REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[**A, L**].—A. D. Booth, "A note on approximating polynomials for trigonometric functions," *MTAC*, v. 9, 1955, p. 21–23.

$J_n(\frac{1}{2}\pi)$ is given to 11D for $n = 0(.5)12.5$.

J. T.

2[**B, C**].—John von Neumann & Bryant Tuckerman, "Continued fraction expansion of $2^{\frac{1}{3}}$," *MTAC*, v. 9, 1955, p. 23–24.

Let $S_n(x)$ be the sum of the first n partial quotients of the continued fraction for x. A table of $n \log n/\log 2$ and $S_n(2^{\frac{1}{3}})$ is given for $n = 100(100)2000$; the first expression is given to 1D; the second is given exactly.

J. T.

3[**D**].—Birger Jansson, *Numerisk fourieranalys för hand-och hälkortsberäkning.* 44 leaves of blue line prints from transparencies, 29.7 cm., deposited in the UMT File.

This contains tables of $\sin\left(vn\frac{360}{100}\right)$ and $\cos\left(vn\frac{360}{100}\right)$ for $v = 0(1)25$ and $n = 1(1)25$ and of $\sin\left(vn\frac{360}{400}\right)$ and $\cos\left(vn\frac{360}{400}\right)$ for $v = 0(1)100$, $n = 1(1)50$, all to 4D. The tables are prepared to assist in Fourier analysis using desk calculators or punched card machinery. A remarkably detailed description of procedures for Fourier analysis included; this is based mainly on an assumption that punched card machinery similar to International Business Machines is available. The discussion leads up to a careful time estimate for the calculation.

The author announces but does not include tables of $\sin\left(vn\frac{360}{1200}\right)$ and $\cos\left(vn\frac{360}{1200}\right)$ for $v = 0(1)300$, $n = 1(1)160$. Tables designed for the same application have been prepared also by L. W. Pollak and by Owen Mock; these are reviewed in *MTAC*, v. 5, 1951, p. 19–21 and p. 149, and in *MTAC*, v. 9, 1955, p. 72 and p. 196. No earlier tables are known to contain as detailed instructions for machine analysis.

C. B. T.

4[**F**].—Andrew S. Anema, *A table of primitive Pythagorean triangles with their generators, having identical perimeter totals.* 13 typewritten pages deposited in the UMT File.

The main table is a set of 107 primitive Pythagorean triangles with identical perimeters. The table contains two primitive Pythagorean triangles with perimeter 1716 (sides 364, 627, and 725; 748, 195, 772), three primitive Pythagorean triangles with perimeter 14280, four primitive Pythagorean triangles with perimeter 3 17460, and 107 primitive Pythagorean triangles with perimeter 60850 05270 54420.

These were computed without aid of machines. The author, an old hand at lightning calculation, includes several comments about the method of calculation.

One example from each of the sets with 3, 4, and 107 entries follows:

From set of three:

$$P = 14280 = 2\cdot 2^2\cdot 3\cdot 5\cdot 7\cdot 17$$
$$3\cdot 5\cdot 7 = 105,\ 2^2\cdot 17 = 68 \text{ (generator } a\text{), 37 is generator } b;$$
$$A = 2ab = 5032,\ B = a^2 - b^2 = 3255,\ C = a^2 + b^2 = 5993.$$

From set of four:

$$P = 3\,17460.$$
$$A = 1\,53868,\ B = 9435,\ C = 1\,54157.$$

From set of one hundred seven:

$$P = 60850\,05270\,54420,$$
$$A = 2763\,24098\,54668,$$
$$B = 28977\,68095\,17195,$$
$$C = 29109\,13076\,82557.$$

C. B. T.

5[F].—N. G. W. H. BEEGER, *Table of the Least Factor of the Numbers that are not Divisible by* 2, 3, 5 *of the Eleventh Million.* 429 tables, 34 pages of form, 33½ cm., deposited in the UMT FILE.

In 1949 D. JARDEN and A. KATZ published, *Page 477 to D. N. Lehmer's "Factor Table for the First Ten Millions"* [1]. It contains the table of the least factor of all the numbers not divisible by 2, 3, 5, and 7, in the interval 10 017 000–10 038 000. I have checked this extension against the part of KULIK's table, "Magnus Canon Divisorum" [2], and against the part of a table constructed by L. POLETTI by means of his printed fasciles, *Neocribrum. No discrepancy was found.* Therefore my construction of the table of the least factors of the eleventh million was commenced at cycle 335, that is to say at 10 030 021.

In the printed fasciles (conceived by L. POLETTI), mentioned above, the least factors 7, 11, 13 are *printed.* I used "*stencils*" to insert the least factors 17, 19, 23, ···, 359. All insertions in the fasciles were made by rubber stamps. Therefore I calculated by means of a calculating machine and using J. GLAISHER's table of the differences of consecutive primes [3]: the products $367q$, $373q$, ···, $3313q$ (q are suitable primes) in the eleventh million. These were then inserted in the fasciles. The resulting table was checked: first against KULIK's table, second against the mentioned table by L. POLETTI. The discrepancies were corrected. This work confirmed the primality of all the numbers of *Liste des nombres premiers du onzième million* (*plus précisément de* 10 006 741 *à* 10 999 997), d'après les tables manuscrites de J. PH. KULIK, L. POLETTI, et R. J. PORTER. Imprimerie "Werto," Amsterdam, 1951.

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1. D. JARDEN & A. KATZ, *Additional page (477) to D. N. Lehmer's Factor Table, Riveon Lemat.*, v. 3, p. 50.
2. J. PH. KULIK, *Magnus Canon Divisorum.* [See UMT **48**, *MTAC*, v. 2, 1946, p. 139–140.]
3. J. GLAISHER, *Factor Table for the Fourth Million*, London, 1879, p. 48–52.

6[F, Z].—EDGAR KARST, *Tables for converting* 4 *digit decimal fractions to periodic octal fractions.* 4 Tables, 13 handwritten sheets, 10.8 cm., deposited in the UMT FILE.

The author lists the complete period of 4, 20, 100, and 500 octal digits for the decimal fractions .2, .04, .008, and .0016. For any other fraction, the period is a cyclic permutation of one of these periods and the proper starting place is tabulated. If the fraction is not a multiple of 5^{-4}, determine the first digit or two in the usual way (multiply by 8) and the table may now be consulted for the periodic digits. The tables are arranged for the convenience of the table maker, rather than the user. A complete description is:

Table V—i

$5^{-i}k \quad \mathrm{Ind}_8\, k \quad n \quad 8^n 5^{-i} \pmod 8$

where $k = 1(1)5^i$ $(k, 5) = 1$ $n = 1(1)4\cdot 5^{i-1}$ $i = 4(1)1$ and indices are with respect to 5^i.

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7[I].—FRANCIS L. MIKSA, *Stirling numbers of the first kind.* 27 leaves reproduced from typewritten manuscript on deposit in the UMT FILE.

After completing a table [4] of Stirling numbers of the second kind, ${}_rS_n$, for $n = 1(1)(50)$ the writer decided to prepare a companion table of Stirling numbers of the first kind, $S(r, n)$, computed for the same range.

Both kinds of Stirling numbers arise in formulating relationships between algebraic factorials and powers. For the Stirling I numbers we need only to express the factorials by means of power series.

It is customary to define the coefficients in these power series as the *Stirling numbers of the first kind*, and this of course includes the algebraic sign. For our purpose of tabulation, however, it is more convenient to consider only the *absolute value* of the coefficients.

The table was computed by means of the recurrence relation

$$S(r, n + 1) = n\cdot S(r - 1, n) + S(r, n)$$

and was checked by means of

$$\sum_{r=1}^{n} S(r, n) = n!.$$

Comparison with the table of *Exact Values of the First 200 Factorials* [1] showed that the agreement was complete.

The most extensive earlier table known to the writer is that published during 1900 by J. W. L. GLAISHER [2], where they are listed up to $n = 23$ in our notation.

A more detailed treatment of applications of the Stirling I numbers appears in C. JORDAN [3].

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1. H. S. UHLER, *Exact value of the first 200 factorials*, New Haven, Conn., *MTAC*, v. 1, **RMT 158**, 1943–45, p. 312.
2. J. W. L. GLAISHER, "Congruences relating to the sums of products of the first n numbers and to other sums of products," *Quart. Jn. Math.*, v. 31, 1900, p. 1–35.
3. C. JORDAN, *Calculus of Finite Differences*, Chelsea Pub. Co., New York, 1947, p. 142–168.
4. FRANCIS L. MIKSA, *Stirling Numbers of the Second Kind*. Deposited in *MTAC*, UMT FILE. See Review **85**, *MTAC*, v. 9, 1955, p. 198.

8[I].—KUO-CHU HO, "Double interpolation formulae and partial derivatives in terms of finite differences," *MTAC*, v. 9, 1955, p. 52–62.

Expressions for partial derivatives of $h^{\beta}k^{\alpha-\beta}(\partial^{\alpha}f/\partial x^{\beta}\partial y^{\alpha-\beta})$ where $\alpha = 1(1)4$, $\beta = 0(1)\alpha$ are given in terms of the differences of f at intervals h in x, k in y. The expressions given include the fourth order terms. These are given in various forms suitable for use at the edges and corners of a table as well as in the center. The corresponding Lagrangian expressions are given for $\alpha = 1(1)3$.

J. T.

9[I].—H. E. SALZER, "Osculatory quadrature formulas," *J. Math. Physics*, 34, 1955, p. 103–112.

Quadrature formulas such as

$$I(r, s) = \int_{x_0+rh}^{x_0+sh} f(x)dx = h \sum A_i f_i + h^2 \sum B_i f_i' + Rh^{2n+1}f^{(2n)}(\theta)$$

can be found by integration of osculatory interpolation formulas. The exact values of the A_i, B_i, R, expressed as rational fractions in their lowest terms, are given in the case of n points for $n = 2(1)7$. For each n the appropriate coefficients are given for all possible sub-intervals: thus for $n = 4$, with f and f' given at points $x_i = x_0 + ih$, $i = -1, 0, 1, 2$, we find expressions for the integrals $I(r, s)$ for the following values of (r, s): $(-1, 0)$, $(0, 1)$, $(1, 2)$; $(-1, 1)$, $(0, 2)$; $(-1, 2)$ in terms of *all* the data.

J. T.

10[I].—T. N. E. GREVILLE & H. VAUGHAN, "Polynomial interpolation in terms of symbolic operators," *Trans. Soc. Actuar.*, v. 6, 1954, p. 413–476.

Table 6, p. 450–451, gives the exact values of the basic function $L(x) = L(-x)$ of the following continuous operators: M^6, μM^5, M^4, $\delta^2 M^4$, $\delta^4 M^4$, μM^3, $\mu\delta^2 M^3$, $\delta^4 M^2$, M^5, μM^4, M^3, $\delta^2 M^3$, for $x = 0(.1)4$ or less (usually around 3), up to the value of x where $L(x) = 0$. The basic function $L(t)$ for the continuous operator K is defined by $K = \int_{-\infty}^{\infty} L(t)E^{-t}dt$ where E denotes the displacement operator of the calculus of finite differences. The operator K is also called the *characteristic operator* for the interpolation formula of which $L(t)$ is the basic function, and it is analogous to the graduation operator which the authors have associated with a discrete interpolation formula.

Preceding Table 6, on p. 443–449 are the following collections of formulas:

Table 1, p. 443–445, *Characteristic Operators of Certain Published Interpolation Formulas*, giving name or originator of formula, place and date of publication, and characteristic operators.

Table 2, p. 446, gives the symbolic expansions of certain operators, namely, combinations or separate powers of M, μ, and δ in terms of $D \equiv \frac{d}{dx}\cdot$ ($\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$, $\mu = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$, and M is a new operator defined by $Mf(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x + t)dt.$)

Table 3, p. 446, Traces of powers M^k and mean powers μM^k, $k = 1(1)8$, in terms of even powers of δ, where the author defines the trace of a continuous operator K with basic function $L(t)$ as the discrete operator $t(K) = \sum_{-\infty}^{\infty} L(n)E^{-n}$.

Table 4, p. 447, Powers and mean powers of M, up to the 10th, in terms of special operators for both endpoint and midpoint formulas.

Table 5, p. 448–449, Various special operators for endpoint, midpoint, or mixed endpoint-midpoint formulas in terms of M, δ, μ, and D.

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11[K].—The RAND Corp., *One Million Random Digits and 100,000 Normal Deviates*, The Free Press, Glencoe, Illinois, 1955, 26.5 cm., xxv + 200 p. $10.00. The tables are also available on punched cards from The RAND Corporation, 1700 Main St., Santa Monica, Calif. $25.00 an order +$3.00 per 1000 cards.

This book contains one million random decimal digits and 100,000 normal deviates, computed from the random digits to 3D. This succeeds [1] as the largest table of random numbers known to be available. The book also contains introductory material describing production of the digits, tests of the digits, production and tests of the normal deviates, and use of the tables.

Random numbers have long been used in various ways for the simulation of random or imponderable effects in various calculations. In recent years a growing popularity of the so-called Monte Carlo type of calculation in various shielding problems in physics and other problems of mathematics and applied mathematics has led to a greatly increased demand for random numbers; some of this recent work is described in [2], [3], [4]. The increasing power of modern computing equipment has also led to higher demands for random numbers in their more classical uses. The present publication was prepared in order to meet these increased demands. The largest earlier publication [1] is reported to have been completely inadequate in size for the needs of The RAND Corporation.

Generally speaking, workers have been content with the generation of numbers conforming with frequency distribution requirements and believed to be essentially independent of the process being simulated in their method of production. Many systematic ways of generating such numbers have been proposed for digital computers; references to some of these are contained in [6], and other schemes may be found in [7]. The present numbers were not generated by any known formula, but they were rather generated by a process which (if current physical theory is correct) was completely random, yielding output which could not have been predicted with any amount of knowledge available before the generation of the numbers. The digits were prepared by driving electronic counters by electronically generated noises. (Another random method of generating binary digits is described earlier in this issue [11].)

The random digits were subjected to numerous tests described in the introductory material. The originally generated digits showed unacceptable biases in their distribution, and a new table was generated systematically by adding pairs of digits modulo 10 to improve (that is, to flatten) the distribution. This was successful. Standard tests of frequency distributions, poker hands, frequencies of digraphs (ordered pairs of digits), and lengths of runs of a single digit were carried out, presumably on the digits as they were punched on the IBM cards.

The printed version was prepared automatically from these IBM cards, and it is unlikely to differ from the punched card version in more than a few digits.

All the standard tests as reported were passed adequately by the digits. However, one test was improperly analyzed, and the authors have requested that the following correction be made in the text. It is believed that the desirability of some such modification was brought to the Corporation's attention by I. J. Good; it is based upon his analysis of a serial test [8].

Replace the paragraph beginning at the bottom of page xv by the following:

Table 5 can be tested by a criterion originally due to Kendall and Smith and revised by Good. Assuming all pairs equally likely we get a normalized sum of squared deviations of 107.8. However, this statistic does not have a χ^2-distribution. On the other hand, it is the sum of the error variation and twice the row (or column) variation, where under the assumption of perfect randomness, the error variation is asymptotically distributed like χ^2 with 81 degrees of freedom. We take the error variation as our test criterion. This gives a χ^2 of $107.8 - 2(7.56) = 92.7$, which is about the 0.18 level for 81 degrees of freedom.

The modification above cannot be interpreted as challenging the randomness of the digits.

It seemed conceivable that the standard tests reported as having been made in the introduction to the book would have failed to detect variable instability of counters (which, in any event, would be unlikely to have a bad effect on the randomness of the digits after the smoothing transformation made above). The reviewer did test a hypothesis that such instability occurred and led to excessive lengths of coupon collector's sequences [9]. Accordingly the digits were transformed from decimally punched card form to binary coded decimal form suitable for introduction to the SWAC computing machine located at the University of California at Los Angeles. The coupon collector's test described by Greenwood was run by the reviewer with the help of F. H. Hollander, J. L. Selfridge, and David A. Pope. The longest single sequence which was required in order to provide at least one of each of the 10 digits had length 115 digits; the total number of sequences tested before the digits were exhausted was 34,248; the longest sequence of length 115 was not inordinately long. Based on the total number of sequences observed and the probabilities reported by Greenwood [9], the following comparison is possible between the numbers of sequences observed of various lengths and the numbers expected.

Length of Run i	Number Observed ν_i	Number Expected μp_i
10	14	12.428
11	63	55.926
12	152	143.542
13	284	276.832
14	462	446.786
15	640	638.190
16	813	834.267
17	1017	1020.064
18	1264	1184.234
19	1342	1319.539
20	1415	1422.517
21	1480	1492.752
22	1528	1532.020
23	1515	1543.501
24	1576	1531.128
25	1488	1499.087
26	1431	1451.483
27	1375	1392.115
28	1299	1324.359
29	1189	1251.116
30	1269	1174.806
31	1136	1097.397
32	977	1020.446
33	972	945.150
34	839	872.401
35	792	802.832
36	689	736.868
37	695	674.765
38	618	616.647
39	554	562.532
40	494	512.363
41	490	466.026
42	408	423.364
43	386	384.196
44	357	348.322
45	338	315.534
46	304	285.622
47	265	258.376
48	235	233.594
49	196	211.081
50	196	190.651
51	188	172.129
52	148	155.352
53	139	140.166

Length of Run i	Number Observed ν_i	Number Expected μp_i
54	114	126.428
55	117	114.009
56	104	102.788
57	84	92.652
58	74	83.502
59	70	75.243
60	67	67.793
61	53	61.072
62	62	55.012
63	53	49.549
64	42	44.624
65	47	40.186
66	37	36.186
67	22	32.583
68	41	29.337
69	21	26.413
70	28	23.780
71	20	21.408
72	15	19.273
73	18	17.349
74	14	15.618
75	11	14.059
76 and more	102	126.631

The reviewer cannot find any significant indication in these data to support a hypothesis that the digits are not random. A calculation of χ^2 based on the assumption that 34,248 samples were divided into 67 categories with 66 degrees of freedom gave a value of χ^2 about 0.135σ below the expected mean value. More detailed results of this coupon collector's test may be obtained by addressing the reviewer.

The normal deviates were computed from the random digits according to a standard procedure, see [6]. In particular, if D is a five-digit number taken from the table of random digits and read as an integer, then the numbers listed are values of x solving the equation

$$(D + 0.5)10^{-5} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt.$$

For the right hand number of this equation the National Bureau of Standards tables of the normal distribution [10] were used.

The printing is satisfactory. Reproduction was by photographic offset printing from pages printed by an International Business Machine Model 856 Cardatype. Each page of the table of random digits contains fifty lines; each line contains a serial number and ten groups of five decimal digits. The digits are easily legible. Each page of the table of normal deviates contains fifty lines; each line contains a serial number and ten deviates. The material was proofread only on a sample basis, but the probability of serious error in printing seems low.

C. B. T.

1. M. G. Kendall & B. B. Smith, *Random Sampling Numbers*, Cambridge University Press, 1939.
2. National Bureau of Standards, *The Monte Carlo Method* (Proceedings of a Symposium held in 1949), NBS AMS 12, U. S. Gov. Printing Office, Washington, D. C., 1951.
3. J. H. Curtiss, "Sampling methods applied to differential and difference equations," *Seminar on Scientific Computation*, International Business Machines Corp., New York, 1949.
4. H. Kahn, Applications of Monte Carlo, The RAND Corp. (to be published).
5. P. Davis & P. Rabinowitz, "Some Monte Carlo experiments in computing multiple integrals," *MTAC* (this issue), v. X, 1956, p. 1.
6. D. L. Johnson, "Generating and testing pseudo random numbers on the IBM Type 701," *MTAC* (this issue), v. X, 1956, p. 8.
7. L. Sacco, *Manuel de Cryptographie*, Payot, Paris, 1951, Chap. XI, p. 74–119.
8. I. J. Good, "The serial test for sampling numbers and other tests for randomness," Camb. Phil. Soc., *Proc.*, v. 49, 1953, p. 276–284.
9. Robert E. Greenwood, "Coupon collector's test for random digits," *MTAC*, v. IX, 1955, p. 1–5. (Note also Corrigendum, *MTAC*, v. IX, 1955, p. 229.)
10. *Tables of Probability Functions*, 2nd ed., U. S. Gov. Printing Office, Washington, D. C., 1948.
11. Z. Pawlak, "Flip-flop as Generator of Random Binary Digits," *MTAC* (this issue), v. X, 1956, p. 28.

12[K].—NBS Applied Mathematics Series, No. 44, *Table of Salvo Kill Probabilities for Square Targets*, U. S. Gov. Printing Office, Washington, D. C., 1954, ix + 33 p., 26 cm. Price $0.30.

Let $\{x, y, \sigma\}$ denote the circular normal distribution centered at (x, y) with standard deviation σ in each direction. A salvo of N bombs is "centered" at (ξ, η), which has the distribution $\{0, y_0, \sigma_A\}$. The bombs of the salvo are independently distributed according to $\{\xi, \eta, \sigma_R\}$. If a bomb hits the square target $(x^2 \leqq 1, y^2 \leqq 1)$, there is chance P_K of a kill. Assuming the bombs act independently, the chance P_{SK} that the salvo kills the target is computed for $P_K = .1, .4, .7, 1$; $y_0 = 0, 1, 2, 4, 7, 11, 16, 22$; $\sigma_A, \sigma_R = 1, 2, 4, 7, 11, 16, 22$; $N = 1, 5, 10, 25, 50, 100, 150, 200$. The entries are 4D with possible error of two in the last place. In an introduction, A. D. Hestenes warns the user against interpolating in this quintuple-entry table, except in the P_K direction for small N.

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13[K].—K. C. S. Pillai & K. V. Ramachandran, "On the distribution of the ratio of the ith observation in an ordered sample from a normal population to an independent estimate of the standard deviation," *Ann. Math. Stat.*, v. 25, 1955, p. 565–572.

Tables to 2D are given for (1), the 95th percentile of distribution of $q_n = x_n/x$; (2), the 5th, and (3), the 95th percentiles of the distribution of $u_n = |x_n/s|$ where x_n is the largest of a sample of n observations from a normal population with zero mean and unit variance and s is independently distributed with ν degrees of freedom, for (1): $n = 1(1)8$: $\nu = 3(1)10(2)20, 24, 30, 40, 60, 120, \infty$; (2): $n = 1(1)10$: $\nu = 1(1)5(5)20, 24, 30, 40, 60, 120, \infty$; (3): $n = 1(1)8$: $\nu = 5(5)20, 24, 30, 40, 60, 120, \infty$, respectively.

A table used in the derivation of the above tables is given to 8S for coefficients $a_i^{(k)}$ which are defined by

$$\left(\int_{-\infty}^{x} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt\right)^k = e^{-kx^2/6}\left(\sum_{i=0}^{\infty} a_i^{(k)} x^i\right)$$

for $i = 0(1)30$; $k = 1(1)7$.

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14[K].—P. B. PATNAIK, "A test of significance of a difference between two sample proportions when the proportions are very small," *Sankhyā*, v. 14, 1954, p. 187–202.

The paper is concerned with the following problem: "if there is a random sample of N_1 individuals from a population, of which x_1 have the characteristic A, and a random sample of N_2 from a second population, of which x_2 have A, then it is desired to test whether the chance of possessing A is the same in the two populations." Consideration is restricted to cases in which Poisson distributions are acceptable approximations to the binomial distributions which give the probabilities for various values of X_1 and of X_2. The problem thus becomes one of testing, using one observation from each Poisson distribution, the hypothesis that the ratio of the parameters of the two Poisson distributions is a given constant. Two test procedures are proposed. Randomized tests are not considered. (A) It is pointed out that, if the conditional probabilities of falling in the critical region for fixed sum of the two observations is kept below some preassigned value, the size of the critical region is, for small sum of the parameters, very much below this preassigned value. One of the many possible methods of interpolating is proposed to keep conditional probabilities near, but not necessarily below, the preassigned number and to make the size of the critical region near this number. Some tables illustrate the effects on conditional and unconditional sizes of critical regions. (B) The idea of a "test with minimal bias" is introduced. The power curve must be at or below the nominal size at the parameter value for the hypothesis under test, derivatives at this point must satisfy certain conditions, and among tests satisfying these conditions, that one is chosen which minimizes the length of the interval of parameter values for which the power curve is below the nominal size. The requirement of minimal bias is imposed on the conditional regions, making the test that of a binomial variate. Critical values for a binomial variate are given (in Table 6) for α(nominal size) $\equiv 0.05, 0.10$; ρ(parameter of the binomial) $= \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}$, n(index at the binomial) $= 6(1)25$. Several typographical errors in titles are obvious.

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15[K].—BENJAMIN EPSTEIN, "Truncated life tests in the exponential case," *Ann. Math. Stat.*, v. 25, 1954, p. 555–564.

With n items on a life test, it is decided in advance that the experiment is terminated either at a fixed truncation time T_U or at the time $X(r_0, n)$ necessary for the r_0th failure, whichever is smaller (stopping rule), and it is assumed that the distribution of lives $X(X \geq U)$ is exponential with unknown mean θ. If $X(r_0, n) < T_0(> T_0)$ the null hypothesis $\theta = \theta_0$ is rejected (accepted). Two cases are considered, non-replacement and replacement from the same population. Formulae are derived for $E_\theta(r)$, the expected number of observations, and $E_\theta(T)$, the expected waiting time necessary to reach a decision. In the replacement case they are related simply by $E_\theta(T) = \frac{\theta}{n} E_\theta(r)$. The probability $L(\theta)$ of accepting $\theta = \theta_0$ where θ is true is calculated as the probability of reaching a decision requiring up to r failures. However, in EPSTEIN's formulae (2), (13), (21), the sign $=$ should read $\geq$.

Table 1 gives the values of r and $\chi^2_{1-\alpha}(2r)/(2r)$ such that the test based on using $\hat{\theta}_{r,n} > C = \theta_0\chi^2_{1-\alpha}(2r)/2r$ as acceptance region for $\theta = \theta_0$ will have the probabilities $L(\theta_0) = 1 - \alpha$ and $L(\theta_1) \geq \beta$ for the errors of the first type with $\alpha = .01, .05, .10$ and of the second type with $\beta = .01, .05, .10$ for $\theta_0/\theta_1 = \frac{3}{2}, 2, \frac{5}{2}$, 3, 4, 5, 10. (The heading of the last column should be $\beta = .10$ instead of $\beta = .01$.) Table 2 gives some integer values of $n = [\theta_0\chi^2_{1-\alpha}(2r_0)/2T_0]$ to be used in truncated non-replacement procedures for $\alpha = .01, .05$; $\beta = .01, .05$ for $\theta_0/\theta_1 = 2$, 3, 5 and $\theta_0/T_0 = 3, 5, 10, 20$.

Excellent practical illustrations are given as solutions of problems of the following type: Find a truncated replacement plan for which $T_U = 500$ hours which will accept a lot with mean life $\theta = 10{,}000$ hours at least 95% of the time and reject a lot with mean life $\theta = 2000$ hours at least 95% of the time. Then $\alpha = \beta = .05$. The values $L(\theta)$, $E_\theta(T)$ and $E_\theta(r)$ are computed for both values of θ.

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16[**K**].—D. B. DUNCAN, "Multiple range and multiple F tests," *Biometrics*, v. 11, 1955, p. 1–42.

Let $q(p, n_2) = w/s$ where w is the range of p independent normal variables having the same mean and unit standard deviation, and $n_2 s^2$ is distributed independently of w as chi square with n_2 degrees of freedom.

Table II labeled "Significant studentized ranges for a 5% level new multiple range test" lists the $(.95)^{p-1}$ percentiles of the sampling distributions of $q(p, n_2)$ for $p = 2(1)10(2)20, 50, 100$ and $n_2 = 1(1)20(2)30, 40, 60, 100, \infty$. Table III lists the $(.99)^{p-1}$ percentiles for the same distributions. These tables are used in performing comparisons among the means of equal sized samples from p populations. Tables of the 95th and 99th (same percentiles for all p) percentiles of $q(p, n_2)$ are given in Biometrika Tables [1].

Care should be taken not to confuse the 5% level heading in the table (what the author calls 95% two mean protection level) with 5% level of significance in an analysis of variance test since they actually refer to different probabilities. If an analysis of variance test is to be performed at the 5% level to test the hypothesis that the p populations have equal means then, if the p means are equal, there is a 5% probability that the data will be such that the hypothesis is rejected. For the tests suggested in this paper there is a 5% least upper bound to the probability that any one pre-specified pair of populations having equal means will be judged to have unequal means. Using the suggested test for a case where all p populations have equal means the probability that at least one pair will be declared significantly different is $1 - (.95)^{p-1}$.

This is an attempt to make the suggested tests more comparable in overall level of significance to the case in analysis of variance where single degrees of freedom are used, and repeated tests at the 5% level are used.

Tables I and IV describe an example.

Table V classifies several discussed test procedures according to the measure of variability used among the means (range or variance), whether having other observed means between two specified means changes the difference labeled significant, and whether these differences depend on a fixed level α or a level which changes with the number of means between the two specified ones.

Table VI lists for comparison some "significant ranges for 5% level tests" of four discussed tests. Table VII compares the power of two classification of tests ($p = 20$, 5% level) to recognize a difference in the means of two pre-chosen populations.

Due to the fact that "5% level" means different things for the different tests Tables VI and VII are likely to give a somewhat slanted comparison of the various discussed tests.

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1. E. S. PEARSON & H. O. HARTLEY, *Biometrika Tables for Statisticians*, v. I, Cambridge, 1954.

17[K].—E. C. FIELLER & H. O. HARTLEY, "Sampling with control variables," *Biometrika*, v. 41, 1954, p. 494–501.

Let x and y be jointly distributed, and let $f(x, \theta)$, $g(y, \phi)$ be the respective marginal distributions. Information about the control variable x is available, *e.g.*, $f(x, \theta)$ may be completely known or θ may be known. The authors are concerned with using this information in the estimation of ϕ for the case where $X_1, \cdots, X_5$ are observations from an $N(\mu, 1)$ population, $x = \sum(X_i - \bar{X})^2$, $y = X_{\max} - X_{\min}$; the distribution of y is assumed unknown and that of x known. In the determination of the variance of the proposed estimate, the expectation of the reciprocal of a binomial variate, excluding the zero class, is needed.

Values of (1) $E'(np, n) = \sum_{r=1}^{n} \frac{1}{r}\binom{n}{r} p^r q^{n-r}$ for $m \equiv np = 1(1)10$, $n = 25$, 50, ∞ are computed directly. For $m = 12(2)20$, $n = 25, 50, 100$,

$$(2)\quad E'(m, n) \simeq \frac{1}{m+p} + \frac{1}{(m+p)(m+2p)} + \frac{2!}{(m+p)(m+2p)(m+3p)} + \cdots$$

is used; for $m = 25(5)45$, $n = 50, 100$ a control is used with (2). $E'(m, \infty)$ may be calculated from $E'(m, \infty) = e^{-m}\{\mathrm{Ei}(m) - \log_e m - \gamma\}$ using available tables

[1, 2]. For large np, the approximation $E' \simeq \frac{1}{m-q} + \frac{q}{(m-q)^3} + \frac{q(q+1)}{(m-q)^4}$ is is given. Values of (1) for $m = 1(1)10$ are given to 4D, all others to 5D.

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1. BAAS *Math. Tables*, v. I, London, 1931.
2. E. JAHNKE & F. EMDE, *Tables of Higher Functions*, 5th ed. Leipzig: 1952

18[K].—P. N. SOMERVILLE, "Some problems of optimum sampling," *Biometrika*, v. 41, 1954, p. 420–429.

Let

$$f(y) \equiv f(y_1, \cdots, y_k) = (2\pi)^{-k/2} |\Sigma|^{-k/2} \exp - \tfrac{1}{2} \operatorname{tr} y\Sigma^{-1}y',$$

where $y = (y_1, \cdots, y_k)$, $\Sigma = (\sigma_{ij})$, $\sigma_{ii} = 1$, $\sigma_{ij} = \frac{1}{2}$ and let $F_k(x) = \int_R f(y)\pi dy_i$, where R: $-\infty < y_i < x$, $i = 1, \cdots, k$. In Table 3.1 the author computes $F_k(x)$, $x = 0(.1)2(.5)3$ to 5D for $k = 1, 2$; 4D for $k = 3, 4$; 3D for $k = 5$. $F_1(x)$ is available in a number of tables. $F_2(x)$ was computed using Table VIII of part II of the Pearson Tables [1], and $F_1(x)$. For $F_k(x)$, $k = 3, 4, 5$, an expansion of $f(y)$ in terms of Hermite polynomials was used [2].

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1. K. PEARSON, *Tables for Statisticians and Biometricians*, London, 1931, part II.
2. W. F. KIBBLE, "An extension of a theorem of MEHLER's on Hermite polynomials." Camb. Phil. Soc., *Proc.*, v. 41, 1945, p. 12–15. See p. 15.

19[K].—H. F. DODGE, "Skip-lot sampling plan," *Industrial Quality Control*, v. XI, no. 5, 1955, p. 3–5.

For derivations, formulae, and basic procedure, see the author's earlier paper [1]. The procedure developed there is as follows: (1) inspect 100% of units consecutively as produced until i successive good units are found (each unit is good or bad), (2) thereupon, inspect only an unbiased proportion f, (3) if in the proportion f a bad unit is found, revert to 100% inspection until i successive good units are found, etc., (4) replace bad by good units. For given f and i, the average outgoing proportion defective (AOQ) as a function of the incoming proportion defective (p), and (consequently) the maximum AOQ ($AOQL$) are determined.

In the current paper, the series of units are replaced by series of lots, inspection of a unit by analysis of a sample, bad unit by bad lot (lot whose sample fails to meet a standard). Corresponding changes of meanings of p, AOQ, and $AOQL$ to apply to lots rather than units.

Figure 1 gives the $AOQL = .01(.01).06, .08, .10$ as a function of $i = 1(1)50, (2)100$ and $f = .01(.001).05(.002).10(.01).50$. Figure 1 is essentially an abridgement of Figure 3 of his 1943 paper cited above (p. 272) though the $AOQL$ curves are now fully shown for low i.

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1. H. F. DODGE, "A sampling plan for continuous production," *Ann. Math. Stat.*, v. 14, 1943, p. 264–279.

20[K].—J. AITCHISON & J. A. C. BROWN, "An estimation problem in quantitative assay," *Biometrika*, v. 41, 1954, p. 338–343.

A popular model for the quantitative response u to a stimulus of concentration x is: $u = HP(\alpha + \beta x) + \epsilon$, where H is the maximum expected response; $P(\alpha + \beta x)$ is the normal cumulative based on a linear function of x; and ϵ is $N(0, \sigma^2)$, independent of x. The authors consider a model of the form:

$$\ln u = v = \ln H + \ln [P(\alpha + \beta x)] + \epsilon = K + \ln P + \epsilon,$$

i.e., Var (u) is proportional to $[E(u)]^2$. Iterative equations, familiar to probit users, are set up to estimate α, β, and K.

A table of working probits, to facilitate the solution of these equations, is given for initial guessed probits, $Y = a_0 + b_0 x = 1.0(0.1)9.0$. This table contains values of the minimum working probit to 4D; of the auxiliary variable, P/Z to 4D or 5S; and of the weighting factor, Z^2/P^2 to 4D. The working probit for this model is $y = \left(Y - \frac{P}{Z} \ln P \right) + (v - K) \frac{P}{Z}$.

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21[K].—A. R. JONCKHEERE, "A distribution-free K-sample test against ordered alternatives," *Biometrika*, v. 41, 1954, p. 133–145.

Let X_i, $i = 1, 2, \cdots, k$, be independent random variables with the continuous cumulative probability functions $F_i(x)$, and let X_{ij}, $j = 1, 2, \cdots, m_i$, be samples of size m_i of the X_i. To test the hypothesis that $X_1, \cdots, X_k$ have the same distribution (H_0): $F_1(x) = F_2(x) = \cdots = F_k(x)$ against the alternative that these random variables are stochastically increasing (A): $F_1(x) > F_2(x) > \cdots > F_k(x)$ (on p. 134 the sense of these inequalities is inverted, as it is in the footnote on p. 135), the author considers the following statistic: Let p_{ij} = number of pairs X_{ir}, X_{js} such that $X_{ir} < X_{js}$, among all possible $m_i m_j$ pairs, for given $i = 1, \cdots, k - 1$; $j = i + 1, \cdots, k$. The test statistic is $S = 2 \sum_{i=j}^{k-1} \sum_{j=1+1}^{k} p_{ij} - \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} m_i m_j$, identical with a statistic considered by M. G. KENDALL [1] and closely related to the rank correlation coefficient.

In his study of the probability distribution of S under (H_0) the author obtains the first four cumulants, studies their extreme values, and obtains asymptotic distributions under several assumptions. He finds in particular that if at least two of the sample sizes m_i tend to infinity, then the limit distribution of S is normal, and he proposes a somewhat better approximation by STUDENT's distribution. Finally, the exact distribution for small samples is given.

Table 3 gives Pr $\{S \geq S_0\}$ to 3 or 4S for k samples, each of size m, covering the range: $k = 3$ and $m = 2, 3, 4, 5$; $k = 4$ and $m = 2, 3, 4$; $k = 5$ and $m = 2, 3$; $k = 6$ and $m = 2$; $S_0 = 0(2)\cdots$ until Pr $\{S \geq S_0\}$ becomes negligible.

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1. M. G. KENDALL, *Rank Correlation Methods*, London, 1948.

22[K].—R. A. Bradley, "Rank analysis of incomplete block designs II. Additional tables for the method of paired comparisons," *Biometrika*, v. 41, 1954, p. 502–537.

The present article extends the 1952 tables of Bradley and Terry [1] to include 7 and 8 repetitions for 4 treatments ($t = 4; n = 7, 8$) and 1 to 5 repetitions for 5 treatments ($t = 5; n = 1, 2, 3, 4, 5$). The problem considered is to estimate true treatment ratings ($\pi_1, \cdots, \pi_t; \Sigma\pi_i = 1$) and test the hypothesis H_0: all $\pi_i = 1/t$. The tables give, for all combinations of total rank sums, estimates of the π_i to 2D, the value of the testing statistic, B_1 to 3D, and the significance probability for B_1 to 4D. Only two treatments are tested at a time, giving $\binom{t}{2}$ pairs per repetition. The favored treatment in each pair is given a rank of 2 and the other a rank of 1; hence, the total rank sum for any treatment can fall between $n(t - 1)$ and $2n(t - 1)$.

The author includes (i) a study of the usefulness of an approximate statistic, (ii) a discussion of various uses of B_1, and (iii) two errata for the original tables.

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1. R. A. Bradley & M. E. Terry, "Rank analysis of incomplete block designs. I. The method of paired comparisons," *Biometrika*, v. 39, 1952, p. 324–345 [*MTAC*, v. 8, 1954, p. 17].

23[K].—Edward Walter, "Über die Ausnutzung der Irrtumswahrscheinlichkeit," *Mitteilungsblatt für Math. Stat.*, v. 6, 1954, p. 170–179.

Tests of significance based on statistics which can assume only a finite number of different values usually do not attain their asserted level of significance. It is always possible to modify the test (by introducing a randomized test) in order to attain the asserted level of significance. The author is interested, however, in a modification based on the observations. He considers the sign test for testing the null hypothesis that the distribution function is symmetric about zero. The modified sign test is defined as follows. Let y equal the number of negative observations in the sample of n; let y' be the rank of the negative observation with largest absolute value among the absolute values of all the observations; let y'' be the rank of the negative observation with second largest absolute value among the absolute values of all the observations; etc. The null hypothesis is to be rejected if $y < y_0$, or $y = y_0$ and $y' < y_0$, or $y = y_0$, $y' = y_0'$ and $y'' < y_0''$, etc. In table 1 values of $y_0, y_0', y_0'', \cdots$ are given for $n = 5(1)25$, $\alpha = .01, .05$, and for one- and two-sided tests. The two-sided test is the same as the one-sided test except that one considers either the number of positive observations or the number of negative observations, according to which is the smaller.

In table 2, empirical results are tabulated to compare the power of the sign test, modified sign test, and the t-test for $\alpha = .01, .05$ in the case where the distribution is normal with variance 1 and mean $\mu = .6, 1, 1.4$.

In table 3, the effective levels of significance of the sign test and modified sign test are given for $n = 5(1)30$.

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24[K].—SHOZO SHIMADA, "Power of R-charts," *Reports of Statistical Application Research*, Union of Japanese Scientists and Engineers, v. 3, 1954, p. 70–74.

Let x_1, x_2, x_3, x_4 be independent and normally distributed random variables with means m_1, m_2, m_3, m_4 and common variance σ^2. The range R is defined as the difference between the largest and smallest value among the four successive observations.

If $m_1 = m_2 = m_3 = m_4$, then the range provides an estimate of σ. On the other hand, if not all the m_i's are equal then the range may include the effect of the variability of the m_i's in its estimation of σ.

The author finds the distribution of R based on four observations when the m_i's are not all equal. He then plots the power curves of the range chart for three special cases, approximating a two fold integral by using "Circular Probability Paper" [1].

He considers the following three cases:

Case 1. $m_i = \alpha + (i - 1)\eta$ where α and η are constants.
Case 2. $m_i = m_j = m_k + \delta = m_l + \delta$ (i, j, k, or l represents one of four integers 1, 2, 3, 4, with no two equal).
Case 3. $m_i = m_j = m_k = m_l + \delta$.

Table 1 gives the probability that a point plotted on a range chart falls outside a specified control limit for the three different cases. Table 2 gives the probability that x_i and x_j take on the smallest and largest value for Case 1 where $\eta = .05$.

Some typographical errors are listed below:

1. Fig. 2 Case 1 should be Fig. 3 Case 2.
2. Fig. 3 Case 2 should be Fig. 2 Case 1.
3. In Table 2, the headings Max. and Min. should be interchanged.

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1. F. C. LEONE & C. W. TOPP, "Circular probability paper," *Industrial Quality Control*, v. 9, 1952, p. 10–16.

25[K].—C. T. FAN, "Note on the construction of an item analysis table for the high-low-27-per-cent method," *Psychometrika*, v. 19, 1954, p. 231–237.

The table whose construction is described in this article provides a means of translating the observed proportions of success in the high and low 27 percent groups (denoted by p_H and p_L) into measures of item difficulty (denoted by p and Δ) and of item discrimination (denoted by r). Values are tabled for the difficulty index, p, and the discrimination index, r, as functions of p_H and p_L. These results are based on KARL PEARSON's tables of the normal bivariate surface. A second item difficulty index, Δ, can be determined from p and is expressed in terms of a normal curve deviate with mean 13 and standard deviation 4. The final complete table is not given in this paper but has been published by the Educational Testing Service, Princeton, N. J. However, a graphical version of the final results is given as figure 2 in this paper. Preliminary computations in the table construction are presented in table 3, which furnishes p_H and p_L values to 4D for $.5000 \leq p \leq .9713$ and $r = .05(.05)1.00$. Figure 2 furnishes a simplified version of the final chart for use in graphical estimation of p and r for given values of p_H and p_L. This graph contains a square grid for p, r = .00(.10)1.00 and curves of corresponding p_H and p_L values for $p_H p_H$ = .10(.10).90, .99 and p_L = .01, .10(.10).90.

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26[K].—W. T. SHARP, J. M. KENNEDY, B. J. SEARS, & M. G. HOYLE, *Tables of Coefficients for Angular Distribution Analysis*, Atomic Energy of Canada, Ltd., Chalk River, Ontario, A.E.C.L. Report No. 97, 1954, xxxix + 38 p., $2.00

The functions of angular momentum quantum numbers introduced by Racah have related closely the studies of the various correlations in angle of particles and radiation absorbed or emitted by nuclei. In the Chalk River tables the authors have compiled and calculated values of the Racah W-function, the associated coefficients Z and Z_1, and the 9-j symbol, X, for ranges of parameters which occur in low energy nuclear reactions.

The coefficients $W(ljl'j';sk)$ and $Z(ljl'j';sk)$ are in the main taken from the tables of BIEDENHARN [1] and OBI *et al* [2]. The range of s, the channel spin parameter, is extended to 7/2; a few values for $s = 4$ not given by Obi are newly computed. Except in a few cases, $j' = j$; the range of j is $0(\frac{1}{2})5$. l and l' are restricted to integers less than 4, whether or not the selection rules give non-zero coefficients for higher integers. A table of $W(jj_1jj_1;Lk)$ is provided for $j, j_1 = \frac{1}{2}(1)\frac{11}{2}$ and $L = 1, 2$.

The coefficients $Z_1(LjL'j';Ik)$ are for $j' = j$ closely related to the $F_k(LIj)$ and $G_k(LL'Ij)$ of BIEDENHARN and ROSE [3]. The present tabulation, for L, $L' = 1, 2, 3$, $j' = j = 0(\frac{1}{2})6$, $I = 0(\frac{1}{2})6$, extends previous tables [4] to the parameter values $I = 5, 6$ and $j = 6$.

In all cases k is restricted to the even integers.

New tables of the coefficient $X\begin{vmatrix} abc \\ def \\ ghk \end{vmatrix}$ are given for a, $d = 1, 2$, $e = b$, $f = c$, b, $c = 1(\frac{1}{2})5$ and g, h, $k = 0(2)8$. A second specialization is made to $k = 1$, $a = d = 1(\frac{1}{2})3$, $g = h = 2, 4$, with the other parameters in the range $1(\frac{1}{2})3$.

Finally, auxiliary tables are provided for the triangle coefficient $\Delta(a, b, c)$, and for the CLEBSCH-GORDON coefficients $(ll'00|k0)$, $l, l' = 0(1)6$, and $(LL' - 11|k0)$, $L, L' = 1, 2, 3$.

The square of a coefficient is in each case a rational fraction. The quantities tabulated are the (positive or negative) powers of the prime factors of each square, and the sign of the coefficient.

A useful introductory section lists many of the important properties of the coefficients and discusses their application in the theory of nuclear reactions, heavy particle-gamma ray correlation, gamma-gamma correlation, transformations between angular momentum coupling schemes, triple correlation, and transition probabilities for electromagnetic radiation.

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1. L. C. BIEDENHARN, Oak Ridge National Laboratory, Reports No. 1098 and 1501.
2. S. OBI *et al.*, Annals of the Tokyo Astronomical Observatory III, 1953, p. 89.
3. L. C. BIEDENHARN & M. E. ROSE, *Rev. Mod. Phys.*, v. 25, 1953, p. 729.
4. W. T. SHARP, GOVE, & TAUL, Chalk River Reports. PD. no. 254, unpublished.

27[K].—J. M. KENNEDY, B. J. SEARS, & W. T. SHARP, *Tables of X Coefficients*, Atomic Energy of Canada, Ltd., Chalk River, Ontario, 1954, A.E.C.L. Report No. 106, ix + 16 p., $1.00.

This work extends the authors' table of X coefficients in "Tables of Coefficients for Angular Distribution Analysis" [1]. The following tables are provided:

1. $X^2\begin{vmatrix} abc \\ abc \\ ghk \end{vmatrix}$ for $a = 1, 2, b, c = 1(1)5$. Here and throughout g, h, k run over all even integers compatible with the selection rules for X.

2. $X^2\begin{vmatrix} 1bc \\ 2bc \\ ghk \end{vmatrix}$ for $b, c = 1(1)5$.

3. $X^2\begin{vmatrix} abc \\ abc \\ ghk \end{vmatrix}$ for $a = 1, 2, b, c = \frac{1}{2}(1)\frac{9}{2}$; and for $a = 3, 4, b = \frac{1}{2}(1)\frac{5}{2}, c = \frac{3}{2}(1)\frac{9}{2}$.

4. $X^2\begin{vmatrix} 1bc \\ 2bc \\ ghk \end{vmatrix}$ for $b, c = \frac{1}{2}(1)\frac{9}{2}$.

5. $X^2\begin{vmatrix} a\frac{1}{2}c \\ abc \\ ghk \end{vmatrix}$ for $b = \frac{3}{2}, \frac{5}{2}, a = 1(1)4, c = \frac{1}{2}(1)\frac{9}{2}$.

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1. W. T. SHARP, J. M. KENNEDY, B. J. SEARS, & M. G. HOYLE, A.E.C.L. Report No. 97, 1954, Chalk River, Ont. See preceding review.

28[L].—N. W. MCLACHLAN, *Bessel Functions for Engineers*, Oxford University Press, New York, 1955. Second edition. xii + 239 pages, 24 cm. Price, $5.60.

This second edition is considerably larger than the first edition, which appeared about twenty years ago. It still is a satisfactory text on the use of Bessel functions in classical physics and engineering problems. A little more has been added on the mathematical properties of the functions and a number of different applications have been added. The tone is strictly practical throughout, fundamental equations are quoted, discussed and applied, but not always proved. The style is condensed but readable. Short five-place tables, included at the back of the book, are for $J_n(x)$ for $n = 0, 1, x = 0(0.1)16$; for $n = 2, 3, 4, x = 0(0.1)5$; for zeros of these J's; for $Y_n(x)$ and $H_n(x)$, $n = 0, 1, x = 0(0.1)16$; for I_0, I_1, K_0, K_1 for $x = 0(0.1)10$; and for similar ranges of Ber and Bei, Ker and Kei, and their derivatives.

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29[L].—MORIO ONOE, "Formulae and tables, the modified quotients of cylinder functions," Report of the Institute of Industrial Science, University of Tokyo, v. 4, 1955, No. 5 (Serial No. 32), 22 p.

$C_\nu(z)$ being any cylinder function [1], the author sets

$$\mathfrak{C}_\nu(z) = zC_{\nu-1}(z)/C_\nu(z), \quad \tilde{\mathfrak{C}}_\nu(z) = zC_{\nu+1}(z)/C_\nu(z),$$

and similarly for Bessel functions of the first, second, and third kinds. He gives a collection of formulas regarding these functions, and also numerical tables to 4 to 7S, each accompanied by graphs.

Table 1. $\mathfrak{J}_n(x)$ for $n = 1(1)5$, $x = 0(.1)5$ and the first zero and the first pole of $\mathfrak{J}_1(x)$ with corresponding values of $\mathfrak{J}_n$.

Table 2. $\mathfrak{J}_n(iy)$ for $n = 1(1)5$, $x = 0(.2)6$.

Table 3. $\mathfrak{Y}_n(x)$ for $n = 1(1)5$, $x = 0(.1)5$ with the first two zeros of $\mathfrak{Y}_1$ and the first zeros of $\mathfrak{Y}_2$, $\mathfrak{Y}_3$, $\mathfrak{Y}_4$.

Table 4. $\mathfrak{H}_n^{(1)}(iy)$, $n = 1(1)5$, $x = 0(.1)5$.

Table 5. $\mathfrak{J}_n(n)$, $\mathfrak{Y}_n(n)$ for $n = 1(1)50$.

Appendix. Some coefficients in power series expansions of $\mathfrak{J}_n$ and asymptotic expansions of $\mathfrak{H}_n^{(1)}$, $n = 1(1)5$.

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1. G. N. WATSON, *Bessel Functions*, Cambridge, 1922, p. 82.

30[L].—H. E. SALZER, "Complex zeros of the error function," *J. Franklin Institute*, 260, 1955, p. 209–211.

Let $f(z) = \int_0^z e^{-u^2}du$. The zeros z_n of $f(z)$, for which $\frac{1}{4}\pi < \arg z_n < \frac{1}{2}\pi$, and $0 < |z_1| < |z_2| < \cdots$ are tabulated as follows: $n = 1$ to 12D, $n = 2(1)10$ to 9D. This table extends and corrects one of T. LAIBLE (*Z. ang. Math. u. Phys.*, 1951, p. 484–486).

J. T.

31[L].—HAROLD OSTERBERG & GORDON L. WALKER, *Table of* $\int_0^z [J_1(x)/x]dx$, Research Center, American Optical Company, Southbridge, Massachusetts, Communication No. 1, September 1, 1955. 7 multilithed pages with cover. 28 cm., deposited in the UMT FILE.

This table lists the function $f(z) = \int_0^z x^{-1}J_1(x)dx$ for $z = .01(.01)3.85$ and $z = 4(1)25$ plus about seven special values of the argument. This is the function $g(z)$ of "A guide to tables of Bessel Functions," by R. C. ARCHIBALD, *MTAC*, v. 1, 1944, p. 247. The numbers shown are usually 7S, and accuracy is claimed through 5S or 6S.

For the early range, the calculations were carried out on an International Business Machines Card Programmed Calculator, and for the later range they were carried out on a desk machine, using tables of Bessel Functions already published.

A similar, less extensive table has been published by R. GANS, "Mikroskopische probleme," *Annalen der Physik*, s. 4, v. 78, 1925, p. 1–34. The authors note that their function is equal to $\int_0^z J_0(x)dx - J_1(z)$. The function $\int_0^z J_0(x)dx$ has been tabulated to 10D by A. N. LOWAN and M. ABRAMOWITZ in "Tables of integrals of $J_0(x)$ and $Y_0(x)$," *J. Math. and Phys.*, v. 22, 1943, p. 2–12, and this table has been reprinted as AMS **37** by the National Bureau of Standards; its range is $z=0(.01)10$.

C. B. T.

32[L].—B. ZONDEK, "The values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their logarithms accurate to 28 decimals," *MTAC*, v. 9, 1955, p. 24–25.

TABLE ERRATA

Reviews in this issue mention errata in the following works:

THE RAND CORPORATION, *One Million Digits and 100,000 Normal Deviates*, Review **11**, p. 39–43.

BENJAMIN EPSTEIN, "Truncated life tests in the exponential test," *Ann. Math. Stat.*, v. 25, 1954, p. 555–564, Review **15**, p. 44–45.

R. A. BRADLEY & M. E. TERRY, "Rank analysis of incomplete block designs. I. The method of paired comparisons," *Biometrika*, v. 39, 1952, p. 324–345, [*MTAC*, v. 8, 1954, p. 17], Review **22**, p. 49.

SHOZO SHIMADA, "Power of *R*-charts," *Reports of Statistical Application Research*, Union of Japanese Scientists and Engineers, v. 3, 1954, p. 70–74, Review **24**, p. 50.

T. LAIBLE, "Höhenkarte des Fehler-integrals," *Z. ang. Math. u. Phys.*, 1951, p. 484–486, Review **30**, p. 53.

247.—GIUSEPPE PALAMA & L. POLETTI, "Tavola dei numeri primi dell'intervallo 12 012 000–12 072 060," Unione Matematica Italiana, *Bollettino*, s. 3, v. 8, 1953, p. 52–58. (*MTAC*, v. 7, 1953, p. 173, Review **1101[F]**.)

The following errata have been found.

Entry	Division	Probably intended prime
12 019 307	277	—
12 020 023	1901	—
12 023 381	31	12 023 383
12 028 813	131	12 028 817
12 045 149	457	—
12 047 023	107	—
12 071 881	2081	—

In addition the following primes should be added to the list.

12 047 309
12 069 919

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Editor's note: Primality of each number listed above as prime has been verified on the SWAC computer by J. L. SELFRIDGE.