region under discussion is bounded by $\alpha = 0$, $\beta = k$, and the generating curve of the cylinder. In this region the exact flow is irrotational since the shock wave is straight.

The shoulder of the cone-cylinder is mapped onto the characteristic $\beta = 0$, and PRANDTL-MEYER flow is computed along this characteristic, using k points. The generating line of the cylinder is mapped onto the line $\beta = \alpha - k$ in the α, β plane. In the computation of the Taylor-Maccoll flow the partial differential equations were reduced to a set of four simultaneous ordinary differential equations, with

$$\Delta\beta = k(v/u)/[(v/u)\text{cone} - (v/u)\text{shock}].$$

Likewise the equations were reduced to ordinary differential equations for the Prandtl-Meyer flow, with $\Delta\alpha = k(v/u)/(v/u)$cone.

The first computations carried out were for $M = 2.1297$ and a cone cylinder of 20° semi-angle, with $k = 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40$. Taylor-Maccoll flow was computed with $k = 960$ and Prandtl-Meyer flow was computed with $k = 16, 32, 64, 128, 256$. The partial derivatives were replaced by difference quotients of first, second and third order approximations, and the results were extrapolated to grid size zero ($k \doteq \infty$). By using the extrapolation formula $z = z_2 + (z_2 - z_1)/3$, where $z = x, y, u, v$, and the subscript 2 indicates that the k for that computation is *twice* the k for the other, it was found that results could be obtained to a much higher accuracy in a short time.

Accordingly, in the next calculation $k = 8$ and 4, and the values of x, y, u, v were calculated at all intersections of the characteristics as well as the values of $p/p_1, \rho/\rho_1, T/T_1$ at all surface points of the body, where p, ρ, T stand local pressure, density, and temperature, and the subscript 1 indicates free stream values.

The various combinations of cone-angle θ and free stream Mach number M for which computations were made are as follows:

$\theta = 5°$ $M = 1.3, 1.5, 1.7, 2, 3, 4, 5, 7$
$\theta = 9.5°$ $M = 1.3, 1.5, 1.7, 2, 3, 3.8$
$\theta = 10°$ $M = 1.3, 1.5, 1.7, 1.72, 2, 3, 4, 5, 7$
$\theta = 12°$ $M = 1.7$
$\theta = 15°$ $M = 1.3, 1.5, 1.7, 2, 2.13, 2.3, 2.7, 3, 4, 5, 7$
$\theta = 20°$ $M = 1.5, 1.7, 2, 2.13, 3, 4, 5, 7$
$\theta = 25°$ $M = 1.5, 1.7, 2, 2.13, 3, 4, 5, 7$
$\theta = 30°$ $M = 1.7, 2, 2.13, 2.3, 3, 4, 5, 7$
$\theta = 35°$ $M = 2, 2.13, 3, 4, 5, 7$
$\theta = 40°$ $M = 3, 4, 5, 7$
$\theta = 45°$ $M = 3, 4, 5, 7$
$\theta = 50°$ $M = 4, 5, 7$

The results are in some places good to 4 significant figures and it is believed that they are everywhere good to three significant figures.

R. F. CLIPPINGER

Ballistic Research Laboratories
Aberdeen Proving Ground, Md.

114. NBSCL TABLES.—Eleven of these tables have appeared in second editions. We have already referred in *MTAC* to two of these: *Tables of the Bessel Functions $J_0(z)$ and $J_1(z)$ for Complex Arguments*, 1943 (v. 3, p. 25) in 1947; and *Tables of the Exponential Function e^x*, 1939 (v. 3, p .173), in 1947. There were also a second printing of *Table of Circular and Hyperbolic Tangents and Cotangents for Radian Arguments*, 1939 (v. 3, p. 88), in 1947; a second edition of *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*, 1939 (v. 1, p. 45), in 1949; and a second edition of *Tables of Probability Functions*, v. 2, 1942 (v. 1, p. 48), in 1948.

With the approval of the editor of the *Journal of Mathematics and Physics*, the following seven short tables in the MT series were reissued by the Government Printing Office to meet a continuing demand:

MT 19, "On the function $H(m, a, x) = \exp(-ix)F(m + 1 - ia, 2m + 2; ix)$," 1942 (v. 1, p. 156) on 15 June 1949;

MT 20, "Table of integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$," 1943 (v. 1, p. 154), 10 Sept. 1948;

MT 21, "Table of $Ji_0(x) = \int_x^\infty J_0(t)dt/t$, and related functions," 1943 (v. 1, p. 155), on 25 Feb. 1949;

MT 22, "Table of coefficients in numerical integration formulae," 1943 (v. 1, p. 157), on 15 Dec. 1949;

MT 23, "Table of Fourier coefficients," 1943 (v. 1, p. 192), on 25 Nov. 1949;

MT 25, "Seven-point Lagrangian integration formulae," 1943, on 1 June 1949;

MT 27, "Table of coefficients for inverse interpolation with central differences," 1943 (v. 1, p. 126, 359), on 1 June 1949.

R. C. A.

115. ON LARGE PRIMES AND FACTORIZATIONS.—The following results of extensive calculations have been reported.

I. The number $N = 3 \cdot 2^{129} + 1$ is composite.
It was found that

$$2^{N-1} \equiv 7949\ 31660\ 65193\ 00342\ 32702\ 86136\ 93493\ 91372 \pmod{N}.$$

If N were a prime, we would have $2^{N-1} \equiv 1 \pmod{N}$.

The number N was suggested as a likely candidate for primality by Mr. THOROLD GOSSET of Cambridge, England. Thus another attempt to discover a larger prime than $2^{127} - 1$ ends in disappointment.

H. S. UHLER

206 Spring St.
Meriden, Conn.

II. The number $6^{26} + 1$ is completely factored.

$$6^{26} + 1 = 37 \cdot 313 \cdot 2341 \cdot 629\ 19466\ 95217.$$

The largest factor was proved to be a prime by a method based on the converse of FERMAT's theorem, as described in *MTAC*, v. 3, p. 496–497; v. 4, p. 54–55.

N. G. W. H. BEEGER

Nicolaas Witsenkade 10
Amsterdam Z, Holland

III. (a) The number $N = (2^{89} + 1)/3 \cdot 179$ is composite
(b) $3^{36} + 1 = 2 \cdot 41 \cdot 6481 \cdot 28\ 24290\ 05041$
(c) $3^{34} + 1 = 2 \cdot 5 \cdot 956353 \cdot 1743831169$
(d) $2^{61} + 15$ is a prime.

The result (a) follows from the fact that if $y = 2^{89}$,

$$3^{536} - 3^y \equiv 3303\ 41699\ 82572\ 42322\ 50798 \pmod{N}.$$

If N were a prime this remainder would have been zero. The complete factorization of $2^{89} + 1$ would be of interest since it would probably lead to some new multiply perfect numbers.

The results (b) and (c) fill in two blank entries in CUNNINGHAM & WOODALL, p. 11. The former result was found by the method of *MTAC*, v. 3, p. 96–97. The latter result was found by expressing $(3^{34} + 1)/10$ as the difference of two squares.

The result (d) was another example of the converse of Fermat's theorem. It is easy to see that the 15 integers between the two primes

$$p_1 = 2^{61} - 1 \qquad \text{and} \qquad p_2 = 2^{61} + 15$$

are composite, being multiples of small primes $\leqslant 13$. Hence, p_1 and p_2 are consecutive primes, and constitute the largest pair of consecutive primes known.

A. Ferrier

Collège de Cusset
Allier, France

[1] A. J. C. Cunningham & H. J. Woodall, *Factorization of $y^n \pm 1$*, London, 1925.

QUERY

34. Archimides Cattle Problem.—Has any attempt been made to use one of the modern electronic computing devices to get a solution of the famous cattle problem[1] of Archimedes?

E. P. Adams

105 Plimpton St.
Walpole, Mass.

[1] See L. E. Dickson, *History of the Theory of Numbers*, Washington, 1920 and New York, 1934, v. 2, p. 342–345.

QUERIES—REPLIES

44. Tables of $\sin nx/\sin x$ (Q16, v. 2, p. 61).—A table of this function "for large integral values of n, say up to 100, and for values of x in radians" would be impossibly large if it were interpolable in x. Millions of entries would be necessary to carry the table as far as $x = 10$ with $8D$. Since isolated values of this function can be obtained with a little trouble from a good table of $\sin x$ it is not surprising that there is no such table in print. For fixed x, however, there are a number of small tables of what is essentially $\sin nx/\sin x$. Two examples may be cited, though they do not correspond to real values of x. For $x = \arccos(-i/2)$ we have

$$\sin nx/\sin x = i^{1-n} F_n,$$

where F_n is the n-th term of the Fibonacci series, which is tabulated as far as $n = 128$ [*MTAC*, v. 2, p. 343]. For $x = \arccos(-3i/\sqrt{2})$ we have

$$\sin nx/\sin x = (i\sqrt{2})^{1-n}(2^n - 1)$$

the values of which can be found easily from a table of powers of 2.

As is well known, $U_n = \sin nx/\sin x$ is a special kind of Lucas[1] function and can be computed recurrently by the formula

$$U_{n+1} = 2 \cos x\, U_n - U_{n-1}.$$

D. H. L.

[1] É. Lucas, *Théorie des Nombres*, Paris 1890, p. 319.

CORRIGENDA

V. 3, p. 362, l. −1 *for* RMT **593** *read* RMT **592**.
V. 3, p. 554, l. 3, *for* 4–17, *read* 3–16.
V. 3, p. 559, l. 4, *for* 454 *read* 554.
V. 3, p. 562, l. 30, *for* Nicholas De Cusa *read* Nicholas de Cusa.
V. 3, p. 563, l. 6, *for* fifteenth *read* thirteenth.
Nos. 28, 29, cover 3, interchange lines J and I.