construction and arrangement of the charts can be understood by explaining the case of Δ_4. For degree 5 it is found that

$$\Delta_4 = a_4\Delta_3 - a_5B$$

where

$$\Delta_3 = a_3\Delta_2 - a_1A, \qquad B = a_2\Delta_2 - a_0A$$

and

$$\Delta_2 = a_1a_2 - a_0a_3, \qquad A = a_1a_4 - a_0a_5.$$

Each of these relations is represented by a chart with three parallel scales, two of which are binary (entered by perpendicular projection from corresponding triangular networks). These charts are in part superimposed along the common scales.

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[1] A. Hurwitz, "Über die Bedingungen unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt," *Math. Ann.*, v. 46, 1895, p. 273–284; *Math. Werke*, v. 2, p. 533–545.

NOTES

121. Production of Tables of Multiplicative Functions by Punched Card Equipment.—Numerical functions $f(n)$ for which the functional equation

(1)
$$f(m)f(n) = f(mn)$$

holds for every pair (m, n) of relatively prime integers are called multiplicative and constitute a conspicuous class of functions. Examples are Euler's totient function $\phi(n)$ (enumerating the numbers not exceeding n and prime to n), the sum $\sigma(n)$, and the number $\nu(n)$ of divisors of n, extensive tables of which were computed by Glaisher.[1] The purpose of this note is to point out that punched card tables of such functions can be produced easily by means of IBM equipment consisting of the sorter, the collator and any one of the 600 type machines.

The most general solution of the equation (1) is obtained by assigning arbitrarily the values of $f(p^\alpha)$ for every prime p and every positive integer α. The value of $f(n)$ for $n = p_1^{\alpha_1}p_2^{\alpha_2} \cdots p_i^{\alpha_i}$ is then defined by $f(n) = f(p_1^{\alpha_1}) \cdots f(p_i^{\alpha_i})$. Hence one begins by producing a table of $f(p^\alpha)$, where f is the given function. This may present some difficulties in case $f(p^\alpha)$ is a complicated function of p and α. Indeed in some cases one may be stopped by the fact that $f(p)$ is an unknown function of the prime p. This occurs for example in the case of Ramanujan's function $\tau(n)$. In many other cases, however, $f(p^\alpha)$ is a polynomial in p^α or some equally simple function which can be computed on a 600 type machine, or otherwise.

Let us suppose that we wish to produce a table of $f(n)$ for $n = 1(1)N$. Let p_k be the greatest prime less than $N^{\frac{1}{2}}$. The table is constructed in various steps as follows.

We begin with the set $S(p_k)$ of cards punched with the values

$$n, f(n)$$

where $n = p$ with $p \leqslant N$ and $p > p_k$; in this set we include also the card for $n = 1 (f(1) = 1)$. The set of cards is arranged according to increasing values of n. From the set $S(p_k)$ we select the set $S'(p_k)$ of cards for which $n \leqslant N/p_k$. With the collator we interfile a blank card between each member of $S'(p_k)$. This deck of cards is now inserted in the hopper of a 600 type machine, which multiplies n by p_k and $f(n)$ by $f(p_k)$ and punches these values into the blank card following the card corresponding to n. The fields for this punching are of course the same as those of the cards already produced. The output of this operation is returned to the collator which now "demerges,"[2] i.e., separates out those cards which have just been punched from the cards of the original set $S'(p_k)$. The former cards are then filed in order among the cards of $S(p_k)$. The latter cards are also returned to the set $S(p_k)$. Finally the card for $n = p_k^2$ is also filed in the set $S(p_k)$. The result is a set $S(p_{k-1})$ of cards corresponding to all integers $n \leqslant N$ divisible by no prime less than p_k.

The set $S(p_{k-1})$ is dealt with in the same manner to produce $S(p_{k-2})$ and so on. Eventually, however, the newly punched cards become too numerous to file with efficiency in the main set $S(p_n)$, and the sorter is used instead to arrange the cards of $S(p_{n-1})$ in their proper order. Also as soon as we descend to the first prime p_n for which $p_n \leqslant N^{\frac{1}{2}}$, it will be necessary to make a (small) additional run multiplying by p_n^2 and $f(p_n^2)$ and to include in $S(p_{n-1})$ the card for $n = p_n^3$.

Continuing the process we have in the final step a table of $f(n)$ for all odd values of $n \leqslant N$. This is the set $S(2)$. The first half of this set $(S'(2))$ is multiplied by $f(2)$, the first quarter by $f(2^2)$, etc. Finally the cards for $n = 2^{\alpha-1}$ and $n = 2^\alpha$, where $2^\alpha \leqslant N < 2^{\alpha+1}$ are filed in to complete the table. From time to time a card count by the sorter and a sequence check by the collator on the argument n is a good precaution.

Finally, if the table is listed on a tabulator, the sums

$$\sum_{n \leq x} f(n)$$

may be obtained for several values of $x \leqslant N$ without extra effort. These may be compared with predicted values, which, in most cases of f, are obtainable from theoretical considerations, in order further to check the table.

The time required to complete the table, once the cards for $f(p^\alpha)$ are produced, is governed by the time required by the 600 type machine, since the collator and sorter are relatively fast and can be operated during the multiplication process after the whole procedure is started.

The writer constructed a table of $\sigma_5(n)$, the sum of the fifth powers of the divisors of n for $n = 1(1)5000$ by this method, using a 602A calculating punch. [See UMT 114.]

D. H. L.

[1] J. W. L. GLAISHER, *Number-Divisor Tables* (BAASMT, v. 8), Cambridge, 1940.
[2] This demerging can also be done on the sorter if the 600 type machine is programmed to emit a distinguishing punch in an otherwise unused column of the newly punched card, this punch being characteristic of the particular multiplication program. This scheme is useful also in case it is found, at some later time, that mistakes were made at a particular stage of the process. These cards and all other cards affected by these mistakes can then be sorted out for correction.

122. Correction to the article, "Matrix Inversion by a Monte Carlo Process."—In the proof of Theorem 1 of the above article [*MTAC*, v. 4, p. 127–129] it was tacitly assumed that the sum given for $E(G_{ij})$ was absolutely convergent, since otherwise the first absolute moment of G_{ij} and therefore $E(G_{ij})$ fail to exist. We must therefore replace assumption (*L*) of the article by a stronger hypothesis, namely

$$\max_r |\lambda_r(A^*)| < 1,$$

where A^* is the matrix with non-negative elements $|A_{ij}|$.

George E. Forsythe
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123. On the number $2^{151} + 1$.—I have made a study of the number

$$N = (2^{151} + 1)/3$$

with a view of establishing its prime or composite character. A search for a prime factor less than $6 \cdot 10^6$ was unsuccessful. On the other hand if N were a prime we should have

$$3^{3N-1} \equiv 9 \ (\text{mod } N).$$

Actually, I find

$$3^{3N-1} \equiv 54302\ 73773\ 60852\ 63755\ 11740\ 55612\ 78194\ 90019\ 88969\ (\text{mod } N).$$

Hence N is composite.

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QUERIES

36. Exponential Integrals for Complex Argument.—Are there tables of the integrals

$$\int_x^\infty t^{-1}e^{-at} \cos t\,dt, \qquad \int_x^\infty t^{-1}e^{-at} \sin t\,dt,$$

or of related functions from which these integrals may be evaluated? The parameters a and x are positive.

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CORRIGENDA

V. 4, p. 156, l. 22, p. 251, *for* Arenburg *read* Arenberg.
V. 4, p. 179, l. 2, *for* C = 2 *read* C = − 2.
V. 4, p. 180, l. −14, *for* 54 *read* 554.
V. 4, p. 238, l. −11, p. 256, *for* P. A. Morton *read* P. L. Morton