[1] G. FROBENIUS, Preuss. Akad., Berlin, *SitzBer.*, 1900, p. 516. A list of Frobenius' relevant papers may be found in ref. 4 or 5.

[2] F. J. MURNAGHAN, *Am. Journal of Math.*, v. 59, 1937, p. 437. See also the paper of LITTLEWOOD & RICHARDSON.[3]

[3] D. E. LITTLEWOOD & A. R. RICHARDSON, Roy. Soc., *Phil. Trans.*, v. 233A, 1934, p. 99.

[4] M. ZIA-UD-DIN, London Math. Soc. *Proc* (2), v. 39, 1935, p. 200; *ibid.*, v. 42, 1937, p. 340.
K. KONDO, *Phys. Math. Soc. Japan*, v. 22, 1940, p. 585.

[5] D. E. LITTLEWOOD: *The Theory of Group Characters*, 2nd edition. Oxford, 1950.

[6] F. J. MURNAGHAN: *The Theory of Group Representations*. Baltimore, 1938.

[7] The partition conjugate to (λ) is obtained by interchanging rows and columns in the Ferrers-Sylvester graph of (λ).

[8] $P_s(n) = P(n) - 2P(n-2) + 2P(n-8) - 2P(n-18) + \cdots$. This was pointed out to us in a private communication from Professor N. J. FINE.

[9] The following question is of some practical interest: given n, for what partition (λ) does (9) take its largest value, and how does this value vary with n? The authors have been unable to find any discussion of this problem in the literature.

RECENT MATHEMATICAL TABLES

1234[A].—M. LOTKIN & M. E. YOUNG, *Table of Binomial Coefficients. Exact Values.* Ballistic Research Laboratories *Memo. Report* No. 762 Aberdeen Proving Ground, 1954, 49 p., 21.6 × 27.9 cm. Mimeographed from typescript.

This table is a sequel to RMT **1123.** It gives exact values of the coefficients

$$\frac{n!}{r!(n-r)!}$$

for $r \leq (n+1)/2$ and $n = 0(1)100$, whereas the previous table rounds these values to 20 figures. This new table will be of use in studies involving congruence properties and other theoretical properties of binomial coefficients.

D. H. L.

1235[C,D,E].—R. A. HIRVONEN, "Nutshell tables of mathematical functions for interpolation with calculating machines," *Bull. Géod.* No. 30, 1953, p. 369–392.

The tables presented here are one page tables. Interpolation is by means of Taylor's development which the author writes in the nested form

$$f(a + th) = \{[(Dt + C)t + B]t + A\}t + f(a)$$

so that

$$A = hf'(a), \quad B = h^2f''(a)/2!, \ldots$$

The tables therefore give the functions at coarse argument intervals together with the coefficients A,B,C,D. The tables are as follows:

$$\begin{aligned} \ln x \text{ for } x &= 1(.02)1.6;\ 10D \\ e^x \text{ for } x &= 0(.01).2;\ 10D \\ \sin x \text{ for } x &= 0°(2°)90°;\ 10D \\ \arctan x \text{ for } x &= 0(.02)1;\ 8D. \end{aligned}$$

The latter table is in decimal parts of a degree. Three special geodesic tables are used to illustrate Taylor interpolation.

D. H. L.

1236[F].—H. Gupta, M. S. Cheema & O. P. Gupta, "On Möbius Means," Panjab Univ. *Research Bulletin* No. 42, 1954, 16 p.

The table occupying all but the first page of this work was computed in order to study conjectures of V. Brun and C. L. Siegel regarding the second numerical integral of the Möbius function $\mu(n)$, the multiplicative function for which

$$\mu(p^\alpha) = -[1/\alpha].$$

If we define

$$\mu_1(n) = \sum_{k=1}^{n} \mu(k) \quad \text{and} \quad \mu_2(n) = \sum_{k=1}^{n} \mu_1(k),$$

then the conjecture of Brun is that "on the average"

$$\mu_2(n-1) = -2n + 12 + \beta n^{-1}$$

where $\beta = -18$. Siegel considered the value

$$\beta = -2\pi^2/\zeta(3) = -16.42119.$$

The authors tabulate

$$F(n) = T_0(n) = n\mu_2(n-1) + 2n^2 - 12n$$

whose average value should be β, and the successive means T_k defined by

$$nT_k(n) = \sum_{\nu=1}^{n} T_{k-1}(\nu)$$

for $k = 0(1)5$, $n = 1(1)750$. Exact values of $T_0(n)$ are given. The other T's are given to 3D. Graphs of T_3, T_4 and T_5 are given. For $375 \leq n \leq 750$, $T_5(n)$ descends nearly monotonely from -14.3 to -15.0.

D. H. L.

1237[K].—J. H. Caldwell, "Approximating to the distributions of measures of dispersion by a power of χ^2," *Biometrika*, v. 40, 1953, p. 336–346.

It is assumed that all observations are from normal populations. If the measure of dispersion is u, then for suitably chosen c and λ, cu^λ has approximately the χ^2 distribution with ν degrees of freedom. Table 1 of this paper gives values of ν, λ, $\log c$ for the range and mean deviation for sample size n, $n = 2(1)20$ and for the first quasi-range with $n = 10(1)30$. ν is given to 2S, λ and $\log c$ to 5S. The first quasi-range is the difference between the largest-but-one and the smallest-but-one observations in a sample of n. Table 2 contains the same constants for the mean of m ranges $m = 2(1)5$ and $n = 2(1)10$, where n is the size of each of m samples. Tables 3 and 4 give the approximate upper 5% and 1% points of the ratio of maximum value to minimum value in a set of k independent ranges (each range for a sample of size n from the same population). Values are given to 3S for $k = 2(1)12$ and $n = 3(1)10, 12, 15, 20$. Tables 5 and 6 give to 3S approximate upper 5% and

1% points for the ratio of maximum to minimum mean deviation for $k = 2(1)12$ and $n = 3(1)10, 12, 15, 30, 20, 60, \infty$. Tables 3 through 6 are based on the approximate distribution of Table 1 and the fact that the ratio of two independent χ^2's has an F-distribution. Adequate indications of accuracy of the approximation are stated.

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1238[K].—E. J. Gumbel, *Statistical Theory of Extreme Values and Some Practical Applications.* NBS Applied Math. Series, no. 33, U. S. Gov. Printing Office, Washington, 1954, viii + 51 p., 20.1 × 25.9 cm. Price 40 cents.

This booklet contains four lectures given at the Bureau of Standards which are an excellent account of the theory of extreme values and their applications by the leading expert in this field. For the exponential type of distribution law for extreme values there is tabled (p. 29) for samples of $N = 15(5)50(10)100(50)300, 400, 500, 750, 1000$, extremes, the expected mean of the reduced (standardized) extremes, the ratio of these to the population mean, their standard deviation, and the ratio of this to the population standard deviation, all to 5D. Interpolated values for the means and standard deviations for $N = 20(1)49$ to 4D are also given (p. 31), these values being obtained from an empirically established linear relationship.

C. C. C.

1239[K].—H. C. Hamaker, "'Average confidence' limits for binomial probabilities," International Stat. Inst., *Review*, v. 21, 1953, p. 17–27.

Given a random sample of n from a binomial population with true percentage of defectives, p. If k items are defective, the usual upper and lower $(1 - \alpha)$ confidence limits for p are computed so that

$$\Pr\{p_k' < p < p_k\} \geq 1 - \alpha, \tag{1}$$

where the equality holds for only a few isolated cases and the confidence probability is generally greater than $1 - \alpha$. The limits are computed from incomplete Beta or F-tables, so that the upper limit is

$$p_k = 100 v_1 F/(v_2 + v_1 F), \tag{2}$$

where $v_1 = 2(k + 1)$ and $v_2 = 2(n - k)$ are the degrees of freedom for F. A similar result holds for p_k'.

This article shows how the confidence probability varies as p changes, for $n = 25$ and $\alpha = .20$ (Figures 3A and 5B′). Another set of confidence limits is then presented for which the confidence probability is generally less than $1 - \alpha$ (Figures 3B and 5A′). In this case the upper confidence limit is simply p_{k-1}.

The author proposes to use as average confidence limits:

$$\begin{aligned} p_{uk} &= (p_k + p_{k-1})/2; \quad p_{lk} = (p_k' + p_{k-1}')/2 \\ p_{u0} &= 100 - p_{ln} = p_{ul}/2 = (p_1 - p_0)/4 \\ p_{l0} &= 100 - p_{un} = 0. \end{aligned} \tag{3}$$

Tables of p_{lk} and p_{uk} to 1D are given for $n = 5(5)30, 40, 50, 75, 100(100)500$; $\alpha = .02, .10, .20$; $k = 1(1)20$, unless $n < 20$, when k goes to n. These confidence limits give confidence probabilities which do not fluctuate much from $1 - \alpha$; however, the confidence probability will sometimes be less than $1 - \alpha$.

It should be mentioned that SIMON[1] presents a Bayesian method of averaging the probabilities; his limits are even narrower than those in the present article. It would be desirable to compare the true confidence probabilities for various p, using (2), (3) and the Simon charts.

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[1] L. E. SIMON, *An Engineer's Manual of Statistical Methods.* New York, 1941.

1240[K].—N. L. JOHNSON, "Some notes on the application of sequential tests in the analysis of variance," *Annals Math. Stat.*, v. 24, 1953, p. 614–623.

In the analysis of variance one often desires to test a linear hypothesis in some systematic model and such hypotheses are usually composite in nature. This paper considers the applicability of certain basic work by BARNARD[1] and COX[2] in sequential tests for composite hypotheses to such cases. Such a sequential situation could be encountered if one treated the number of observations taken in each class as a random variable or if the number of classes studied in an experiment could be increased in a sequential fashion. In order to obtain a sequential test a restriction upon the manner of increasing the number of observations is needed. With such restrictions one can evolve a sequential test which is in a form similar to the usual sequential type, but the lack of adequate tables of required distributions makes it difficult to apply the theory in the general case.

As a special case illustration the author considers a random model in one-way classification, namely

$$x_{ti} = a + u_i + z_{ti}$$

in which the u's are normal variables, each with 0 means and standard deviations σ_R. Here the hypothesis is concerned with the value of $\delta = \sigma^2{}_R/\sigma^2$. If one is interested in a sequential test based on $\delta'' > \delta' = 0$, such a test exists, but to determine appropriate intervals for the tests, values of the criteria $\underline{G}_R$ and $\overline{G}_R$ are required. With the number n of groups of size k as the variable, Table Ia gives values of $\underline{G}_R$ and $\overline{G}_R$ to 3D for $\delta'' = 1$, $\delta' = 0$, $\alpha = \beta = .05$ and $k = 2(1)12, 15, 20, 30, 60, \infty$ and $n = 2(1)12, 15, 20, 30, 60$. Table I_b is similar but with $\alpha = \beta = .01$.

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[1] G. A. BARNARD, "The frequency justification of certain sequential tests," *Biometrika*, v. 39, 1952, p. 144–150.
[2] D. R. COX, "Sequential tests for composite hypotheses," Cambridge Philos. Soc., *Proc.* v. 48, 1952, p. 290–299.

1241[K].—Rand Corp., *Offset Circle Probabilities*. Santa Monica, Calif., 1953, 18 p., 16.5 × 24.1 cm.

Consider a circle of radius r_d with center located a distance D from the origin in the x,y-plane. Let q represent the probability that a sample value from the normal bivariate distribution with $\mu_x = \mu_y = 0$, $\sigma_x = \sigma_y = \sigma$, $\rho = 0$ falls outside this circle. 3D values of q are tabled as a function of D/σ and $(r_d - D)/\sigma$. Here $D/\sigma = 0(.1)6(.5)10(1)20$, ∞ and $(r_d - D)/\sigma = -3.9(.1)4$. This table was obtained from an unabridged 6D table of q which was computed jointly by the Institute for Numerical Analysis, National Bureau of Standards, and the RAND Corporation. Card copies or printed listings of the unabridged table may be obtained by writing to the RAND Corporation.

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1242[K].—S. Rosenbaum, "Tables for a nonparametric test of dispersion," *Annals Math. Stat.*, v. 24, 1953, p. 663–668.

Given a sample of n values of a continuous random variable X, these tables give the upper 5% and 1% points for $m,n = 1(1)50$ of the distribution of the number of values in an additional sample of m values which lie outside the extreme values of the original sample. This distribution was obtained by Wilks[1] in connection with the problem of two-sided distribution-free tolerance limits. The suggested use in this paper is as a non-parametric test of dispersion, which places the emphasis on smaller values of m and n. However, if neither the sample nor the unsampled portion of the population exceed 50, these tables can also be used to establish the level of tolerance limit provided by the extreme values of the sample.

An approximation to the distribution for large and approximately equal values of m and n is also obtained. Since there is an apparent discrepancy between the 5% and 1% values obtained from this limiting distribution, and the values given by Wilks[1] for $m = n = 100$ (Table II, p. 406; n and N in Wilks notation), the reviewer computed exact values of the 5% and 1% points for $m = n = 10(10)200(100)900$. From these results it was noted (a) that the values given by Wilks are in error, and (b) that, while the 5% point of the limiting distribution is correct for $m = n \geqq 25$ (as can be seen from the tables), the exact 1% point does not reach that of the limiting distribution until at least $m = n > 170$.

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[1] S. S. Wilks, "Statistical prediction with special reference to the problem of tolerance limits," *Ann. Math. Stat.*, v. 12, 1942, p. 400–409.

1243[K].—Herbert Solomon, "Distribution of the measure of a random two-dimensional set," *Annals Math. Stat.*, v. 24, 1953, p. 650–656.

$N(= 1,2)$ random circles of equal radius W are dropped according to a bivariate distribution with circular symmetry specified by the parameter σ on a fixed circle of radius T with an aiming point at distance R from its center. The measure of interest is the ratio of the covered area to the total area of the fixed circle. The author calculates for $N = 1$ the probability P_C of getting at least C/W fraction coverage for specified values of T, R, N, σ. Until now only the first two moments were known.

Fig. 1 gives for $N = 1$ the contours of equal probability $P_C = .05(.05).95$ as a function of R for given values of $W + aT$, both in σ units. The factor a is eliminated by the use of the second figure, which shows the relation between coverage 0.05 to 1.00 and the factor $a(-.1 < a < .1)$ for fixed values of the quotient $W/T = .5, .6, .75, .85, 1, 1.2, 2, 3, 5, 10, \infty$. Thus the two figures represent the probability P_C of coverage C for given values of W, T and R (in σ units).

Four short tables give for $N = 2$ the lower and upper bounds $\underline{P}_C = 1 - (1 - P_C)^2$ and $\bar{P}_C = P(c_1 + c_2 \geqq C)$ to 3D. Table I holds for $W/\sigma = 1$, $T/\sigma = 1$, $R = 0$ and $C = .1(.1).9$. Tables II and III deal with median coverage for $W = T$ and $W = 2T$ respectively. Table IV holds for $W = .5T$, $C = .2$. The graphs and tables demonstrate that a realistic decision can be made without resorting to random number devices.

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1244[K].—G. TAGUTI, "Tables of 5% and 1% points for the Polya-Eggenberger's distribution function," Union of Japanese Scientists and Engineers, *Reports of Statistical Application Research*, v. 2, no. 1, 1952, p. 27–32.

Table I gives to 3S the minimum values of h such that

$$F(k;h,d) = \sum_{n=0}^{k} (1/n!)h(h + d)\cdots(h + (n - 1)d)(1 + d)^{-(h+nd)/d}$$

is $\geq a$, where $a = .95$ or $.99$, $h/d = .5(.5)15, 20, 30, 60, \infty$ and $k = 1(1)25$. Table II gives to 3S the maximum values of h^{-1} such that

$$F(k;h,d) \geqslant a$$

where $a = .95$ or $.99$, $d/h = 2., 1., .5(-.1).1, .05, 0$ and $k = 25(5)40(10)60, 75, 100, 200, 500, \infty$.

The method of preparing the tables and their accuracy is not given.

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1245[L].—ADMIRALTY RESEARCH LABORATORY, "Tables of $f(q,\alpha)$," A.R.L./T.3/Maths. 2.7, 13 p., Teddington, Middlesex, England.

4D tables of

$$\exp\left(-\frac{q^2 + k^2}{2}\right)\left\{I_0(kq) + 2\sum_{m=1}^{\infty}\left[\frac{\Gamma(m/2 + 1)}{\Gamma(m + 1)} \times u^{m/2}{}_1F_1(m/2; m + 1; -u)\, I_m(kq) \cos m\alpha\right]\right\}$$

for $u = 0, 1, \infty$, $k = .5(.5)3$, $\alpha = 0(10°) 180°$, $q = 0(.2) q_0$ where q_0 varies between 2 and 6 and is so chosen that the tabulation covers in each case the whole significant range of q.

A. E.

1246[L].—D. R. Bates, Kathleen Ledsham & A. L. Stewart, "Wave functions of the hydrogen molecular ion," Royal Soc. London, *Phil. Trans.*, v. 246 A, 1953, p. 215–240.

When the variables are separated in the Schrödinger equation for the electronic wave functions of H_2^+ in elliptic coordinates, there result the two differential equations

$$(1) \qquad \frac{d}{d\mu}\left\{(1-\mu^2)\frac{dM}{d\mu}\right\} + \left\{-A + p^2\mu^2 - \frac{m^2}{1-\mu^2}\right\} M = 0$$

$$(2) \qquad \frac{d}{d\lambda}\left\{(\lambda^2-1)\frac{d\Lambda}{d\lambda}\right\} + \left\{A + 2R\lambda - p^2\lambda^2 - \frac{m^2}{\lambda^2-1}\right\} \Lambda = 0.$$

In the above, A is a separation constant, and

$$(3) \qquad p^2 = -\tfrac{1}{4}R^2E,$$

where R is the distance between the nuclei in atomic units, and E is the electronic energy in Rydbergs.

Equation (1) is the familiar (oblate) spheroidal wave equation which has been studied.[1] Corresponding to a countable set of eigenvalues $A(l,m,p)$ equation (1) has a solution of the form

$$(4) \qquad M(l,m,p;\mu) = \sum_{s=0}^{\infty}{}' f_s(l,m,p)\, P_{m+s}^m(\mu),$$

where $P_{m+s}^m(\mu)$ is the associated Legendre function and s is either always odd, or always even. Tables of the eigenvalues A and the coefficients f_s have been published in [1] as functions of p. The authors give corresponding tables for the parameter R, which are presumably of direct interest to researchers in the field, and the tables are arranged in such a manner that the various *orders* of the eigenvalues are directly identified with the "united atom designations." Ten such "states" are given in the tabulation, namely

$$ns\sigma_g, \quad n = 1,2,3; \quad np\sigma_u, \quad n = 2,3,4; \quad 3d\sigma_g, \quad 4f\sigma_u, \quad 2p\pi_u, \quad \text{and} \quad 3d\pi_g.$$

All ten correspond to parameters $m = 0,1$; $l = 0,1,2,3$. Corresponding to these states, the authors tabulate the functions p, σ, $-A'$, and $-E$, for $R = 0(.2)5(.5)9$ or 10; 5D (Table 1), and the coefficients f_s (Table 2). The latter are given to five significant figures for the dominant coefficients, and to six decimals (usually) for the others. According to the authors, these were obtained by interpolation from the tables in[1]. The parameters A' and σ are defined below.

The authors also tabulate eigenvalues and coefficients corresponding to equation (2). Following Jaffé,[2] the authors write

$$(5) \qquad \Lambda(\lambda) = (\lambda^2-1)^{m/2}(\lambda+1)^{\sigma}\exp(-p\lambda)y(\zeta)$$

with

$$\sigma = \frac{R}{p} - m - 1; \quad \zeta = (\lambda - 1)/(\lambda + 1)$$
$$A' = A - p^2.$$

The solution is written in the form

$$y = \sum_{t=0}^{\infty} g_t \zeta^t. \tag{6}$$

The solution exists corresponding to the eigenvalues of σ, for given p, m, and A'. In Table 3, the authors give the coefficients g_t for the same range of R as in Tables 1 and 2; four decimals are given in the tabulation. The paper also contains contour diagrams of H_2^+, as well as some comparisons between the exact solution for certain of the wave functions and the "L.C.A.O" approximation.

For theory and some tabulation relating to the general spheroidal wave function in the more accessible publications, see also references [3,4,5].

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[1] J. A. STRATTON, P. M. MORSE, L. J. CHU & R. A. HUTNER, *Elliptic Cylinder and Spheroidal Wave Functions.* John Wiley and Sons, Inc., New York, 1941.
[2] G. JAFFÉ, *Zeit. Phys.*, v. 87, 1934, p. 535 (author's citation).
[3] A. LEITNER & R. D. SPENCE, "The oblate spheroidal wave functions," Franklin Inst., *Jn.*, v. 249, no. 4, 1950, p. 299–321.
[4] J. MEIXNER, *Lamé's Wave Functions of the Ellipsoid of Revolution*, translated by MARY L. MAHLER, NACA, June, 1944.
[5] C. J. BOUWKAMP, "On spheroidal wave functions of order zero," *Jn. Math. Physics*, v. 26, 1947, p. 79–92.

1247[L].—S. CHANDRASEKHAR & DONNA ELBERT, "The roots of $Y_n(\lambda\eta)J_n(\lambda) - J_n(\lambda\eta)Y_n(\lambda) = 0$," Cambridge Phil. Soc., *Proc.*, v. 50, 1954, p. 266–268.

The authors tabulate to 5D the first root, λ_1, of the equation mentioned in the title, together with 7 to 9D values of

$$J_n(\lambda_1),\ Y_n(\lambda_1),\ J_n(\lambda_1\eta),\ Y_n(\lambda_1\eta),\ \frac{2}{\pi^2\lambda_1^2}\left\{\frac{J_n^2(\lambda_1\eta)}{J_n^2(\lambda_1)} - 1\right\}$$

for $\eta = .2$, $n = 1(1)5$; $\eta = .3(.1).5$, $n = 1(1)6$; $\eta = .6$, $n = 1(1)8$; and $\eta = .8$, $n = 1(1)12$. For $\eta = .5$, $n = 1(1)6$ they also give the corresponding quantities for the second and third zeros.

A. E.

1248[L].—CH. VITAL DUNSKI, "Les fonctions de Bessel d'argument complexe $x\sqrt{j}$ et les fonctions de Kelvin d'ordre zéro et 1," Soc. Roy. Sci. Liége *Bull.*, v. 23, 1954, p. 52–59.

Table I gives ber x, bei x, ker x, kei x, $\text{ber}_1\, x$, $\text{bei}_1\, x$, $\text{ker}_1\, x$, $\text{kei}_1\, x$, and Table II gives the real and imaginary parts of $J_0(xi^{\frac{1}{2}})$, $H_0^{(1)}(xi^{\frac{1}{2}})$, $i^{\frac{1}{2}}J_1(xi^{\frac{1}{2}})$, $i^{\frac{1}{2}}H_1^{(1)}(xi^{\frac{1}{2}})$. Each of the two tables occupies one page. The entries have been computed, to various degrees of accuracy, from the asymptotic expansions for 25 values of x ranging from 10 to 72.

A. E.

1249[L].—CARL-ERIK FRÖBERG & PHILIP RABINOWITZ, "Tables of Coulomb wave functions," NBS Report 3033, 1954, 3 + 40 p., 20.5 × 27 cm.

F and G being the regular and the irregular Coulomb functions as defined in RMT **982** [*MTAC*, v. 6, p. 92], in this report

$$A_0(\eta) = [(1 - e^{-2\pi\eta})/(2\pi\eta)]^{\frac{1}{2}}, \quad A_L(\eta) = \tfrac{1}{2}(L^2 + \eta^2)^{-\frac{1}{2}} A_{L-1}(\eta)$$
$$F_{L,\eta}(\rho) = A_L(\eta)\rho^{L+1} f_{L,\eta}(\rho)$$
$$G_{L,\eta}(\rho) = A_L(\eta)\rho^{L+1} g_{L,\eta}(\rho).$$

The report contains tables of f, f', g, g' and of the first five "reduced" derivatives $\frac{1}{k!}\frac{\partial^k}{\partial\eta^k}$ of these quantities for $L = 0, 5$, $\eta = 1(1)10$, $\rho = 1(1)10$. For other NBS tables of Coulomb wave functions see RMT **1091** [*MTAC*, v. 7, p. 101–102]. The present table was first announced as UMT **186** [*MTAC*, v. 8, p. 97], where further details are given.

A. E.

1250[L].—J. M. HAMMERSLEY, "Tables of complete elliptic integrals," NBS *Jn. of Research*, v. 50, 1953, p. 43.

The functions tabulated are K, E, and $\frac{1}{2}\pi/K$. The argument used is $p = 1/k$. The tables are for $p = 1(.01)1.3(.02)2$; 10S. The last figure is doubtful.

D. H. L.

1251[L].—ALDO MUGGIA, "Sul calcolo dell' integrale di Poisson," Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat., *Atti*, v. 87, 1953, p. 116–126.

5D tables of

$$A(\tau) = 2\ln\left|\sin\frac{\tau}{2}\right|, \qquad B(\tau) = \int_{-\pi}^{\tau} t\cot\frac{t}{2}\,dt$$
$$C(\tau) = -\cot\frac{\tau}{2}, \qquad D(\tau) = -\tau\cot\frac{\tau}{2} + 2\ln\left|\sin\frac{\tau}{2}\right|$$

for $\tau = -\pi$, $-3.12(.04)-.04(.01).04(.04)4.6$. The tables were computed by LIA ERRERA of the computing center of the Politecnico di Torino.

A. E.

1252[L].—UNO OLSSON, "Non-circular cylindrical gears," *Acta Polytechnica*, Mechanical Engineering Series, v. 2, no. 10, Stockholm, 1953, p. 1–215.

The author not only develops the theory for the construction of different types of non-circular gears but also furnishes directions for the production of the wheels. As a by-product of the investigation, he gives the following tables:

Table I: The arc length of the hyperbola,

Table II: The arc length of the ellipse with imaginary axes or the arc length of the sinh-curve,

Table III: The arc length of the parabola,

Table IV: The arc length of the exponential curve,

Table V: The hyperbolic argument u as a function of the hyperbolic amplitude φ.

Tables I and II are given for φ (the amplitude angle) $= 0°(1°)90°$ and α (the eccentric angle) $= 0°(5°)90°$; Table III for $\varphi = 0°(1°)90°$ with $\Delta'_{\frac{1}{2}}$ and $\Delta_0'' + \Delta_1''$; Table IV for $\varphi = 0°(.1°)10°$, $0°(1°)90°$ and Table V for $\varphi = 0°(.1°)90°$. All tables are given for the most part for modified functions to about 5 significant figures. A short section describing the use of higher degree interpolation, necessary in the tables, is also given.

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NBSCL

1253[L].—L. J. SLATER, "Some new results on equivalent products," Cambridge Phil. Soc., *Proc.*, v. 50, 1954, p. 394–403.

Table 1 gives values, mostly to 8D or 8S, of

$$\left[\prod_{n=1}^{\infty} (1 - aq^n)\right]^{-1}$$

for $a = -.9(.05).95$ and $q = 0(.05)1$.

Table 2 gives 8D or 8S values of the same quantity for $a = 1, q = 0(.005).89$.

The first table is stated to be accurate to 7D except where fewer figures are given, the second, to 8D. The tables were computed on the EDSAC.

A. E.

1254[L].—L. J. SLATER, "The evaluation of the basic confluent hypergeometric functions," Cambridge Phil. Soc., *Proc.*, v. 50, 1954, p. 404–412.

The functions discussed and tabulated in this paper are

$$\sum_{n=0}^{\infty} \frac{(q^a)_n\, y^n}{(q^b)_n\, (q)_n} \tag{1}$$

$$\sum_{n=0}^{\infty} \frac{(q^a)_n\, y^n\, q^{\frac{1}{2}n(n+1)}}{(q^b)_n\, (q)_n} \tag{2}$$

where

$$(q^a)_n = (1 - q^a)(1 - q^{a+1}) \cdots (1 - q^{a+n-1})$$

Table 1 is of the function (2) for $a = 0(.2)2$, $b = .2(.2)1$, $q = .9$, $y = (1 - q)x = .1(.1)1$.

Table 2 is of the same function, for the same values of a and b, and for $q = .99$, $y = .01(.01).1$.

Table 3 is of the function (1) for the same values of a and b, and for $q = .9$, $y = .1(.1).9$.

The values are mostly given to 7 or 8S: they were computed on the EDSAC.

A. E.

1255[L].—ROBERT L. STERNBERG, "A general solution of the two-frequency modulation product problem. II. Tables of the functions $A_{mn}(h,k)$," *Jn. Math. Phys.*, v. 33, 1954, p. 68–79.

"The purpose of this paper is to provide tables and evaluation methods for the functions

$$A_{mn}(h,k) = (2/\pi^2) \int\int_R (\cos u + k \cos v - h) \cos mu\, du \cos nv\, dv$$

for $m,n = 0,1, |h| \leq 2, 0 < k \leq 1$ where R is the region defined by

$$R: \cos u + k \cos v \geq h, \quad 0 \leq u,v \leq \pi$$

and subsequently also to tabulate briefly the first six higher order functions $A_{mn}(h,k)$."

The functions arise in the problem indicated in the title and have been investigated in an earlier paper[1] where there are also further references. The tables are to 6D.

A. E.

[1] R. L. STERNBERG & H. KAUFMAN, "A general solution of the two-frequency modulation product problem. I," *Jn. Math. Phys.*, v. 32, 1953, p. 233–242.

1256[L].—A. WALTHER & H. UNGER, "Mathematische Zahlentafeln, numerische Untersuchung spezieller Funktionen," *Naturforschung und Medizin in Deutschland*, 1939–1946, Band 3. Angewandte Mathematik, Teil I, p. 167–183. Verlag Chemie, Weinheim, 1953.

This valuable report covers numerical tables of special functions computed in Germany during the period under review, and also analytical work of importance in connection with the numerical computation of special functions. The report consists of three parts: A. Introduction, B. Survey of developments and results, C. Bibliography. Each of parts B and C is divided in four sections: 1. elementary functions, 2. tables pertaining to astronomy and geodesy, 3. Bessel functions, 4. other higher transcendental functions. The bibliography lists some eighty items. Many of these have been published (and reviewed in *MTAC*). In the case of most of the tables which have not been published a brief description (in the style of the FMR Index) is appended.

A. E.

1257[L].—M. W. WILKES, "A table of Chapman's grazing incidence integral $\mathrm{Ch}(x,\chi)$," Phys. Soc. *Proc.*, v. 67 B, 1954, p. 304–309.

This table gives

$$\mathrm{Ch}(x,\chi) = x \sin \chi \int_0^{\chi} \exp\,(\chi - x \sin \chi / \sin \lambda) \operatorname{cosec}^2 \lambda \, d\lambda$$

to 3D for $x = 50(50)\,500(100)1000$, $\chi = 20°(1°)\,100°$, excluding values of $\mathrm{Ch}(x,\chi)$ which exceed 100. The table was prepared on EDSAC, using a Gauss 5-point formula repeatedly, with automatic adjustment of the length of the interval. A more complete discussion of the machine techniques used is to appear elsewhere. Various checks were applied: recalculation for $\chi = 20°, 21°, 70°$, differencing in both directions, and use of the relation

$$\mathrm{Ch}(x,\chi) + \mathrm{Ch}(x,\pi - \chi) = 2 \exp\,(\chi - x \sin \chi)\, \mathrm{Ch}(x \sin \chi, \tfrac{1}{2}\pi).$$

The values are expected to be correct to within a unit in the last place.

J. T.