5. Specific Example of Error Bounds on the Approximate Solution. As yet, no mention has been made of how γ should be chosen once D is given. Generally speaking, the choice would be to minimize k, but this indeed is a serious problem.[1] Without looking for the optimum choice, inspection of D may give some guide. However, a simple choice for γ is $e = \sum_{j=1}^{n} e_j$. For example, with $D = (d_{ij})$, let

$$k = \max_j \sum_i |d_{ij}|,$$

$$f_i = \max_j |d_{ij}|.$$

It is then seen that

$$e'\alpha(D) \leqslant ke',$$

$$e_i'\alpha(D) \leqslant f_i e'.$$

If $k < 1$, (7) gives

$$\alpha\left[\sum_{h=1}^{\infty} e_i' D^h\right] \leqslant \frac{f_i}{1-k} e',$$

whence from (9)

$$\alpha(e_i'\epsilon) \leqslant \frac{f_i}{1-k} e'\alpha(x). \tag{10}$$

Clearly, with the methods presented, many other error bounds such as (10) are possible.

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[1] The referee makes the following comments:

"If all $|d_{ij}| > 0$, the optimum choice of γ is unique and is the eigenvector u_1 of $\alpha(D)$ whose components are all positive, and k is then the dominant eigenvalue λ_1 of $\alpha(D)$. This follows from a lemma that, since all $g_i > 0$, λ_1 lies strictly between the minimum and maximum of the ratios $e_i'\alpha(D)\gamma/e_i'\gamma$, unless the ratios are all equal (and hence equal to λ_1). The lemma is a slight extension of Theorem I of Hazel Perfect, 'On matrices with positive elements,' *Quart. Jn. of Math.*, s. 2, v. 2, 1951, p. 286–290.

"The vector $\alpha^p(D)e$, which is asymptotically a multiple of u_1 as $p \to \infty$, may be a useful approximation to γ for sufficiently large p."

RECENT MATHEMATICAL TABLES

1065[A].—H. S. Uhler, "Many-figure approximations for $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$, and $\sqrt[3]{9}$ with χ^2 data," *Scripta Math.*, v. 18, 1952, p. 173–176.

Values of the constants mentioned in the title are given to 710, 709, 478, and 478 decimals respectively.

The author reports errata in the values of the cube roots of 2 and 3 as given by Boorman.[1] These are correct only to 198D and 54D instead of the 305D and 241D given.

There is given also data on the distribution of the digits of each of the four cube roots. Values of χ^2 and the associated probabilities are given for the first 50 k digits for $k = 1, 2, \cdots$. No unusual distributions are revealed.

D. H. L.

[1] J. M. Boorman, "Square-root notes," *The Mathematical Magazine*, v. 1, 1887, p. 208.

1066[D, R].—HEINZ WITTKE, (1) *Geodätische Registertafeln. Schnell-Rechentafel. Fünfstellige Winkelfunktionen. 400ᵍ mit direkter zweistelliger Interpolation und Fehlergrenzen.* Goslar-Harz, Selbstverlag, 1952, 60 p., 16.2 × 24 cm., 9.80 Marks bound. (2) *Geodätische Registertafeln Schnell-Rechentafel. Sechsstellige Winkelfunktionen. 400ᵍ zur Maschinenrechnungen.* Goslar-Harz, Selbstverlag, 1953, 59 p., 16.7 × 25.5 cm., 14.00 Marks bound.

These very neatly prepared geodesy tables, each with many label flap guides, contain tables of sin, tan, cot, cos, for interval hundredth of a grade, differences being printed in red. In each is introductory material with illustrative examples. In (1) are a few useful supplementary tables; and in (2) are sine and cosine tables for [$0(0^g.1)100^g$; 11D].

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1067[F].—H. GUPTA, "A table of values of $N_2(t)$," East Panjab Univ., *Research Bulletin*, no. 20, 1952, p. 13–93.

The table gives for each possible integer $t \leq 20000$ all non-negative integral solutions x, y of

$$x^2 + y^2 = t. \tag{1}$$

There is also given the sum function

$$N_2(t) = \sum_{j=0}^{t} n_2(j)$$

where $n_2(t)$ denotes the number of solutions (x, y) of (1), the solutions (x_0, y_0) and (y_0, x_0) being counted as different when $x_0 \neq y_0$.

This table is mentioned as existing in manuscript in *MTAC*, v. 3, p. 22, and was used to prepare the author's table of $N_3(t)$, the number of solutions of

$$x^2 + y^2 + z^2 \leq t.$$

The author states that

$$N_2(t) = \sum_{0 \leq j \leq t^{\frac{1}{2}}} [(t - j^2)^{\frac{1}{2}}] \tag{2}$$

but this is in contradiction with the definition of $N_2(t)$ and its tabulated values. In fact the terms

$$1 + [t^{\frac{1}{2}}]$$

should be added to the right hand side of (2).

The table can be used to find the exact number of lattice points inside or on a circle of radius r by means of the formula

$$4N_2(r^2) - 4r - 3.$$

Thus the number of lattice points inside and on a circle of radius 140 is

$$4N_2(19600) - 560 - 3 = 61529.$$

The area of the circle is greater than this by the factor 1.000751.

D. H. L.

1068[F].—OVE HEMER, *On the Diophantine Equation* $y^2 - k = x^3$. Uppsala, 1952, 101 p., 23.5 × 17.6 cm.

The booklet is primarily concerned with the solutions of the equation

$$y^2 - k = x^3$$

for positive $k < 100$.

Table 1 (p. 94) purports to give all solutions for 47 of the 59 values of $k < 100$ for which solutions exist (10 of these were cases which were known previously), together with the number N of solutions. For $k = 37$, however, N is given as 2, but besides the two solutions which are listed, the solution (243, 3788) can be read from ROBINSON'S tables, which are on deposit in the UMT File [*MTAC*, v. 5, p. 162], and of which the author is apparently unaware. The discussion pertaining to the case $k = 37$ (p. 75–78) is therefore incomplete. In ten of the remaining 12 cases one solution is given and it is stated that $N \leq 2$. In the other 2 cases $k = 63$ and $k = 76$, two and one solutions are given respectively with the remark that "in all probability there are no more solutions."

There is only a one page discussion (p. 88–89) for negative k. A statement is made on p. 88 that $y^2 + 56 = x^3$ has no solutions. However, Robinson's table lists (18, 76) which is indeed a solution. Table 2 (p. 95) lists the 44 values of $k < 0$ for which solutions exist, together with the known solutions and their number. In 22 of the cases all solutions are known, in the remaining cases one or two solutions are given. An extra solution can be read from Robinson's table for $-k = 7, 28, 39, 47, 53, 55, 60$, and 63.

Table 3 gives solutions of $y^2 \pm 27k = x^3$ for $|k| \leq 50$. These were obtained with the help of CASSELS'[1] table of rational solutions [*MTAC*, v. 4, p. 202]. All solutions are supposed to be given, except for 11 values of k. For $k = -11$, however, the number of solutions is given as 8, while a ninth solution (1362, 50265) can be read from ROBINSON'S table. In two other cases, $k = 24$ and 37 which are not complete, Robinson gives an extra solution.

Table 4 gives the discriminants $-D$, the fundamental rings and units ϵ in the cubic field corresponding to 36 square free values of k between 2 and 100. In 10 of these cases ϵ is not "definitely proved to be a fundamental unit."

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[1] J. W. S. CASSELS, "*The rational solutions of the Diophantine equation* $y^2 = x^3 - D$," *Acta Math.*, v. 82, 1950, p. 243–273.

1069[F].—M. KRAITCHIK, "On the factorization of $2^n \pm 1$," *Scripta Math.*, v. 18, 1952, p. 39–52.

This expository article contains tables giving the complete factorization of $2^n \pm 1$ for those values of n for which such information was known to the author. There are four tables. The first is for $2^n - 1$ and for

$$n = 1(2)99, 105, 107, 111(2)117, 123, 127, 135.$$

The second is for $2^n + 1$ and for

$$n = 1(2)99, 105, 111, 123, 135.$$

The third is for $2^n + 1$ and for

$$n = 4(4)100, 108, 120.$$

The fourth is for $2^n + 1$ and for

$$n = 2(4)130, 138(4)162, 170(4)190, 198, 210, 222, 234, 250, 258, 270, 330.$$

The tables separate the algebraic factors from the primitive ones. The fourth table has column headings for the two factors in

$$2^{4k+2} + 1 = (2^{2k+1} - 2^{k+1} + 1)(2^{2k+1} + 2^{k+1} + 1).$$

These two factors are inadvertently interchanged for $n = 4k + 2 = 94$, 114, and 150.

An additional entry can be given, namely

$$2^{89} + 1 = 3 \cdot 179 \cdot 6202089 \cdot 1858477404602617.$$

The factor 6202089 was discovered by the SWAC. The large factor was proved prime by A. L. Brown and the reviewer independently.

Other additional entries may be made from Notes **131**, **138**, **142**, and **146**.

D. H. L.

1070[I].—J. M. Hammersley, "Lagrangian integration coefficients for distance functions taken over right circular cylinders," *Jn. Math. Phys.*, v. 31, 1952, p. 139–150.

The author considers the problem of evaluating the sixfold integral of the form

$$I = \int t(r) dv_1 dv_2 / r^2$$

where dv_1 and dv_2 are two volume elements, a distance r apart, in a cylinder of diameter a and length ca and $t(r)$ is an arbitrary function of r. The integral is approximated by

$$I = (a/2)^4 \pi^2 (cy_1 + y_2)$$

where

$$y_1 = \int_0^c t(a\xi) g(\xi) \xi^{-2} d\xi,$$

$$y_2 = \int_0^c t(a\xi) h(\xi) \xi^{-2} d\xi + t(ac) k(c).$$

The auxiliary functions g, and k involve the hypergeometric function

$$F(\alpha, 1/2; 2; x) \qquad (\alpha = -1/2, -3/2).$$

The integrals y_1 and y_2 are to be evaluated by the Lagrangian integration formulas

$$y_1 = \sum t(a\xi) G(\xi),$$
$$y_2 = \sum t(a\xi) H(\xi),$$

the sums being taken over equally spaced points ξ.

The author tabulates $G(\xi)$, $-H(\xi)$ and (for checking purposes) $G(\xi) + H(\xi)$ for

$$\xi = 0(.01)2(.05)3(.25)10; 8D$$

with the last place uncertain.

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NBSINA

1071[K].—T. W. ANDERSON & D. A. DARLING, "Asymptotic theory of certain 'goodness of fit' criteria based on stochastic processes," *Annals Math. Stat.*, v. 23, 1952, p. 193–212.

The table contained in this paper (p. 203) gives the argument z to 5D which satisfies $a_1(z) = \lim_{n\to\infty} \Pr(n\omega^2 \leqq z)$ for $a_1(z) = .01(.01).99, .999$. This is the limiting distribution for large samples of $n\omega^2$, a statistic attributed to VON MISES for testing the hypothesis that n independent, identically distributed random variables have a specified distribution function $F(x)$. ω^2 is defined by $\omega^2 = \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 dF$ where $F_n(x)$ is the sample cumulative distribution function for a sample of size n.

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1072[K].—Z. W. BIRNBAUM, "Numerical tabulation of the distribution of Kolmogorov's statistic for finite sample size," Amer. Stat. Assn., *Jn.*, v. 47, 1952, p. 425–441.

Let $X_1, X_2, \cdots, X_N$ be a random sample from a population having a continuous cumulative distribution function $F(x)$. Define the empirical distribution function $F_N(x)$ by $F_N(x) = j/N$ where j is the number of observed values of $X_1, \cdots, X_N$ which are less than or equal to x, and let

$$D_N = \operatorname*{l.u.b.}_{x} |F(x) - F_N(x)|.$$

Table I (p. 428–430) gives values to 5D of $P(D_N < c/N)$ for $N = 1(1)100$ and $c = 1(1)15$. Table II (p. 431) gives, for $N = 2(1)5(5)30(10)100$, values to 4D of the 95th and 99th percentiles of the distribution of D_N and compares these with the limiting values.

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1073[K].—A. H. BOWKER & H. P. GOODE, *Sampling Inspection by Variables.* McGraw-Hill Book Co., Inc., New York, 1952. xi + 216 p., 15.6 × 23.3 cm. Price $5.00.

This book is built around a set of single and double variables sampling inspection plans of the non-central t type. For normal variables the cases of known and unknown standard deviation, and two-sided as well as one-sided specifications, are included. No sequential variables plans are described, for reasons of incomplete theory and elaborate calculation.

For roughly corresponding discussion of single, double (and sequential) attributes sampling inspection plans, see Statistical Research Group, Columbia University, *Sampling Inspection*, New York, McGraw-Hill Book Company, Inc., 1948.

The heart of this book lies in Chapter 11 (pages 125–141) on the mathematics of the plans, and in tables and charts (pages 164–209), together with a number of other tables and charts scattered throughout the text. In addition, there is much material on sampling inspection generally, such as the installation and administration of sampling inspection plans, tightened and reduced plans, disposition of rejected product, non-normal populations, control charts in sampling inspection, comparisons of variables and attributes plans.

A few of the principal tables: Table A (necessary for later tables): for each of fourteen inspection lot sizes and for each of three inspection levels, a sample size letter (42 entries, 14 different). Table B: for single sampling, for each of the fourteen sample size letters (each here corresponding to a single sample size) and for each of fifteen AQL classes, ranging unevenly from .024–.035 to 8.5–11.0, the value of k to 3D in the acceptance criterion $\bar{x} + ks \leqq U$ or $\bar{x} - ks \geqq L$, where U and L are the upper and lower specification limits respectively, and AQL, $\bar{x}$, and s have the usual meanings. (The acceptable quality level, AQL, as here used, is the percentage of defective items in an inspection lot such that the inspection plan will accept 95% of all lots submitted containing that percentage of defects.) Table C: as in Table B, for double sampling, showing k_a, k_r, and k_t, all to 3D, corresponding to k in Table B, but for acceptance on the first sample, rejection on the first sample, acceptance on the second sample. Table D: for each of the fourteen sample size letters and for each of the fifteen AQL classes, the AQL and the LTPD. (The lot tolerance percent defective, LTPD, as here used is the percentage of defective items in an inspection lot such that the sampling plan will reject 90% of all lots submitted containing that percentage of defects.) Table E (the major table of the book): for each of the fourteen sample size letters, the operating characteristic curves of each of from 9 to 16 single and double sampling plans, each pair of plans showing the AQL class, the AOQL class, and the criteria for acceptance (163 curves in all). (The average outgoing quality limit, AOQL, is the maximum percent defective remaining in lots subjected to a plan if all rejected lots are screened, no matter what may be the quality of the lots submitted.)

This work is a sturdy accomplishment by the authors and a credit to all who pioneered in this area. Among the latter are H. Romig, H. Dodge, N. L. Johnson, B. L. Welch, W. J. Jennett, C. C. Craig, W. A. Wallis, K. J. Arnold, J. Wolfowitz, and the two authors.

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1074[K].—K. Freudenberg, "Die Grenzen für die Anwendbarkeit des Gesetzes der kleinen Zahlen," *Metron*, v. 16, 1951, p. 285–310.

The principal objective of this paper is to determine the smallest sample size required in order to obtain a given degree of approximation to the binomial distribution by means of the Poisson distribution and by means of the following, so to speak, second order approximating function

$$B_{\lambda,m} = \frac{e^{-\lambda}\lambda^m}{m!}\left[1 - \frac{(m-\lambda)^2 - m}{2s}\right], \tag{1}$$

where s is sample size, $\lambda = sq$ is the mean number of occurrences of an event of probability q in random samples of size s; λ is also the mean of the approximating Poisson distribution and $m \equiv$ an integer. The mean of $B_{\lambda,m}$ is also independent of s, and its variance is $\lambda(1 - \lambda s^{-1})$. In spite of the fact that the zeroth moment of $B_{\lambda,m}$ is unity independent of s, it is not a distribution, since some of its values can be negative. Since these negative probabilities are, however, exceedingly small, they do not vitiate the practical usefulness of $B_{\lambda,m}$ as an approximation to the Poisson and binomial distributions. In fact, the $B_{\lambda,m}$ require much smaller sample sizes for equivalent approximation to the binomial distribution than does the Poisson distribution. The Poisson and binomial are in this notation denoted as follows:

$$A_{\lambda,m} = e^{-\lambda}\lambda^m/m! \tag{2}$$

$$C_{\lambda,m} = \binom{s}{m}\left(\frac{\lambda}{s}\right)^m\left(1 - \frac{\lambda}{s}\right)^{s-m}. \tag{3}$$

The following tables are of principal interest: on p. 307 values of $s_{p,A}$ and $s_{p,B}$ as functions of $\lambda = \frac{1}{2}(\frac{1}{2})10$, which are respectively lower bounds for s such that the deviations of $A_{\lambda,m}$ and $B_{\lambda,m}$ from $C_{\lambda,m} \leqq p = .001$. $s_{p,A}$ varies from 141.5 to 640.8, where correspondingly $s_{p,B}$ goes from 7.8 to 78.8. On p. 308 $s_{p,A}$ and $s_{p,B}$ are given as functions of $q = .01(.01).10$, again for $p = .001$. Here $s_{p,A}$ varies from 406.0 to 4912.2 while $s_{p,B}$ goes from 0 to 34.1.

Auxiliary tables of some interest are found on p. 292–294. $A_{\lambda,m}$, $B_{\lambda,m}$, $C_{\lambda,m}$, $(A_{\lambda,m} - C_{\lambda,m})$, and $(B_{\lambda,m} - C_{\lambda,m})$ are given for $s = 50$ and 100 and for $\lambda = \frac{1}{2}, m = 0(1)7, > 7; \lambda = 1, \frac{3}{2}, m = 0(1)10, > 10; \lambda = 2, m = 0(1)11, > 11; \lambda = 3, m = 0(1)12, > 12; \lambda = 4, m = 0(1)13, > 13$.

For the approximation $B_{\lambda,m}$ to be of practical use in sampling, extensive tables would be required; the present table has limited usefulness. The reader's attention may be called to errors in formula (11) p. 289 and the subsequent argument. The right side of (11) should be λ and the subsequent argument can be corrected so as to yield this result. The findings of the paper are not affected by these errors.

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1075[K].—C. H. Goulden, *Methods of Statistical Analysis.* John Wiley & Sons, New York. 2nd ed., 1952, vii + 467 p., 15.2 × 23.7 cm. $7.50.

The tables in this volume present no novelty, consisting of areas and ordinates of the normal curve, percentage points for the Student-Fisher t, χ^2, and the variance ratio, values of the bias factor c_2 for the standard deviation in samples from normal, and an extract from the Fisher & Yates random numbers.

C. C. C.

1076[K].—A. HALD, *Statistical Tables and Formulas*. London, Chapman & Hall; New York, John Wiley & Sons, 1952. 97 p., 27.8 × 21.9 cm., $2.50.

The actual tables are preceded by 22 pages of pertinent formulas, explanations and illustrations. Tables I and II of ordinates and cumulative frequencies of the normal frequency function has the unusual feature of giving all values to 4S for the standardized argument $u = 0(.01)4.99$. Table III is a 3D probit table slightly abridged from FISHER & YATES.[1] The usual percentage points of the Student-Fisher t distribution are replaced by "fractiles" (Table IV) which are percentage points of the cumulative distribution. These are taken from the Fisher & Yates tables with the omission of 55% fractiles and the addition of 99.9% values as well as the set of values for 50, 80, 100, 200 and 500 degrees of freedom. Table V, fractiles of the χ^2 distribution, is reproduced from the HALD & SINKBAEK table[2] and gives values of χ_0^2 satisfying $P(\chi^2 < \chi_0^2) = p$ to 1D for $100p = .05$, .1, .5, 1, 2.5, 5, 10(10)90, 95, 97.5, 99, 99.5, 99.9, 99.95 and the degrees of freedom $f = 1(1)100$. Table VI gives fractiles of χ^2/f to 4D for $100p = .05$, .1, .5, 1, 2.5, 5, 95, 97.5, 99, 99.9, 99.95 and $f = 1(1)100(5)200(10)300(50)$-$1000(1000)5000$, 10000. Table VII, fractiles of the variance ratio, $\chi_1^2 f_2/\chi_2^2 f_1$, is an extension both of the Fisher & Yates tables and of those of MERRINGTON & THOMPSON.[3] Values are given to 3S for $100p = 50$, 70, 90, 99.9, 99.95 with $f_1 = 1(1)10$, 15, 20, 30, 50, 100, 200, 500, ∞; $f_2 = 1(1)20(2)30(10)60$, 80, 100, 200, 500, ∞ and for $100p = 95$, 97.5, 99, 99.5 with $f_1 = 1(1)20(2)$-$30(5)50$, 60, 80, 100, 200, 500, ∞; $f_2 = 1(1)30(2)50(5)70(10)100$, 125, 150, 200, 300, 500, 1000, ∞. Table VIII for fractiles of the range in samples from a normal universe has been reproduced in part from TIPPETT[4] and PEARSON.[5] Here values are given to 2D for $100p = .05$, .1, .5, 2.5, 5, 10(10)90, 95, 97.5, 99, 99.5, 99.9, 99.95 and the sample size $n = 2(1)20$. Values of the mean, standard deviation and their ratio for the range in samples from normal are given to 3D for the same set of sample sizes.

The next two tables are devoted to functions useful in the estimation of the mean, ξ, and the standard deviation, σ, of a one-sided truncated normal distribution with a known point of truncation according to methods given in HALD's text[6] of which these tables form the companion volume. In the case only a sample of n from the truncated range is used, for arguments y, calculable from the sample, $z = f(y)$ gives an estimate of the standardized point of truncation, ζ; another function, $s = \bar{x}g(z)$, where $\bar{x}$ is the sample mean, estimates σ; and $\bar{x}$ estimates ξ. Table IX gives z to 3D for $y = .55(.001).91$, $g(z)$ with its first differences to 4D for $z = -3(.1)2$, and the variances and covariance of $n^{\frac{1}{2}}\bar{x}/\sigma$ and $n^{\frac{1}{2}}s/\sigma$ to 4S and their coefficient of correlation to 3D also for $z = -3(.1)2$. For a one-sided censored normal distribution the sample consists of $n - a$ known values falling to the right of the known truncation point and a of whose values no more is known than that they fall to the left of the point of truncation. In this case ζ is estimated by $z = f(h, y)$ where $h = a/n$ and y is calculated from the known portion of the sample, σ is estimated from $\bar{x}g(h, z)$, where $\bar{x}$ is the mean of the known portion of the sample and $g(h, z)$ is simply obtained from the tabulated function $\psi'(z)$ and h, and again $\bar{x}$ estimates ξ. In Table X, $f(h, y)$ is given to 3D for $h = .05(.05).5$ and $y = .5(.005).6(.01).8(.05)1.5$; $\psi'(z)$ to 5S with first and second differences, the same variances and covariances as before to 4S and the same coefficient of correlation to 3D, all for $z = -3(.1)2$.

Table XI gives two-sided 95% and 99% confidence limits for the probability ϑ in the binomial distribution if the sample consists of x occurrences in n trials. Values are given to 3D for x and $n - x = 0(1)20(2)30(5)50, 60, 80, 100, 200, 500, \infty$. Table XII gives 2 arc sin $\sqrt{x}$ to 4D for $x = 0(.001)1$. Tables XIII of log $(n!)$ to 4D for $n = 1(1)1000$, XV of n^2 for $n = 0(1)999$, XVI of $n^{\frac{1}{2}}$ and $(10n)^{\frac{1}{2}}$ for $n = 1(.01)9.99$, XVII of n^{-1} to 5D for $n = 1(.01)9.99$, and XVIII a 4D table of common logarithms are all readily found elsewhere. But Table XIV of log $\binom{n}{x}$ to 4D for $x = 1(1)\left(\frac{n}{2}\right)$ and $n = 2(1)100$ appears to be an extension of previous tables. And the final Table XIX is a new set of 15000 random digits compiled from drawings in the Danish State Interest Lottery.

This is an excellent set of statistical tables containing a good deal of new material.

C. C. C.

[1] R. A. Fisher & F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*. London & Edinburgh, 1948.

[2] A. Hald & S. A. Sinkbaek, "A table of percentage points of the χ^2-distribution," *Skandinavisk Aktuarietidskrift*, v. 33, 1950, p. 168–175. [*MTAC*, v. 7, p. 25.]

[3] M. Merrington & C. M. Thompson, "Tables of percentage points of the inverted beta (F) distribution," *Biometrika*, v. 33, 1943, p. 73–88. [*MTAC*, v. 1, p. 78.]

[4] L. H. C. Tippett, "On the extreme individuals and the range of samples taken from a normal population," *Biometrika*, v. 17, 1925, p. 364–387.

[5] E. S. Pearson & H. O. Hartley, "The probability integral of the range in samples of n observations from a normal population," *Biometrika*, v. 32, 1942, p. 301–308. [*MTAC*, v. 1, p. 105.]

[6] A. Hald, *Statistical Theory with Engineering Applications*. New York & London, 1952.

1077[K].—T. Kitagawa, *Tables of Poisson Distribution*. Tokyo, Japan, Baifukan, 1951, 156 p., 18.7 × 26.0 cm., $3.50.

The integer-valued random variable X is said to have a Poisson distribution with parameter m if Pr $(X = x) = e^{-m}m^x/x! = p(x, m)$.

The tables give $p(x, m)$ for $x = 0(1)\cdots$, $m = .001(.001)1$ to 8D (Table I); $m = 1.01(.01)5$ to 8D (Table II); and $m = 5.01(.01)10$ to 7D (Table III).

These tables were prepared directly from their definition. No interpolation methods are suggested.

In the introductory material there is an English translation of a Japanese article by Kitagawa on the subject of double sampling inspection plans which illustrates some of the uses of the above tables.

In Table IV 95% and 99% confidence limits are given to 6D for the population parameter, m, with the sample mean $\bar{x} = .001, .005, .01, .02(.02).1, .2(.2)1(1)10$ and the sample sizes 50(10)100(100)500, 1000, 2000(2000)10000, 20000, 50000, 100000, when large sample theory is applicable.

Kitagawa prepared his tables before it was possible for him to learn about Molina's tables.[1] In these $p(x, m)$ is given to 6D for $m = .001(.001)$-$.01(.01).3(.1)15(1)100$ together with $P(c, m) = \sum_{x=c}^{\infty} p(x, m)$ to 6D for the same values of m and $c = 0(1)153$. Kitagawa's volume does not contain a table of either $P(c, m)$ or $1 - P(c + 1, m)$. In about one hundred comparisons between Molina's and Kitagawa's tables no discrepancies were found.

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[1] E. C. MOLINA, *Poisson's Exponential Binomial Limit*. New York, 1942. [*MTAC*, v. 1, p. 18.]

1078[K].—K. KUNISAWA, H. MAKABE & H. MORIMURA, "Tables of confidence bands for the population distribution function. 1.," Union of Japanese Scientists & Engineers, *Reports of Statistical Application Research*, v. 1, 1951, p. 23–44.

Let $(X_1, \cdots, X_n)$ be a sample of independent variables with the same continuous cumulative distribution function $F(x)$, and let $N(z)$ be the number of X_k's which are $\leqq z$. Setting $F_n^*(x) = N(z)/n$, the maximum D_n of $|F_n^*(z) - F(z)|$ is a random variable. Then KOLMOGOROV,[1] who introduced this statistic, showed that $n^{\frac{1}{2}}D_n$ has the asymptotic cumulative probability function $L(z) = 1 - 2 \sum_{j=1}^{\infty} (-1)^{j-1}e^{-2j^2z^2}$. This function was tabulated by N. SMIRNOV[2] to 6D, for arguments $z = .28(.01)3$. In the present paper the authors publish a table of $L(z)$ to 7D, for $z = .250(.001)2.299$, tabulating also the first differences.

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[1] A. KOLMOGOROV, "Sulla determinazione empirica di una legge di distribuzione," Inst. Ital. Attuari, *Giorn.*, v. 4, 1933, p. 83–91.

[2] N. SMIRNOV, "On the estimation of the discrepancy between empirical curves of distribution for two independent samples," *Bull. Math. de l'Univ. de Moscou*, v. 2, 1939, fasc. 2. 16 p. The table referred to was also reproduced in *Annals Math. Stat.*, v. 19, 1948, p. 279–281.

1079[K].—F. J. MASSEY, JR., "Distribution table for the deviation between two sample cumulatives," *Annals Math. Stat.*, v. 23, 1952, p. 435–441.

Given two ordered samples of sizes N_1 and N_2 from cumulatives $F_1(x)$ and $F_2(y)$. $S_{N_i}(x)$ is the proportion of the members of the sample N_i $(i = 1, 2)$ less than x. Let $d = \max |S_{N_1}(x) - S_{N_2}(x)|$. Table 1 (p. 436–439) presents all possible values of d_α and α, for $N_1, N_2 \leqq 10$, where

$$\alpha = \text{Prob } [d \leqq d_\alpha | F_1(x) = F_2(x)].$$

The table is in terms of N_1, N_2, h, and α; h is an integer, such that $d_\alpha = h/l$, where l is the lowest common multiple of N_1 and N_2 $(h \leqq l)$; and α is given to 5D.

Table 2 (p. 440–441) is the same for additional selected values of N_1 and N_2 except that h usually is not carried out to l.

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1080[K].—J. M. May, "Extended and corrected tables of the upper percentage points of the 'Studentized' range," *Biometrika*, v. 39, 1952, p. 192–193.

This paper contains tables of the upper percentage points of the 'Studentized' range $q = (x_n - x_1)/s$ for the 5% and the 1% levels of significance for $n = 2(1)20$ and $\nu = 1(1)20, 24, 30, 40, 60, 120, \infty$ where s^2 is a mean square estimate of σ^2 independent of the range, n is the sample size for the range, and ν is the number of degrees of freedom of s. Most entries are given to 2D. None contains fewer than 3S. These tables were computed by numerical quadrature as a supplement to tables computed earlier by Pearson & Hartley[1] with the aid of approximation formulas which were found unreliable for small ν and large q. The Pearson-Hartley tables did not include entries for $\nu < 10$ and the authors pointed out that entries exceeding 6 should not be considered reliable for $\nu \geqq 10$. For additional tables of the 'Studentized' range see the review of Pillai's paper in this issue [**RMT 1083**].

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[1] E. S. Pearson & H. O. Hartley, "Tables of the probability integral of the 'Studentized' range," *Biometrika*, v. 33, 1943, p. 89–99.

1081[K].—J. Moshman, "Testing a straggler mean in a two-way classification using the range," *Annals Math. Stat.*, v. 23, 1952, p. 126–132.

In 1948 Nair[1] obtained the distribution of the Studentized extreme deviate from the sample mean, i.e., he obtained the distribution of the sample statistic

$$u_\nu = (x_k - \bar{x})/s_\nu \qquad \text{or} \qquad u_\nu' = (\bar{x} - x_1)/s_\nu$$

where $x_1 \leqq x_2 \leqq \cdots \leqq x_k$, $\bar{x} = k^{-1} \sum_{i=1}^{k} x_i$ and s_ν is an independent estimate of the standard deviation based on ν degrees of freedom. In a two-way classification in the analysis of variance, therefore, s_ν can be taken as the residual standard deviation and the above test can be used to judge whether a row or column mean is high or low relative to the grand mean. In 1950 Patnaik[2] published a paper which indicated that the distribution of the mean range can be approximated with sufficient practical accuracy by a χ^2 distribution, provided an appropriate scale factor is used and an equivalent (usually fractional) number of degrees of freedom employed in the conversion. Tables were published in Patnaik's article which give both the appropriate scale factor and the equivalent number of degrees of freedom for the mean range. These two results have apparently led the author of the present paper to recommend a test for detecting a "straggler" row or column mean in a two-way classification using in effect the mean range instead of the standard deviation in Nair's test. As a matter of fact, the test proposed by Moshman is based on the sample statistic

$$g = \frac{\left| k \sum_{i=1}^{r} x_{ik} - \sum_{i=1}^{r} \sum_{j=1}^{k} x_{ij} \right|}{W_{s,1}} \qquad \text{or} \qquad \frac{\left| r \sum_{j=1}^{k} x_{rj} - \sum_{i=1}^{r} \sum_{j=1}^{k} x_{ij} \right|}{W_{s,1}}.$$

The numerator of the g statistic hence involves only row or column totals, the grand total, the number of rows and columns and a function $W_{k,r}$ of certain "ranges." This latter factor is in fact the sum of the "column" ("row") ranges of the "row" ("column") residuals. Appropriate tables for the 1% and 5% significance levels of the statistic, g, are given to 3S in the article for all cases where the number of rows or columns does not exceed nine. A quick answer concerning whether a particular row or column mean in a two-way classification is a straggler can therefore be obtained directly from these tables. For another method see TUKEY.[3]

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[1] K. R. NAIR, "The distribution of the extreme deviate from the sample mean and its Studentized form," *Biometrika*, v. 35, 1948, p. 118–144.
[2] P. B. PATNAIK, "The use of the mean range as an estimator of variance in statistical tests," *Biometrika*, v. 37, 1950, p. 78–87.
[3] J. W. TUKEY, "Comparing individual means in the analysis of variance," *Biometrics*, v. 5, 1949, p. 99–114.

1082[K].—D. N. NANDA, "Probability distribution tables of the larger root of a determinantal equation with two roots," Indian Soc. Agr. Stat., *Jn.*, v. 3, 1951, p. 175–177.

Define the distribution function $P(x)$ by

$$2P(x)I(1, 2m+2, 2n+2) = 2I(x, 2m+2, 2n+2) \\ - x^{m+1}(1-x)^{n+1}I(x, m+1, n+1)$$

where

$$I(x, p+1, q+1) = \int_0^x u^p(1-u)^q du.$$

The author tabulates $P^{-1}(.95)$ and $P^{-1}(.99)$ to 2D for $m = 0(\frac{1}{2})2, n = \frac{1}{2}(\frac{1}{2})10$. These tables provide tests for a number of statistical hypotheses, including that of the equality of two covariance matrices of bivariate normal populations.

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1083[K].—K. C. S. PILLAI, "On the distribution of the 'Studentized' range," *Biometrika*, v. 39, 1952, p. 194–195.

This paper contains tables of the lower percentage points of the 'Studentized' range $q = (x_n - x_1)/s$ at the 5% and the 1% levels of significance for $n = 2(1)8$ and $\nu = 1(1)9$, where the notation is that of RMT **1080**. All entries are given to 2D. These results further supplement those of PEARSON and HARTLEY.[1] An editorial note explains that PILLAI also computed certain upper percentage points which were not published separately since his paper was submitted while MAY's (see RMT **1080**) more extensive program of computation was in progress. Pillai's upper 5% and 1% points computed for $1 \leqq \nu \leqq 4$ and $2 \leqq n \leqq 8$ are included in the tables by May, and according to May, agreed with his calculations to within a unit of the last figure quoted by Pillai. In order to perform the necessary calculations, the author determined the distribution of q explicitly in the form of an infinite series.

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[1] E. S. PEARSON & H. O. HARTLEY, "Tables of the probability integral of the 'Studentized' range," *Biometrika*, v. 33, 1943, p. 89–99.

1084[K].—ALAN STUART, "The power of two difference-sign tests," Am. Stat. Assn., *Jn.*, v. 47, 1952, p. 416–424.

The author considers a distribution-free test of serial independence of N (unequal) observations with its $N - 1$ first differences ordered in time, a test proposed by MOORE & WALLIS.[1]

Define $D = \sum_{i=1}^{N-1} D_i$ where D_i is unity when the ith difference is positive, and zero otherwise. The null hypothesis is that the observations come from the same continuous population, and under this hypothesis the mean and variance of D is derived and its asymptotic normality for large N proved. The alternative hypothesis, assuming the observations equally spaced, is $y_k = \alpha + \beta k + x_k$ where x_k is a normal variate with mean β and variance σ^2. The mean, variance, and the asymptotic normality under the alternative hypothesis is derived. Assuming exact normality of the statistic D under both the null and alternative hypotheses, the author tabulates the power of the D-test to 2D at the 95% level of significance for $N = 15, 25, 50, 75, 100$ and $R = \beta/\sigma\sqrt{2} = .1(.1).5, 1$. He concludes for $N \geqq 25$, $R \geqq .5$, and $N \geqq 75$, $R \geqq .3$, that the loss of power as compared with the best available test is at most 8 percent. He next considers the case of two series of N unequal observations in time, and to investigate the correlation between them defines D_i for the first series as before, and similarly d_i for the second series. Let $C_i = D_i d_i$, and the correlation statistic $C = \sum_{i=1}^{N-1} C_i$. Under the null hypothesis that the N pairs of observations are drawn from the same continuous bivariate population in which the two variates are independent he finds the mean variance of C and proves its asymptotic normality. The alternative hypothesis is that the N pairs of observations are drawn from a normal bivariate population with $\rho \neq 0$, and under this hypothesis the mean, variance of C and its asymptotic normality is derived. The author provides a table for the power of the C test to 2D against this alternative (under the assumption again of exact normality) at the 95% level for $\rho = .1(.1).7$ and $N = 50, 100, 200$, and 400. This is compared to the power of the usual Z test, $Z = \frac{1}{2} \log \left(\frac{1 + r}{1 - r} \right)$ at the 95% level for $\rho = .1(.1).5$, $N = 50, 100, 200, 400$, with results given to 2D. For $\rho \geqq .5$, $N \geqq 100$, $\rho \geqq .4$, $N \geqq 200$; or $\rho \geqq .3$, $N \geqq 400$, he concludes that the use of the C test is satisfactory since at most 8% of the power is lost.

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[1] G. H. MOORE & W. A. WALLIS, "Time series significance tests based on signs of differences," Amer. Stat. Assn., *Jn.*, v. 38, 1943, p. 153–164.

1085[K].—T. J. TERPSTRA, "A confidence interval for the probability that a normally distributed variable exceeds a given value, based on the mean and the mean range of a number of samples," *Applied Scientific Research*, A, v. 3, 1952, p. 297–307.

The author employs PATNAIK'S[1] method to find the confidence intervals described in the title. For l samples of size k each the author gives four charts from which one can read the central 90% confidence limits on the vertical axis with the ratio of the numerical difference of the given value from the mean of the total sample to the average range on the horizontal axis, for $l = 4(2)10$ and $k = 1(1)5$.

C. C. C.

[1] P. B. PATNAIK, "The use of the mean range as an estimator of variance in statistical tests," *Biometrika*, v. 37, 1950, p. 78–87.

1086[K].—J. WISHART, "Moment coefficients of the k-statistics in samples from a finite population," *Biometrika*, v. 39, 1952, p. 1–13.

TUKEY[1] in order to simplify the problem of finding moment coefficients of Fisher's k-statistics in samples from a finite population introduced the sample statistic $K_{rs}...$ which has the property that its expected value is the same function of the N members of the population that $k_{rs}...$ is of the n members of the sample. The author first gives a conversion table for finding the $k_{rs}...$ statistics in terms of the augmented monomial symmetric functions of DAVID & KENDALL[2] and vice versa through order 6. Then to facilitate the derivation of further sampling formulae he gives a complete table of products and powers of k-statistics in terms of the $k_{rs}...$ through order 6. For orders 7 and 8 he gives the 14 and 21 basic formulae respectively, the remaining results of those weights then being derivable by algebraic substitution. Illustrations of the application of these tables are given.

C. C. C.

[1] J. W. TUKEY, "Some sampling simplified," Amer. Stat. Assn., *Jn.*, v. 45, 1950, p. 501–519.

[2] F. N. DAVID & M. G. KENDALL, "Tables of symmetric functions—part I," *Biometrika*, v. 36, 1949, p. 431–449. [*MTAC*, v. 4, p. 146.]

1087[L].—BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Committee on Mathematical Tables, *Table of Bessel Function*, Part II. Cambridge, 1952. xl + 255 p., 21.5 × 28 cm.

This is the long-awaited "second volume." The functions tabulated are $J_n(x)$, $Y_n(x)$, $I_n(x) = i^{-n}J_n(ix)$ and $K_n(x) = \frac{1}{2}\pi i^{n+1}[J_n(ix) + iY_n(ix)]$ for $n = 2(1)20$ (the first volume[1] gave values of the functions for $n = 0, 1$) and for $x = 0(.1$ or $.01)10(.1)20$ with δ^2 or $\delta_m{}^2$. Tables for J_n are 8D whereas those for Y_n, I_n, and K_n are to 8 figures. In regions where Y, I, or K is not convenient for interpolation, the functions x^nY_n, x^nK_n, $e^{-x}I_n$ or e^xK_n are tabulated. Finally J_n, Y_n, I_n, and K_n are tabulated for $n = 0(1)20$, $x = 0(.1)25$, to 10D for J_n, 10 figures for Y_n, I_n, K_n.

The tables are, in part, based on those of ALDIS[2] but are very considerably expanded and reworked. The checking has been carried out with the scrupulousness typical of this series of tables.

Bessel functions are like the weather, they nearly always intrude somewhere in any research in theoretical physics or applied mathematics. Like the weather also, for a long time very little was done about assembling a set of tables of the functions complete enough to satisfy most needs. Few of the calculations of acoustical and electromagnetic wave radiation, propagation or scattering can be made with the sole use of Bessel functions of the first kind, of real argument, so attempts to use the remarkable HARVARD COMPUTATION LABORATORY[3] tables are usually reminiscent of attempts to skate with only one ice-skate. Here, in the present BAASMTC volume, we have *both* solutions tabulated, for both real and imaginary values of the argument, with enough significant figures and enough range of the argument and order so that most calculations can be carried through. The Bessel function Panel responsible for its production—W. G. BICKLEY, L. J. COMRIE, J. C. P. MILLER, D. H. SADLER, and A. J. THOMPSON—are to be congratulated. Incidentally, this is the last volume of the BAAS tables; the Committee has been reconstituted as the Royal Society Mathematical Tables Committee.

As far as the needs of the theoretical physicist and engineer are concerned, this volume and the earlier one[1] will suffice, in the majority of cases. In a few scattering calculations the functions J and Y for $n = 20(1)50$ for $x = 0(.1)25$ would be useful, for in scattering calculations, orders up to about twice the argument, still contribute in the result. In some electromagnetic and acoustical problems, functions of a complex argument are required. These are available in the NYMTP tables[4] for $n = 0$ and 1. It would be useful to have similar tables for $n = 2(1)10$, but such needs are not urgent. Tables available and desired for half-integer orders will be discussed in RMT **1092**.

The volume under review here is clearly printed and each page is clearly marked for ready reference. It is a pleasure to use the volume.

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[1] BAASMTC, *Bessel Functions*, Part I. Cambridge, 1937, xx + 288 p.
[2] W. S. ALDIS, Roy. Soc. *Proc.*, v. 64, p. 203, 1899, and v. 66, p. 32, 1900.
[3] HARVARD COMPUTATION LABORATORY, *Tables of Bessel Functions of the First Kind for* $n = 0$ to 135, vols. III to XIV, 1947–1951, Harvard U. Press, Cambridge, Mass.
[4] NYMTP, *Tables of* J_0, J_1 *and* Y_0, Y_1 *for Complex Arguments*, 1943 and 1950, Columbia University Press, New York.

1088[L].—H. KOBER, *Dictionary of Conformal Representations*. New York, Dover, 1952. xvi + 208 p., 15.2 × 23.8 cm. Price $3.95.

The book represents a formidable achievement in bringing together a wide variety of explicit conformal maps useful to physicists, engineers and computers. Particularly notable are the numerous clear diagrams, which allow the reader to locate quickly a required mapping. Most of the transformations are by elementary functions, but short theoretical discussions of the Schwarz-Christoffel transformation and of the Schwarz principle of reflection indicate in part the derivation of more complicated formulas. Proofs are altogether omitted due to the dictionary character of the presentation. Mappings are classified according to the nature of the analytic functions involved, with chapters on bilinear transformations, algebraic functions, trigonometric and exponential functions, the Schwarz-Christoffel transformation, and elliptic and modular functions. Although a geometrical index is listed as topological, almost no reference is made to Riemann surfaces. This is particularly to be regretted, since it is quite common in engineering problems, such as those involving the hodograph method, to ignore the usefulness and necessity of exploiting simple conformal mappings onto Riemann surfaces. Accuracy and a notable absence of misprints, of which the reviewer found only one, should make the handbook of permanent and growing value to the applied scientist.

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[*Editorial Note.* A preliminary version of this work was reviewed in *MTAC*, v. 2, p. 296–297, v. 3, p. 103, 368].

1089[L].—N. H. March, "Thomas-Fermi fields for molecules with tetrahedral and octahedral symmetry," Cambridge Phil. Soc., *Proc.*, v. 48, 1952, p. 665–682.

Two types of solutions of the differential equation

$$\frac{d^2\phi}{dx^2} = x^{-\frac{1}{2}}\phi^{\frac{3}{2}}$$

are tabulated in this paper. One of these is of the form

$$\phi = 1 + a_2x + a_3x^{\frac{3}{2}} + a_4x^2 + \cdots$$

and is fully determined once a_2 is given. Table 2 (p. 670) gives $a_3, \cdots, a_{11}$ in terms of a_2, and Table 1 (p. 668) values of ϕ for $-a_2 = 1.55$, 1.56, 1.567, 1.5748, 1.581, 1.5849, 1.5853, 1.5866, 1.58756, 1.58764. x varies from 0 to 10.74. The solutions were started by using the above series and then continued by numerical integration of the differential equation. Table 1 also gives in each case the point x_0 the tangent at which passes through the origin, with the corresponding $\phi(x_0)$. It is suggested that a classification of the solutions by $\phi(x_0)$ is more reliable than that based on a_2.

The second type of solutions is of the form

$$\phi = \frac{144}{x^3}\left(1 - \frac{F_1}{x^c} + \frac{F_2}{x^{2c}} - \frac{F_3}{x^{3c}} + \cdots\right).$$

Table 3 (p. 670) gives expressions of $F_2, \cdots, F_{10}$ in terms of F_1. It turns out that all solutions of this type may be derived from two "master solutions" in which F_1 is, respectively, 1 or -1. These two master solutions and their first derivatives are tabulated in Tables 4 and 5 (p. 671, 672) for values of x which range from .1375 to 20.2 in Table 4 ($F_1 = 1$), and from .5375 to 19.4 in Table 5 ($F_2 = -1$).

In the paper there are also references to solutions computed previously.[1]

A. E.

[1] R. P. Feynman, N. Metropolis, & E. Teller, "Equations of state of elements based on the generalized Fermi-Thomas Theory," *Phys. Rev.*, s. 2, v. 75, 1949, p. 1561.

J. C. Slater & H. M. Krutter, "The Thomas-Fermi method for metals," *Phys. Rev.*, s. 2, v. 47, 1935, p. 559.

1090[L, M].—W. Meyer zur Capellen, *Integraltafeln. Sammlung unbestimmter Integrale elementarer Funktionen.* Springer-Verlag. Berlin/Göttingen/Heidelberg, 1950. viii + 292 p., 23.8 × 16.5 cm.

This book contains probably the largest published collection of indefinite integrals whose integrands are combinations of elementary functions. It was undertaken in 1942 and completed in 1944, publication being delayed by difficulties of the post-war period. There are about 3000 entries, most of them giving the final answer, and some of them stating recurrence relations or other auxiliary formulae for the integrals in question. Many special cases of more general results were included, thus enabling the user to get the final answer quickly. For instance, in section 2.2.2, there are recurrence relations for the integral

$$\int x^p(a + bx)^{n/m}dx \qquad m, n, p \text{ integers,}$$

and there is also an explicit formula for positive integer p. Nevertheless, there are about 7 pages of some eighty particular cases listed, for special values of m, n, p—a great convenience for the user.

Contents. Chap. 1. Preliminaries (5 p.). Rules for differentiation and integration. Chap. 2. Integrals of algebraic functions (138 p.). 2.1. Rational integrands. 2.2. Irrational integrands leading to elementary functions. 2.3. Algebraic integrands leading to elliptic integrals. Chap. 3. Integrals of transcendental functions (80 p.). 3.1. Exponential and logarithmic functions. 3.2. Trigonometric and inverse trigonometric functions. 3.3. Hyperbolic and inverse hyperbolic functions. Chap. 4. Products of algebraic and transcendental functions (45 p.). Arranged according to the transcendental part, similar subdivision to that of Chapter 3. Chap. 5. Products of transcendental functions (11 p.). 5.1. $\int g(x) \ln x dx$. 5.2. $\int e^x g(x)dx$. Addenda (3 p.). Chap. 6. Supplementary material (9 p.). Constants, expansions, definitions of some higher transcendental functions. Chap. 7. References (19 items, 1 p.).

The classification and arrangement of such a large number of integrals presents very formidable difficulties, and only continued and frequent use of the tables will show the excellence, or the imperfections, of the scheme adopted here. The author's aim was to provide tables which can be used with ease by a non-mathematician, and the first impression is decidedly favorable. There is a very detailed table of contents which enables one to locate any integral printed in the book within four or five pages. One will, of course, encounter integrals which are not in the book, yet can be transformed into integrals printed in the book; but a generous duplication of several forms of the same integral will greatly reduce the number of these occurrences.

The only drawback the reviewer found in using these tables is the absence of a list of notations and abbreviations. The user not acquainted with German books will find out sooner or later that $\mathfrak{Sin}$ means sinh and $\mathfrak{ArSin}$ means $\sinh^{-1}$ (although these symbols are defined on p. 286, 287 under hyperbolic functions, meeting them on p. 77 as integrals of irrational functions gives no clue as to the place where one should look for their definition). Ad hoc notations are sometimes explained where they occur (for instance, x on p. 63 in 2.3.1.2.1.3.1) or in a footnote on the same page on which they are used (as U, V, W, on p. 68); yet sometimes one has to turn several pages until one finds an explanation (the same V, W, are used on p. 76, and one has to go back to p. 68 to find a footnote referring one to nos. 2.2.1.1.1 and 2.2.1.1.2 which are then found on p. 73, 74). Also, the numbering of the entries (samples above) is cumbersome, if systematic, and makes it inconvenient to give specific references.

In all other respects this is an eminently useful book, and the printing is excellent.

A. E.

[For errata in this work see in this issue MTE **223**.]

1091[L].—NBS Applied Mathematics Series, No. 17. *Tables of Coulomb wave functions*. U. S. Government Printing Office, Washington, D. C., 1952. xxvii + 141 p., 20.5 × 27 cm. Price, $2.00.

Coulomb wave functions are confluent hypergeometric functions, but the normalisation and notation usual in quantum mechanics differ from the standard notations in mathematical texts.

The "regular Coulomb wave function," $F_L(\eta, \rho)$ is that solution of the differential equation

$$\frac{d^2y}{d\rho^2} + \left[1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\right] y = 0$$

which is regular at $\rho = 0$ and is so normalised that

$$F_L(\eta, \rho) \sim \sin\left(\rho - \eta \ln 2\rho - \tfrac{1}{2}L\pi + \sigma_L\right)$$

as $\rho \to \infty$, where $\sigma_L = \arg \Gamma(L + 1 + i\eta)$. The notation

$$F_L(\eta, \rho) = C_L(\eta)\rho^{L+1}\Phi_L(\eta, \rho)$$

is frequently used, where

$$C_0(\eta) = [2\pi\eta/(e^{2\pi\eta} - 1)]^{\frac{1}{2}}, \quad L(2L + 1)C_L(\eta) = (L^2 + \eta^2)^{\frac{1}{2}}C_{L-1}(\eta),$$

and $\Phi_L(\eta, \rho)$ is a power series in ρ.

Coulomb wave functions have been computed by several authors: on p. 152 of the recent tables by BLOCH *et al* (*MTAC*, v. 6, p. 92), there are references to available tables. The volume under review contains the most extensive, and the most systematic, tables, and it largely (but not entirely) supersedes the earlier tabulations.

Table I (p. 1–111) gives 7D values of $\Phi_L(\eta, \rho)$, and of as many reduced derivatives

$$\frac{1}{k!}\frac{d^k}{d\eta^k}\Phi_L(\eta, \rho)$$

as are needed for interpolation. In this table, $\eta = -5(1)5$, $L = 0(1)5, 10, 11, 20, 21$, and $\rho = 0(.2)5$.

Table II (p. 113–127) gives 10D values of the real part of $\Gamma'(1 + i\eta)/\Gamma(1 + i\eta)$, with modified second central differences, for $\eta = 0(.005)2(.01)6(.02)10(.1)20(.2)60(.5)110$.

Table III (p. 129–135) gives 8D values of σ_0, with modified second central differences, for $\eta = 0(.01)1(.02)3(.05)10(.2)20(.4)30(.5)85$.

Table IV (p. 137–141) gives 8S values of $C_0(\eta)$ for $\eta = 0(.01)5(.05)10$.

The Foreword contains an account of the applications of Coulomb wave functions, and the Introduction (by M. ABRAMOWITZ) gives definitions of, and formulas, Bessel function expansions, and asymptotic expansions for, Coulomb wave functions. The Introduction also describes the method of computation, and gives instructions for interpolation and for the use of recurrence relations.

In view of the fact that this volume deserves, and is likely, to become the principal work of reference for Coulomb wave functions, without entirely superseding some of the earlier tables, it is regrettable that there is no systematic account of all (although there are references to some) available tables.

A. E.

1092[L].—ROYAL SOCIETY MATHEMATICAL TABLES COMMITTEE, *Short Table of Bessel Functions* $I_{n+\frac{1}{2}}$, $K_{n+\frac{1}{2}}$. Cambridge, 1952, 20 p., 21.5 × 28 cm.

This is a preliminary table of spherical Bessel functions of imaginary argument. The functions actually tabulated are

$$\begin{array}{rllll} x^{-n-\frac{1}{2}}I_{n+\frac{1}{2}}(x) & \text{for} & n = 0(1)10 & \text{and} & x = 0(.1)5, \\ (2/\pi)x^{n+\frac{1}{2}}K_{n+\frac{1}{2}}(x) & \text{for} & n = 0(1)10 & \text{and} & x = 0(.1)5, \\ e^{-x}I_{n+\frac{1}{2}}(x) & \text{for} & n = 0(1)10 & \text{and} & x = 5(.1)10, \\ (2/\pi)e^{x}K_{n+\frac{1}{2}}(x) & \text{for} & n = 0(1)10 & \text{and} & x = 5(.1)10, \end{array}$$

in most cases to eight significant figures, together with δ_m^2, for interpolation by Everett's formula, when $I_n(x) = i^{-n}J_n(ix)$ and

$$(2/\pi)K_{n+\frac{1}{2}}(x) = (-1)^{n+1}[I_{n+\frac{1}{2}}(x) - I_{-n-\frac{1}{2}}(x)].$$

These tables begin the completion of the spherical Bessel functions as the table in RMT **1087** completes the main tabulation for cylindrical Bessel functions. The NBS tables[1] have covered the two independent solutions, for real values of the argument, for $n = 0(1)30$ and $x = 0(.01 \text{ or } .1)25$ to 8 or 10 significant figures. The present table is a start toward a similar table for imaginary arguments.

Spherical Bessel functions of imaginary argument are useful in the quantum mechanical calculation of atomic and molecular properties. In particular, an atomic wave function for a bound state, about one center, when expanded about another center, appears as a sum of such Bessel functions, multiplied by spherical harmonics.[2] Calculations of interatomic forces in a molecule have been quite difficult to carry out, chiefly because of the lack of adequate tables. Now, with the advent of high speed computing machines, to remove the burden of routine details, such computations should become relatively easy to perform and our insight into molecular structure can begin to be satisfactorily complete. Values of $I_{n+\frac{1}{2}}(x)$ and $K_{n+\frac{1}{2}}(x)$ for $n \simeq 0(1)20$ and $x \simeq 0(.1)20$ will be needed as a basis for the calculation of the component integrals. The present table is a promising start in this direction; let us hope that the RSMTC, or some other group, will carry on.

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[1] NBSCL, *Tables of Spherical Bessel Functions.* Vols. I and II, 1947, Columbia University Press, New York.
[2] See, for example, N. ROSEN, "Interaction between Atoms with *s*-Electrons," Phys. Rev., v. 38, p. 255, 1931.

1093[V].—AERONAUTICAL RESEARCH COUNCIL, *A Selection of Tables for Use in Calculations of Compressible Airflow.* Prepared by the Compressible Flow Tables Panel (L. ROSENHEAD, Chairman; W. G. BICKLEY, C. W. JONES, L. F. NICHOLSON, C. K. THORNHILL, R. C. TOMLINSON) of the ARC. Oxford, 1952, viii + 143 p., 21.5 × 27.0 cm. Price $8.00.

The earlier and more familiar tables for compressible flow in this country are H. W. EMMONS' "Gas Dynamics Tables for Air" (Dover publications, 1947, *MTAC*, v. 3, p. 36) and J. H. KEENAN & J. KAYE's "Gas Tables" (Wiley, 1945, *MTAC*, v. 2, p. 20). Compared with these tables, the present publication is certainly more detailed and more accurate. It is stated that the two earlier tables have numerous errors, mostly affecting the final place of decimals. As in Emmons' table, the value of the ratio of specific heats, γ, is chosen to be 1.4. The authors try to remedy this limitation in part by listing the derivatives of a few important quantities with respect to γ at $\gamma = 1.4$. But nevertheless, this restriction will make the table unsuitable for calculations of gas flow at higher temperatures, such as the flow in high pressure compressors and gas turbines.

There are three main groups of tables: The first group is concerned with isentropic one-dimensional flow. The argument used is the local Mach number M, the ratio S of speed to the critical sonic speed, or the square of the ratio of speed to maximum speed τ. The M-tables have the ranges $M = 0(.01).5(.001)1; 1(.01)5$. The S-tables have the ranges $S = 0(.01)2.44$ and $\sqrt{6}$; $1(.005)1.5(.01)2.44$ and $\sqrt{6}$. The τ-table has the range $\tau = 0(.01)1$.

The second group of tables is for the solution by the characteristics method. The argument used is either the angular characteristics variable ω or the Mach angle μ. The ranges of these tables are $\omega = 0°(.5°)100°(1°)130°$; $0°(.001°).05°$; $0°(.01°)1°$, $0°(.1°)30°$. $\mu = 0°(1°)90°$.

The third main group of tables gives the ratio of quantities after the shock to that before the shock. The argument of the tables is either the Mach number M_1 or the speed ratio S_1 before the shock. For normal shock waves, $M_1 = 1(.01)5$; $S_1 = 1(.01)2.44$ and $\sqrt{6}$. For oblique shock waves only the maximum flow deflection angle and angle for sonic flow after shock are given for $M_1 = 1(.01)2(.1)5(1)20$.

Other tables in this volume are tables for obtaining Mach number from the dynamic pressure ratio or the Pitot pressure ratio, tables of pertinent powers of x and $(1 - x^2)$, tables for calculating the Reynolds number, the pressure coefficient, etc.

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