Moore School of Electrical Engineering, University of Pennsylvania.—The University is offering a graduate course for 1950–1951 in electrical engineering with emphasis on large-scale computing devices. The course of study includes the following subjects: transient circuit analysis, engineering physics, electronics, introduction in digital computing machines, engineering techniques for solving differential equations, servomechanisms and feedback control, continuous variable computers, digital computers-logic, digital computers-engineering principles, advanced topics in numerical methods for digital computers, and advanced engineering mathematics.

Swedish Board for Computing Machinery.—The Board for Computing Machinery was appointed by the Swedish Government in November 1948. At present its members are:

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The Board has the authority to make decisions concerning the development and acquisition of computing machinery for the Swedish State within a budgetary frame fixed by the Parliament. Its aim is to provide computational service to Swedish State agencies as well as to private institutions and enterprises.

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Under the supervision of the Board, Dr. CONNY PALM, Stockholm, is head of the Projects Group, within which the experimental, design, and construction work is carried out. The secretary of the Board is Mr. GÖSTA MALMBERG, Stockholm.

The Board will issue from time to time communications in English, which will be sent to interested persons in other countries. Anyone desiring to receive these communications should write to:

Matematikmaskinnämnden
Drottninggatan 95 A
Stockholm 6.

OTHER AIDS TO COMPUTATION

BIBLIOGRAPHY Z–XIV

14. R. BEREIS, "Mechanismen zur Verwirklichung der Joukowsky-Abbildung," *Archiv für Math.*, v. 2, 1949–1950, p. 126–134.

The author considers two mechanisms which transform the $z = x + iy$ plane into itself according to the JOUKOWSKY transformation. As an introduction, some of the well known properties of the Joukowsky transformation are summarized. By appropriate use of his mechanisms, the author shows how these transformation properties may be realized. The mechanisms discussed consist of modifications of two types of inversors. First, the author considers two PEAUCELLIER inversors, each of which consists of a rhombus linkage. By cross-knotting these inversors, the author obtains the desired mechanism (called a "Zwillings-inversors"). The second mechanism is built up from the HART inversor. This inversor consists of an antiparallelogram.

By adjoining to this mechanism an ordinary parallelogram linkage, the author obtains the second mechanism.

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15. Z. W. BIRNBAUM & H. S. ZUCKERMAN, "A graphical determination of sample size for Wilks' tolerance limits," *Ann. Math. Stat.*, v. 20, 1949, p. 313–316.

This note contains a graph for the solution of the equation $N\beta^{N-1} - (N - 1)\beta^N = 1 - \epsilon$ for N, given ϵ and β.

F. J. M.

16. A. R. BOOTHROYD, E. C. CHERRY & R. MAKAR, "An electrolytic tank for the measurement of steady-state response, transient response, and allied properties of networks," Inst. Elect. Eng., *Proc.*, v. 96, sect. 1, 1949, p. 163–177.

Suppose that an infinite thin sheet of conducting material corresponds to the plane of the complex variable λ and that electrodes carrying equal currents are placed at points $\mu_1, \cdots, \mu_k$ and electrodes with equal but oppositely directed currents are placed at $\gamma_1, \cdots, \gamma_l$. Then, except for certain scale constants, the potential $V(\lambda)$ is given by

$$V(\lambda) = \sum \log|\lambda - \mu_i| - \sum \log|\lambda - \gamma_i| = \log|Z(\lambda)|$$

where

$$Z(\lambda) = \frac{(\lambda - \mu_1) \cdots (\lambda - \mu_k)}{(\lambda - \gamma_1) \cdots (\lambda - \gamma_l)}.$$

The conjugate to the harmonic function $V(\lambda)$ is the phase angle of $Z(\lambda)$ and this represents a powerful method of representing the rational function $Z(\lambda)$.

The authors discuss various applications of this analogy and present two methods for eliminating the difficulty inherent in the requirement that the sheet be infinite in extent. In one of these, two circular sheets of conducting material are joined on their outer rim. One circular sheet corresponds to a circle around the origin in the complex plane, the other circular sheet corresponds to the rest of the complex plane with the point at infinity in the middle of the circular sheet. The other solution to the difficulty takes advantage of the fact that in the majority of applications, the roots and poles of $Z(\lambda)$ are symmetrically placed relative to the real axis so that no current flows across this axis in the analogue. Consequently attention can be confined to either upper or lower half plane, and this in turn can be conformally mapped on a finite circle.

A variety of applications are discussed in the paper, especially those associated with an impedance function $Z(\lambda)$. For instance, such an analogue can be used to find the roots of a polynomial $f(\lambda)$ by the method of LUCAS. If f is of the n-th degree, a polynomial $g(\lambda)$ with $n + 1$ known real roots is introduced in order to form the rational function $F(\lambda) = f(\lambda)/g(\lambda)$. Since the roots of $g(\lambda)$ are real, the residues of $F(\lambda)$ at the corresponding poles are real and

$$F(\lambda) = \sum a_j(\lambda - \alpha_j)^{-1}$$

where a_j are real. Electrodes are set up at the roots α_j of $g(\lambda)$ and fed currents of strength a_j. The potential function is then, if $\lambda = x + iy$, $\alpha_j = \alpha_j' + i\alpha_j''$,

$$V(x, y) = \sum a_j \log|\lambda - \alpha_j| = \sum a_j \tfrac{1}{2} \log((x - \alpha_j')^2 + (y - \alpha_j'')^2).$$

The roots of $f(\lambda) = 0$ which are also those of $F(\lambda) = 0$ are located by finding the points where both $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$ are zero. For

$$\frac{\partial V}{\partial x} = \sum a_j \frac{x - \alpha_j'}{(x - \alpha_j')^2 + (y - \alpha_j'')^2} = \sum a_j \frac{x - \alpha_j'}{(\lambda - \alpha_j)(\bar{\lambda} - \bar{\alpha}_j)}$$

and similarly

$$\frac{\partial V}{\partial y} = \sum a_j \frac{y - \alpha_j''}{(\lambda - \alpha_j)(\bar{\lambda} - \bar{\alpha}_j)}.$$

From these, one readily shows that $F(\lambda) = \frac{\partial V}{\partial x} - i\frac{\partial V}{\partial y}$ and thus the simultaneous vanishing of $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$ yields a root. The paper also describes the methods used to find the phase angle of $Z(\lambda)$ as represented above and the residues of $Z(\lambda)$ at poles.

Two models of the double sheet representation have been set up in the form of electrolytic tanks with a glass insulating disk for separating the electrolyte into two sheets. (A correction for the thickness of the electrolyte is described.) An illustration shows five electrodes and a probe. In addition a second version having fixed probes along the frequency axis is mentioned. The voltages from these fixed probes are measured in rapid succession by a stepping relay and displayed on a long persistence cathode ray tube in order to plot frequency characteristics of an impedance function $Z(\lambda)$.

F. J. M.

17. E. C. Cherry, "The analogies between the vibrations of elastic membranes and the electromagnetic fields in guides and cavities," Inst. Elec. Eng., *Proc.*, v. 96, part III, 1949, p. 346–360.

In addition to the analogies mentioned in the title, the use of lumped circuit elements is also discussed.

F. J. M.

18. E. A. Goldberg, "Stabilization of wide-band direct-current amplifiers for zero and gain," R. C. A. *Rev.*, v. 11, 1950, p. 296–300.

In analogue computers, d.c. feedback amplifiers are used both as integrating and summing amplifiers. The major problem in these has always been drift. In the present paper a method of stabilizing a d.c. amplifier is described which does not adversely affect the frequency characteristics. The summing point voltage of a direct coupled feedback amplifier is chopped and the result is a.c. amplified, rectified and fed back to a zero set point of the direct coupled amplifier. It is shown that this prevents drift and variation of gain due to drift, and for low frequencies yields a very high loop gain.

F. J. M.

19. C. E. Grosser, "Involute-gear geometry. Nondimensional analysis and design of tooth forms, tooth thickness, and mesh relationships." A.S.M.E., *Trans.*, v. 71, 1949, p. 535–554.

The basic properties of the involute permit the design of involute gears based on the equivalence of their action to a crossed belt drive. The author constructs a "belt-length-ratio" chart "to simplify the computations necessary for the analysis of involute gears and for design for optimum performance."

F. J. M.

20. J. F. Heiss & James Coull, "Nomographs speed flow calculation," *Chem. Eng.*, v. 56, April 1949, p. 104–107.

"Three charts are presented to facilitate determining flow rate or time of discharge from a tank through a pipe."

21. J. A. Hrones & J. B. Reswick, "The electronic analogue—a design tool," *Machine Design*, v. 21, Sept. 1949, p. 115–124.

The electronic analogue is a device which produces repeating solutions of the problem under investigation at the rate of sixty per second; these solutions may be conveniently viewed by oscillographic means.

The analogue is essentially a linear device, but non-linearities of one variable of a special type (similar to saturation effects) may also be simulated. The small amplitude response of systems non-linear in one or two variables may be determined quickly by exploring over the range of the parameters.

A typical servo system is chosen as an example to be analyzed by the electronic analogue. The block diagram of the servo system is transformed to a form suitable for setting the problem up on the analogue. The equivalence between mechanical systems and electrical circuits is indicated. Solutions which were obtained from the analogue of output angle, angular error, and motor torque versus time for different types of disturbances, and different system gain settings, are indicated. The effect of non-linear motor characteristics is also demonstrated.

The article demonstrates how the electronic analogue can be useful in understanding and analyzing the performance of a servo system, as a specific example.

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22. C. F. Kayan, "Heat-transfer temperature patterns of a multicomponent structure by comparative methods," A.S.M.E., *Trans.*, v. 71, 1949, p. 9–16.

A complex problem in heat flow is solved in three different ways. In one of these, an electrical analogue involving slabs of conducting materials is used; in the second an electrical network, and in the third a computational network procedure. Greatest accuracy is claimed for the first method, while the second is considered to be the most convenient when appropriate equipment is available.

F. J. M.

23. D. B. LANGMUIR, "An automatic plotter for electron trajectories," *R. C. A. Rev.*, v. 11, 1950, p. 143–154.

The electric potential is simulated in an electrolytic tank, and the gradient of this potential and the velocity of the electron determine the radius of curvature of the path. The radius of curvature is used to obtain a mechanical plot of the path. A detailed description of the technical difficulties in this set up is given.

F. J. M.

24. G. D. MCCANN & C. H. WILTS, "Application of electric-analog computers to heat-transfer and fluid-flow problems," *Jn. App. Mech.*, v. 16, 1949, p. 247–258, bound with v. 71, A.S.M.E., *Trans.*

A brief description of the Caltech electric analog computer is given which contains a list of available components [*MTAC*, v. 3, p. 501–513]. The analogy upon which certain heat flow problems have been solved is described. Certain of these involve ordinary differential equations with step function coefficients, in others a grid pattern is used to solve a non-linear partial differential equation, linear in the partial derivatives but with coefficients dependent upon the unknown function.

F. J. M.

25. J. K. MICKELSEN, "Automatic equipment and techniques for field mapping," *Gen. Elect. Rev.*, v. 52, no. 11, Nov. 1949, p. 19–23.

Equipment for plotting the equipotential lines in an electrolytic tank is described. Problems of either the LAPLACE or POISSON type may be considered. Three electrodes, set up in a straight line are positioned by servomotors in such a way that their line is approximately tangent to an equipotential line. Thus the slope of the line is determined and the electrode assembly is moved along this line. The relative amount of x and y motion is obtained from this slope by means of resolvers. However there is an additional servo signal which permits a more accurate positioning of the electrode assembly and also permits one to choose the potential of the equipotential line that is being plotted.

F. J. M.

26. K. I. TIKHOTSKIĬ, "Nomogrammy dli͡a vychisleni͡a opredeliteleĭ Gurvitsa" (Nomograms for the calculation of Hurwitz determinants), *Avtomatika i Telemekhanika*, v. 9, 1948, p. 152–155 + one folding plate.

The real parts of the roots of $a_0 + a_1x + \cdots + a_nx^n = 0$ are negative if and only if[1] $a_0 > 0, \Delta_1 > 0, \cdots, \Delta_n > 0$ where Δ_m is the Hurwitz determinant

$$\begin{vmatrix} a_1 & a_0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{2m-1} & a_{2m-2} & a_{2m-3} & & a_m \end{vmatrix}$$

in which $a_k = 0$ for $k > n$. The author has constructed charts to evaluate the Δ_m for equations up to and including degree 6. In the paper they are reproduced for Δ_2 (deg 3), Δ_3 (deg 4), Δ_3 (deg 5) and in part Δ_4 (deg 6). The construction and arrangement of the charts can be understood by explaining the case of Δ_4. For degree 5 it is found that

$$\Delta_4 = a_4\Delta_3 - a_5B$$

where

$$\Delta_3 = a_3\Delta_2 - a_1A, \qquad B = a_2\Delta_2 - a_0A$$

and

$$\Delta_2 = a_1a_2 - a_0a_3, \qquad A = a_1a_4 - a_0a_5.$$

Each of these relations is represented by a chart with three parallel scales, two of which are binary (entered by perpendicular projection from corresponding triangular networks). These charts are in part superimposed along the common scales.

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[1] A. Hurwitz, "Über die Bedingungen unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt," *Math. Ann.*, v. 46, 1895, p. 273–284; *Math. Werke*, v. 2, p. 533–545.

NOTES

121. Production of Tables of Multiplicative Functions by Punched Card Equipment.—Numerical functions $f(n)$ for which the functional equation

$$f(m)f(n) = f(mn) \tag{1}$$

holds for every pair (m, n) of relatively prime integers are called multiplicative and constitute a conspicuous class of functions. Examples are Euler's totient function $\phi(n)$ (enumerating the numbers not exceeding n and prime to n), the sum $\sigma(n)$, and the number $\nu(n)$ of divisors of n, extensive tables of which were computed by Glaisher.[1] The purpose of this note is to point out that punched card tables of such functions can be produced easily by means of IBM equipment consisting of the sorter, the collator and any one of the 600 type machines.

The most general solution of the equation (1) is obtained by assigning arbitrarily the values of $f(p^\alpha)$ for every prime p and every positive integer α. The value of $f(n)$ for $n = p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_t^{\alpha_t}$ is then defined by $f(n) = f(p_1^{\alpha_1})\cdots f(p_t^{\alpha_t})$. Hence one begins by producing a table of $f(p^\alpha)$, where f is the given function. This may present some difficulties in case $f(p^\alpha)$ is a complicated function of p and α. Indeed in some cases one may be stopped by the fact that $f(p)$ is an unknown function of the prime p. This occurs for example in the case of Ramanujan's function $\tau(n)$. In many other cases, however, $f(p^\alpha)$ is a polynomial in p^α or some equally simple function which can be computed on a 600 type machine, or otherwise.

Let us suppose that we wish to produce a table of $f(n)$ for $n = 1(1)N$. Let p_k be the greatest prime less than $N^{\frac{1}{2}}$. The table is constructed in various steps as follows.

We begin with the set $S(p_k)$ of cards punched with the values

$$n,\ f(n)$$