

In a previous discussion of a paper by CARRUS and TREUENFELS (CT), a difference test indicated that some of the early zeros of the associated Legendre function $P_n^1(\cos \theta) = 0$ as a function of n were incorrect (*MTAC*, v. 5, p. 152–153). The investigation of the present article also reveals some errors. The authors give an alternative proof of an equation due to MACDONALD² for determining the early zeros of $P_n^m(\cos \theta) = 0$ where θ is near π . For $m = 1$, $\theta = 165^\circ$, this formula gives 1.035 as an approximation to the first zero. Employing power series, it is shown that the first zero must be between 1.0316 and 1.0321. The value reported by CT is 1.053 and so is in error. Application of the Macdonald formula shows that for $165^\circ \leq \theta \leq 180^\circ$, the corresponding values of n decrease with increasing θ . For $m = 1$, $\theta = 170^\circ$, the first zero given by CT is 1.05 and thus is also incorrect. Numerical analysis of early zeros for values of θ other than those cited above is not given, but sufficient evidence now exists to show that the CT tables should be used with caution.

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¹ P. A. CARRUS & C. G. TREUENFELS, "Tables of roots of incomplete integrals of associated Legendre functions of fractional orders," *Jn. Math. Phys.*, v. 29, 1951, p. 282–299 [*MTAC*, v. 5, p. 152–153].

² H. M. MACDONALD, "Zeros of the spherical harmonic $P_n^m(\mu)$ considered as a function of n ," *London Math. Soc., Proc.*, s. 1, v. 31, 1900, p. 264–278.

MATHEMATICAL TABLES—ERRATA

In this issue references to Errata have been made in RMT's 989, 990, and 991.

203.—AKADEMIĀ NAUK, SSSR. *Tablitsy znachenii Funktsii Besseliâ ot mnimogo Argumenta.* [*MTAC*, v. 5, p. 151–152.]

p.	x	Function	For	Read
10	.444	ΔiH_0	2367	2267
19	.899	ΔiH_0	667	657
42	2.031	H_1	593738	493738
42	2.032	H_1	481922	381922
106	5.237	iH_0	153939	132939
114	5.650	ΔH_1	788	782
115	5.700	x	5.605	5.700
118	5.815	ΔH_1	476	469
118	5.816	ΔH_1	449	456
163	8.061	ΔH_1	949	849
166	8.235	ΔH_1	001	81991
195	9.654	iH_0	276029	276022
205	.074	ΔK_1	82290	81290
206	.139	ΔK_0	76	876
220	.815	ΔK_0	693	683
220	.848	ΔK_0	645	685
230	1.312	K_0	380745	381745

p.	x	Function	For	Read
238	1.719	$J_{\frac{1}{2}}$	20163	30163
249	2.295	$J_{\frac{1}{2}}$	889777	869777
251	2.369	$J_{-\frac{1}{2}}$	942091	941991
253	2.491	$J_{\frac{1}{2}}$	318266	308266
254	2.538	$J_{\frac{1}{2}}$	480753	490753
254	2.539	$J_{\frac{1}{2}}$	506447	516447
257	2.663	$J_{\frac{1}{2}}$	864568	884568
260	2.803	K_1	926830	926820
264	3.044	ΔK_1	565	505
264	3.047	ΔK_1	394	334
269	3.262	$J_{-\frac{1}{2}}$	739537	739587
269	3.293	$J_{-\frac{1}{2}}$	271376	271326
276	3.629	ΔK_0	122	128
293	4.480	$\Delta J_{\frac{1}{2}}$	779	784
294	4.502	ΔK_0	588	582
304	5.029	ΔK_0	145	140
374	8.533	K_0	300300	300250
375	8.576	K_1	118097	118197
392	9.423	ΔK_0	283	263

Besides the above errata, 112 errors of less than five units in the last place were noticed. For further errata see *MTAC*, v. 5, p. 152.

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204.—BAASMTTC, *Mathematical Tables*, v. 1. Cambridge, 1946, 2nd ed.

Table II—Circular Functions

page 7, $x = 48.6$

for .0945447099 79701 read .0945447098 79701

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205.—N. W. McLACHLAN & P. HUMBERT, *Formulaire pour le Calcul Symbolique*. Mémorial des Sciences Mathématiques, fasc. 100, 1941.

p. 4, formula 2; upper limit of second integral: for t read $\pi^2/2$.

p. 5, formula 11; omit the index t on the left hand side.

p. 7, formula 3; same correction to second integral as on p. 4.

p. 14, formula 2; omit b^{2r+1} on the right hand side.

p. 14, last formula; for 1 on the r.h.s. read p .

p. 32, formula 10; to the l.h.s. add $(1/\pi) \log [(t-b)/(t+b)] J_0(ay)$.

p. 34, formulae 3, 4; to the l.h.s. add $\pm(i/\pi) \log [(t-b)/(t+b)] J_0(ay)$, respectively.

p. 34, formulae 5, 6; to the l.h.s. add $\pm(it/\pi) \log [(t-b)/(t+b)] J_0(ay)$, respectively.

p. 34, delete formulae 7, 8.

p. 55, formula 1; for n above \sum read m . After the formula add $m = \frac{1}{2}n$, n even; $m = \frac{1}{2}(n-1)$, n odd.

- p. 58, formula 1; should be on p. 19 in § 3, since $|\sin t|$ is not discontinuous.
 p. 61, line below Règles; for $f_2(x)$ read $f_2(y)$, and for $\phi_2(p)$ read $\phi_2(q)$.

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206.—N. W. MCLACHLAN, P. HUMBERT & L. POLI, *Supplément au Formulaire pour le Calcul Symbolique*, *Mémoires des Sciences Mathématiques*, fasc. 113, 1950.

- p. 4, formula 4; upper limit of second integral; for x read $\pi x^2/2$.
 p. 6, third formula from bottom; for \sqrt{ix} read $i\sqrt{ix}$.
 p. 7, last formula; same correction as on p. 4.
 p. 8, formula 4; for $0 \leq x < \infty$ read $0 < x < \infty$.
 p. 10, line 19; see correction to p. 14 in Fascicule 100.
 p. 11, line 11; see corrections to p. 34 in Fascicule 100.
 p. 19, last line but one; for K read k .
 p. 29, formulae 3, 4; for $\frac{\sin a\sqrt{3}t}{2}$ read $\sin(a\sqrt{3}t/2)$.
 p. 30, formula 3; for 1 in parentheses read p .
 p. 44, formulae 9, 10; for $H_0^{(1),(2)}$ read $H_{2\nu}^{(1),(2)}$.
 p. 46, formula 2; for e^{-b} read $e^{-b\pi}$.
 p. 47, formula 4; the argument of the second gamma function should be $(-\mu - \nu + \frac{1}{2})$.

N. W. MCLACHLAN

207.—E. OBERG & F. D. JONES, *Machinery's Handbook*. 14th edition, New York, 1950 (and earlier editions).

Table of Gear Ratios and Decimal Equivalents, p. 708–711 (different page number in earlier editions)

Insert in appropriate positions

Decimal Equivalent	Gear Ratio
.6944	25/36
.7956	34/45

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208.—K. PEARSON, *Tables of the Incomplete Beta-Function*. Cambridge, 1934.

On pages 2 and 3, line 1, $p = q = \frac{1}{2}$ in the value of the complete Beta function

for 3.14159245 read 3.14159265

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209.—J. V. USPENSKY, *Introduction to Mathematical Probability*, 1937.

On page 407, Table of the Probability Integral
for $\phi(z) = .499997$ read $.500000$

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UNPUBLISHED MATHEMATICAL TABLES

In this issue an Unpublished Manuscript Table is referred to in RMT 990.

144[F].—A. GLODEN & J. BONNEAU, *Factorization of $N^4 + 1$ for isolated values of N between 30000 and 40000*. One page typewritten manuscript. Deposited in UMT FILE.

The table contains 88 factorizations, all complete. No primes are given. [For previous tables of this kind see *MTAC*, v. 2, p. 211, 252, 300; v. 3, p. 21, 118–119, 486; v. 4, p. 24; v. 5, p. 133–134.]

145[D, F].—D. H. LEHMER, *Table of Cyclotomic Cosines*. Ten manuscript pages tabulated from punched cards. On deposit in the UMT FILE. Also available on punched cards.

The table gives 20D values of

$$2 \cos 2\pi k/p \quad \text{for } k = 1(1)(p - 1)/2$$

for every odd prime $p < 100$. There are 517 values in all. Thus the table gives twice the real parts of the p -th roots of unity.

146[F, L].—D. H. LEHMER, *Table of Kloosterman Sums*. Twenty manuscript pages tabulated from punched cards. On deposit in the UMT FILE. Also available on punched cards.

The table gives 19D values of

$$S_p(k) = \sum_{n=1}^{p-1} \exp \{2\pi i(kn + \bar{n})/p\} \quad (n\bar{n} \equiv 1 \pmod{p})$$

for $k = 1(1)p - 1$ and for every odd prime $p < 100$. The table was computed from UMT 145, and contains 1034 entries. These sums appear in Fourier coefficients of many elliptic modular functions.

AUTOMATIC COMPUTING MACHINERY

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TECHNICAL DEVELOPMENTS

THE SERIAL-MEMORY DIGITAL DIFFERENTIAL ANALYZER

Introduction. In January, 1950, the first model of a digital differential analyzer became a working reality. This machine was entirely contained