$B_1 = c + fx_6 = .00006\ 98850\ 68541\ 53731,$
$B_2 = f - cx_6 = .99999\ 99993\ 98930\ 27060.$
$A_1B_2 = .62493\ 99996\ 24367\ 48331, A_2B_1 = .00006\ 88298\ 04006\ 56010,$
$A_2B_2 = .98489\ 99994\ 08006\ 42351, A_1B_1 = .00004\ 36739\ 74734\ 34833,$
$A_1B_2 + A_2B_1 = .62500\ 88294\ 28374\ 04341,$
$A_2B_2 - A_1B_1 = .98485\ 63254\ 33272\ 07518,$

$$t = \tan x \doteq \frac{A_1B_2 + A_2B_1}{A_2B_2 - A_1B_1} = .63461\ 92975\ 44148\ 10040,$$

which is correct to about 3 units in the 19th decimal place.

B. To obtain arctan t, for $t = .59139\ 83513\ 99471\ 09817$:

Since $a_1 = .5$, we have

$$t_1 = (t - a_1)/(1 + a_1t) = \frac{.09139\ 83513\ 99471\ 09817}{1.2956\ 99175\ 69973\ 55491},$$

$$\text{or} \quad t_1 = .07053\ 97928\ 11176\ 17252, a_2 = .07;$$

$$t_2 = (t_1 - a_2)/(1 + a_2t_1) = \frac{.00053\ 97928\ 11176\ 17252}{1.0049\ 37785\ 49678\ 23321},$$

$$\text{or} \quad t_2 = .00053\ 71405\ 26474\ 81117, a_3 = 0;$$

$$t_3 = t_2, a_4 = .0005;$$

$$t_4 = (t_3 - a_4)/(1 + a_4t_3) = \frac{.00003\ 71405\ 26474\ 81117}{1.0000\ 00268\ 57026\ 324},$$

$$\text{or} \quad t_4 = .00003\ 71405\ 16499\ 97288, a_5 = .00003;$$

$$t_5 = (t_4 - a_5)/(1 + a_5t_4) = \frac{.00000\ 71405\ 16499\ 97288}{1.0000\ 00001\ 11421\ 55},$$

$$\text{or} \quad t_5 = .00000\ 71405\ 16492\ 01681, a_6 = .000007;$$

$$t_6 = (t_5 - a_6)/(1 + a_6t_5) = \frac{.00000\ 01405\ 16492\ 01681}{1.0000\ 00000\ 04998\ 36},$$

$$\text{or} \quad t_6 = .00000\ 01405\ 16492\ 00979.$$

$$\begin{aligned}\arctan t = \arctan .5 + \arctan .07 + \arctan .0005 + \arctan .00003 \\ + \arctan .000007 + .00000\ 01405\ 16492\ 00979,\end{aligned}$$

$$\text{or} \quad \arctan t = .53407\ 07511\ 10264\ 85054,$$

which happens to be correct to 20 decimals.

H. E. SALZER

NBSCL

RECENT MATHEMATICAL TABLES

834[E].—L. PRANDTL & F. VANDREY, "Fliessgesetze normal-zäher Stoffe im Rohr. Ein Beitrag zur Rheologie," *Zeit. angew. Math. Mech.*, v. 30, 1950, p. 169–174.

This article gives two tables of functions having to do with viscous flow. The function $\phi(a)$ is defined by

$$\phi(a) = 8 \sum_{n=1}^{\infty} n(2n + 1)a^{2n-2}/(2n + 2)!$$

or by

$$\phi(a) = 2\{(a^2 - 2a + 2)e^a - 4 + (a^2 + 2a + 2)e^{-a}\}a^{-4},$$

the latter definition being given incorrectly by the authors.

The first table gives 4S values of ϕ for $a = 0(.1)5(.2)10$.

The second table gives, to 3D, values of $\phi(a\xi)/\phi(a)$ for $\xi = 0(.2).6(.1)1$ and $a = 1(1)10(2)14$.

D. H. L.

835[F].—N. G. W. H. BEEGER, "On composite numbers n for which $a^{n-1} \equiv 1 \pmod{n}$ for every a prime to n," *Scripta Math.*, v. 16, 1950, p. 133–135.

According to a theorem of FERMAT if a is any integer then $a^n - a$ is divisible by n, when n is a prime. The converse is false, however, that is, there exist composite numbers n dividing $a^n - a$ for all a. The smallest such anomalous composite number is $561 = 3 \cdot 11 \cdot 17$ and any number of this sort must be the product of at least three primes. The author studies the case of $n = pqr$. If p is given, there exist only a finite number of q's and r's for which pqr is an anomalous composite number. For each prime $p \leqslant 43$, there is given a table of all possible q's and r's. There are in all 52 such numbers given.

D. H. L.

836[F].—A. GLODEN, "Résolution de la congruence $X^4 + 1 \equiv 0 \pmod{p^3}$ avec une table," *Euclides*, v. 10, 1950, p. 74.

The table gives two solutions $< p^3/2$ of the congruence mentioned in the title for each prime $p < 200$. Since the congruence is solvable if and only if $p = 8n + 1$, the values of p considered are $p = 17, 41, 73, 89, 97, 113, 137$, and 193.

D. H. L.

837[F].—A. VAN WIJNGAARDEN, "A table of partitions into two squares with an application to rational triangles," Nederl. Akad. Wetensch., *Proc.*, v. 53, 1950, p. 869–881 = *Indagationes Math.*, v. 12, 1950, p. 313–325.

The table, p. 872–881, gives all integers (x, y) such that $0 \leq x \leq y$ and

$$n = x^2 + y^2 \tag{1}$$

for each integer $n \leq 10000$ for which (1) has solutions.

The table extends the previously published table of BICKMORE & WESTERN[1] for $n = 1(1)1000$.

It is used to study the question of triangles with integral sides and rational medians, but is applicable to a number of other questions also.

The table was produced on a National machine, 12 columns at a time, by constant second difference procedure. Then the table was carefully rearranged. An alternative procedure would have been to use punched card equipment, in particular the summary punch and sorter.

D. H. L.

[1] C. E. BICKMORE & O. WESTERN, "A table of complex prime factors in the field of 8th roots of unity," *Messenger Math.*, v. 41, 1911, p. 52–64.

838[G].—H. W. BECKER, "Discussion of Problem 4277," *Amer. Math. Monthly*, v. 56, 1949, p. 697–699.

The author gives a table for $n = 1(1)25$ of N_n and N_n' respectively the number of non-commutative and commutative, non-associative products of n factors, and suggests the asymptotic formula

$$N'_{n+1} = .812k^n/(n + 1)\sqrt{n}$$

with $k = 2.48$.

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839[G].—K. YAMAMOTO, "An asymptotic series for the number of three-line latin rectangles," Math. Soc. Japan, *Jn.*, v. 1, no. 4, 1949, p. 226–241.

KERAWALA'S tables[1] for the number of reduced three-line latin rectangles, [that is, rectangles such that each line contains the numbers 1 to n, the first in natural order, and each column unlike numbers] for $n = 3(1)15$ is extended to $n = 15(1)20$. The last entry has 36 digits.

The reviewer, in his turn, by a new recurrence relation has carried this on to $n = 25$; the results are not yet published.

J. RIORDAN

[1] S. M. KERAWALA, "The enumeration of the latin rectangle of depth three by means of a difference equation," Calcutta Math. Soc. *Bull.*, v. 33, 1931, p. 119–127.

840[G].—R. ZURMÜHL, *Matrizen. Eine Darstellung für Ingenieure.* Berlin, 1950, Springer, xv, 427 p. 15.2 × 20.3 cm. Price 25.50 marks.

This book is a welcome addition to the literature in the field. Apart from a large number of numerical examples which illustrate the theories and their application, this book contains an extensive chapter (chapter VI) of 80 pages on numerical methods for the solution of linear systems of equations and the determination of the characteristic roots of finite matrices. It is with this chapter only that the present review is concerned. The author does not claim to present a complete enumeration of the known methods; those which are mentioned were chosen because of their suitability for engineering problems.

The methods are discussed in great detail and a complete work sheet is given many times.

For the solution of linear systems the author discusses on one hand the GAUSS elimination process and its more recent variations by CHOLESKY and BANACHIEWICZ and, on the other hand, some of the well known iteration processes. It is generally agreed to-day that the elimination process is the most convenient method, unless the system in question is of a special form which makes it more suitable for other methods, in particular if its matrix has a dominant principal diagonal. The matrix formulation employed in the Cholesky and Banachiewicz treatment makes it very suitable for calculating machines since some of the results can be obtained without recording all the intermediate steps. The related problem of inverting a matrix is also discussed.

The iteration processes have great drawbacks through convergence uncertainties. These methods have been studied extensively, e.g. in a now classical paper by VON MISES and POLLACZEK-GEIRINGER. There it is shown that for symmetric positive definite matrices the GAUSS-SEIDEL iteration process will converge always so that "normal" equations can, theoretically at least, be treated by it. This process, with error estimates by COLLATZ, is discussed as well as SOUTHWELL'S relaxation method (which the author traces back to Gauss). The latter gave a method for checking as well as a device for improving convergence in difficult cases. This device does not seem to be known to modern relaxers. It consists in replacing the n unknowns $\bar{x}_i$ by a system of $n + 1$ unknowns, x_0 and $x_i = \bar{x}_i - x_0$, $i = 1, 2, \cdots, n$. The coefficients of x_0 are given by $a_{i0} = -\sum_{k=1}^{n} a_{ik}$ $(i = 1, 2, \cdots, n)$ while the other coefficients are unaltered. An $(n + 1)$-st equation is added: it is formed by the negative sum of the first n equations.

In the report of BODEWIG[1] it is stated that the combination of Gauss elimination and application of iteration afterwards is the most efficient method known so far. This was verified by the author in the case of 8 systems of 42 equations with small determinant.

Next, characteristic roots: the usual iteration process for the determination of the dominant root when it is real is discussed, particularly for the case of real symmetric matrices. Next, matrices with a complex dominant root are treated, both in the case of linear and non-linear elementary divisors. The latter case is based on the results of H. WIELANDT and for both cases the methods of DUNCAN and COLLAR are used. Collatz's "inclusion" theorem for the characteristic roots of real symmetric and positive (not necessarily symmetric) matrices is explained. It is a certain generalization of the iteration procedure for finding the dominant roots. Several methods are given for the determination of all the characteristic roots. Some of them were developed in Germany in recent years only and have not been published in easily accessible places before. There is the method (by KOCH) to apply the iteration process used to determine the dominant root by starting with a vector orthogonal to the vector that corresponds to the dominant root. Next the method of reducing the matrix to a matrix of one dimension less, but having the same roots as the original one apart from the dominant root. The methods of Duncan and Collar and of Wielandt are reported here. Wielandt's "fractional iteration" does not require the knowledge of the dominant root, but requires the knowledge of a suitable approximation to the next one. The process of Frazer, Duncan and Collar leads to the determination of the characteristic equation by applying powers of the matrix to an arbitrary vector. The method of HESSENBERG also leads to the characteristic polynomial by building it up from polynomials of lower degree.

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NBS

[1] E. Bodewig, "Bericht über die verschiedenen Methoden zur Lösung eines Systems linearer Gleichungen mit reellen Koeffizienten I–V," Nederl. Akad. Wetensch., *Proc.*, v. 50, 1947, p. 930–941, 1104–1116, 1285–1295, v. 51, 1948, p. 53–64, 211–219 = *Indagationes Math.*, v. 9, p. 441–452, 518–530, 611–621, v. 10, p. 24–35, 82–90.

841[I, L].—G. BLANCH & R. SIEGEL, "Table of modified Bernoulli polynomials," NBS, *Jn. of Research*, v. 44, 1950, p. 103–107.

The polynomials to which the title refers may be defined by their Fourier series as follows

$$b_{k+1}(x) = -\sum_{n=1}^{\infty} n^{-k} \cos (nx + \tfrac{1}{2}\pi k)$$

and are related to the Bernoulli polynomials

$$B_k(x) = (B + x)^k$$

by the relation

$$2k!b_k(2\pi x) = (-2\pi)^k B_k(x)$$

so that $b_1(x) = (\pi - x)/2$, $b_2(x) = \dfrac{x^2}{4} - \dfrac{\pi x}{2} + \dfrac{\pi^2}{6}$, etc. The polynomials are given explicitly for $k = 1(1)11$ and $\left[x = 0\left(\dfrac{\pi}{36}\right)\pi;\ 17D\right]$. The values were computed from differences using the IBM 405 tabulator and checked by summation.

D. H. L.

842[I].—E. T. FRANKEL, "A calculus of figurate numbers and finite differences," *Amer. Math. Monthly*, v. 57, 1950, p. 14–25.

Figurate numbers are, effectively, taken as defined by generating functions

$$(1 - t)^{-n} = \sum F_r{}^n t^r$$

and thus are essentially binomial coefficients with sign convention reversed. Their relation to finite differences and sums depends essentially on the following results.

If $V(t) = \sum u_r t^r$ is the generating function of u_r $(r = 0, 1, \cdots)$, then $(1 - t)^{-1}V(t)$ is the generating function of $u_0 + u_1 + \cdots + u_r$ and $(1 - t)V(t)$ of $u_r - u_{r-1}$. The author writes $Su_r = u_0 + u_1 + \cdots + u_r$ and $S^{-1}u_r = u_r - u_{r-1}$ and defines their iterates in the usual way, which of course involves figurate numbers. The function generated by the product of two generating functions, now commonly called the convolution, he calls the criss-cross product. For n-th degree polynomials, special attention is given to numbers $S^{-(n+1)}u_r$, which the author calls d_r, because $d_r = 0$, $r > n$, and all other sums (or differences) of the given number sequence u_r can be expressed in terms of them. Other than illustrative tables, there are two main tables, one of figurate numbers $F_r{}^n$ for $n = -7(1)7$ and $r = 0(1)7$ and one of $d_r = S^{-(n+1)}r^n$ for $n = 1(1)11$ and $r = 1(1)11$. The last have a long history (back to LAPLACE) and have lately been called cumulative numbers (DWYER), KUMMER numbers (PIZA), triangular permutation numbers (KAPLANSKY & RIORDAN).

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843[I].—E. PFLANZ, "Allgemeine Differenzenausdrücke für die Ableitungen einer Funktion $y(x)$," *Zeit. angew. Math. Mech.*, v. 29, 1949, p. 379–381.

This paper gives an expression for the m-th derivative of a function $y(x)$ at a point x_0, i.e. $y^{(m)}(x_0)$, in terms of the functional values at $(n + 1)$ points x_0 and $x_\rho \equiv x_0 + \alpha_\rho h$, $\rho = 1(1)n$. In general the points x_0, x_ρ may be spaced irregularly, but must be distinct. Also $y(x)$ is assumed to have a continuous $(n + 1)$st derivative. The general formula is

$$y^{(m)}(x_0) = (-1)^m h^{-m} m! S_m y(x_0)$$
$$- (-1)^n h^{-m} m! \prod_{\nu=1}^{n} \alpha_\nu \sum_{\rho=1}^{n} \alpha_\rho^{-m-1} \prod_{\nu=1}^{n}{}' (\alpha_\rho - \alpha_\nu)^{-1} F(m, n, \alpha_\rho) y(x_\rho)$$
$$+ R_{m,n} \qquad (1 \leq m \leq n).$$

Here $F(1, n, \alpha_\rho) = 1$ and

$$F(m, n, \alpha_\rho) = \sum_{\nu=0}^{m-1} (-1)^\nu \alpha_\rho^\nu S_\nu \qquad (2 \leq m \leq n).$$

S_λ denotes the sum of all possible products of λ distinct factors from the set $\alpha_1^{-1}, \alpha_2^{-1}, \cdots, \alpha_n^{-1}$ and the $\prod_{\nu=1}^{n}{}' (\alpha_\rho - \alpha_\nu)$ denotes the fact that the case $\nu = \rho$ is excluded. The remainder term $R_{m,n}$ is a rather involved expression which is given explicitly.

For $n + 1 = p + q + 1$ equi-distant points of interpolation, denoted by $x_0 + \rho h$, with $\rho = -p(1)q$, where p and q are integers $\geqslant 0$ and $p + q \geqslant 1$, the general formula is given in this special case. All formulas are given without proof, the only indication of their origin being the statement that they were obtained from LAGRANGE's interpolation formula. To facilitate the computations for equally spaced points x_ρ, the exact fractional values of S_λ are tabulated for $\lambda \leq p + q \leq 7$, $(p, q \geqslant 0)$, $\lambda = 1(1)7$.

The expression which is given for the derivatives for equally spaced points x_ρ, even with the author's auxiliary table of S_λ is far from being in the simplest form for computational purposes. The present article should be compared with a similar paper by BICKLEY,[1] which is omitted from the list of references. Bickley tabulates the exact integral quantities ${}_{mn}A_{pr}$ in the formula (retaining Bickley's notation)

$$n! w^m D^m y_p \doteq m! \sum_{r=0}^{n} {}_{mn}A_{pr} y_r$$

for $n = 2(1)6$, $m = 1(1)n$, $p = 0(1)n$, $r = 0(1)n$; for $n = 8, 10$, $m = 1(1)4$, $p = 0(1)n$, $r = 0(1)n$. Bickley also gives error terms. Although Bickley's formulas are much more direct and involve only a fraction of the computational work arising in the use of PFLANZ's formulas, final judgment of the value of this note should be reserved until his expressions for the remainder may be compared with other forms given by Bickley and other writers in textbooks on finite differences.

In the heading of the table of $\sum_\lambda$ *for* $1 \leq p + q \leq 7$ *read* $\lambda \leq p + q \leq 7$.

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[1] W. G. Bickley, "Formulas for numerical differentiation," *Math. Gazette*, v. 25, 1941, p. 19–27.

844[K].—F. J. ANSCOMBE, "Table of the hyperbolic transformation $\sinh^{-1} \sqrt{x}$," Roy. Stat. Soc., *Jn.*, A, v. 113, 1950, p. 228–229.

The function tabulated was proposed by the author[1] for use in transforming highly skewed distributions of counts to a more nearly normal form. In a forthcoming book on statistical methods[2] he also proposes its use in normalizing a variable obeying a STUDENT-FISHER t-distribution. The tabulation values are given to 3D for $x = 0(.01)1(.1)10(1)200(10)500$.

C. C. C.

[1] F. J. ANSCOMBE, "The transformation of Poisson, binomial and negative-binomial data," *Biometrika*, v. 35, 1948, p. 246–254.
[2] No title or publisher is given.

845[K].—W. G. COCHRAN & G. M. COX, *Experimental Designs.* ix + 454 p. New York, Wiley & Sons, 1950. 15.8 × 23.7 cm. Price $5.75.

The tables presented are listings of plans for experimental designs and tables of random permutations of nine and of sixteen numbers.

Plan 4.1 gives one latin square arrangement each of sides 3(1)12, except that a set of four 4 × 4 squares is given. A randomization procedure is suggested which selects one square at random from all possible 3 × 3 squares or 4 × 4 squares. (FISHER & YATES[1] give complete representations up to 6 × 6 squares.) Plan 4.2 gives graeco-latin squares of orders 3, 4, 5, 7, 8, 9, 11 and 12.

Plans 6.1 and 6.2 present 2^3 and 2^4 factorial designs with the highest order interaction confounded in each and plan 6.3, a 2^6 factorial in 4 blocks with three four-factor interactions confounded. Plan 6.4 is a balanced group of partially confounded 2^4 factorial designs in blocks of 4 units, and plans 6.5 and 6.6, the same for 2^5 and 2^6 factorials in blocks of 8 units. Plan 6.7 gives a balanced group of four replications of a 3^3 factorial design in blocks of 9 units which partially confounds the highest order interaction (ABC), and plan 6.8, a balanced group for a 3^4 factorial which partially confounds each of the three-factor interactions. Plan 6.9 partially confounds ABC and BC of a 3×2^2 factorial design in blocks of 6 units. Plan 6.10 is a balanced group which partially confounds, for a 3×2^3 factorial, the two-factor and three-factor interactions involving two of the three factors at two levels each. This plan and the next also involve blocks of 6 units. Plan 6.11 is a balanced group for a $3^2 \times 2$ factorial, confounding partially AB and ABC. Plans 6.12 and 6.13 are balanced groups for a 4^2 factorial in blocks of 4 units and a 4×2^2 factorial in blocks of 8 units, both of which partially confound the highest order interaction. Plan 6.14 is a balanced group of nine replications of a $4 \times 3 \times 2$ factorial in blocks of 12 units in which components of AC and ABC are partially confounded.

Plans 8.1a and 8.1b are 2^3 factorial designs in two 4 × 4 squares with AB, AC, BC and ABC partially confounded and with ABC completely confounded. Plan 8.2 is a 2^4 factorial in an 8 × 8 quasi-latin square with all three- and four-factor interactions confounded. Plan 8.3 gives a 2^5 factorial in an 8 × 8 quasi-latin square which completely confounds four of the four- and eight of the three-factor interactions. Plan 8.4 is a 2^6 factorial in five 8 × 8 quasi-latin squares so that each three- and four-factor interaction is confounded in two of the squares. Plan 8.5 gives a 3^3 factorial in two 9 × 9

quasi-latin squares with four of the degrees of freedom for ABC confounded in each. Plan 8.6 is a 3^4 factorial in two 9×9 quasi-latin squares in which all three-factor interactions are partially confounded. Plan 8.7 gives a 4×2^2 factorial in an 8×8 quasi-latin square with 2/3 confounding of ABC. Plan 8.8 is a $2 \times (2^2)$ factorial in a 4×4 half-plaid square with ABC completely confounded. Plan 8.9 gives a $2 \times (3 \times 2)$ factorial in a 6×6 half-plaid square with BC and ABC partially confounded. Plan 8.10 is a $3 \times (3 \times 2)$ factorial in two 6×6 half-plaid squares with AB and ABC partially confounded in each. Plans 8.11 and 8.12 are $2 \times (2^3)$ and $2 \times (2^4)$ factorials in 8×8 half-plaid squares with ABCD confounded in the first and ABD, BCE, ACDE, BCDE and ABCDE confounded in the second. Plans 8.13 and 8.14 are $2 \times 2 \times (2^3)$ and $2 \times 2 \times (2^4)$ factorials in 8×8 plaid squares with four and twelve third- and higher-ordered interactions confounded.

Plans 10.1 to 10.6 give $n \times n$ balanced lattice designs in n blocks and $n + 1$ replicates for $n = 3, 4, 5, 7, 8$ and 9. Plans 10.7 and 10.8 give the three replicates each for $n = 6$ and 10 to complete, with the first three sets of each of plans 10.1 to 10.6, the triple lattices. Plan 10.9 is a 12×12 quadruple lattice. Plans 10.10 to 10.16 give the three replicates of the $k \times (k + 1)$ rectangular lattices in blocks of k units for $k = 3(1)9$.

Plans 11.1 to 11.46 present in detail balanced incomplete block designs for all combinations (of treatments t, units per block k, replications r, blocks b, and λ, the number of times two treatments appear in the same block) that are considered by Fisher & Yates[1] with the exception of the $n \times n$ balanced lattices given in plans 10.1 to 10.6. There is included, as plan 11.5, one design *not* given by Fisher & Yates: (6, 3, 10, 20, 4).

Plans 12.1 to 12.8 are the $k \times k$ balanced lattice squares for $k = 3$ (prime powers) 13.

YOUDEN squares (incomplete latin squares) are given in plans 13.1 to 13.15 for $r < t$ in the following combinations:

t	7	7	11	11	13	13	15	15	16	16	19	19	21	31	37
r	3	4	5	6	4	9	7	8	6	10	9	10	5	6	9

Extensions of these squares for $r > t$ are given in plans 13.16 to 13.26 with

t	3	3	3	3	4	4	4	5	5	6	7
r	5	7	8	10	5	7	9	6	9	7	8

The relative efficiency of each design as compared with a randomized block layout is given, starting with plan 11.1.

Table 15.6 gives 1000 random permutations of the numbers 1(1)9 and table 15.7, 1000 random permutations of 1(1)16. The first table was constructed by reducing pairs of random digits modulo k and neglecting replicates of previously obtained residues. 200 permutations were obtained for each of $k = 9(1)13$ with numbers above 9 omitted. The authors do not point out a source of bias in that the digits 1 to 9 are not equally likely in this process. (The probability of residue = 1 is .1165.) The second table was also constructed by a mixture of methods but, this time, without bias. Sixteen pairs of random digits were ranked numerically with additional digits used to break ties in rankings. Permutations produced by the order of the ranks

gave 800 of the permutations; the remaining 200 were obtained by drawing from an urn. Both tables were tested for randomness by testing the distributions of numbers in positions and positions for numbers. A further test of the number of inversions in order in each permutation was made; none of the tests indicated significant deviation from random order.

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[1] R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural, and Medical Research.* Edinburgh, 3rd ed., 1948 [*MTAC*, v. 3, p. 360–361].

846[K].—M. G. KENDALL, "Tables of autoregressive series," *Biometrika*, v. 36, 1949, p. 267–289.

The author gives further examples of autoregressive series calculated from $u_{t+2} + au_{t+1} + bu_t = \epsilon_{t+2}$, in which a and b are constants and ϵ is a random element, in addition to 4 he previously published.[1] There are 8 series of 400 terms each with ϵ rectangularly distributed with $a = -1.1$ in all 8 and $b = .5$ in the first 4 and .8 in the second 4. There are 5 series of 500 terms each constructed from $u_{t+1} - cu_t = \epsilon_{t+1}$ in which $c = .1(.2).9$ respectively and ϵ is normally distributed. The final set of 5 series of 500 terms each obeys the same difference equation as in the first 4 series but with the terms of each of the preceding 5 series taken as the values of the ϵ's in turn. The final set of 4 tables gives the product sums about zero, $\sum_t u_t u_{t+k}$ and estimates of the serial correlation to 3D for $k = 0(1)50$ for the first and for $k = 0(1)30$ for the remaining 3 of the 4 previously published series; to 4D and $k = 0(1)4$ (for the serial correlation) for the first 8 of the present series and also for the 2 series of 1600 terms obtained from each of the 2 sets of 4, and to 3D for $k = 0(1)30$ for the 5 series in which ϵ is normally distributed.

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[1] M. G. KENDALL, *Contributions to the study of oscillatory time-series*, National Institute of Economic and Social Research. *Occasional Papers*. IX. Cambridge and New York, 1946.

847[K].—L. W. POLLAK, assisted by U. N. EGAN, *Eight-Place Supplement to Harmonic Analysis and Synthesis Schedules for Three to One Hundred Equidistant Values of Empiric Functions.* (Dublin Institute for Advanced Studies, School of Cosmic Physics, Geophysical Memoirs No. 1, Parts 1 and 2), Dublin, 1949. Parts 1 and 2, separately bound: xix, 43 p; 72 p., 23.8 × 32.6 cm, 7s 6d each.

This work is supplementary to the *Harmonic Analysis and Synthesis Schedules for Three to One Hundred Equidistant Values of Empiric Functions, by* L. W. POLLAK, assisted by C. HEILFRON [*MTAC*, v. 2, p. 306–307] and is intended to be used in conjunction with them or with the *All Term Guide for Harmonic Analysis and Synthesis using* 3 *to* 24; 26, 28, 30, 34, 36, 38, 42, 44, 46, 52, 60, 68, 76, 84 *and* 92 *equidistant values*, by L. W. POLLAK and U. N. EGAN, although it is also independently useful in harmonic analysis and synthesis. Its purpose is to provide more accurate values of sines, cosines, and angles pertinent to the harmonic analysis or synthesis of equidistant

values of empiric functions than were given in the publication to which it is supplemental.

For the preponderance of problems in harmonic analysis or synthesis, such as those commonly encountered in geophysics, economics, or ordinary physics laboratory work, the tables provided in the former publication are more than amply sufficient in accuracy. The present work, however, should be extremely valuable to astronomers and others having need for unusually high accuracy in computation, since angles are given to 10^{-12} degree and 10^{-10} second (although reliable to only 10^{-8} second), and sines and cosines to eight decimal places in one of the included tables, and to ten decimal places in another.

Part 1 consists of an "Introduction," and two tables called "Appendix" and "Register"; Part 2 consists of a table known as the "Index." The "Register" and "Index" contain the same information as the portions of the previous publication having the same captions, the difference being in the greater number of significant figures given for values in the later work.

The "Introduction" includes description of the method of computing the tabulated values, illustrated by three short tables, the reliability and accuracy of the values, and a brief discussion of the method of using the tables, either with or without the use of the previously published *Schedules*, or the *All Term Guide*. Three short additional tables illustrate suitable work forms and methods, respectively, for harmonic analysis and synthesis using the tables of this publication and the *Schedules*, and the construction of a schedule by means of this *Supplement* only. An extremely brief bibliography (7 items) is given.

The "Appendix" provides sine or cosine values to ten decimal places, listed according to the 2068 identification numbers ("T-numbers") given in the *Schedules*, which indicate the sine or cosine value to be used as a multiplying factor of each empiric value in harmonic analysis. Since the same values as multipliers are used repeatedly at various places in any series of empiric values, their identification by such serial T-numbers (that of their order of appearance in schedules for analysis of a series of n empiric values, n increasing from 3 to 100, sine or cosine values equal to 0, $\frac{1}{2}$ or 1 being so stated rather than identified by a serial T-number), is a convenience in saving labor and space. Pollak has previously published[1] a set of tables giving multiples of each of the first 120 "T-numbers." The sine or cosine values of the "Appendix" were computed by Pollak, using the angles to twelve decimals of a degree, as presented later in the "Index," and interpolating linearly, extending the interpolation to the second differences, with the ten-place values given in E. ENGEL's tables of sines, cosines, and tangents [*MTAC*, v. 1, p. 131, 170–171].

The "Register" presents values of angles in increasing order of magnitude, accompanied by their identifying T-number. These angles are given both in degrees, minutes, and seconds to the 10^{-10} second (reliable to only 10^{-8} second), and in degrees and decimal fractions of a degree to 10^{-12} degree. Each angle given in degrees and decimal parts of a degree was derived individually by dividing the product $360°i$ by n, the number of equidistant empiric values in a period. The last figure retained, in each case, was rounded off. Accuracy of these values was insured by the authors, using an inde-

pendent check process. The rounded off values thus obtained were then converted into minutes and seconds. Since the rounding-off process limits significance to $\pm 5 \cdot 10^{-13}$ degree, or 1.8×10^{-9} seconds, the presentation of values to 10^{-10} second in this table is misleading.

The "Index" presents the same angular values (iz), together with their complements ($\lambda z'$), and the multiplying factors $\sin iz = \cos \lambda z'$ given to the eighth decimal place, arranged in ascending magnitude of n, and also, consequently, of their identifying T-numbers, both of which are furnished. All the cosines and sines were computed twice, once by Miss NUALA O'BRIEN, using PETERS'[2] Eight-Place Tables of Trigonometric Functions, and independently by Pollak (the values given in the Appendix), and, in addition, a schedule-by-schedule check was made, so that the error is less than 5×10^{-9}.

The present work fills an apparent need for a set of such tables, giving sine, cosine, and angular values to greater than usual accuracy [cf. *MTAC*, v. 2, p. 307]. Because of the frequent practice of those having limited library funds, who usually purchase only one—the most accurate—set of a given type of tables, and therefore, in the present case, might choose to purchase only the *Supplement*, without the *Schedules*, it seems to the reviewer that repetition of much of the material included in the "Introduction" to the *Schedule* might have been advisable. This included an excellent, condensed, presentation of the procedures used in analysis and synthesis, for grouped data, non-equidistant values, mean values, and values having non-cyclic change, together with WALKER's short-cut method, and formulae for computation of error. It also seems that a much more comprehensive bibliography would have been appropriate.

The quality of printing and of the paper used in this *Supplement*, although somewhat inferior to that used for the *Schedules*, is good.

MARCELLA L. PHILLIPS

NBS

[1] L. W. POLLAK, *Rechentafeln zur Harmonischen Analyse*. Leipzig, 1926.

[2] J. PETERS, *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*. Berlin, 1939.

848[K].—J. WESTENBERG, "A tabulation of the median test for unequal samples," Nederl. Akad. Wetensch., *Proc.*, v. 53, 1950, p. 77–82, = *Indagationes Math.*, v. 12, 1950, p. 8–13.

Given that in a pooled sample of $N_1 + N_2$ from a univariate statistical universe $2n$ observed values of the variate do not and $N_1 + N_2 - 2n$ do exceed a given critical value, the author writes the joint probability that in the sample of $N_1, n + \Delta$ values and in the sample of $N_2, n - \Delta$ values do not exceed this value. The critical value is taken to be the median of the pooled sample. Then the number of values in the sample of N_1 lying between the median of this sample and the median of the pooled sample is $\frac{1}{4}(N_2 - N_1) + \Delta = \delta$. The author remarks that the above probability is maximal for $\delta = 0$ and is symmetrical with respect to N_1 and N_2. He then tabulates the values of $|\delta|$ to 1D that will be exceeded in such samples with probabilities .001, .005, .01(.01).05 for N_1 and $N_2 = 6, 10, 20, 50, 100, 200, 500, 1000, 2000$. The mode of calculation is not described but evidently $|\delta|$ is treated as a continuous variable whereas in actual samples $|\delta|$ is always an integer.

The experimenter can then tell from the table where his δ lies with respect to the given percentage points. A second table and logarithmic chart give for the same set of values of N_1 the minimal values of N_2 to 1D (A remark similar to that with regard to fractional values of δ could be made here.) for which the two samples could result in a $|\delta|$ at the significance levels given above. The author gave the results for $N_1 = N_2$ in a previous paper.[1]

C. C. C.

[1] J. WESTENBERG, "Significance tests for median and interquartile range in samples from continuous populations of any form," Nederl. Akad. Wetensch., *Proc.*, v. 51, 1948, p. 252–261.

849[K].—A. VAN WIJNGAARDEN, "Table of the cumulative symmetric binomial distribution," Nederl. Akad. Wetensch., *Proc.*, v. 53, 1950, p. 857–868, = *Indagationes Math.*, v. 12, 1950, p. 301–312.

The function tabled is

$$P(n, c) = 1 - 2^{1-n} \sum_{s=0}^{c} \binom{n}{s} = 2^{-n} \sum_{s=c+1}^{n-c-1} \binom{n}{s}$$

to 5D for $n = 1(1)200$ and $c = 0(1)\left[\frac{n-1}{2}\right]$. Noting that for n even

$$P\left(n, \frac{n-2}{2}\right) = 2^{-n}\binom{n}{n/2},$$

we see that the table enables one to find all the partial sums in $(\frac{1}{2} + \frac{1}{2})^n$ for $n \leq 200$. In order to save space the tabulation is made with the arguments c and $n - 2c$. Values of $P(n, c)$ to 7D for $n = 1(1)49$ are also available in the tables recently published by the National Bureau of Standards.[1] The reviewer compared the two tables for $n = 48$ and 49 as a check of the present author's assertion that his accuracy "is well under one unit of the last (fifth) decimal place." The differences were less than 1 unit in this place but for $n = 48$, $c = 9, 16$; $n = 49$, $c = 16, 17$ the final digit differed by unity from the rounded off value from the NBS tables. In the present tables no signs are indicated for final 5's.

C. C. C.

[1] NBS, *Tables of the Binomial Probability Distribution*. NBS Applied Math. Series, No. 6, Washington, 1950 [*MTAC*, v. 4, p. 208–209].

850[L].—M. ABRAMOWITZ, "Tables of integrals of Struve functions," *Jn. Math. Phys.*, v. 29, 1950, p. 49–51.

The standard notations[1] (p. 328–329) are used, and the integrals in question are

$$\bar{H}_n = \int_0^x H_n(t)dt, \qquad \bar{L}_n = \int_0^x L_n(t)dt.$$

$\bar{H}_0$, $\bar{H}_1$ are tabulated for $x = 0(.1)5$ to 6D and for $x = 5(.1)10$ to 5D; $\bar{L}_0$, $\bar{L}_1$ for $x = 0(.1)5$ to 6D, for $x = 5(.1)10$ to 6S. The tabulated values were obtained by numerical integration, using WATSON's tables[1] (p. 666–685) for $H_n(t)$ and tables of the Computation Laboratory of the NBS for $L_n(t)$. Spot checks were made by using the series expansion of the integrals in powers

of x. It is stated that in general, interpolation with five-point Lagrangean interpolation coefficients will yield the full accuracy of the tables.

The integrals arose in a paper by LEVINE & SCHWINGER.[2]

A. E.

[1] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. Cambridge, 1922.
[2] H. LEVINE & J. SCHWINGER, "On the theory of diffraction by an infinite plane screen. I," *Phys. Rev.*, s. 2, v. 74, 1948, p. 958–974.

851[L].—S. CHANDRASEKHAR & G. MÜNCH, "On the integral equation governing the distribution of the true and the apparent rotational velocities of stars," *Astrophys. Jn.*, v. 111, 1950, p. 142–156.

The authors discuss the numerical solution of the integral equation

$$g(y) = y \int_y^\infty x^{-1}(x^2 - y^2)^{-\frac{1}{2}} f(x)dx. \tag{1}$$

The analytical solution involves differentiation and hence is not very reliable when $g(y)$ is given in the form of a histogram, and the authors hold that any direct numerical solution of (1) is likely to encounter the same difficulty. They advocate the use of either of two methods. (i) Determine $f(x)$ from its moments by means of the formula (given in the paper) for converting moments of g into moments of f. (ii) Assume a shape for $f(x)$ and determine the parameters by fitting the corresponding g to the observed $g(y)$.

In following up the second alternative, they use

$$f(x, x_1) = \pi^{-\frac{1}{2}}\{e^{-(x-x_1)^2} + e^{-(x+x_1)^2}\}.$$

For the corresponding $g(y, x_1)$ they give an integral representation, several expansions, and some numerical material.

Table 1(p. 148) gives

$$\bar{x} = \pi^{-\frac{1}{2}}e^{-x_1^2} + x_1\Phi(x_1) \text{ to 4S}, \quad \overline{x^2} = x_1^2 + \tfrac{1}{2} \text{ to 2D},$$

$$\overline{x^3} = \pi^{-\frac{1}{2}}e^{-x_1^2}(1 + x_1^2) + (\tfrac{3}{2} + x_1^2)x_1\Phi(x_1) \text{ to 3S},$$

and

$$\bar{x}/[2(\overline{x^2} - \bar{x}^2)]^{\frac{1}{2}} \text{ to 4 or 5S for } x_1 = 0(.1)1(.2)3.$$

Here

$$\Phi(x_1) = 2\pi^{-\frac{1}{2}} \int_0^{x_1} e^{-t^2}dt.$$

Table 2 (p. 150) gives $F_n(y)$ to 4 to 6D for $n = 0(1)6$ and $y = 0(.2)2.6$, where

$$F_n(y) = \pi^{-\frac{1}{2}}2^{2n+1}I_n(y)/(2n)!,$$

$$I_0(y) = \tfrac{1}{2}\pi\{1 - \Phi(y)\}, \qquad I_1(y) = \tfrac{1}{2}\pi^{-\frac{1}{2}}ye^{-y^2},$$

and

$$I_{n+1} = (n - \tfrac{1}{2} + y^2)I_n - (n - 1)y^2I_{n-1}, \qquad n = 1, 2, \cdots.$$

Table 3 (p. 153) gives

$$g(x_1, x_1) = \pi^{-\frac{1}{2}}x_1 \int_{x_1}^\infty x^{-1}(x^2 - x_1^2)^{-\frac{1}{2}}e^{-(x-x_1)^2}dx \text{ to 4D}$$

for $x_1 = 2(.5)5(1)10$.

Table 4 contains the results of astronomical computations.

A. E.

852[L].—V. N. FADDEEVA & M. K. GAVURIN, *Tablit͡sy Funkt͡siĭ Besselia͡ $J_n(x)$ t͡selykh nomerov ot* 0 *do* 120 [Tables of Bessel Functions $J_n(x)$ of integral order from 0 to 120]. Pod redakt͡sieĭ L. V. KANTOROVICHA. (*Matematicheskie Tablit͡sy*, no. 2.) Akad. Nauk, SSSR, Matematicheskiĭ Institut imeni V. A. Steklova, Moscow and Leningrad Gostekhizdat, 1950, 440 p. An errata slip containing 23 corrections is inserted in this volume. 17 × 26 cm. Cloth, 21 roubles. An edition of 4000 copies was issued in April.

The attractive appearance of this publication of the Academy of Science is in marked contrast to no. 1 of the series [RMT**854**]. It contains the following four tables:

Table 1, p. 9–371: $J_n(x)$ for $n = 0(1)120$, $x = [0(.1)124.9;\ 6D,\ \delta^2]$. In $J_{120}(x)$ the first significant value .000001 is for $x = 95.4$.
Table 2, p. 373–382: Zeros <125 of $J_n(x)$ to 5D. There are 40 zeros for $J_0(x)$, 39 for $J_1(x)$, and the last is a single zero for $J_{115}(x)$. Most of the values given here are new.
Table 3, p. 383–388: Interpolation coefficients.
Table 4, p. 389–439: $J_n(x)$, $x = [0(.01)14.99;\ 8D]$, $n = 0(1)13$.

The authors state that it was not until their tables had been completed that they became acquainted with the first 8 volumes of the Harvard Bessel Function tables. With the twelfth and final volume of the Harvard tables we have at our disposal the values of $J_n(x)$, $n = 0(1)135$, $x \leq 100$, at interval $\leq .1$, to 10D at least.

For $x \leq 100$, all the values of $J_n(x)$ in the two new Russian tables are in the Harvard tables, which do not, however, give any explicitly stated zero values. Thus there is an appreciable amount of new results here. Not alone on account of difference in cost (less than 37 roubles as compared with $96) many workers will probably often find it convenient to turn to the two volumes of Russian tables, if it is found that they are reliable.

Since the Russians appear to have started the publication of a series of mathematical tables, already including two tables of Bessel functions, let us hope that the series will include the Tables of Bessel Functions with Complex Argument, announced in *Matematicheskiĭ Sbornik*, v. 51, 1941 [*MTAC*, v. 3, p. 66], but, as far as we know, never published.

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853[L].—C. W. HORTON, "A short table of Struve functions and of some integrals involving Bessel and Struve functions," *Jn. Math. Phys.*, v. 29, 1950, p. 56–59.

J_n is the Bessel function, H_n the Struve function,

$$C_n = \int_0^z t^n J_n(t)dt, \qquad D_n = \int_0^z t^n H_n(t)dt,$$

Tables of J_n, H_0, H_1 are known.[1] The present paper gives H_n for $n = 2(1)4$, C_n for $n = 1(1)4$, and D_n for $n = 0(1)4$, all for $z = 0(.1)10$ mostly to 4D. Values of H_n have been computed to 7D by means of the power series and

the recurrence relations, and then rounded off to 4D because of the coarseness of the interval.

Values of C_0 were taken from a table by LOWAN & ABRAMOWITZ,[2] and values of C_n obtained by means of numerical integration and a recurrence relation. For $n = 3$, 4 and $z \geq 6$ only 3D are given.

The situation with regard to D_n is similar, except that here a table by J. W. WRENCH[3] was the point of departure.

It is believed that the maximum error is .6 units of the last decimal.

A. E.

[1] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. Cambridge, 1922, p. 666–685.

[2] A. N. LOWAN & M. ABRAMOWITZ, *Jn. Math. Phys.*, v. 22, 1943, p. 3–12 [*MTAC*, v. 1, p. 154].

[3] *MTAC*, v. 3, p. 66.

854[L].—L. A. LIÛSTERNIK, I. Îa. AKUSHSKIĬ & V. A. DITKIN, *Tablitsy Besselevykh Funktsiĭ*. (*Matematicheskie Tablitsy*, no. 1.) [Tables of Bessel Functions. (Mathematical Tables, v. 1)]. Moscow and Leningrad, Gostekhizdat, 1949, 430 p. 14.6 × 22 cm. Boards, 15.70 roubles. An edition of 10000 copies published in March.

This volume contains the following seven tables, several of which are new:

Table 1, p. 8–345: $J_0(x)$, $J_1(x)$ for $x = [0(.001)25; 7D]$, Δ^2.
Table 2, p. 349: 8D values of zeros α_k of $J_0(x)$, β_k of $J_1(x)$, γ_k [except 7D for $k = 1(1)10$] of $J_1'(x)$ for $k = 0(1)40$.
Tables 3–6, p. 350–429: $J_0(\alpha_k x)$, $J_0(\beta_k x)$, $J_1(\beta_k x)$, $J_1(\gamma_k x)$ for $x = [.01(.01)1; 7D]$, $k = 0(1)40$.
Table 7, p. 430: $2[J_1^2(\alpha_k)]^{-1}$, $2[J_0^2(\beta_k)]^{-1} = 2[J_2^2(\beta_k)]^{-1}$, $2\gamma_k^2(\gamma_k^2 - 1)^{-1} \times [J_1^2(\gamma_k)]^{-1}$, for $k = 1(1)40; 7S]$.

These last four tables are new and are for calculating terms in the development of a function in a Fourier-Bessel series. From data in the volume it is easy to verify the accuracy of the second and third of these functions, but in the case of the first it is not nearly so easy since the values of $J_1(\alpha_k)$ are not here given—but are, of course, readily available elsewhere.

Most of the values of γ_k are also new; but all the other values of the functions are implied in the Harvard and BAAS tables. The typography of the volume is very unattractive, the paper poor, and the proofreading bad. For example β_{40} is given as 126.1461387, instead of 126.4461387, on pages 388, 389, 408, 409; and there are other errors on pages 6, 7, 340, 347, 348, 374.

R. C. ARCHIBALD

855[L].—J. P. STANLEY & M. V. WILKES, *Table of the Reciprocal of the Gamma Function for Complex Argument*. Computation Centre, University of Toronto, 1950. i + 100 p., 35.4 × 25.1 cm. Price $4.50.

The table is that briefly described in UMT 102 (*MTAC*, v. 4, p. 162).

$1/\Gamma(x + iy)$ was computed for $x = -.5(.01).5$, $y = 0(.01)1$ from the infinite series in powers of $x + iy$: twenty-one terms of the series were used. The computations were carried out on the EDSAC in the Mathematical Laboratory at Cambridge University, and checked by differencing in both

directions on a National accounting machine. In order to minimize rounding-off errors, ten decimals were carried throughout the process. In the present volume values are given to 6D, and the authors estimate that the maximum error does not exceed .7 units of the sixth decimal place.

Values of the reciprocal gamma function outside the range of tabulation can be obtained by one or the other of the functional equations satisfied by the gamma function.

The preface (1 p.) gives the following references to available numerical values of the gamma function in the complex domain:

J. G. BECKERLEY, *Indian Jn. Physics*, v. 15, 1941, p. 229–232 [RMT **195**, *MTAC*, v. 1, p. 419–420].
H. T. DAVIS, *Tables of the Higher Mathematical Functions*, 1933, p. 269f.
A. GHIZETTI, Accad. Naz. Lincei, *Atti Rend.*, s. 8, v. 3(2), 1947, p. 254–257 [RMT **617**, *MTAC*, v. 3, p. 415–416].
M. E. LONG, Radar Research Development Establishment, *Memorandum* no. 96, 1945 [RMT **234**, *MTAC*, v. 2, p. 19].
W. MEISSNER, *Deutsche Mathematik*, v. 4, 1939, p. 537–555 [RMT **140**, *MTAC*, v. 1, p. 177].
C. P. WELLS & R. D. SPENCE, *Jn. Math. Phys.*, M.I.T., v. 24, 1945, p. 51–64 [RMT **228**, *MTAC*, v. 1, p. 446].

The preface states that the present table has been checked against all the available values, but it does not state whether any discrepancies were found.

It should be noted that in connection with the tabulation of certain parabolic cylinder functions, a table of $\ln \Gamma(x + iy)$ for $x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ is being prepared at the Scientific Computing Service, London, by J. C. P. MILLER [*MTAC*, v. 2, p. 62–63, 147–148, v. 4, p. 90].

A. E.

856[R].—New Zealand Department of Lands and Survey, *Geodetic and Transverse Mercator Projection Tables. Latitudes* 34° *to* 48°. (*International Spheroid*), Wellington, 1946, 96 p. 22 × 33.3 cm.

These are tables of special functions for the Transverse Mercator projection for an area included between latitudes 34° and 48° and longitude 3° each side of a chosen central meridian.

The formula for the x-coordinate in the Transverse Mercator projection may be written

$$x = m + G(\Delta\lambda)^2 + I(\Delta\lambda)^4$$

where m is the meridional distance from the equator to latitude ϕ. G and I are functions of ϕ and of the constants of the meridian ellipse. These functions are tabulated as well as analogous functions for the Transverse Mercator y-coordinate, and for the various inverse formulas expressing latitude and longitude in terms of x and y, etc.

Notable points concerning these tables are:

1. The meridian distance, though tabulated only from 34° to 48°, is given for one minute intervals accurately to 1/1000 link [1 link = 7.92 in.].

2. Detailed examples are given of the geodetic computations involved with the formulas and of the interpolation procedure for the tables.
3. All quantities are in multiples or submultiples of links which would make it necessary to use conversion factors for application to areas where triangulation distances are in meters or feet.

The only detected tabular errors occur on p. 21 and p. 31 and are noted in the volume.

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MATHEMATICAL TABLES—ERRATA

In this issue references have been made to Errata in RMT **854** (Liûsternik, Akushskiĭ & Ditkin).

179.—G. F. BECKER & C. E. VAN ORSTRAND, *Smithsonian Mathematical Tables, Hyperbolic Functions*, Washington, fifth reprint, 1942 [*MTAC*, v. 1, p. 45].

On p. 314, in the table of the anti-gudermannian, the value of 43°3′,

for 2667.20 *read* 2867.20

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180.—A. M. LEGENDRE, *Traité des Fonctions Elliptiques*, v. 2, Paris, 1826. In Chapter 3, p. 56 and 58, corresponding to $n = 4$, the coefficient of $\delta^6 f_0$

for $421/(4725 \cdot 2^{10})$ *read* $1/3024$

On p. 58, corresponding to $n = 5$ and $n = 6$ the coefficients of $\delta^4 f_0$,

for $-5/384$ *read* $1/384$
for $-23/1440$ *read* $1/120$

H. E. SALZER

NBSCL

UNPUBLISHED MATHEMATICAL TABLES

110[E].—RICHARD R. KENYON, *Table of* $x^n e^{-x}$. 3 leaves and a graph deposited in the UMT FILE. Photostat.

This is a table of $x^n e^{-x}$ to 5S or 6S for $n = 0(1)8$ and $x = 0(.01).1(.1)5(1)30(5)60$. A graph is included with the tables to show the behavior of the function. It allows rough graphical interpolation to be made for non-integral values of n.