$$J_0(x) = \sum_{n=0}^{\infty} A_n T_n^*\left(\frac{x^2}{100}\right), \qquad J_1(x) = x \sum_{n=0}^{\infty} A_n T_n^*\left(\frac{x^2}{100}\right),$$

$$-10 \leq x \leq 10. \qquad -10 \leq x \leq 10.$$

n	$T_n^*(x)$
0	1
1	$2x - 1$
2	$8x^2 - 8x + 1$
3	$32x^3 - 48x^2 + 18x - 1$
4	$128x^4 - 256x^3 + 160x^2 - 32x + 1$
5	$512x^5 - 1280x^4 + 1120x^3 - 400x^2 + 50x - 1$
6	$2048x^6 - 6144x^5 + 6912x^4 - 3584x^3 + 840x^2 - 72x + 1$
7	$8192x^7 - 28672x^6 + 39424x^5 - 26880x^4 + 9408x^3 - 1568x^2 + 98x - 1$
8	$32768x^8 - 1\ 31072x^7 + 2\ 12992x^6 - 1\ 80224x^5 + 84480x^4 - 21504x^3 + 2688x^2 - 128x + 1$
9	$1\ 31072x^9 - 5\ 89824x^8 + 11\ 05920x^7 - 11\ 18208x^6 + 6\ 58944x^5 - 2\ 28096x^4 + 44352x^3 - 4320x^2 + 162x - 1$
10	$5\ 24288x^{10} - 26\ 21440x^9 + 55\ 70560x^8 - 65\ 53600x^7 + 46\ 59200x^6 - 20\ 50048x^5 + 5\ 49120x^4 - 84480x^3 + 6600x^2 - 200x + 1$
11	$20\ 97152x^{11} - 115\ 34336x^{10} + 273\ 94048x^9 - 367\ 65696x^8 + 306\ 38080x^7 - 164\ 00384x^6 + 56\ 37632x^5 - 12\ 08064x^4 + 1\ 51008x^3 - 9680x^2 + 242x - 1$
12	$83\ 88608x^{12} - 503\ 31648x^{11} + 1321\ 20576x^{10} - 1992\ 29440x^9 + 1905\ 13152x^8 - 1203\ 24096x^7 + 506\ 92096x^6 - 140\ 57472x^5 + 24\ 71040x^4 - 2\ 56256x^3 + 13728x^2 - 288x + 1.$

RECENT MATHEMATICAL TABLES

1202[A,P].—M. L. Clinnick, (a) *Gear Ratios No.* 43. (b) *Gear Ratios No.* 59. Privately printed, 3211 School Street, Oakland 2, California, 1953. Each book has 84 unnumbered pages 8.9 × 11.4 cm. and 8.26 × 14.0 cm. respectively, photo-offset from typescript. Price $1.00 each.

These pocket-sized tables are designed for use in selecting appropriate sprocket gears in motorcycle racing events. They are triple entry tables giving 2D values of

$$R = kr/(ec)$$

for $c = 10(1)23$, $e = 15(1)25$, $r = 46(1)75$, $k=43$, 59. Unrealistic values of $R \geq 15$ are omitted. In the intended application r, e and c are the numbers of teeth in the rear, engine, and countershaft sprockets respectively. The clutch sprocket is assumed to have 43 teeth or 59 corresponding to certain popular British and American makes of motorcycle. Instructions are given in (b).

D. H. L.

1203[A,B,P].—J. K. Lynch, *Kilocycle-Radian Frequency Conversion Tables.* Commonwealth of Australia, Postmaster-general's Dept., Research Laboratories, *Report* No. 3726. Melbourne, 1953, 24 p. 20.2 × 25.4 cm.

This table gives 6S values of

$$\omega = 2000\pi F$$

and ω^2, ω^{-1} and ω^{-2} for

$$F = .1(.1)100(10)1000, \quad 10^k \text{ for } k = 6(1)11.$$

This allows the electrical engineer to pass easily from a frequency F in kilocycles per second to the corresponding angular velocity ω in radians per second, or to its square, reciprocal or reciprocal square.

For example the inductance L and capacitance C for resonance of frequency F are related by $L = \omega^{-2}/C$.

D. H. L.

1204[A,D].—G. E. Reynolds, *Conversion Tables of Tangents or Cotangents to Sines and Cosines of Three Decimals.* Air Force Cambridge Research Center, *Technical Report* 53–29, Cambridge, Mass., 1953. 26 p. 18.4 × 27.3 cm.

This is a short handy table of sines and cosines as functions of tangents or cotangents. Values of sines and cosines are given to 3D as consecutive multiples of 10^{-3}. Corresponding 4S values of tangents and cotangents are listed alongside. The table is intended to be entered via the more rapidly moving functions tan and cot to read out corresponding values of sine and cos without interpolation.

The table is in fact a table of $r/(1 - r^2)^{1/2}$, $r - .0005 = 0(.001)1$. Applications are mentioned to the layout of parallel plate surfaces and the path of a milling cutter.

D. H. L.

1205[B,F].—D. R. Kaprekar, *Cycles of Recurring Decimals*, v. II. (*From* N = 167 *to* 213 *and many other numbers.*) Khare Wada, Deolali, India, 1953. Published by the author, iv + 47 p., 24.3 × 16.7 cm. Price 6 rupees.

This is an extension of v. I, reviewed in *MTAC*, v. 7, p. 238, to the extent mentioned in the title. Besides the table of cycles there are other tables as follows.

P. 25. Table of 2^n and 5^n for $n = 1(1)33$

P. 36–41. Table of exponents of 10 (mod p) for all primes $p \leq 13709$.

This table is taken from a previous table of Kraitchik.[1] This reprint contains three errata.

p. 36 *For* $P = 669$ *read* $P = 659$
p. 40 $P = 9619$, *for* $C = 8$ *read* $C = 3$
p. 41 *For* $P = 12901$ *read* $P = 12907$

P. 43–47. Table of factors of $(10^e - 1)/p$ where e is the exponent of 10 (mod p) for 23 primes p mostly less than 100. In connection with $p = 47$ the author claims the discovery of the prime

$$1934235976789168278 53.$$

However on the next page two factors of this number are given, namely 2531 and 549797184491917, the last being misprinted.

D. H. L.

[1] M. Kraitchik, *Recherches sur la Théorie des Nombres*, v. 1. Paris, 1924, p. 131–145.

1206[C,D].—Akademiiâ Nauk SSSR. Institut Tochnoĭ mekhaniki i vychislitel'noĭ techniki. Matematicheskie Tablitsy. *Desiâtiznachnye Tablitsy logarifmov kompleksnykh chisel i perekhoda ot dekartavykh koordinat k poliârnym. Tablitsy funktsiĭ.* [Ten place tables of logarithms of complex numbers and of the transformation from cartesian to polar coordinates. Tables of functions] $\ln x$, arctg x, $\frac{1}{2}\ln(1 + x^2)$, $(1 + x^2)^{\frac{1}{2}}$. Moscow, 1952. 116 p. 17 × 26 cm. 10.8 roubles.

There are four 10D tables each at interval .001 and with Δ^2:

$$A.\ \ln x,\ 1 \leq x < 10;$$
$$B.\ \tfrac{1}{2}\ln(1 + x^2),\ 0 \leq x \leq 1;$$
$$C.\ \text{arc}\tan x,\ 0 \leq x \leq 1;$$
$$D.\ (1 + x^2)^{\frac{1}{2}},\ 0 \leq x \leq 1.$$

From such tables $\ln z = \ln(A + iB) = \ln A + \frac{1}{2}\ln[1 + (B/A)^2] + i\,\text{arc}\tan(B/A)$, $|B| < |A|$, $r = |A|[1 + (B/A)^2]$, $\theta = \frac{1}{2}\pi - \arctan(B/A)$ may be found; similar expressions exist for $|B| > |A|$, since $\ln(A + iB) = \ln i + \ln(B - iA)$.

Illustrative numerical examples in the use of the tables are given on pages 4–6, and there is an errata slip with 15 corrections in the following tables. An interpolation sheet is in a pocket of the volume, of which there were 3000 copies in the edition.

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1207[D].—H. E. SALZER, "Radix table for obtaining hyperbolic and inverse hyperbolic functions to many places," *Jn. Math. Phys.*, v. 32, 1953, p. 197–202.

These tables are intended to give data from which 18D values of hyperbolic functions and their inverses can be found with an ordinary desk calculator. A similar table for circular functions has been given by the author in *MTAC*, v. 5, p. 9–11. As in the previous radix table, the author chooses the inverse tangent function. In this case it is

$$\text{Arc}\tanh(k\cdot 10^{-\lambda}) \quad k = 1(1)9, \quad \lambda = 1(1)6.$$

These values are given to 20D. He also gives 20D values of

$$\tanh A \text{ for } A = 1(1)24$$

and

$$\tfrac{1}{2}\ln x \text{ for } x = 1.2(.2)2(1)10.$$

By using the addition theorems for tanh and arc tanh and approximations to these functions for small values of the argument any desired value can be built up. Of course, the other hyperbolic functions and their inverses can be expressed in terms of the tangent and its inverse.

D. H. L.

1208[F].—OVE HEMER, "Note on the diophantine equation $y^2 - k = x^3$," *Arkiv för Matematik*, v. 3, 1954, p. 67–77.

This paper contains corrections and additions to the author's dissertation [*MTAC*, v. 7, p. 86].

Tables 1 and 2 of this dissertation are reprinted here with the corrections called for in our previous review. Table 1 is now supposed to be complete in the sense that it gives all solutions of the equation $y^2 - k = x^3$ for $0 < k < 100$, whenever the equation is soluble, together with the number N of solutions. The largest $N = 8$ is for the well known case of $k = 17$ discussed by MORDELL, Table 2 for $-100 < k < 0$ still contains 22 incomplete cases. For these, the coefficients and solutions of the corresponding cubic forms are given. Table 3 of the dissertation is not reproduced here. Table 4 is slightly enlarged giving the discriminant $-D$, the fundamental rings and units in the cubic field corresponding to all square free values of $1 < k < 50$, and for the values of k corresponding to soluble equations for $50 < k < 100$. There still remain 7 cases in which the unit is "not definitely proved to be fundamental."

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1209[F].—D. H. LEHMER, EMMA LEHMER, & H. S. VANDIVER, "An application of high speed computing to Fermat's Last Theorem," Nat. Acad. of Sci., *Proc.*, v. 50, 1954, p. 25–33.

Kummer defined a prime l to be regular if it does not divide any of the first $(l - 3)/2$ Bernoulli numbers and showed that

$$x^l + y^l = z^l, \quad l > 2$$

is impossible in non-zero integers if l is a regular prime. In this paper, the irregular primes less than 2000 are tabulated, together with the accompanying least prime p of the form $1 + kl$, as well as three other associated constants; $2a$, Q_a, Q_a^k. On the basis of this table not only is Fermat's Theorem established for $l < 2000$, but data are given which will greatly simplify and facilitate the study of units and ideals in cyclotomic fields.

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1210[G,K].—F. N. DAVID & M. G. KENDALL, "Tables of symmetric functions, Part IV," *Biometrika*, v. 40, 1953, p. 427–446.

In this set of tables the authors continue a series of fundamental symmetric function tables for functions of weight not exceeding twelve. When the series is complete the user will be able to relate any two of the following kinds of symmetric functions of x_i:

(U) unitary, or elementary, $a_k = \sum x_1x_2\cdots x_k$
(S) sums of like powers, $S_k = \sum x_1^k$
(M) monomial, such as $(3, 2, 1, 1) = \sum x_1^3x_2^2x_3x_4$
(H) homogeneous product-sums h_r generated by

$$\prod_i (1 - x_it)^{-1} = \sum_{r=0}^{\infty} h_rt^r.$$

The first three parts[1] have already been published and are described in RMT **769** and **1020** [*MTAC*, v. 4, p. 146, v. 6, p. 224–5, see also corrigendum, v. 8, p. 188]. These relate S and M, M and U, and U and H respectively. The present Part IV relates M and H. Thus one may use the table of weight 3 to read such results as

$$h_1h_2 = (3) + 2(2, 1) + 3(1, 1, 1)$$

and

$$(2, 1) = -2h_1^3 + 5h_1h_2 - 3h_3.$$

In the arrangement of the tables advantage is taken of Hirsch's law of symmetry which states that the coefficient of $(p, q, r, \cdots)$ in the expansion of $h_ah_bh_c\cdots$ is the same as the coefficient of $(a, b, c, \cdots)$ in the expansion of $h_ph_qh_r\cdots$. Thus although each expanded symmetric function of weight w requires w terms each coefficient may be used twice. The whole set of tables for Part IV occupies no more space than the previously published parts which are triangular in nature. The printing is in the same small but elegant type of the earlier tables.

Part V which will relate symmetric functions of types U and S will complete the set since a Part VI relating H and S would be identical with Part V except for obvious sign changes.

D. H. L.

[1] F. N. David & M. G. Kendall, "Tables of symmetric functions Part I; Parts II and III," *Biometrika*, v. 36, 1949, p. 431–449; v. 38, 1951, p. 435–462.

1211[K].—R. S. Burington & D. C. May, Jr., *Handbook of Probability and Statistics with Tables*. Handbook Publishers, Sandusky, Ohio, 1953. ix + 332 p., 14.3 × 20.3 cm. Price $4.50.

This book is principally a rather complete and up-to-date handbook of statistical methods and theory which will prove very useful; reviews concerned with this aspect will be found elsewhere. Scattered through the text are some 17 tables and at the end 24 tables and a nomogram occupy 62 pages.

The tables in the text, mostly short, begin with four giving ordinates, areas, and percentage points for the normal frequency function. The following four are devoted to the normal bivariate frequency function. The first, if this function be written $f(x, y) = k \exp(-c^2/2)$, gives c to 4S for the $P\%$ probability ellipse for $P = 25, 50, 75, 90, 95, 99$. The next three deal with the circular case, giving radii $c\sigma$ of $P\%$ probability circles with c to 4S for $P = 25, 39.3, 50, 54.4, 75, 90, 95, 99$; the radii R_1 to 4D in standard units of a circular disk centered at the means over which the probability integral is P for $P = 0(.05)1$ as well as the integrand evaluated at R_1 to 4D; and values of the same probability integral to 3 or 4D over a circular disk of radius R/σ centered at the distance d/σ from the means for $R/\sigma = .1(.1)1(.2)3$ and $d/\sigma = 0(.1)3(.2)6$. The next two tables are concerned with the trivariate normal frequency function with zero correlations, the first being the exact analog of the first bivariate table and the second, for the spherical case, the radius $c\sigma$ (c to 3D) of the sphere within which the total frequency is p for $p = .25, .5, .75, .9, .95, .99$.

All but one of the remaining tables in the text are related to sampling distributions. Those for which credit is given to other sources are here designated by (R). If s is the standard deviation in a sample of n from a normal universe with standard deviation σ, the next table (R) gives b for which $P(s > b\sigma) = P$ to 3D for $n = 5(1)30$ and $P = .01, .05, .95, .99$.

If $r = r_x\sqrt{n-1}$ where r_{xy} is the coefficient of correlation in a sample of n) from an$_y$ uncorrelated bivariate normal universe, the following table (R gives values of r_1 to 3D for which $P(r \geqq r_1) = P$ for $P = .001, .01, .05, .1$ and $m = n - 2 = 1(1)30(5)50(10)100, 120, \infty$. Then comes a table ($R$) concerned with the distribution of ranges, R, in samples of n from the same universe. Values of $E\left(\frac{R}{\sigma}\right)$ are given to 4S for $n = 2(1)20$, to 3S for $n = 30$, 50, 75, 100, and to 2S for $n = 150, 200$ and $\sigma_{R/\sigma}$ is given to 3S for $n = 2(1)20$. Also R'/σ is given to 2D for which $P(R \leqq R') = P$ for $P = .001, .005, .01, .025, .05, .1, .9, .95, .975, .995, .999$ and $n = 2(1)20$. Next the probability p that the fraction h of any continuous universe is included in the range of a sample of n drawn from it is tabulated to 2S (more in a few cases) for $h = .8, .9, .95, .99$ and $n = 5(5)30, 40, 50, 75, 100$. The next pair of tables (R) give equal-tail confidence intervals for the probability of success p in a binomial universe from which n trials have yielded S successes. The end-points are given to 2D for $n = 10, 15, 20, 30, 50, 100$, $S = 0(1)50$ and the confidence coefficients .95 and .99. Then follows a table of Stirling's numbers of the first kind, S_i^n, for $n = n(1)10$ and $i = 0(1)9$. The final table (R) in the text gives the numerical coefficients for estimating center lines and control limits for $\bar{X}$ and σ or R charts in quality control, both for process means and standard deviations known or unknown. These are given to 3 or 4S for samples of $n = 2(1)10(2)20$ for averages and standard deviations and $n = 2(1)10(2)14$ for ranges.

The section devoted to tables (p. 247–309) include:

Table I. $\binom{n}{x} p^x(1-p)^{n-x}$ to 4D for $p = .05(.05).5$ and $n = 1(1)20$.

Table II. $\sum_{x=x'}^{n} \binom{n}{x} p^x(1-p)^{n-x}$ to 4D for the same values of p and n as in Table I.

Table III. p for which the incomplete β-function, $I_p(x, n-x+1) = .005, .01, .025, .05, .1$ for $x = 1(1)15, 20, 30, 60, \infty$ and $n - x + 1 = 1(1)6, 10, 15, 20, 30, 60$.

Table IV. $\binom{n}{r}$ for $n = s(1)20$ and $r = 0(1)10$.

Table V. $[np(1-p)]^{\frac{1}{2}}$ to 4D for $n = 1(1)20$ and $p = .05(.05).5$.

Table VI. $(pq)^{\frac{1}{2}}$ to 4D for $p = .005(.005).5$.

Table VII. $e^{-m}m^x/x!$ for $x = 0(1)\cdots$, to 4D for $m = .1(.1)10(1)20$.

Table VIII. $\sum_{x=x'}^{\infty} e^{-m}m^x/x!$ for the same ranges of the arguments as in Table VII.

Table IX. $\psi(t) = (2\pi)^{-\frac{1}{2}}e^{-t^2/2}$, $\alpha/2 = \int_0^t \psi(\tau)d\tau$, and $\psi^{(n)}(t)$ to 4D or 4S for $t = 0(.01)4(.05)5$ and $n = 1(1)6$.

p. 275. Selected important constants to 8D and their common logarithms to 7 or 8D.

Table X(R). 1% and 5% points of the F distribution to 3 or more S for the degrees of freedom 1(1)12, 14, 16, 20, 24, 30, 40, 50, 75, 100, 200, 500, ∞ for the larger estimated variance and 1(1)30(2)50(5)80, 100, 125, 150, 200, 400, 1000, ∞ for the smaller estimated variance.

Table XI(*R*). 1%, 1% and 5% points of the *z*-distribution. ($Z = \frac{1}{2} \ln F$) to 4D for the degrees of freedom 1(1)6, 8, 12, 24, ∞ for the larger estimated variance and 1(1)30, 60, ∞ for the smaller estimated variance.

Table XII(*R*). The percentage points 100ϵ of the Student-Fisher t distribution to 3D for $\epsilon = .001, .01, .02, .05, .1(.1).9$ and the degrees of freedom 1(1)30, 40, 60, 120, ∞.

Table XIII. The cumulative Student-Fisher t distribution to 3D for $t = 0(.2)5$ and the degrees of freedom 1(1)6, 8, 10, 15, 20.

p. 285. A nomogram for $1 - (1 - p)^n$.

Table XIV. $(R)\chi_0^2$ satisfying $P(\chi^2 \geqq \chi_0^2) = \epsilon$ to 3D for $\epsilon = .001, .01, .02, .05, .1, .2, .3, .5, .7, .8, .9, .95, .99$ and the degrees of freedom 1(1)30.

Table XV. e^{-x} to 5D for $x = 0(.01)3(.05)4(.1)6(.25)7(.5)10$.

Table XVI. $n!$ to 5S and $\log_{10} (n!)$ to 5D for $n = 1(1)100$.

Table XVII. $\Gamma(n)$ to 4D for $n = 1.01(.01)4.99$.

Table XVIII. $\log_{10} \Gamma(n)$ to 4D for $n = 1.01(.01)2$.

Table XIX. $n!$ exact and $(n!)^{-1}$ to 5S for $n = 1(1)20$.

Table XX. n^2 exact and $n^{\frac{1}{2}}$, $(10n)^{\frac{1}{2}}$ and $1000\ n^{-1}$ to 5S for $n = 1(1)999$.

Table XXI. The natural trigonometric functions and the radian measure to 4D for the angle $\alpha = 0(1°)90°$.

Table XXII. $\ln N$ to 3D for $N = 0(.01).99$ and to 5D for $N = 1(.01)10.09$.

Table XXIII. $\log_{10} N$ to 4D for $N = 100(1)999$.

C. C. C.

1212[K].—J. H. Cadwell, "The distribution of quasi-ranges in samples from a normal population," *Annals Math. Stat.*, v. 24, 1953, p. 603–613.

Let $x_1 \leqq x_2 \leqq \cdots \leqq x_n$ be the ordered values in a random sample of size n drawn from a normal population. The statistic $w_r = x_{n-r} - x_{r+1}$ is called the range for $r = 0$, and a quasi-range for $r \geqq 1$. The author gives an approximate, but apparently very accurate, method for evaluating the probability distribution function of w_r. Of interest are the following results: (1) in section 3, the approximation for the p.d.f. of w_r is given; (2) in section 4, a comparison is made between the exact and approximate values of $E(w_0)$ and $E(w_1)$ for $n = 2, 8, 20, 30, 60, 100$; (3) in section 5, the mean, variance, skewness, and flatness of the range are given for $n = 20$; (4) in section 7, there is a brief discussion of the efficiency of w_0, w_1, and w_2 as estimators of the population standard deviation σ; (5) in section 8, a table of values of moment constants to 3 or 4D and the .1, 1, 2.5, 5, 95, 97.5, 99, and 99.9 percentage points of w_1 are given for $n = 10(1)30$ to 2D; (6) in section 9, there is a brief consideration of the efficiency of estimates of σ based on a linear combination of two quasi-ranges. The paper fills in a gap between published work for cases where $n \leqq 10$ on the one hand and n large on the other.

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1213[K].—R. E. CLARK, "Percentage points of the incomplete beta function," Am. Stat. Assn., *Jn.*, v. 48, 1953, p. 831–843.

The table presented here gives to 4S the value of

$$p = P(N, X, \alpha)$$

defined by

$$\alpha = \sum_{r=X}^{N} \binom{N}{r} p^r (1 - p)^{N-r} = I_p(X, N - X + 1)$$

for

$$N = 10(1)50, \quad X = 1(1)N, \quad \alpha = .005, .01, .025, .05,$$

where $I_p(X, Y)$ is KARL PEARSON's incomplete beta function. Applications of the table are indicated.

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1214[K].—W. J. DIXON, "Processing data for outliers," *Biometrics*, v. 9, 1953, p. 74–89.

Samples of size N are drawn from a mixture of normal distributions $(1 - \gamma)N(\mu, \sigma^2) + \gamma N(\mu + \lambda\sigma, \sigma^2)$ and $(1 - \gamma)N(\mu, \sigma^2) + \gamma N(\mu, \lambda^2\sigma^2)$, respectively, in which γ measures the contamination of the $N(\mu, \sigma^2)$ universe. Comparisons are made between the mean and median as estimates of μ; range and variance as estimates of dispersion. The bias and the MSE (mean squared error) for untreated (treatment refers to processing the data in a sample to remove outliers) samples (Table I), the ratios of the MSE of the mean and median of untreated data to the MSE of the mean for uncontaminated data (Table II), and minimum MSE of four procedures: use of $\bar{X}$ after treating for rejection at the significance levels, $\alpha = .01, .05, .10$ and the use of median (Table III), serve as a basis for comparing the mean and median. The appropriate α to remove bias in s^2 and range (Tables IV, V, VI, VII) are used to compare range and variance. For most of the Tables $N = 5, 15$; $\gamma = .01, .05, .10, .20$, $\lambda = 0, 2, 3, 5, 7$ with some modifications. The values known to be correct are those for $\lambda = 0$, the quantities for the mean in Table I, the results for no contamination in Tables II and III and the first line in Table V. Other results for $N = 5, 15$ are based on 100 and 66 samples, respectively. The appendix gives critical values and criteria for testing for extreme values based on $r_{ij} = (X_N - X_{N-i})/(X_N - X_{j+1})$, $\alpha = .005, .01, .05, .10, .20, .30$, $N = 3(1)7$ for r_{10}, $N = 8, 9, 10$ for r_{11}, $N = 11, 12, 13$ for r_{21}, $N = 14(1)25$ for r_{22} to 3D.

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1215[K].—ABRAHAM GOLUB, "Designing single-sampling inspection plans when the size is fixed," Amer. Stat. Assn., *Jn.*, v. 48, 1953, p. 278–288.

Consider the sample size n fixed for a given single sampling plan. The problem is to choose an acceptance number s which yields the "best" protection against misclassifying submitted lots. Two cases are considered: (1) single sampling-plans for placing a lot into one of two categories defined by fraction defectives p_1 and p_2, (2) single sampling-plans for placing a lot into one of m categories defined by fraction defectives $p_1, p_2, \cdots, p_m$. In case (1), the "best" plans are defined as those which minimize the sum of the producer's and consumer's risks. In case (2), the "best" plans are defined as those which minimize the sum of the probabilities of misclassifying the submitted lots. Given m categories defined by fraction defectives $p_1, p_2, \cdots, p_m$, Tables 1 through 8 provide $m - 1$ acceptance numbers, c_i $(i = 1, 2, \cdots, m - 1)$ for single sampling-plans based on given fixed sample sizes 5(5)40, for acceptable lot qualities .01(.01).20, and for objectionable lot qualities .01(.01).40 respectively. The above tables should be very useful in quality control applications in which it is desired to divide the lot quality into more than two categories.

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1216[K].—E. L. Grab & I. R. Savage, "Tables of the expected value of $1/x$ for positive Bernoulli and Poisson variables," Am. Stat. Assn., *Jn.*, v. 49, 1954, p. 169–177.

Let X be a random variable having the probability distribution

$$\text{(a)} \quad P\{X = k\} = \binom{n}{k} p^k q^{n-k}(1 - q^n)^{-1},$$

where $k = 1, 2, \cdots, n; q = 1 - p; 0 < p \leqq 1;$

$$\text{(b)} \quad P\{X = k\} = e^{-m}(1 - e^{-m})^{-1}m^k/k!,$$

where $k = 1, 2, \cdots; m > 0$. For case (a) Table I gives the values of

$$E(1/X \mid n, p) = (1 - q^n)^{-1} \sum_{k=1}^{n} \binom{n}{k} k^{-1}p^k q^{n-k}$$

to 5D for $n = 2(1)20$, $p = .01, .05(.05).95, .99$; $n = 21(1)30$, $p = .01, .05(.05).50$. The authors give an empirical approximation to $(np - q)^{-1}$ for certain values of the parameters which is accurate to at least 2S. For case (b) Table II gives the values of

$$E(1/X \mid m) = e^{-m}(1 - e^{-m})^{-1} \sum_{k=1}^{\infty} m^k k^{-1}/k!$$

to 5D for $m = .01, .05, (.05)1(.1)2(.2)5(.5)7(1)10(2)20$.

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1217[K].—F. E. Grubbs & H. J. Coon, "On setting test limits relative to specification limits," *Industrial Quality Control*, v. 10, No. 5, 1954, p. 15–20.

This paper is concerned with the problem of setting test limits when specification limits are fixed and test measurements are subject to error. The authors consider test limits established in accordance with the following criteria: (1) producer's and consumer's risk are made equal; (2) the sum of producer's and consumer's risk is minimized; and (3) the cost of incorrect decisions is minimized. As used here, consumer's risk and producer's risk do not have quite the same meaning that is usually associated with these terms in connection with sampling inspection. For the purposes of this paper, the consumer's risk, designated as A, is defined as the joint probability of selecting a defective unit and judging it acceptable. Similarly the producer's risk, designated as B, is defined as the joint probability of selecting an acceptable unit and judging it defective. With specification limits expressed as $\mu \pm k\sigma_x$ where σ_x is the product standard deviation, test limits are expressed as $\mu \pm (k\sigma_x - b\sigma_e)$ where σ_e is the standard deviation of the errors of measurement. For criteria (1) and (2) above, tables of b, A, B, and $A + B$ are given to 4D with $k = 1.5(.5)4$, and $r = .5(.25)1(1)10$, where $r = \sigma_x/\sigma_e$. Examples are given which illustrate the use of these tables.

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1218[K].—J. M. HAMMERSLEY, "On estimating restricted parameters," Roy. Stat. Soc., *Jn.*, s.B v. 12, 1950, p. 192–240.

This paper contains a table of solutions of the following distribution problem: A philanthropist with u coins gives the first beggar he meets a number of coins which with equal probabilities may be $1, 2, \cdots$, or u. If the first beggar receives $u - r$ coins, the next beggar will receive $1, 2, \cdots$, or r coins, each number being equally probable. If this is continued until the coins are exhausted, what is the probability that exactly ν beggars will receive a gift? The probability $P(u, \nu)$ satisfies the equation

$$P(u, \nu) = \sum_{r=\nu-1}^{u-1} u^{-1}P(r, \nu - 1)$$

with $P(u, 1) = 1/u$. $P(u, \nu)$ is tabled for u and $\nu = 1(1)13$.

C. C. C.

1219[K].—HANNES HYRENIUS, "On the use of ranges, cross-ranges, and extremes in comparing small samples," Am. Stat. Assn., *Jn.*, v. 48, 1953, p. 534–545.

A procedure is proposed using ranges, cross-ranges, and lower-extreme-differences as alternatives to the variance in testing variation homogeneity. The universe sampled is rectangular. Let sample 1 have N_1 items with lower extreme u_1 and upper extreme v_1, and let sample 2 have the corresponding N_2, u_2, and v_2. Designate the samples so that $u_1 \leqq u_2$. The ranges are $R_{11} = v_1 - u_1$ and $R_{22} = v_2 - u_2$. The cross-range is $R_{21} = v_2 - u_1$; and the lower-extreme-difference is $S_{21} = u_2 - u_1$.

Distributions of the test quotients $T = S_{21}(R_{11})^{-1}$, $U = R_{22}(R_{11})^{-1}$ and $V = R_{21}(R_{11})^{-1}$ are studied. With means and variances derived, tables are prepared showing upper 1, 5 and 10 percentage points of T, and showing upper .5, 2.5, and 5 and lower 95, 97.5 and 99.5 percentage points of U and V for $N_1, N_2 = 1(1)10$ to 2D.

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1220[K].—R. Latscha, "Tests of significance in a 2 × 2 contingency table: Extension of Finney's table," *Biometrika*, v. 40, 1953, p. 74–86.

Consider a 2 × 2 contingency table arranged so that the sum of the two values in the first row is A and the sum of the two values in the second row is B where $A = B$. Let a be value in the first row and first column while b is the value in the second row and first column. For given A, B, a, b, the table presented is entered in the section for A, the sub-section for B and the line for a; then the main body of the table shows in bold type the appropriate significance points for b. If the observed value b is equal to or less than the bold-faced integer in the column headed .05, .025, .01 or .005 (only values considered), then a/A is significantly greater than b/B (single-tail test) at these probability levels. This same condition on b furnishes a two-tail test of whether a/A differs from b/B at twice the probability value heading the column. A dash or absence of entry for specified A, B, a indicates that no 2 × 2 table in that class can show a significant effect at that level. The true significance level is at most equal to the probability value heading the column and in some cases may be noticeably smaller. For each table entry considered, the true probability value is given to 3D in small type next to the bold-faced integer with which b is compared. The table presented in this article covers the cases $A = 16(1)20$, $B = 2(1)A$, $A \geqq a \geqq$ (smallest integer not yielding all dashes). Previously Finney[1] obtained a similar table for $A = 9(1)15$.

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[1] D. J. Finney, "The Fisher-Yates test of significance in 2 × 2 contingency tables," *Biometrika*, v. 35, 1948, p. 145–156 [*MTAC*, v. 3, p. 359].

1221[K].—M. Masuyama & Y. Kuroiwa, "Table for the likelihood solutions of gamma distribution and its medical applications," Union of Japanese Scientists and Engineers, *Reports of Statistical Application Research*, v. 1, 1951, p. 18–23.

For the Γ-type frequency function,

$$f(y) = \begin{cases} (y/a)^{p-1} \exp(-y/a)\, dy[a\Gamma(p)]^{-1} & \text{if } y \geqq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

the maximum likelihood estimates, $\hat{p}$ and $\hat{a}$, of the two parameters, p and a, are the solutions of the equations,

$$\ln \hat{p} - \psi(\hat{p}) = \ln(A/G) = g(\hat{p})$$
$$\hat{a} = A/\hat{p}$$

in which ψ is the digamma function and A and G are the arithmetic and geometric means of the sample respectively. To facilitate the approximate solution of the equations, the authors have tabulated $x \log(A/G)$, $g(x)$, $\log(A/G)$ and A/G to 8S or 7D for $x = .1(.05)3(.1)5(.5)10, 15, 20(10)50$. For large samples of N, $\frac{N\sigma_{\hat{a}}^2}{a^2}$ and $N\sigma_{\hat{p}}^2$ are given to 8D for the same values of x.

C. C. C.

1222[K].—NBSCL *Probability Tables for the Analysis of Extreme-Value Data*. NBS Applied Math. Ser. No. 22. U. S. Gov. Printing Office, Washington, D. C., 1953. iii + 32 p., 22 × 20 cm. Price $0.25.

There are many engineering and scientific problems in which the random variable being observed is a largest (smallest) or more generally *m*'th largest (*m*'th smallest) value. The 15 page introduction by E. J. GUMBEL, who pioneered in the field of extreme values, is a succinct and excellent survey of current knowledge in the field. In the first 9 pages of this introduction he sketches the theory and carries out the analysis of a problem in detail. The remainder of the introduction is devoted to listing the functions tabulated, giving a history of the genesis of the tables, and describing methods of computation. A useful bibliography completes the introduction.

If one carries out a suitable transformation on the largest value in a sample one gets a random variable which is called the reduced largest value. The asymptotic distribution of this reduced largest value is (for a wide class of common distributions such as the exponential, normal, and chi-square distributions) given by the cumulative distribution function $\Phi_y \equiv \exp(-e^{-y})$ and associated probability density function $\varphi_y \equiv \Phi_y' \equiv \exp(-y - e^{-y})$. Table 1 gives Φ_y and φ_y for $y = -3(.1) - 2.4(.05)0(.1)4(.2)8(.5)17$ to 7D. Table 2 gives the inverse of the cumulative probability function of extremes $y = -\log_e(-\log_e \Phi_y)$ for $\Phi_y = .0001(.0001).005(.001).988(.0001)$ $.9994(.00001).99999$ to 5D. Table 3 gives the probability density function φ_y as a function of the cumulative distribution function $\Phi_y[\varphi_y = -\Phi_y \log \Phi_y]$ for $\Phi_y = 0(.0001).01(.001).999$ to 5D. The reduced *m*'th largest value has associated cumulative distribution function $\Phi_m \equiv \int_\infty^{y_m} \frac{m^m}{(m-1)!} \times \exp(-my - me^{-y})dy$. Table 4 gives probability points y_m for the reduced *m*'th largest value, $y_m = y_m(\Phi_m)$, for $m = 1(1)15(5)50$; $\Phi = .005, .01, .025, .05, .1, .25, .5, .75, .9, .95, .975, .99, .995$ to 5D. Tables 5 and 6 deal with an appropriately reduced range R. The cumulative probability distribution function $\Psi(R)$ is given by $\Psi(R) \equiv 2e^{-\frac{1}{2}R}K_1(2e^{-\frac{1}{2}R})$ and the probability density function $\psi(R) \equiv \Psi'(R)$ is given by $\psi(R) \equiv 2e^{-R}K_0(2e^{-\frac{1}{2}R})$. K_0 and K_1 are the modified Bessel functions of the second kind of orders 0 and 1, respectively. Table 5 gives $\Psi(R)$ and $\psi(R)$ for $R = -4.6(.1) - 3.3(.05)11(.5)20$ to 7D. In Table 6, the reduced range R is considered as a function of Ψ, i.e., as the solution of the equation $\Psi = 2e^{-\frac{1}{2}R}K_1(2e^{\frac{1}{2}R})$. This is given in Table 6 for $\Psi = .0001(.0001).001(.001).01(.01).95(.001).999(.0001).9999$ to 4D, except for $\Psi = .0001$ and $\geqq .999$ to 3D.

The tables are the fruits of the painstaking labors of many people. Chief among these are E. J. GUMBEL who took part in the detailed planning, J. ARTHUR GREENWOOD, JULIUS LIEBLEIN, and H. E. SALZER. The preparation of the tables in their final form was carried out under SALZER's direction at the Computation Laboratory of the National Bureau of Standards. LIEBLEIN assisted in the preparation of the Introduction.

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1223[K].—NBSCL *Tables to Facilitate Sequential t-Tests.* NBS Appl. Math. Ser. No. 7. U. S. Gov. Printing Office, Washington, 1951. xix + 52 p. Price $0.45.

The sequential t-test is designed to test the hypothesis that the mean of a normally distributed variate with unknown mean and unknown variance has a given value. The present tables are designed for use with a probability ratio test based on the likelihood ratio for the non-central t to the central t, which thus differs from the one originally proposed by WALD.[1] However by a simple adjustment of the parameters these tables can also be used for Wald's test.

The test criterion after the n-th observation is $Z_n = [\sum_{\alpha=1}^{n} (x_\alpha - l_0)]^2 \times [\sum_{\alpha=1}^{n} (x_\alpha - l_0)^2]^{-1}$ in which l_0 is the value of the mean specified by the null hypothesis. The critical values of Z_n are solutions of the equation $L = \ln F\left(n/2, \frac{1}{2}; \frac{\delta^2 Z}{2}\right) - \frac{n\delta^2}{2}$ in which $F(a, b; x)$ is the confluent hypergeometric function,

$$\sum_{j=0}^{\infty} \frac{\Gamma(b)\Gamma(a+j)}{\Gamma(a)\Gamma(b+j)} \frac{x^j}{j!};$$

δ is the amount in standard units by which the mean under the alternate hypothesis differs from l_0 and L is either $\ln\{(1-\beta)/\alpha\}$ or $\ln\{\beta/(1-\alpha)\}$ in which α and β are the risks of errors of the first and second kinds respectively. Values of Z_n are given to 1D for $\delta = .1$, to 2D for $\delta = .2(.1).5$ (to 3D for $L < 0$ and $\delta = .2$) and to 3D for $\delta = .6(.1)1(.2)2, 2.5$ for $\pm L = 2$, ln 19, 3, 4, ln 99, 5, 6, 7 and $n = 1(1) \leqq 200$. (Values of n are carried as far in each case as is judged likely to be useful in practice but never beyond 200.) There are some smaller auxiliary tables. One of these gives the values of α and β to 3S for the L's listed; others are to assist in reaching a decision if n goes beyond the values in the table; and still others give approximate upper bounds for the error in linear interpolation with respect to L or S.

C. C. C.

[1] A. WALD, *Sequential Analysis.* New York, 1947, p. 204–207.

1224[K].—B. M. SEELBINDER, "On Stein's two-stage sampling scheme," *Annals Math. Stat.*, v. 24, 1953, p. 640–649.

STEIN[1] has proposed a two-stage plan for sampling from a normal population of unknown variance σ^2 in order to estimate the expectation with a confidence interval of preassigned length $2d$ and confidence coefficient $1 - \alpha$. The experimenter sets the size n_1 of the first sample, while the size $n - n_1$ of the second sample depends on the variability observed in the first. The present paper is concerned with the problem of choosing n_1 with an eye to minimizing $E(n)$. SEELBINDER tables $E(n)$ for some of the following values: $\alpha = .1, .05, .02, .01$; $d/\sigma = .01(.01).1(.1)1$; $n_1 - 1 = 5, 10(10)60, 80, 120,$

240. An approximation based on the normal distribution is found to be adequate.

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[1] CHARLES STEIN, "A two-sample test for a linear hypothesis whose power is independent of the variance," *Annals Math. Stat.*, v. 16, 1945, p. 243–258.

1225[K,N].—H. STEINHAUS, *Table of shuffled four-digit numbers. Rozprawy Matematyczne*, No. 6, Warsaw 1954, 46 p. The introduction is in Polish, Russian and English.

This table of 10,000 random 4-digit numbers differs from the usual random number lists in that no two 4-digit numbers are equal. In other words the table, when read in the usual order, gives one of the 10,000! permutations of the numbers 0000–9999. This feature allows the list to be used in problems in which samples are drawn without replacement. One may also read the digits vertically by fours and obtain samples which are drawn with replacement.

The table was produced by hand. To begin with a table of 10,000 four-digit numbers $U_n(n = 0(1)9999)$ was produced by

$$U_n \equiv 4567 U_{n-1} \pmod{10{,}000}$$

with $U_0 = 0000$. This table was arranged in a square 100×100 and then subjected to a number of randomizing transformations fully described in the introduction. Whether the 25 pages which finally resulted pass any of the standard tests of randomness is not indicated. Apparently the author has made no tests whatever. The reviewer subjected the first page to a frequency test which gave a χ^2 with a probability of only 2 percent.

The list could be useful for ordering the retirement of bonds.

D. H. L.

1226[K].—G. M. THOMPSON, "Scale factors and degrees of freedom for small sample sizes for χ approximation to the range," *Biometrika*, v. 40, 1953, p. 449–450.

Consider m independent samples of size n from a normal population, mean μ, variance σ^2. Let ω be the range of a sample, $\bar{\omega}_{m,n}$ the mean range. PATNAIK[1] found the approximation

$$\bar{\omega}_{m,n} = c\chi\nu^{-\frac{1}{2}}\sigma$$

where c is a scale factor, ν the degrees of freedom for χ. Evidently $\bar{\omega}_{m,n/c}$ is equivalent to the usual standard deviation estimator with ν degrees of freedom. Thompson tables c and ν to 4D for $m = 1$, $n = 2(1)10$. He also gives the 95% confidence limits on μ using ω, i.e. $\bar{X} \pm k_{.05}\omega$ to 4D, for $n = 2(1)10$. Agreement with results obtained by the use of Student's t is excellent.

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[1] P. B. PATNAIK, "The use of mean range as an estimator of variance in statistical tests," *Biometrika*, v. 37, 1950, p. 78–87.

1227[L].—F. DiMaggio, A. Gomza, W. E. Thomas & M. G. Salvadori, "Instabilità laterale di travi inflesse e compresse," Accad. Naz. dei Lincei, *Atti Rend.*, s. 8, v. 12, 1952, p. 524–529.

This paper summarizes the results of a number of technical reports published by the Dept. of Civil Engineering, Columbia University. The critical loads for the lateral buckling of several types of beams and loadings are given. These are the lowest eigenvalues of $\delta V = 0$, where

$$V = \tfrac{1}{2}B\int_0^l (\mu'')^2dz + \tfrac{1}{4}Dh^2\int_0^l (\beta'')^2dz + \tfrac{1}{2}H\int_0^l (\beta')^2dz + \int_0^l M\beta\mu''dz - \tfrac{1}{2}P\int_0^l (\mu')^2dz.$$

B, D, h, H and P are constants. Table 1 gives the results for I-beams under unequal end moments, tabulates eigenvalues $K = M_2l/(BC)^{\frac{1}{2}}$ to 2D as a function of $l^2/a^2 = 2l^2C/h^2D = .1$ (irreg.) ∞ and $r = M_1/M_2 = -1(.1)1$, where $H = C - PI_p/A$, $M(z) = M_2[r + (1 - r)z/l]$, $M_1 = rM_2$, $\mu(0) = \mu''(0) = \mu(l) = \mu''(l) = 0$, $\beta(0) = \beta''(0) = \beta(l) = \beta''(l) = 0$. Table 2 gives the solution of rectangular beams under bending and compression, corresponding to $D = 0$, $\mu(0) = \mu(l) = \mu''(0) = \mu''(l) = 0$, $\beta(0) = \beta(l) = 0$. $K = M_2l/(BH)^{\frac{1}{2}}$ given to 2D for $p = Pl^2\pi^{-2}/B = 0(.2)1$; and $r = M_1/M_2 = -1(.1)1$. Table 3 gives solution to rectangular cantilever beam with shear and axial load at the free end corresponding to $D = 0$; $r = 0$; $M(z) = Qz = M_2z/l$ and $\mu(l) = \mu'(l) = \mu''(0) = B\mu''(0) + P\mu'(0) + Q\beta(0) = 0$, $\beta(l) = \beta'(0) = 0$, $q = Ql^2/(4.013(BH)^{\frac{1}{2}})$ given to 4D for $p = 4Pl^2\pi^{-2}/B = 1(-.2) - .4, -1$. Solutions of this problem (originated by Prandtl and Michell) for many other end conditions have been discussed in Timoshenko.[1]

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[1] S. Timoshenko, *Theory of Elastic Stability*. New York, 1936.

1228[L].—Stig Ekelöf, "Theory of electromagnetically delayed telephone relays," *Ericson Technics*, v. 9, 1953, p. 141–224.

On p. 221–224 there is a 4D table of the functions

$$\cosh x, \quad \frac{\sinh x}{x}, \quad \frac{3\cosh x}{x^2} - \frac{3\sinh x}{x^3}, \quad \frac{2(\cosh x - 1)}{x^2},$$
$$\frac{12\sinh x}{x^3} - \frac{24(\cosh x - 1)}{x^4}$$

for $x = 0(.01)1{\cdot}5$. Graphs of these functions (in the same range) are given on p. 161.

A. E.

1229[L].—ERNST GLOWATZKI, "Tafel der Jacobischen elliptischen Funktion $\phi = \text{am}\,(mK/n)$," Bayer. Akad. Wiss., *math.-naturw. Kl.*, *Abhandlungen*, Neue Folge, heft 61, 1953, 27 p.

3D tables of $\text{am}(u, k)$ in degrees for $u = mK/n$, $k = \sin\theta$, $n = 2(1)12$, $n = 1(1)n - 1$, $\theta° = 0(1)90$. First differences are also given. The entries were computed from Legendre's tables, and were checked against the *Smithsonian Elliptic Function Tables* (see *MTAC*, v. 3, 1948–49, p. 89, RMT 485).

A. E.

1230[L].—LOUIS ROBIN & ALFREDO PEREIRA-GOMES, "L'antenne biconique, symmétrique, d'angle quelconque," *Ann. d. Télécommunications*, v. 8, 1953, p. 382–390.

The authors discuss the equations

$$P_n(\cos\theta) + P_n(-\cos\theta) = 0 \quad \text{or} \quad F\left(-\frac{n}{2}, \frac{n+1}{2}; \tfrac{1}{2}; \cos^2\theta\right) = 0$$

$$P_n(\cos\theta) - P_n(-\cos\theta) = 0 \quad \text{or} \quad F\left(\frac{1-n}{2}, \frac{n+2}{2}; \tfrac{3}{2}; \cos^2\theta\right) = 0$$

where P_n is the Legendre function, F the hypergeometric series, θ is given, and n is to be determined.

On p. 390 they give tables (mostly to 3D) of the first nine roots of the first equation, and of the first eight roots of the second equation, when $\theta = \pi/12, \pi/6, \pi/4, \pi/3, 5\pi/12$.

A. E.

1231[L].—M. ROTHMAN, "The problem of an infinite plate under an inclined loading, with tables of the integrals of $\text{Ai}(\pm x)$ and $\text{Bi}(\pm x)$," *Quart. Jn. Mech. Appl. Math.*, v. 7, 1954, p. 1–7.

For the definition of the Airy integrals $\text{Ai}(x)$ and $\text{Bi}(x)$ see *MTAC*, v. 1, 1944, p. 236.

Table 1. 7D values of $\int_0^x \text{Ai}(t)dt$ for $x = 0(.1)7.5$ with $\delta_m{}^2$. 7D values of $\int_0^x \text{Bi}(t)dt$ for $x = 0(.1)2$ with $\delta_m{}^2$ and γ^4.

Table 2. 7D values of $\int_0^x \text{Ai}(-t)dt$ for $x = 0(.1)10$ with $\delta_m{}^2$ and γ^4.

Table 3. 7D values of $\int_0^x \text{Bi}(-t)dt$ for $x = 0(.1)10$ with $\delta_m{}^2$ and γ^4.

A. E.

1232[L].—F. G. TRICOMI, "On the statistical distribution of mutant bacteria," *Bull. Mathem. Biophysics*, v. 15, 1953, p. 277–292.

The author gives 5D tables of

$$G(t) = \sum_{n=1}^{\infty} \frac{1}{2^n - 1}\frac{t^n}{n!}, \quad G'(t) = \frac{dG}{dt}, \quad G^*(t) = \int_0^t G(u)du$$

for $t = -2(.1)2$. Graphs are given of $G(t)$, and also of the curve represented parametrically by

$$x = G(t), \quad y = \left[\frac{\alpha}{2\pi G'(t)}\right]^{\frac{1}{2}} \exp\{\alpha[G^*(t) - tG(t)]\}$$

for $\alpha = 1$ and $\alpha = .01$.

A. E.

1233[L].—E. F. M. van der Held, "The contribution of radiation to the conduction of heat. II. Boundary conditions," *Appl. Sci. Research* A, v. 4, 1953, p. 77–99.

The Appendix (p. 92–99) contains a collection of formulas and some numerical tables concerning the function

$$K_n(x) = \int_1^\infty t^{-n} e^{-xt} dt$$

which is closely related to the complementary incomplete gamma function.

Table VII gives values of K_1, K_2, $2K_3$, $3K_4$, $4K_5$ to 4D or 4S for $x = 0(.01).2(.02)2(.5)5(1)8$.

Table VIII gives values to 2 to 4S of

$$\int_1^\infty t^{-1} e^{-xt} \log t \, dt, \quad \int_0^x K_n(x - t) K_2(t) dt, \quad n = 1(1)5$$

for $x = 0, .01, .05, .1, .25(.25)1(.5)5(1)8$.

Table IX gives values to 2 to 4S of

$$\int_0^x K_n(D - x + t) K_2(t) dt, \quad \int_0^\infty K_n(x + t) K_2(t) dt, \quad D = 2, 4, \quad n = 2(1)5$$

in varying ranges, and at varying intervals, of x.

V. Kourganoff (see *MTAC*, v. 3, p. 307, RMT **569**) has tabulated K_2, $2K_3$, $\frac{4}{3} - 2K_4$, $\frac{8}{3}x + 4K_5$ for $x = 0(.02)2$. Earlier tables of $K_n(x)$ are listed in FMR *Index*, sec. 14.83.

A. E.

1234[S].—Otto Emersleben, "Das elektrostatische Selbstpotential äquidistanter Ladungen auf einer Kreislinie," *Math. Nachr.*, v. 10, 1953, p. 135–167.

If $2n$ charged particles are equally spaced on a circle of radius r with a charge $(-1)^k$ at P_k, $1 \leq k \leq 2n$, then the self-energy of the system is given by the finite sum

$${}^{0}_{2n}\Phi = \frac{n}{r} \sum_{k=1}^{(n)} (-)^k \csc\left(\tfrac{1}{2}\pi k/n\right),$$

where (n) indicates that the last term must be multiplied by $\frac{1}{2}$. In a unit lattice $n = \pi r$ and the power series for the cosecant leads to the decomposition

$${}^{0}_{n}\Phi = {}_{n}\overline{\Phi}_0 + {}_{n}\overline{\Phi}_1,$$

where ${}_{n}\overline{\Phi}_0$ comes from the first term of the power series and is similar to the potential of a linear configuration. Both expressions ${}_{n}\overline{\Phi}_0$ and ${}_{n}\overline{\Phi}_1$ involve the sums $\sum_{k=1}^{(n)} (-)^k k^{-s}$ for various integer values of s and their asymptotic behavior for large n is studied. The term ${}_n\bar{\Phi}_1$ arises from the curvature of the circle and is $O(n^{-1})$ as $n \to \infty$, while ${}_n\bar{\Phi}_0 = -(2 \log 2)n + O(n^{-1})$. The O-term of ${}_n\bar{\Phi}_0$ is also analyzed in somewhat more detail and a table of ${}_n\bar{\Phi}_0$ is given for $n = 1(1)12$.

Similar results are obtained for the problem of n equally charged and equally spaced particles on a circle. The self-energy in this case is

$$ {}_n^0\Phi = \frac{n}{4r} \sum_{k=1}^{n-1} \csc(\pi k/n), $$

and a table is given for $n = 1(1)30$ when $n = 2\pi r$.

The alternating sums $\sum_{k=1}^{(n)} (-1)^k k^m$ reduce to polynomials in n with rational coefficients when m is an integer. These are given explicitly for $m = 0(1)7$. Finally, the graphs of the seven sums $\sum_{k=1}^{(n)} (-1)^k k^s$, $n = 1(1)7$, are plotted as functions of s for $-2.6 \leq s \leq 3$.

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MATHEMATICAL TABLES—ERRATA

In this issue references have been made to errata in RMT **1205.**

239.—NBSCL, *Tables of* 10^x. (*Antilogarithms to the Base* 10.) Applied Math. Ser. No. 27. Washington, 1953.

In working with 10-figure logarithms recently, I noticed a discrepancy between a value given in this new table of 10^x, and a value from Guillemin.[1] Since Guillemin gives 13 figures for 10^x for $x = 0(.0001).6999$, and 12 figures for $x = .6999(.0001).9999$, and Deprez[2] gives 14 places 1(1)9999, every tenth value in v. 27 may easily be checked directly from these tables.

In checking every tenth value for the block from $x = .40000$ to $.50000$, I find 128 values in error in the last figure; 127 values should have the last figure raised by one, and one value, that for .49270, should have the last figure reduced by one.

It seems rather surprising that a corps of experienced computers, working with the most modern calculating instruments, should spend the amount of time and labor represented by this table simply to smooth out a 200-year-old table without even checking the accuracy of that table. Only the expenditure of a very little extra time would have been necessary to use the Sundstrand machine to subtabulate Deprez's table to tenths and thus to get a 14-place table of 10^x, which, if rounded to ten places would yield a table far superior to Dodson. Reference is made in the introduction of this volume to the Deprez table, so it is certainly known and available to the personnel of the laboratory.

In line with this reasoning, by using the Deprez values for the block .4680 to .4700 in conjunction with Comrie's simple, accurate, and very