The quantities I_n for $a = 1$ are given in Table 1 while arguments and weights computed for $n = 2(1)7$ are listed in Table 2.

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[1] G. SZEGÖ, *Orthogonal Polynomials*, Amer. Math. Soc., *Colloquium Publs.*, v. 23, 1939, chap. III & XV.
[2] E. J. NYSTRÖM, "Über die praktische Auflösung von Integralgleichungen mit Anwendungen auf Randwertaufgaben," *Acta Math.*, v. 54, 1930, p. 185–204. Integral equations defined within an infinite interval have been discussed from the numerical point of view by A. REIZ, "On the numerical solution of certain types of integral equations," *Arkiv Mat. Astr. Fysik*, v. 29A, no. 29, 1943, 21 p.
[3] G. C. WICK, "Über ebene Diffusionsprobleme," *Z. Physik*, v. 121, 1943, p. 702–718.
[4] S. CHANDRASEKHAR, "On the radiative equilibrium of a stellar atmosphere, II," *Astrophys. Jn.*, v. 100, 1944, p. 76–86 and following volumes; see also the same author's *Radiative Transfer*, Oxford Univ. Press, 1950.
[5] E. B. CHRISTOFFEL, "Sur une classe particulière de fonctions entières et de fractions continues," *Annali di Mat.* S.2, v. 8, 1877, p. 1–10.
[6] E. W. HOBSON, *The Theory of Spherical and Ellipsoidal Harmonics*, Cambridge Univ. Press, 1931, particularly chap. II: § 34.

New Information Concerning Isaac Wolfram's Life and Calculations

1. Introductory.—We have already noted certain items[1–4] regarding Wolfram, an eighteenth century Dutch artillery officer, one of his mathematical tables, and his contacts with LAMBERT.[3] Hardly anything else is to be found in any mathematical history, periodical, or bibliography—to practically all of which we refer. No one has previously remarked that in two volumes of correspondence of Lambert, edited by one of the Bernoullis (1784, 1785–87), more than 200 pages of material,[9] including many of Wolfram's letters, tell us much concerning him and his mathematical activities for over 35 years. In what follows my main objects shall be to give some idea of the nature of the new material, and also to include some interesting recently discovered additional facts, supplied by J. H. B. KEMPERMAN, research worker at the Mathematisch Centrum, Amsterdam. Bernoulli refers to the "great calculator Wolfram"[9] (v. 5, p. 464), who was notified that the Prussian Academy of Sciences would be glad to receive as a legacy the complete collection of his logarithmic calculations for preservation in its archives.

2. Mr. Kemperman's Report[29] (24 January 1950).—"According to information received from the Royal Military Academy at Breda and the General Public Record Office at The Hague, the following data are certain—

(i) His full name is ISAAC WOLFRAM (according to an army list from 1781). He is always indicated by the name J. Wolfram.

(ii) Before 1747 Wolfram was not yet an officer.

(iii) On August 3, 1747 he became "onderlieutenant" (artillery).

(iv) On November 27, 1764 he was promoted to be "ordinairlieutenant."

(v) On September 1, 1779 he became "captain-lieutenant."

(vi) Before 25 August 1788 Wolfram was no longer in the army, for on that date his substitute A. VAN HOEY VAN OOSTEE was sent out (according to the consignment book of the Council of State). In the army-list of 1786 Wolfram's name still appears, but in that of 1789 it is no longer found.

(vii) In 1778 he was stationed at Nijmegen.

(viii) About 1780 he was stationed among other places at Axel, Steen-bergen, Hellevoetsluis, and Brielle.

From the artillery regiment in which J. Wolfram served, there are no confidential reports left, nor other documents from which the date of his birth or his birthplace might appear. The archivist of the town of Nijmegen could not find any data concerning Wolfram."

3. Comments.—In July and September 1774 Wolfram was in Danzig and in each of these months wrote letters from there to Lambert[9] (XXVII, XXX). In the first letter he refers to Danzig as his "Vaterstadt," p. 509. Writing to Lambert from Nimwegen (so the name is always spelled in the German correspondence; the English form is Nimeguen) in 1776 (XXXII) he tells us that his factor table, up to 126000 (giving the least divisors greater than 5, of the numbers), was made at Danzig in 1743 and enlarged to 300000 in Holland.

The postscript to a letter of 6 September 1774 to Lambert[9] (XXX) (in translation of the German original) is as follows: "Should the Royal Prussian ambassador at The Hague, Herr von Thulemayer, mention my name to the Prince of Orange, it might well happen that I should not have to wait 17 years and some months, as I did in 1764, when I became "Ordinairlieuten-ant"; that I was in the campaigns of 1746 and 1747, and did my duty in the battles of Roucoux and Laffeld [places in Belgium], has doubtless been long since forgotten." As we have seen, it was not until five years after this that Wolfram received his promotion.

Wolfram was an officer in 1747. On the assumption that he was then 22 years old (recall that 4 years earlier he had completed a factor table) he was born about 1725 and did not leave Danzig for Holland before 1743. The accuracy of Mr. Kemperman's statement "no longer in the army," in August 1788, is readily checked, since we know that Wolfram was dead by 1787. Writing in this year[9] (v. 5, p. 344) Bernoulli tells us that "only in recent months have I learned with certainty that this industrious and skill-ful calculator, this really extraordinary man in his field, is no longer living. But I am without particulars of time, place, etc., where his life ended, or what may have been the fate of his many painstakingly handwritten calcu-lations, as one must very much desire to learn, when one recalls from his letters to Lambert all his labors and intended plans."

4. First Publication, et al.—Wolfram's first mathematical publication seems to have been a portion of his 1743 factor table[17,20] (3), namely: "Proeve van eene tafel ter ontledinge der getallen" [specimen of a table for the factorization of numbers], which appeared in Hollandse Maatschappy de Wetenschappen, Haarlem, Verhandelingen, v. 2, 1755, p. 622–624. This table gives the least divisors greater than 5 for numbers less than 6000. After three pages of text the three-section table is on a folding plate 16.7 × 44 cm. in size. Although the rest of the table up to 126000 was promised by the editor for publication in the next volume (1757), the promise was never kept. In a letter to Lambert[9] in 1772 (XX), p. 453, Wolfram refers to the matter and quotes the reason for nonpublication of the rest of his table, as given in a letter dated 6 March 1761, from VAN DER AA, the secretary of the Society; in translation from the Dutch this passage is as follows: "because one can easily find the divisors of a number, when they exist, by means of artifices of VAN SCHOOTEN, KRAFFT, EULER, etc." Wolfram then wrote that

this judgment did not in the least deter him from extending his table, "Vollständige Zergliederung der ersten 300000 Zahlen in einem kurzen Begriffe: oder eine Tafel der kleinste Factor, wenn er grösser als 5, zu finden ist." By 1756 he had already laid plans for extending his table yet further, to a million.

That the scientific editing of the above-mentioned Haarlem publication of 1755 was not of a very high order was also illustrated by two tables of W. O. Reitz in a long-drawn-out article "De berekening van Kunsttallen," p. 166–224 + 4 pages. The tables are of log N, $N = [101(1)1000;18D]$, Δ, and of log $(1 + a \cdot 10^{-n})$, $a = 1(1)9$, $n = [3(1)18;18D]$, with Δ up to $n = 9$. In a letter to Lambert[9] in 1773 (XXII), p. 483, Wolfram states that van der Aa told him in 1759 that the Society had awarded a prize for this paper. Wolfram then points out that the first table has only four more decimal places than that of Briggs, and that the logs of 16 of the prime numbers and 9 of the composite numbers, are erroneous. I checked the accuracy of this last statement. Long before this table was published Abraham Sharp gave (1717) log N, for $N = [1(1)100$ and primes to $1100; 61D]$, every logarithm apparently correct to at least 57D.

5. Lambert and his first contact with Wolfram.—During the years 1756–1758 Lambert spent more than a year at the Dutch Universities of Utrecht and Leyden. But, as we have already noted,[3] from 1764 until his death 25 September, 1777, he was almost wholly at Berlin, where he received many favors from Frederick the Great, and was in 1765 elected a fellow of the Royal Academy of Sciences and Belles-Lettres. In this year he published the first part of his *Beyträge*,[6] but the two sections of the second part (which has more immediate interest for our inquiry) came out in 1770. Shortly afterwards, in the same year, his valuable volume of tables, *Zusätze*,[7] appeared. Among the *Beyträge* items is a complete factor and prime table up to 10200 (p. 42–53), and the evaluation of π^{-1} by continued fractions (p. 156–158). In the *Zusätze* there is a table similar in results to Wolfram's, up to 102000, with a separate table of primes; an appeal was made for more extended computations.

On 5 March 1772 Wolfram wrote his first letter to Lambert[9] (XVIII). He reported that on the previous 28 February he had acquired a copy of the *Zusätze* and noted there the request for more extensive factor tables. Hence he described: (a) his own factor table up to 300000 on 25 folio pages; and then also mentioned (b) his 39D common logarithmic table, based on 42D computation, of all primes less than 10000; and (c) his recomputation of J. P. Buchner, *Tabula Radicum, Quadratorum & Cuborum ad Radic. 12000 extensa*, Nuremberg, 1701, since its tables of squares and cubes contained so many errors. W. offered to make this material available to L. for publication if he so desired and if L's. exact address were furnished. W. added that he would then send also a list of 70 errors in L's. factor table which he noted during the few days he had the *Zusätze*.

W. then goes on to report that, in the logarithms of A. Sharp in Sherwin's *Mathematical Tables*, 1726 [second edition], except for the first 16 digits of log 131, and the first 20 digits of log 163, all other digits of these logarithms are incorrect. He also states that in other logarithms of this table one or four or seven digits are incorrect. This is the table on p. 28–29 of the logarithms of 1(1)100 and of all prime numbers less than 200, 51–61D. In

his *Geometry Improv'd*, London, 1717, p. 56–60, Sharp gave a table of the logarithms of [1(1)100, primes less than 1100; 61D]. In this table, for Sherwin's 1726 range, only two errors have been found, namely, unit errors in the 61st decimal places for log 127, and log 149. If Sherwin was authorized to use Sharp's name and computations, it is not a little curious that the 1726 Sherwin, published nine years after the Sharp volume, is so inaccurate and incomplete.[32] The third carefully revised and corrected edition was in 1741, (also 1742) and the fourth in 1761.

W's. great interest in extended logarithmic computation led him to compute logarithms of primes from 63D to 88D,[9] v. 5, p. 460. There are several later references to the Sharp table in Sherwin's tables[9]: v. 4, p. 451, 482, 485–486, 493–496, 518, 522; v. 5, p. 462–463. W. had gained access to the third (1742) edition, containing Sharp's complete table as given in 1717, and completed the checking of this table in October 1780. In a letter dated 2 March 1781, W. wrote that he found Sharp's table "correct except for a few digits in the 61st. decimal place." As a matter of fact, however, there were, apparently, more serious errors in the logarithms of 227, 839, 1009, and also of 643 (1761 Sherwin) where a printer's slip has 42, instead of 24, in the 18th and 19th decimal places. I do not have the third edition for checking.[32]

W. reported also that he had found 8 erroneous values in the 100 logarithms of the "*Clausbergischen Rechenkunst.*" This is C. von Clausberg, *Demonstrative Rechenkunst*, Leipzig, 1732, which I have not seen; there were also later editions by C. A. Hausen, second, 1748; third, 1767; fourth 1771 (see XXIV, p. 495); also p. 453, 489.

And in concluding his first letter to L., W. refers to his discovery in 1754 of a numerical series for π by means of which he computes π correct to 9D. The series is simply Newton's expansion of arcsin x, when $x = \frac{1}{2}$.

6. Lambert's Reply to Wolfram's First Letter and Later Correspondence.—Lambert replied to W. promptly and cordially. He referred to having received, in response to his appeal, several other contributions of tabular material, including one table by "Ober-Finanz Buchhalter Oberreit" of Dresden, which contained the complete factorization of the first half-million numbers; the author planned to complete the table for the first million. In September 1770 L. published a note about this table in order to prevent useless work by others. L. states that in his table collection, planned for some future date, he may possibly use W's. tables of logarithms and of squares and cubes. He also mentions that for a time one of his Berlin friends had been calculating a table of hyperbolic logarithms to many places of decimals, but had put the work aside. W. had already in 1772 commenced the preparation of his great table of hyperbolic logarithms. L. naturally stated that he would be glad to receive lists of errata in his work; he noted various corrigenda reported to him by others. L. suggested that W. should make a table of $u = \ln \tan (45° + \frac{1}{2}w)$, for $w = 45°(1')\ 90°$, instead of at interval 1° as in Table 32 of the *Zusätze*.

W's. reply[9] (XX) occupies 24 printed pages. He thanked L. for his suggestion as to a computation; sent L. not only the 70 errors in *Zusätze*, referred to earlier (printed in *Beyträge*,[6] v. 3), but also at least 28 more given in this letter; sent also 29 errors in *Beyträge* factor table (5). Among these corrections were a number in the *Zusätze*, Table 2 of primes, in the value for $(\frac{1}{6}\pi)^{\frac{1}{2}}$, and for convergents of the continued fraction for π^{-1}. Here W. carried

the work further even to the extent of WALLIS [*MTAC*, v. 2, p. 340–341; v. 3, p. 172]. W. explained also, with numerical examples, his method (discovered in 1756) for finding the cube root of large numbers, and in particular for small ones, 1(1)125, values, which were necessary on artillery bore sticks.[34] W. found also in terms of square roots the length of sides of regular n-sided polygons, $n = 4(4)24(8)48,15,20(10)40(20)80,120$, and therefrom finds sides for n up to $n = 512$, in order to get approximations to π. In this connection W. refers to T. SIMPSON, *Éléments de Géométrie*, Paris, 1755. Elsewhere in XX he quotes SARGANECK, *Geometrie in Tabellen*,[30], C. MACLAURIN, *Traité des Fluxions*, Paris, 1749, KÄSTNER's, *Anfangsgründe*,[31] and works of C. v. Clausenberg, L. Euler, G. W. Krafft, A. F. MARCI, J. PELL, W. O. Reitz, F. v. Schooten, A. Sharp. Throughout the correspondence there are many similar references.

And so the correspondence between W. and L. continued up to a period less than seven weeks before L's. death in 1777. In 1772–73 W. wrote from Namur; in 1774 from Danzig, his birthplace, where he spent the months June–September while on leave of absence from his regiment; in 1775–77 from Nimwegen. The letters are written in a very pleasing style and are clearly expressed, numerical examples being frequent. In the two v. (4–5) of the *Briefwechsel*[9] there are references to 20 dated W. letters; we shall later refer to more of the contents of the interesting final letter written from "Helvoetsluys" in 1781 from which we have already quoted (5). The material of v. 5, mainly supplementary to that in v. 4, gives details of computation, often very extended and displaying originality and power, and a number of new tables. He touched on a considerable variety of topics. I shall now refer to only two.

He was interested in the number of primes less than a million[9] (v. 4, p. 497–498, 533; v. 5, p. 380–382, 459) and gave the total as 78461 instead of the correct number 78499. He listed also the totality of numbers under a million having the smallest prime factor p for each p up to 997.

The second topic which interested W. a good deal was the periodicity of rational fractions expressed as decimals[9] (v. 4, p. 523–524, 529, 533–534; v. 5, p. 449–459, 463). In his letter of 1781 W. tells us that already in 1776 he happened on the proof by means of periods of decimal numbers, that the Quadrature of the Circle cannot be expressed by either rational or irrational numbers.[35] He states that he reported his thought in this regard to L. who replied on 8 March 1777, according to W., "Es wird wohl auch— lassen" (apparently an unintelligible extract from an unprinted part of letter[9] XXXVII). L. discovered his proof of the irrationality of π in 1766.

Continuing in his 1781 letter W. reported that finally in 1779, in an investigation of the periods of decimal numbers and their applications to the infinite series of the hyperbola and circle, he had shown that these series cannot be expressed as either rational or irrational numbers,[35] and hence the same results hold for hyperbolic logarithms and the ellipse, which depend upon the hyperbola and circle. W. states that on 28 April 1779 he sent a description of this investigation to Schulze, but received no reply. When Bernoulli later wrote to S. regarding this paper, S. acknowledged its receipt, and stated that use would be made of it when the opportunity presented itself.

Discussion of the periodicity of rational numbers expressed as decimals

seems to have originated with Wallis, as L. mentions. In this connection we have referred to tables of H. Goodwyn [*MTAC*, v. 1, p. 22–23, 67, 100, 372].

7. Wolfram Tables in the Correspondence.—Of the many tables, other than those mentioned above, we shall here refer to only three.

I. In XXV W. sent to L. a table printed on a folding sheet, p. 499–500, indicating 5000 possible "Raketensätze" (rocket mixtures of saltpeter, sulphur, and carbon) for use in artillery firing. As a result of comments by L. in XXV, W. made an entirely new table[9] (v. 5, p. 369–371) for 9402 mixtures, accompanied by discussion of air conditions, angles of fire, calibres of guns, and meal-powder additions, in connection with which latter items a new table is added on p. 377 of v. 5.

II. Levelling tables for France and Rhineland, latitude 50°, the radius of the earth according to La Lande being 3 271 200 toises or 1 692 000 Rhineland Ruthen,[9] v. 5, p. 366–367. These tables are to accompany L's. letter XXIII where he tells W. (p. 491) that the tables were "very well arranged and somewhat similar to tables which he himself introduced while preparing in 1770 a new edition of Picart's paper on hydrostatic balances.

III. In XXXIII W. told L. that he was sending him "something on calculating sines and cosines." This material is in form sort of a table, with part of the calculations,[9] v. 5, p. 443–448. The values of sines and cosines, 18D to 28D, are given for the same 18 angles, namely: 6″, 18″, 54″, 1′, 3′, 5′24″, 6′45″, 9′, 10′, 30′, 54′, 1°7′30″, 1°30′, 1°40′, 5°, 9°, 11°15′, 15°. W. remarks that by means of these results a good number of other values of sines and cosines may be calculated.

In Davisson's correspondence with L. there are enthusiastic references[9] to W., XIII, XIV, v. 4, p. 417–419, 422; and on p. 426 L. writes to D.: "In calculation he possesses an uncommon facility and always observes thereby an admirably planned form." D. remarked, p. 416, that W. had checked his values of sine and cosine for 1″ to 34D.

8. Wolfram's Calculations and the Prussian Academy.—In a brief letter from W. to L. (XXVII), dated Danzig, 29 July 1774, the following sentences occur: "I am now again sending to you a continuation of 320 logarithms, and a third specimen of the same calculation. In such a form have I also since 1750 calculated the common logarithms and I have written out the calculations themselves completely in three volumes, not counting one volume which contains merely preparatory calculations therefor. In the same way I began also two years ago the calculation of hyperbolic logarithms because hitherto I have discovered no other way of calculating *correct* logarithms and preserving them, than by calculating the same in at least two or three ways, and then writing these calculations down in complete form. If this idea appeals to you now, or as you think it over, and if you will indicate a place where one preserves manuscripts of this kind then I shall state in writing that the above volumes after my death shall be transferred without cost to such a place."

At a meeting of the Royal Prussian Academy on 27 August 1774 L. presented the Wolfram plan for consideration, and it was voted that if Wolfram's papers were received as a legacy, they should be carefully preserved in the Academy's archives[9] (see v. 4, p. 511–513, 515–516, 519–521). W. was delighted when L. reported that the Academy approved this idea,

and proceeded to describe three manuscript volumes of his computations (see also v. 4, p. 507–508). In these he had the common logarithms of all primes $< 10^4$, to 42D (39D accurate), work which had been done 1752–1761. In 1762 he began the calculation of logarithms of primes < 200, to 63D. W. estimated in 1774 that there might eventually be six or seven manuscript volumes. In 1776 W. wrote to L. (XXXII) that he had on 17 June 1775 signed the document setting forth the legacy to the Royal Prussian Academy, and remarked that all manuscript volumes already prepared were carefully marked as part of the legacy, which was to include his factor table of 1743, to 126000 (later enlarged to 300000), and his revision of the Buchner volume (5). W. sent to L. a sample of his factor table,[9] v. 5, p. 438–442, showing that its form was quite different from that of the table in the *Beyträge*.[6] W. stated also that he had notified the "Auditeur militaire," of the Nimwegen garrison, concerning his legacy.

Bernoulli remarks that nothing more is to be found in correspondence concerning this remarkable and praiseworthy legacy. He expresses the hope that the donor will not take it amiss that he had informed the learned world where the fruits of his unremitting industry and rare natural gifts may later be found. From Bernoulli's quoted statement of three years later it seems clear that upon the death of W. there was no disposition of his manuscripts by his executors in accordance with the legacy provisions.

9. **Schulze and his Recueil.**—J. C. Schulze (1749–1790), was born in Berlin and became a pupil of Lambert during 1770–1772. A Lambert statement concerning his exceptional abilities is contained in the interesting Éloge of Schulze by PAUL ERMAN.[10] He was elected a fellow of the Prussian Academy in 1787 and became a favorite of Frederick the Great. But for the moment we are interested only in Schulze the table-maker, and his notable work, *Recueil de Tables Logarithmiques, Trigonométriques et autres nécessaires dans les Mathématiques Pratiques. Neue und erweiterte Sammlung logarithmischer, trigonometrischer und anderer zum Gebrauch der Mathematik unentbehrlicher Tafeln.* 2 v., Berlin, 1778. The 20 pages of introductory material were in both French and German. In this material Wolfram is referred to on two pages [xii, xvi]. In the last reference we learn that Wolfram had pointed out erroneous results in the trigonometric table of natural sines, tangents, and secants, which Schulze had lifted from the *Opus Palatinum* of RHETICUS. The reason for the difficulty will be apparent from what we have already published [*MTAC*, v. 3, p. 556–557].

But the other page of the Introduction refers to the serious illness of Wolfram when the first volume of the *Recueil* was published. This contained (p. 189–258) Wolfram's extraordinary table of ln x to 48D. We know from his correspondence with Lambert that he had seen many pages of earlier proof of this table but just when final checking was most desirable, and before he had finished the computation of the values of ln x for $x = 9769$, 9781, 9787, 9871, 9883, 9907, illness intervened. But Wolfram's computation of the values was published two years later,[8] in 1780. BIERENS DE HAAN thought[17] that possibly this illness had been fatal and that the table was unfinished for this reason.

There is another page of the *Recueil*, v. 1, p. 260, which seems as if it might have been prepared by W. (We know that he prepared p. 188 and most, if not all, of p. 259). On this page are given formulae with coefficients

of the series, for determination of 20D values of hyperbolic logarithms of sines and cosines. All that Schulze stated was (Introd., p. [xiii]) that the formulae were inserted there because of their relation to the preceding table, and because one needed exact logarithms to this extent. There is no reference to these formulae in the W.–L. correspondence.

The way that the W. table came to appear in Schulze's work may be noted. Writing to W. on 30 Nov. 1776 (XXXIV) L. states that having heard of Schulze's work being in the press, he had found, upon consultation with the publisher, that room could be found for a 30D table of hyperbolic logarithms of numbers 1(1)2080, by W. But at this time W. had apparently not only completed his 30D table for all primes and many composites < 10,000 (1772–1776), but also started his 51D table for this same range—which was later published as a 48D table. It is evident that it was Abraham Sharp's 61D table (which W. finally checked completely by 63D computations,[9] v. 5, p. 462–463) in Sherwin's volume, which got W. interested (1772 +) in extended calculations. For certain ln x, W. made more extended calculations. In[9] v. 5, p. 460 we find (when combined with 48D previously given): 57D for $x = 8191$; 63D for $x = 7499$; 64D for $x = 5827, 6263, 7331, 7487, 9283, 9421$; 65D for $x = 5399, 5471, 5563, 7681$; 66D for $x = 9437$; 71D for $x = 9091$; and 72D for $x = 9973$. Then there is a very curious thing; W. gives 64D to 84D for 12 numbers $x = 193, 199, 251, 503, 521, 643, 997, 1999, 2083, 2699, 4001, 4111$, for which we find in his main table only 48D. I know of no table which gives from the 49th through the 63rd decimals for the last five numbers, or the 62nd and 63rd decimals for the remaining seven. DHL wrote to me, "The 12 numbers are just primes whose simple multiples lie close to numbers whose reciprocals have very simple decimals so that the series which W. used (11) was particularly easy to apply."

10. **Pages 188 and 259 of the Recueil.**—With reference to p. 188 there are two passages in the correspondence, v. 4, p. 533–534, and v. 5, p. 464. For changing from common to hyperbolic logarithms and conversely, W. computed tables of n/M, and of nM, $n = 1(1)9$ to 66D; but these were abridged to 48D. on p. 188. Next are 44D values of e(42 correct), and of e^{-1} (43 correct); of course $M = \log e$ to 48D is in the table just noted. Then follows a table giving numbers N for which ln N has successively the values 1(1)25, 30, 60. To the value 1, $N = e$(to 27D); to the value 60, N is a 33 figure-6D number. The page inappropriately concludes with 23D values of ln π and log π, values put there by L. to fill an empty space,[9] v. 4, p. 534.

On p. 259, W. (or Schulze) first gives W's. 12 values to 42D of log x, for $x = 9769, 9781, 9787, 9851, 9859, 9871, 9883, 9887$ (only 41D), 9907, 9923, 9967, 10009, that is, for 12 primes. These values seem to have been available to W. since 1761 (8). It is noticeable that these primes include the six that are missing in the ln x published table of 1778 (made good in 1780).[8] Indeed Schulze has told us, 9, p. [xii], that these logarithms were put there because of the blanks on the opposite page, 258. See the comment of KULIK,[12] col. 48. Professor UHLER of Yale University kindly calculated the values of log 9887 and log 10009 to over 60D, and found perfect agreement with W's. results. It turned out that the 42nd and 43rd digits of log 9887, repeated the 40th and 41st.

Why any but the values of logs of the missing primes were here given is not obvious. One would naturally infer that by means of only such values

one might readily compute the missing lns to about 40D by using a table of p. 188.

Already in a letter to L. (XXXI) p. 517, W. remarked that there are 1229 primes < 10000 [correct if 1 be not counted as prime], and that given the logarithms of such primes, the logarithms of each of 627549 numbers less than a million might then be found, merely by the addition of two numbers. This statement is elaborated on p. 259, with an auxiliary table.

11. Wolfram's Method of Calculating Hyperbolic Logarithms.—W. has told us that he always used two, and sometimes three, distinct methods for calculating each of his lns (see XXVII, p. 508; XXX, p. 514). We have seen (8) that by 1761 W. had calculated to 42D the common logarithms of all primes, N, less than 10^4, and that in 1762 he began the extension of these calculations of 63D for primes less than[33] 200, which he later continued to $N = 1097$ at least, since he checked the whole of Sharp's table less than 1100 (9). It seems highly probable that a similar extension (or an extension to at least 51D) was calculated for all primes less than 10^4 (note results in 10). We know also that W. had 66D modulus tables (10). Thus one method of making a 51D table of lns (afterwards rounded off to 48D) would be by multiplication of the common logarithms by the modulus reciprocals. But we know that calculation from the following two expansions was basic[9] (XXXI, and v. 5, p. 383–385, 428–437):

$$\ln(1 + x) = \sum_{m=1}^{\infty} (-1)^{m+1} m^{-1} x^m$$

$$\ln(1 - x)^{-1} = \sum_{m=1}^{\infty} m^{-1} x^m$$

but the expansion for $\ln [(1 + x)/(1 - x)]$ was never used. In finding $\ln 3343$ he gives details of computation. In $\ln (1 + x)$, x may be taken as $1/234 \cdot 10^2$, or $1/692 \cdot 10^3$, or $1/2075 \cdot 10^4$, or $1/2528 \cdot 10^6$, or $1/159 \cdot 10^8$; while in $\ln(1 - x)^{-1}$, $x = 1/2651 \cdot 10^3$, or $1/4611 \cdot 10^4$, or $1/4806 \cdot 10^5$. The subsequent details, and dozens of auxiliary tables used in logarithmic calculations in general, are interesting. Here, then, are the three independent methods of calculating the lns.

12. Wolfram's Table and its Accuracy.—DE MORGAN characterized this table[14] as "one of the most striking additions to the *fundamenta* of the subject which has been made in modern times." Since I know of no wholly correct published statement as to what is in Wolfram's table, I shall now set this forth in some detail. For the most part there are 50 logarithms on a page, arranged in 10 groups of 5. But on 12 pages (238–248, 258), there are 51 logarithms since each of the tenth groups contains 6 entries. Since the table occupies 69 pages we find, therefore, that the values of 48D hyperbolic logarithms are given for 3462 numbers, the last one being the prime number 10 009. [I am assuming that the 6 missing numbers, given by W. two years later, are in place.] These include all values of $x = 1(1)2201$, the 928 following primes and 533 composite odd numbers greater than 2203. Up to 3439 (beyond 2201) the count of composite and prime numbers is exactly the same; but thereafter, primes are the more numerous. In all, there are 1232 primes and 2230 composites in Wolfram's work.

The values of Wolfram's logarithms are each arranged in 8 hexads. I shall now list every reported error, of which I have knowledge, by rewriting the corrected hexads with italicized corrected digits. The number of any particular hexad is indicated by (n), where n is one of the digits 1(1)8; p denotes that the number is a prime:

1. ln 390   (4) 466724
2. ln 829p (3) 294974
3. ln 1087p (5) 011345; (8) 366597
4. ln 1099   (1) 002155
5. ln 1409p (4) 961900
6. ln 1900   (6) 581952
7. ln 1937   (2) 663406
8. ln 1938   (2) 792450
9. ln 2022   (4) 317343
10. ln 2064   (8) 280145
11. ln 2093   (4) 965593

12. ln 2173   (8) 222311
13. ln 2174   (8) 500957
14. ln 2175   (8) 189283
15. ln 2194   (8) 055117
16. ln 3481   (6) 395248
17. ln 3571p (3) 448444
18. ln 3763   (8) 279622
19. ln 3967p (5) 791389
20. ln 4033   (8) 470671
21. ln 4321   (7) 350597
22. ln 4357p (8) 464180

23. ln 4757   (3) 281480
24. ln 4891   (4) 762180
25. ln 5123   (6) 375359
26. ln 6343p (2) 121633
27. ln 7247p (6) 251021
28. ln 7853p (1) 968650
29. ln 8023   (1) 990067
30. ln 8837p (3) 004423
31. ln 8963p (6) 381531
32. ln 9409   (5) 243407
33. ln 9623p (4) 054318

Thus in the 166176 printed digits in the values of logarithms of the table there are only 38 digits known to be in error. Various considerations, however, suggest that Wolfram's original manuscript may have been almost entirely free from error. While 390 has an incorrect logarithm (no. 1), it is notable that the lns for four of its multiples, and for 195, are correct. So also ln 1658, ln 2174 (except in final digit) and ln 2198 are correct, in spite of the errors in ln 829, ln 1087 and ln 1099 (nos. 2, 3, 4). Hence these facts suggest accuracy of the original manuscript, but bad proofreading perhaps during W's. illness. Three errors (nos. 2, 5, 29) were manifestly caused by the proofreader not recognizing that two inverted 9s should be 6s and an inverted 6 a 9. Similar neglect allowed interchanged numbers to pass in two cases (nos. 3, 11). The four unit-errors in the 48th decimal place (nos. 3, 12–14) are naturally somewhat trivial. Such must suffice by way of justification of our suggestion concerning the most extraordinary accuracy of W's. manuscript. In his letter to Davisson,[9] v. 5, p. 462, W. records "4 Schreib- und Rechnungsfehler"; thus W. himself was willing to confess to a computation error. No one else has, up to the present, noted the error he announced in no. 22.

The first discoverers of 37 of these 38 digit-errors were as follows: BARZELLINI,[8] 1780 (4891); BURCKHARDT,[10a] 1817 (7853); COSENS,[5] 1939 ((8)1087, 2173, 2174, 2175); DUARTE,[24] 1927 (829); DUARTE;[25] 1933 (3967, 8837, 9623); GRAY,[16] 1865 (1409); KULIK,[12] 1824 (390, 1099, 1937, 1938, 2022, 2064, 3481, 3763, 4033, 4321, 4757, 5123, 9409); NYMTP,[26] 1941 (2093, 8023); Peters & Stein, 1922 ((5)1087, 3571) 14; T. M. SIMKISS,[24] 1874 (829, unpublished); STEINHAUSER,[19] 1880 (6343); WOLFRAM,[9] 1781, published 1787 (1900, 4357, 7247, 8963). I have mislaid my reference for the name to be associated with the remaining error, in ln 2194. Can anyone supply this name?

In all three cases (1099, 7853, 8023) where Wolfram has an error in the first hexad, DASE's 7D values are correct in his *Tafel der natürlichen Logarithmen der Zahlen*, 1850.

13. Vega Reprint of Wolfram.—The first reprint of Wolfram's table was in G. VEGA, *Thesaurus Logarithmorum*. Leipzig, 1794, p. 642–684. It will be recalled that W's. 6 correct missing lns and the Barzellini correction of W's.

ln 4891 were published[8] in 1780. Since all errors in W. (12) except, no. 24 in ln 4891, occur in Vega, it would seem as if V. made use of this publication as well as of Schulze's. In that case he made one proofreading slip, or a slip in his checking computation, since for the final digit in the value of ln 9883 he had a 4 instead of the correct 3[.298]. The lns of 25, 520, 7027, were incorrect in the table, but on the two pages of errata the necessary corrections were supplied.

On p. 641 of the Vega volume, the heading of all pages which follow, through 684 is: "Wolframii Tabula Logarithmorum Naturalium." On this page is a somewhat detailed explanation of series to be used for the computation of hyperbolic logarithms. We have seen that such material did not come from Schulze's Wolfram.

Of Vega's table there were four known later editions in 1889, 1896, 1923, 1946; that there was a 1910 edition has not been proved, see *MTAC*, v. 2, p. 163, 283–284. In the 1889 edition all the corrections of the original errata sheets were made in the text throughout. The same is true of the 1923 and 1946 editions. But curiously the original errata sheets are reprinted in the 1923 edition. The very handy miniature edition of 1946 is still in print [*MTAC*, v. 2, p. 161–165]. In the 1896 edition the errors at 1099, 4891, and 6343 have been corrected.

14. Peters & Stein Edition.—Vega's edition of Wolfram's table was the basis of the PETERS & STEIN, Table 13, p. 127–151, in the *Anhang* of 1922. This table contains ln x to 48D, for $x = 2(1)146$ and all later primes less than 10 000. Ten errors of Vega and Wolfram are continued.[1] (829, 1087 last figure, 1409, 3967, 4357 [R.C.A.], 6343, 7247, 8837, 8963, 9623); and also one error of Vega, where W. was correct (9883). No other error has been found. Thus P. & S. were the first to pick up the errors in the fifth hexad of ln 1087, and in the third hexad of ln 3571. Since P. & S. had ln 7853 correct they may have got the result from Kulik,[12] the only prime for which K. gave a correction; Peters would almost certainly have known about this.

15. Callet Extracts.—A few values of Wolfram's table, taken from either the Schulze or the Vega volumes, appeared in F. CALLET, *Tables Portatives de Logarithmes*. Paris, 1795. We here find ln x, to 48D, for $x = 1(1)99$, and for primes to 1097; but Callet adds 48D values for $x = 999980(1)1\,000\,021$; for all but the last six of these numbers Sherwin gave 61D values of the common logs. Callet gives also lns for $x = [1(1)1200; 20D]$, $[101000(1)101179; 20D]$. Wolfram's errors in ln 829 and ln 1087 are carried over, and also those for ln 829 and ln 1099 in the 20D table.

Of Callet's work there were many reprints or editions.

16. Kulik and Wolfram.—Writing in May 1824 Kulik stated[12] that for two years he had been busy with the revision, extension and publication of Wolfram's 48D table, for all numbers up to 11000. He gave the title of his partially published work as follows: *Canon logarithmorum naturalium in 48 notis decimalibus pro omnibus numeris inter 1 et 11000 denuo in computum vocatus ab* Jac. Phil. Kulik. On January 27, 1825 he reported[12] that the "12th Bogen," presumably through page 192, was being printed (the completed work was to contain 288 pages). He also stated that all printing ought to be finished in the following May, and that the volumes should be on sale by the end of the following September. There is no reference to this work in: (i) any later volume of the *Astronomische Nachrichten*, (ii) any ordinary bibli-

ographies or library catalogues, (iii) even the personal bibliography sent to "Poggendorff" by K. shortly before his death in 1863, nearly 40 years later.

We do know that at Gratz in 1825 Kulik published a xxvi, 286-page factor table—to be found in very few libraries: *Divisores numerorum decies centena millia non excedentium. Accedunt tabulae auxiliares ad calculandos numeri cujuscunque divisores destinatae. Tafeln der einfachern Factoren aller Zahlen unter einer Million nebst Hülfstafeln zur Bestimmung der Factoren jeder grösseren Zahl.*

In the very interesting biography of Kulik (a published portrait is referred to) in C. VON WURZBACH, *Biographisches Lexikon des Kaiserthums Oesterreich*, v. 13, Vienna, 1865, p. 356–359, we find, however, the following entry in a list of Kulik's publications: "*Canon logarithmorum naturalium in notis decimalibus duo de quinquaginta.* (Gratz, 1826), ein logarithmischer Canon mit 48 Decimalen." From this it would appear that this Kulik volume was actually completed and published, but with the title somewhat changed from that of the announcement.

What library has this volume, or any part of it?

17. Thiele and Wolfram.—Elements of mystery are not wholly lacking also in connection with the second attempt to publish an extended edition of Wolfram's great table. Here, at least, four copies are known to exist of: *Tafel|der|Wolfram'schen hyperbolishen|48-stelligen|Logarithmen.|Bearbeitet und erweitert|von|*W. THIELE,*|Herzoglich Anhaltischer Forstmeister a.D.|* Dessau, 1908.|Verlag der Hofbuchdruckerei C. Dünnhaupt.|ii, 108 p. 13 × 20 cm. Bound copies were sold at 6 marks. The volume is clearly printed, but the "Forstmeister" and "Hofbuchdruckerei" combination seems to have prevented the volume from reaching many of the ordinary mathematical publicizing sources. The only references to the work which I could find were in *Jahrbuch ü.d. Fortschritte d. Math.* for 1908, and Deutsche Mathem.— Ver., *Jahresbericht*, 1908. The volume is extraordinarily scarce; the only copies of which I know are at Harvard University (Brown University has a film copy of this), University of Colorado, John Crerar Library, Chicago, and in the private library of C. R. COSENS.[5] I know of no wholly correct published statement as to what is in the table of this volume.

The back of the title page is blank. Then on page 1 the complete introductory text is the following, arranged in fanciful print, diamond-shape, somewhat suggested by the indications for new lines: "Die|natürlichen|oder hyperbolischen|Logarithmen sind in dieser|Tafel bis zu 5000 für jede Zahl| ausgeworfen; von 5000 bis zu 10000|jedoch nur für die dazwischen|liegenden 560 Primzahlen|und für einige andere|ungerade|Zahlen." This statement is wholly correct as far as it goes but there is no reference to the numbers 10001(1) 10014, which are also included in order to fill out 115 pages (2–116), with 50 logarithms on each page. Hence hyperbolic logarithms to 48D are given for 5750 numbers. Thiele took everything from Wolfram's table, that is, the logarithms of 3456 numbers (2230 composites and 1226 primes) the values of six primes being missing (9). Thus Thiele's new work consisted in adding the values of the logarithms of 2288 composite numbers and of 6 prime numbers, in a range where Wolfram had given accurate values for all the 1232 primes less 12 (18). Of the logarithms of these 2288 composite numbers, 2214 are in the first 5000 entries; and therefore beyond 5000, only 74 were added by Thiele. Since copies of Wolfram's table (1946) are still in

print we record just what these 74 numbers are. 5083, 5111, 5267, 5359, 5497, 5543, 5681, 5773, 5899, 5957, 6187, 6233, 6371, 6463, 6593, 6631, 6739, 6859, 6973, 7061, 7199, 7259, 7381, 7511, 7627, 7751, 7859, 7991, 8033, 8113, 8257, 8357, 8479, 8579, 8671, 8723, 8809, 8993, 9089, 9131, 9211, 9367, 9481, 9579, 9683, 9709, 9773, 9889, 9893, 9919, 9937, 9943, 9959, 9979, 9983, 9987, 9989, 9991, 9993, 9997, 9999, and the 13 composite numbers in the range 10000(1) 10014. Thus we can at once tell for what numbers 48D lns may be found in Wolfram (3462), Thiele (5750), Kulik (11000).

There are 562 primes beyond 5000. What about the values furnished by Thiele for the six missing primes? The answer to this question is that the last 6–8 decimal places are incorrect in every one of them. When Cosens wrote his "Note"[5] he had not seen Schulze's work, so that explains part of his following statement: "In the only case I have checked (9883) Thiele is definitely wrong, and I have little doubt that he is wrong in all. Apparently he must have calculated them himself though why he should invariably go wrong at the 40th to 42nd decimal is not clear. I know of no source to 40 decimals, or I should be tempted to suggest that he copied them and made up the last few figures in his head to fill out the line!" How right Cosens is! He did not know of the 42D common logarithms of those six missing primes, which Wolfram had furnished. (8). Hence Thiele simply multiplied these values by ln $10 = M^{-1}$ and rounded them off in the manner indicated. Thus Thiele made no computation by series of any new logarithm and, apart from the multiplications here noticed, his other 2288 logarithms seem to have been each obtained by the addition of two numbers, without other checking.

All errors connected with 32 numbers in Wolfram (12) are repeated. Curiously enough Wolfram's error in no. 28(7853) is corrected. This is indeed a major mystery; the only explanation which I can offer is that the typesetter substituted an 8 for a 7, a "mistake" which Thiele did not observe! As a result of these errors Thiele made 14 other errors in connection with the following lns of composite numbers, 8 of these being found by careful calculation of Cosens, and 6 (through additions) by R.C.A.:

1. ln 2487 C (3) 404665; (8) 451603
2. ln 2818 C (4) 379132
3. ln 3261 C (5) 248268; (8) 924420
4. ln 3297 C (1) 100768; (8) 918268
5. ln 3800 C (6) 758520
6. ln 4044 A (4) 734576
7. ln 4128 A (8) 414505
8. ln 4145 C (3) 395349; (8) 248049
9. ln 4186 A (4) 382825
10. ln 4227 C (4) 357145
11. ln 4346 A (8) 356671
12. ln 4348 A (7) 122908; (8) 635317
13. ln 4350 A (8) 323643
14. ln 4974 C (3) 349975

But Cosens found also the following three other errors of a different type:

ln 18 (3) 164692
ln 81 (4) 580980

ln 3458 (6) 528439; (7) 755232

Thus errors have been noted in connection with 55 numbers, 18 of them being among the 2288 whose lns were added by Thiele. In order to state the case fairly it should be noted in final summary that 38 of these 55 errors noted in Thiele's table were due to errors in Wolfram's printed table. A large amount of checking remains to be done. The kind of error for which Thiele is now known to be responsible makes, however, a highly unfavorable impression.

The page and one-half (117–118) following the table is headed "Einige bekannten Constanten," and its most important parts are taken from p. 188 of Wolfram (10): M and $1/M$ to 48D, log π and ln π to 23D, and an abridgement of the table of N, for which ln $N = 9(1)25$. Among other items is given π to 127D, but there are two errors: *for* 0, *read* 6 in the 108th decimal place; *for* 7 *read* 8 in the 113th.

Following p. 118 is a fly-leaf on the back of which, in a little frame, is:

Hofbuchdruckerei
Weniger & Co.

We now turn to the "mystery" referred to at the beginning of this section. and supplementing the correction mystery. According to Kayser's *Vollständiges Bücher-Lexikon*, v. 36, 1911, and *Hinrichs Halbjahr-Katalog*, v. 221, 1909, the volume we discussed above is referred to as a reprint (Neue Ausg.). In the case of the Harvard University copy, at least, there is no such indication. On the other hand, in the *Halbjahr-Katalog*, v. 217, the details are given for an edition published at Dessau by another printer, "P. Baumann's Nachf.," in 1905; *Bücher-Lexikon*, v. 34, has a less definite reference. This, then, was the original of the work. In no other publication have I been able to find this work mentioned, and scores of inquiries have failed to locate a copy in any library of the world. Such is "mystery" connected not only with Thiele 1905, but also with Kulik 1826. May the publication of this article lead to the discovery of one or both of the volumes.

18. Conclusion.—Such is a summary of the Wolfram information which I have been able to collect. Since some of the books, to which extended reference is made, are not often available in libraries, more details are given than would otherwise be desirable. On the other hand I withstood the temptation to quote passages from Wolfram's letters which definitely suggested a very attractive personality, and a man of high ideals constantly working to the limit of his strength. As a result of this paper I trust that future writers on prominent computers may be able to make concerning Wolfram a more adequate appraisal than was formerly possible. Let us hope that our Dutch friends may succeed in unearthing yet further facts concerning this one of their several outstanding table-makers.

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[1] R. C. A., *MTAC*, v. 1, p. 57; WOLFRAM, and PETERS & STEIN errata.
[2] R. C. A., *MTAC*, v. 2, p. 161–163; Wolfram errata.
[3] R. C. A., *MTAC*, v. 2, p. 340, and v. 3, p. 171; Wolfram and Lambert.
[4] R. C. A., "Wolfram's table," *Scripta Mathematica*, v. 4, 1936, p. 99–100, 293.
[5] C. R. COSENS, "A note on the errors in W. Thiele's table of hyperbolic logarithms to 48 decimals, with some remarks on previous tables taken from the original work of Wolfram." Unpublished manuscript dated August 7, 1939 and sent to me in that month. Mr. Cosens (of the Engineering Laboratory, University of Cambridge) computed a number of ln x, to 52D, in testing the accuracy of the table. In writing my paper, some of Mr. Cosens' material has been useful. He had not seen the 1778 edition of Wolfram when he wrote his note.
[6] J. H. LAMBERT, *Beyträge zum Gebrauche der Mathematik und deren Anwendungen.* Berlin, 3v., 1765, 1770, 1772; v. 3, p. [vi–xi] of the Vorrede contains Wolfram's list of 70 errors in Table I of no. 7 as well as in the SHERWIN-SHARP and CLAUSBERG tables. See no. 9(a)(XVIII–XX); v. 2 contains a complete factor table (p. 42–53 + large folding plate.)
[7] J. H. LAMBERT, *Zusätze zu den logarithmischen und trigonometrischen Tabellen.* Berlin, 1770.

[8] J. C. Schulze, "Zusätze und Verbesserungen der Berliner Sammlung trigonometrischer Tafeln," *Berliner Astronomisches Jahrbuch für das Jahr 1783*, Berlin, 1780, p. 191; six missing logarithms, calculated by Wolfram are given, as well as the note by Barzellini, of a Wolfram error. Barzellini's calculation of the values of the 6 missing lns agreed with that of W.

[9] *Deutscher gelehrter Briefwechsel Joh. Heinr. Lamberts*. Edited by Johann (III) Bernoulli, Berlin, (a) v. 4, 1784; (b) v. 5, 1785, p. 1–242, and 1787, p. 243–502. (a): Correspondence between Wolfram and Lambert, Letters XVIII–XXXIX, 5 March 1772– 9 August 1777, p. 436–536. XVIII W(L), = letter from W. to L., dated Namur 5 March, 1772 (436–440); XIX L(W) Berlin 21 March, 1772 (441–445); XX W(L) Namur 3 August, 1772 (445–469) (see no. 6, above); XXI L(W) Berlin 19 December, 1772 (469–476); XXII W(L) Namur 8 February, 1773 (477–484); XXIII L(W) Berlin 13 March, 1773 (484–492); XXIV W(L) Namur 5 April, 1773 (493–496); XXV W(L) Namur 26 July, 1773 (497–500); XXVI L(W) Berlin 21 August, 1773 (501–507); XXVII W(L) Danzig 29 July, 1774 (507–509); XXVIII L(W) Berlin 11 August, 1774 (509–512); XXIX L(W) 28 August, 1774 (513); XXX W(L) Danzig 6 September, 1774 (514–516); XXXI W(L) Nimwegen 14 April, 1775 (516–519), notes on L(W) (519); XXXII W(L) Nimwegen 2 April, 1776 (520–522); XXXIII Nimwegen 18 October, 1776 (523–524); XXXIV L(W) Berlin 30 Nov. 1776 (528–529); XXXV W(L) Nimwegen (528–530); XXXVI W(L) Nimwegen 24 January, 1777 (530); XXXVII L(W) Berlin 8 March, 1777 (531–534); XXXVIII W(L) Nimwegen 18 April, 1777 (535); XXXIX L(W) Berlin 9 August, 1777 (536). On p. 531–532 and 536 there are references to another letter W(L) dated 21 March, 1777. In editing these letters Bernoulli supplied annotations. The Nimwegen here evidently corresponds to Mr. Kemperman's Nijmegen. Lambert died in the month following the one in which he wrote his last letter to W.

(b); P. 353–464 constitute supplements to (a), p. 474f, 491, 500, 503, 508, 517, 523, 529. There are many new tables, new Wolfram letters and other valuable material. There are references to 5 other W. letters: to L.(14 October, 1773, p. 368; 30 December, 1773, p. 368, 383; 17 March 1774, p. 392, and for all 3 to v. 4, p. 508); to Schulze (28 April, 1779, p. 463–464), and to Geheim Kriegsrath Davisson in Dantzig (Helvoetsluys, 2 March, 1781, p. 461–463). In connection with this last letter W. records errors in his table at 1900, 4357, 7247, 8963.

[10] P. Erman, Académie R. d. Sciences . . . , Berlin, *Mémoires, MDCCXCIV et MDCCXCV*, Berlin, 1799, p. 55–70.

[10a] J. C. Burckhardt, *Tables des Diviseurs*. Paris, 1817, p. [iii]; correction of ln 7853.

[11] J. B. J. Delambre, *Histoire de l'Astronomie Moderne*. V. 1, Paris, 1821, p. 501, 511–513, 519.

[12] J. P. Kulik, "Auszug aus einem Briefe . . . 1824, May 13," *Astron. Nachrichten*, v. 3, 1824, cols. 191–192; list of 17 common errors in Wolfram and Vega tables (K. was incorrect in listing an error in ln 1658) and a description of the partial publication of an extension of this table; also *idem*, v. 4, cols. 47–48, April, 1825; further details of this publication. Earlier recordings of two W. errors noted by K. are 4891 (Barzellini), 7853 (Burckhardt), 1900 (Wolfram).

[13] C. Gudermann, *J. f. d. reine u. angew. Math.*, v. 9, 1831, p. 362; notes Wolfram error at 1099 (earlier reported by Kulik).

[14] A. De Morgan, article "Table," (a) *Penny Cyclopaedia, Suppl.*, v. 2, London, 1846, p. 600–603; (b) *English Cyclopaedia, Arts and Sciences Section*, London, v. 7, 1861, cols. 1000–1001; high praise of Wolfram's work but inaccurate description of its contents.

[15] P. Gray, *Tables for the Formation of Logarithms & Antilogarithms, to Twelve Places*, London, 1865, p. 39; Wolfram error in connection with 1409 noted.

[16] A. F. D. Wackerbarth, R. A. S., *Mo. Not.*, v. 27, 1867, p. 254; Wolfram error in connection with 1099 noted (already published by Kulik and Gudermann).

[17] D. Bierens de Haan, "Bouwstoffen voor de Geschiedenis der wis- en natuurkundige Wetenschappen in de Nederlanden," Akad. v. Wetenschappen, *Afd. Natuurk., Verslagen*, s. 2, v. 10, 1876, p. 189–191, 202–204; also reprinted in a volume with the above title, 1878, p. 199–201, 212–214.

[18] J. W. L. Glaisher, *Report of the Committee on Mathematical Tables*. London, 1873, p. 69, 126. In the Wolfram paragraph on p. 126, line 8, *for* 47 *read* 43; and in the last line, *for* 38, *read* 39.

[19] A. Steinhauser, *Hilfstafeln zur präcisen Berechnung zwanzigstelliger Logarithmen*. . . . Vienna, 1880, p. iii, 1; Wolfram error in connection with 6343.

[20] D. Bierens de Haan, *Bibliographie Néerlandaise Historique—Scientifique des Ouvrages Importants . . . sur les Sciences Mathématiques et Physiques*. Rome, 1883, p. 309–310.

[21] M. Cantor, *Vorlesungen über Geschichte d. Mathematik*. V. 4, Leipzig, 1908, p. 299, 436, 438; notably trivial.

[22] R. Mehmke & M. d'Ocagne, *Encycl. d. Mathématiques*, t. 1, v. 4, fasc. 2, 1908, p. 306.

[23] J. Henderson, *Bibliotheca Tabularum Mathematicarum*. Part I. Cambridge, 1926; p. 138, 178, three inaccurate statements: (i) about Wolfram's table; (ii) that Gray noted the error discovered by Gudermann (see note 13); (iii) that Wolfram calculated the common logarithm table, p. 259, only once (see note 8); p. 191 more than one misleading statement about the Thiele table.

[24] F. J. Duarte, *Nouvelles Tables de Log n! à 33 Décimales depuis n = 1 jusqu'à n = 3000*. Geneva and Paris, 1927, p. iii; errors noted in Wolfram's table in connection with 829, 1087, 1409, 1900. On July 23, 1874, T. M. Simkiss reported the 829 case to J. W. L. Glaisher but his result was unpublished before 1928; earlier recordings 1087 (Peters & Stein), 1409 (Gray), 1900 (Wolfram and Kulik).

[25] F. J. Duarte, *Nouvelles Tables Logarithmiques*. Paris, 1933, p. xxii; eleven Wolfram (1794 table) errors, including 3 of these listed in no. 24—the other 8 being in connection with 3571, 3967, 6343, 7247, 7853, 8837, 8963, 9623—earlier recordings being 3571 (Peters & Stein), 6343 (Steinhauser), 7247 and 8963 (Wolfram), 7853 (Kulik).

[26] NYMTP, *Table of Natural Logarithms*. V. 1–4, Washington, 1941, p. xi, xi, xi, xiii, respectively. W. errors are noted in 829, 1099, 1409, 1937, 1938, 2093, 3571, 4757, 6343, 7853, 8023, 8837, 9623, but the first announcement was made only in connection with 2093 and 8023.

[27] FMR, *Index*. 1946, p. 176–177, 432, 437, 440, 443; inaccurate contents descriptions for Thiele and Wolfram.

[28] J. H. Lambert, *Opera Mathematica*, ed. by A. Speiser. Zürich, v. 1, 1946, p. xiv, xx, 123, 205; v. 2, 1948, p. ix, 70–71. We find in these v. various parts of *Beyträge*,[6] v. 1–3, and the essential parts of the *Zusätze*.[9]

[29] This information was furnished to us through the courtesy of Mr. Eugene Epperson, of Miami University, Oxford, Ohio.

[30] In no bibliography except the *British Museum Catalogue of Printed Books*, and the catalogue of the Royal Observatory Library, Edinburgh, could I find any reference to a Sarganeck: J. J. Schmidt, *Biblischer Mathematicus, Oder Erläuterung der Heil. Schrift aus den Mathematischen Wissenschaften . . . Als ein Anhang ist beygefüget Herrn Georg Sarganeck's Versuch einer Anwendung der Mathematic in dem Articul von der Grösse der Sünden-Schulden*. Zullichau, 1736, 27 plates, 11 ff + 672 p. + 16 ff.

[31] A. G. Kästner published 10 volumes beginning with this word, hence it is not easy to determine which one refers to the Leibniz series; perhaps it was *Anfangsgründe der Analysis des Unendlichen*. Leipzig, 1760.

[32] Henderson's statement[23] concerning Sherwin may be recalled here: "No edition of Sherwin was stereotyped and so some of the later editions are less accurate than the earlier. The third edition in 1742, revised by Gardiner, is probably the most correct, although Hutton [Introduction to his *Tables*, p. 40] says it contains many thousands of errors in final figures. With regard to the fifth edition [1770] Hutton remarks, 'It is so erroneously printed that no dependence can be placed in it, being the most inaccurate book of tables I ever knew.' "

[33] The number 200 was undoubtedly suggested to Wolfram by the fact that in his 1726 edition of Sherwin's *Tables*, log 199 was the last entry in Sharp's table as given there.

[34] What I have written here is not very illuminating. Wolfram's complete statement in this regard, however, is as follows (p. 459): "Auf gleichem Grunde habe ich die Cubicwurzeln von Eins bis auf 125, die man in der Artillerie zum Caliberstabe nöthig hat, ohne wirkliche Ausziehung berechnet."

[35] The German passage on which the first of these statements is based is as follows (v. 5, p. 463): "Ich war schon 1776 auf den Einfall gekommen, durch die Perioden der Dezimalzahlen zu beweisen, dass die Quadratur des Zirkels durch keinen endlichen Werth, weder in Rational- noch Irrationalzahlen ausgedrukt werden könne." The second passage is of very similar construction.

RECENT MATHEMATICAL TABLES

794[B, F].—H. E. Salzer, *Table of Powers of Complex Numbers*. NBS, *Applied Math. Series*, no. 8, Govt. Printing Office, Washington, 1950, iv, 44 p. 18 × 26 cm. For sale by Superintendent of Documents, Washington, price 25 cents.

This short table gives the exact real and imaginary parts of $(x + iy)^n$ for $x = 1(1)10$, $y = 1(1)10$, $n = 1(1)25$. The last page gives x^n for $x = 2(1)9$ and $n = 1(1)25$.

The table is unnecessarily repetitive in that it gives powers of both $x + iy$ and $y + ix$. The essential information of the table can be drawn from that