The calculations were performed on the HEC 1 and the time for calculation and printing of each polynomial was 50 seconds. For convenience of reference, the sign combination of each polynomial was printed as well.

An analysis of the results of this tabulation shows that of the 256 equations considered:

58 have no real roots
190 have two real roots
8 have four real roots.

The distribution of the roots by numerical value is shown in Fig. 1. The detailed results of this tabulation are available for reference at the Computation Laboratory, Birkbeck College.

The author wishes to thank Mr. R. L. MICHAELSON of the British Tabulating Machine Company for providing computing facilities for this calculation.

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166.—NUMERICAL STUDY OF SIGNATURE RANK OF CUBIC CYCLOTOMIC UNITS. In the search for algebraic fields (particularly beyond the quadratic) where unique factorization fails, little work seems feasible on high speed computers for the reason that the straightforward evaluation of forms must be controlled by group-theoretic data-processing.[1] A rare opportunity, however, is afforded by the cubic field $R(\alpha)$ generated by the Gaussian sum

$$\alpha = \sum_{x=0}^{p-1} \exp 2\pi i x^3/p$$

where $p = 6n + 1$ is a prime. Here the so-called *cyclotomic unit*[2] is given by the simple formula (for three conjugates $0 \leq i \leq 2$):

$$\Theta_i = -\prod_{t=0}^{n-1} \sin\left(\tfrac{1}{2}\pi g^{3t+i}/p\right) \csc\left(\tfrac{1}{2}\pi g^{3t+i+1}/p\right)$$

where g is an odd primitive root modulo p (or an even one augmented by p). Now while the decimal accuracy of such a formula would be lost very rapidly the Θ_i have the property that the possible non-unique factorization in $R(\alpha)$ can be of a certain frequent type (namely *even* class number) only when all three Θ_i are positive.[2] This condition is easy to determine by machine.

We accordingly form three tallies A_0, A_1, A_2 for each p as follows: We increment the A_i tally $(0 \leq i \leq 2)$ when the least positive residues of g^{3t+i+1} and g^{3t} modulo $2p$, $0 \leq t \leq n-1$, lie in different halves of the interval $(0, 2p)$ subdivided at p. Such a procedure tallies the possible negative sign of $\sin(\tfrac{1}{2}\pi g^{3t+i+1}/p) \csc(\tfrac{1}{2}\pi g^{3t}/p)$; hence the three Θ_i are seen to be all positive exactly when the A_0, A_1, and A_2 have (final) values that are odd, even, and odd respectively.

Since there are many primitive roots g for each p we find that the A_i depend on the g chosen, except for the fact that the occurrence of three positive Θ_i must be independent[2] of g. Assuming that the powers of g lay in half-intervals "at random" we should expect the A_i to be each approximately $p/12$, while assuming "random parities," (which is more precarious), we should expect the Θ_i to be all positive approximately $\frac{1}{4}$ of the time; (since $\Theta_0\Theta_1\Theta_2 = +1$, or, equivalently, A_2 is always odd).

The computation was performed on the MIDAC starting Jan. 27, 1954, for the first 207 values of p (<3000). The values[3] of p and g in the form of binary-coded decimals were placed as alternate words on high-speed photoelectric input tape. They were read in by pairs (taking approximately .15 sec. for each pair). The conversion to binary form was internal. For the main induction loop the starting point was each g^{3t}, $0 \leq t \leq n - 1$, from which were formed g^{3t+i+1} for $0 \leq i \leq 2$ (always modulo $2p$). The three tally increment decisions (for A_i) were made as described above (taking .04 sec. for each circuit of the loop). In addition, every time the MIDAC chose not to increment an A_i tally it incremented a common D tally. After the loop had been traversed n times two checks on accuracy were made. First the sum of the three A_i and the D tallies was verified to be $(p-1)/2$ (the number of increment decisions), and secondly the last power, $g^{(p-1)/2}$ (reduced modulo $2p$), was verified to be $2p - 1$. The MIDAC was made to examine the three final A_i tallies for the occurrence of alternate parities and indicate such occurrences or non-occurrences in the form of a -0 or 0 respectively in a temporary storage space. The output for each 0 consisted of

$$p\ g\ A_0\ A_1\ A_2\ -0\ \text{(or 0)}$$

where the first five items were converted to decimal internally and shifted for a short-word (4 character) print-out while the sixth item was the signed zero indicating presence or absence of alternate parities. (Print-out time for each p was approximately 5 seconds.)

The total running time was about one hour of which 20 minutes was input-output.

The 21 values of p(<3000) for which the A_i have alternate parity (or for which the Θ_i are all positive) are reproduced below with j their order in the list of primes (of the form $6n + 1$).

j	p	j	p	j	p
18	163	71	853	143	1879
27	277	77	937	145	1951
33	349	81	1009	156	2131
37	397	107	1399	169	2311
48	547	129	1699	191	2689
52	607	135	1777	199	2797
60	709	137	1789	200	2803

It will be seen that the frequency of these p is close to $\frac{1}{10}$, which is not in agreement with "randomness" of parities (as postulated above). The magnitudes of the A_i did come out, however, to agree rather well with the randomness assumption made earlier, and they are omitted here. A spot check[4] of $p = 163, 277, 349, 2803$ by hand calculations revealed that unique factorization does fail in the field $R(\alpha)$, and that the class number is 4 for each of these cases.

The author is indebted to Dr. J. W. CARR III and the staff of the Willow Run Research Center for kindly making the MIDAC available for this study (which is part of Army Ordnance Project DA-20-018-ORD-12332).

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[1] Compare D. H. LEHMER, *Guide to tables in the theory of numbers.* National Research Council, 1941, p. 75–77, O. TAUSSKY, *Some computational problems in algebraic number theory.* National Bureau of Standards *Report* (to appear).

[2] For theoretical background consult H. HASSE, *Arithmetische Bestimmung von Grundeinheit.* Berlin, 1950, p. 70.

[3] These were taken from a table of minimum positive g for $p < 3000$ in I. M. VINOGRADOV, *Osnovy Teorii Chisel* [*Fundamentals of the Theory of Numbers*]. Moscow, 1940, p. 110.

[4] The case $p = 163$ was discovered through another procedure by E. ARTIN, according to a private communication.

167.—CULLEN NUMBERS. These are numbers of the form $n2^n + 1$ and are remarkable in that they seem to be composite for $n > 1$, although there is no *a priori* reason for this. CUNNINGHAM & WOODALL[1] made a study of these numbers and found them all composite with a small factor for $1 < n < 141$. No factor of $141 \cdot 2^{141} + 1$ is known. I have completely factored the following cases left incomplete by Cunningham. The case $n = 46$ is due to R. A. LIÉNARD of Lyons.

n	$n2^n + 1$	n	$n2^n + 1$
33	$47 \cdot 6031230671$	42	$23 \cdot 43 \cdot 83 \cdot 2250270487$
35	$37 \cdot 32502455213$	43	$3 \cdot 5 \cdot 163 \cdot 2633 \cdot 58752797$
37	$3 \cdot 5 \cdot 339016085231$	45	$11 \cdot 47 \cdot 2437 \cdot 1256655529$
38	$3^2 \cdot 20879 \cdot 55586743$	46	$5 \cdot 31 \cdot 47 \cdot 139297 \cdot 3189821$
39	$41 \cdot 3433 \cdot 152326961$	47	$7 \cdot 11 \cdot 43 \cdot 3593 \cdot 556021079$
40	$41 \cdot 131611 \cdot 8150491$	48	$7 \cdot 379 \cdot 997 \cdot 5107973329$
41	$13 \cdot 43 \cdot 1291 \cdot 124932557$	66	$5^3 \cdot 13 \cdot 67 \cdot 107 \cdot 131 \cdot 8353 \cdot 382030403$

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[1] A. J. C. CUNNINGHAM & H. J. WOODALL, "Factorisation of $Q = (2^q \mp q)$ and $(q \cdot 2^q \mp 1)$," *Messenger Math.*, v. 47, 1917, p. 1–38.

CORRIGENDA

V. 6, p. 225, l. 11, *for* monomial *read* elementary.
V. 7, p. 34, l. 6, *for* 6 *read* 1.
V. 7, p. 175, l. 17, *for* 9 *read* 8.