

The numerical result for $S(x)$ is 7.131×10^{-5} , compared to the rigorous value of 7.1314×10^{-5} . Since the error introduced by the quadrature rule is only of the order $e^{-2x} = e^{-20}$, the main error in this procedure is introduced in the summation process, during which significant figures are lost. Thus, the numerical result obtained here agrees with the analytical result to as many significant figures as were retained during the numerical procedure. However, three of the seven figures to which $\phi(k)$ was originally evaluated were lost during integration.

The same integral has been treated numerically by Simpson's and Filon's methods, and it has been found that in order to obtain accuracy comparable to that obtainable by the method described here the integrand must be evaluated at at least 10 times as many points.

The tables and graph in this paper were prepared by Miss D. M. Keaveney.

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Formulas for the Partial Summation of Series

1. Introduction. The paper gives a table of the coefficients $A_m(n)$ in the "partial" summation formula $S_n \doteq \sum_{m=4}^{10} A_m(n) S_m$. The coefficients $A_m(n)$, $m = 4(1)10$, are tabulated exactly in the fractional form $C_m(n)/D(n)$ for $n = 11(1)50(5)100(10)200(50)500(100)1000$. For every n , except 47, $D(n)$ is the least integer containing no more than ten digits exclusive of final zeros, to permit ready division on a ten-bank calculating machine using $S_n \doteq \left[\sum_{m=4}^{10} C_m(n) S_m \right] / D(n)$.

The purpose of $A_m(n)$ is the calculation of the sum of n terms of a slowly convergent series, or, more generally, the evaluation of the n -th term of a sequence S_m which is either slowly convergent or asymptotically characterized by $S_m \sim f(m)$ (convergent or divergent), in such a manner that the auxiliary sequence $S_m/f(m)$ is slowly convergent. The formula for S_n was obtained by Lagrangian extrapolation upon S_m considered as a polynomial in $1/m$, based upon the last 7 values of

the first 10 terms of the sequence S_m . Examples of the calculation of S_n from the initial values S_4, S_5, \dots, S_{10} are given for $S_m = \sum_{r=1}^m 1/r^2$, $\sum_{r=1}^m 1/r$, the smallest zero of the m -th order Laguerre polynomial, and the m -th zero of the Bessel function $J_0(x)$, all the results being of high accuracy.

2. Relation to earlier work. In a previous article [1] the writer gave a method, with formulas, for the summation of a slowly convergent series or, more generally speaking, for the evaluation of the limit of a slowly convergent sequence. Those formulas were concerned only with the "complete" sum or limit, i.e., limit S_m . But in many practical problems we wish to have S_m for some larger finite value of m , say $m = n$. For example we may need to estimate the sum of 100 or more terms of a series from the sums of the first few. Then we may have to know the smallest zero of a sequence of polynomials or transcendental functions for some large order from a tabulation of those zeros for only lower orders. The production of this newer table of formulas for "partial" or "incomplete" summation was motivated by the extreme accuracy of the method in the preceding tables of formulas for complete summation, namely, Lagrangian extrapolation for S_m considered as a polynomial in $\frac{1}{m}$. Only here, instead of extrapolation for $\frac{1}{m} = 0$ ($m = \infty$) from the last tabulated values up to a fixed m , we extrapolate for $\frac{1}{m} = \frac{1}{n}$ ($m = n$) when we require the value of S_n . But in employing this method, the sequence S_m must always be examined, usually in the roughest way, to see whether it converges slowly as it is, or whether it diverges in some regular manner, in which case it is essential to calculate with some auxiliary sequence obtained by dividing through by some simple function of m , often m itself, to insure convergence. Even when the sequence S_m does not diverge, knowledge of its asymptotic behavior, say $S_m \sim f(m)$ as $m \rightarrow \infty$, enables one to obtain much more accurate results by extrapolating upon the auxiliary sequence $S_m/f(m)$ instead of S_m (as will be apparent from some of the examples below).

3. Choice of arguments. The earlier article [1], which was devoted only to complete summation, gave sets of multipliers depending upon the number of terms of S_m that were available, and also the degree of accuracy that was desired. But since "incomplete" summation requires a separate set of multipliers for every desired S_n , where n should assume a number of values to be generally useful, to save excessive space it was decided to choose a single number of available values of S_m , as well as just one degree of approximation, which would serve best in an all-purpose capacity. Careful scrutiny of the examples for complete summation in the previous article [1] showed that the best single all-purpose choice of the total number of available values of S_m and the number of end values upon which to base the approximate formulas was 10 and 7, respectively. Other considerations for the choice of 10 and 7, besides adequacy of the approximation, are:

A. One does not often go beyond the computation of the first 10 terms of a sequence (especially when the calculation may be involved, as in the computation of the roots of transcendental equations) and

B. From the examples in [1] where the results of the 7-point formula were comparable with those of the 4-point formula, the substantially greater accuracy

in the former justifies the somewhat larger table of formulas that is needed for 7 instead of 4 points.

4. Range of table. This present table gives the coefficients $A_m(n)$, more simply written as A_m , the n being understood, in the following summation formula:

$$(1) \quad S_n \doteq \sum_{m=4}^{10} A_m(n) S_m.$$

The coefficients A_m , $m = 4(1)10$, are tabulated exactly in the fractional form $C_m(n)/D(n)$, more simply written as C_m/D , the n being understood, for $n = 11(1)50(5)100(10)200(50)500(100)1000$. The left-to-right direction of the columns of C_m as m goes from 10 down to 4 was chosen to comply with the natural tendency in extrapolation to start the multiplication with the last or 10th term of S_m and to proceed backwards. The denominator $D(n) \equiv D$ was chosen as the least common denominator of the $A_m(n)$, and nearly every D had no more than ten digits exclusive of final zeros, so that use of (1) in a ten-bank desk calculating machine in the convenient form

$$(1') \quad S_n \doteq (1/D) \sum_{m=4}^{10} C_m S_m$$

permits ready division by D . In the few cases where the l.c.d. of the $A_m(n)$ had more than ten digits for division, both numerators and denominator were multiplied by 2, 4, or 8, resulting in larger values of C_m and D , but with no more than ten digits in D exclusive of final zeros, with the sole exception of $n = 47$, where division by $D = 47^6$ is conveniently managed by two successive divisions by $47^3 = 103823$.

At the suggestion of the referee, decimal values of A_n have been computed and listed by R. B. Horgan [4].

5. Error terms. In addition to the theoretical error in formula (1) or (1'), the user should always bear in mind the size of the coefficients $A_m(n)$ and the fact that in most practical examples the quantities S_m are inexact, so that if the error σ_m in S_m satisfies $|\sigma_m| \leq e$, an upper bound for the error in (1) or (1') besides the theoretical one, is $e \sum_{m=4}^{10} |A_m|$. In many cases the actual error due to inexact values of S_m is a small fraction of that upper bound.

6. General formula for $A_m(n)$. The arguments n were chosen to fulfill most of the needs in summation or evaluation of sequences. For all other values of n the user may calculate $A_m(n)$ from the formula

$$(2) \quad A_m(n) = \frac{m^6}{n^6} \frac{\prod_{j=4}^{10} (n - j)}{\prod' (m - j)},$$

where $j = m$ is absent from \prod' .

7. Method of computation. The computation of this table used (2), the procedure being to first get the seven multipliers $m^6/\prod'(m - j)$, $m = 4(1)10$, which are independent of n , and to multiply each by the corresponding $\prod_{j=4}^{10} (n - j)/(n - m)n^6$.

For the convenience of the computer, we list those multipliers:

m	10	9	8	7	6	5	4
$\frac{m^6}{\prod'(m-j)}$	$\frac{12500}{9}$	$\frac{-17714}{40}$	$\frac{7}{16384}$	$\frac{-11764}{36}$	$\frac{972}{1}$	$\frac{-3125}{24}$	$\frac{256}{45}$

A preliminary functional check upon the $A_m(n)$ was performed by using

$$(3) \quad \sum_{m=4}^{10} A_m(n) = 1,$$

and a final functional check made use of the formula

$$(4) \quad \sum_{m=4}^{10} \frac{A_m(n)}{m^6} = \frac{1}{n^6}.$$

The author is grateful to Miss Isabelle Arsham for her assistance in the calculation and checking of the table of $A_m(n)$.

8. Illustrations. The following examples are intended to illustrate the wide applicability and great accuracy of this table of $A_m(n)$ for many different types of problems:

Example 1: As the most elementary type of problem consider the sum $S_m = \sum_{r=1}^m 1/r^2$, where the 4th to 10th partial sums are, to 10D:

$$\begin{aligned} S_4 &= 1.42361 \ 11111 \\ S_5 &= 1.46361 \ 11111 \\ S_6 &= 1.49138 \ 88889 \\ S_7 &= 1.51179 \ 70522 \\ S_8 &= 1.52742 \ 20522 \\ S_9 &= 1.53976 \ 77312 \\ S_{10} &= 1.54976 \ 77312 \end{aligned}$$

Suppose that we want both S_{15} and S_{24} , and employ (1), using the table of $A_m(n)$ for $n = 15$ and $n = 24$. The calculated value of S_{15} is 1.58044 02838, which agrees with the true value of $S_{15} = 1.58044 \ 02834$ to around 4 units in the 10th decimal. The calculated value of S_{24} is 1.60412 34045, which agrees with the true value of $S_{24} = 1.60412 \ 34036$ to within a unit in the 9th decimal.

Example 2: Consider the partial sums of the divergent series $S_m = \sum_{r=1}^m 1/r$, whose values from $m = 4$ up to $m = 10$ are, to 10D:

$$\begin{aligned} S_4 &= 2.08333 \ 33333 \\ S_5 &= 2.28333 \ 33333 \\ S_6 &= 2.45000 \ 00000 \\ S_7 &= 2.59285 \ 71429 \\ S_8 &= 2.71785 \ 71429 \\ S_9 &= 2.82896 \ 82540 \\ S_{10} &= 2.92896 \ 82540 \end{aligned}$$

Suppose that we wish to calculate S_{26} , whose true value is 3.85441 97162. Although S_m diverges, the sequence $s_m = S_m - \log m$ converges slowly to Euler's constant, so that one bases the use of (1) upon s_m , $m = 4(1)10$, to calculate s_{26} and to obtain S_{26} from $s_{26} + \log 26$:

$$\begin{aligned}s_4 &= 0.69703\ 89722 \\ s_5 &= 0.67389\ 54209 \\ s_6 &= 0.65824\ 05308 \\ s_7 &= 0.64694\ 69939 \\ s_8 &= 0.63841\ 56012 \\ s_9 &= 0.63174\ 36767 \\ s_{10} &= 0.62638\ 31610\end{aligned}$$

Employing s_m in (1), we find for $n = 26$, $s_{26} = 0.59632\ 31116$. Since $\log 26 = 3.25809\ 65380$, $S_{26} = s_{26} + \log 26$ is found to be 3.85441 96496, which agrees with the true value to $\frac{2}{3}$ of a unit in the 8th significant figure.

Example 3: From the known values of the smallest zero of the first ten Laguerre polynomials $L_m(x)$, one wishes to calculate the smallest zero of the fifteenth Laguerre polynomial [2]. It is true that the v -th zero of $L_m(x)$, or $x_v^{(m)}$, converges to zero as $m \rightarrow \infty$ in accordance with the asymptotic approximation $2(x_v^{(m)})^{\frac{1}{2}} = m^{-\frac{1}{2}}(v\pi + 0(1))$. But a much smoother approximation can be had from the sequence $S_m = mx_1^{(m)}$; these values from $m = 4$ to 10 are, to 9D:

$$\begin{aligned}S_4 &= 1.2901\ 90758 \\ S_5 &= 1.3178\ 01599 \\ S_6 &= 1.3370\ 79625 \\ S_7 &= 1.3513\ 05736 \\ S_8 &= 1.3622\ 37058 \\ S_9 &= 1.3709\ 00049 \\ S_{10} &= 1.3779\ 34705\end{aligned}$$

Use of (1) for $n = 15$ yields $S_{15} = 1.3996\ 17169$, from which $S_{15}/15 = 0.09330\ 78113$, agreeing with the true value of $x_1^{(15)} = 0.09330\ 78120$ to within a unit in the 9th decimal place.

Example 4: From the first ten tabulated zeros of the Bessel function $J_0(x)$ [3], to calculate both the 11th zero and the 42nd zero. The sequence $J_{0,m}$ of zeros of $J_0(x)$ diverges, but (from the asymptotic formula), $j_{0,m}/m$ converges, and $S_m = j_{0,m}/m$ yields a good approximation; the values for $m = 4$ to $m = 10$, are, to 9D:

$$\begin{aligned}S_4 &= 2.9478\ 83610 \\ S_5 &= 2.9861\ 83542 \\ S_6 &= 3.0118\ 43995 \\ S_7 &= 3.0302\ 33804 \\ S_8 &= 3.0440\ 58941 \\ S_9 &= 3.0548\ 31015 \\ S_{10} &= 3.0634\ 60647\end{aligned}$$

TABLE OF

n	$C_{10}(n)$	$-C_9(n)$	$C_8(n)$	$-C_7(n)$
11	70 00000	111 60261	91 75040	41 17715
12	17 50000	37 20087	34 40640	16 47086
13	840 00000	2008 84698	1981 80864	988 25160
14	18 75000	47 82969	49 15200	25 21050
15	308 00000	818 41914	865 07520	452 94865
16	577 50000	1578 37977	1703 11680	905 89730
17	17160 00000	47877 51969	52481 22880	28263 99576
18	625 62500	1773 24147	1968 04608	1070 60590
19	50050 00000	1 43632 55907	1 61021 95200	88324 98675
20	500 50000	1450 83393	1640 03840	905 89730
21	17680 00000	51677 32284	58825 11360	32686 25360
22	5801 25000	17075 19933	19552 66560	10920 18018
23	2 71320 00000	8 03347 47324	9 24623 83104	5 18708 55855
24	4037 50000	12015 88101	13891 58400	7823 65850
25	1 08528 00000	3 24428 78727	3 76543 64160	2 12803 51120
26	23316 56250	69974 83647	81497 29280	46200 76230
27	3 36490 00000	10 13339 29851	11 83855 41120	6 72991 10417
28	12017 50000	36302 73471	42529 25952	24237 37470
29	17 71000 00000	53 64737 46270	63 00631 04000	35 98882 91000
30	5755 75000	17479 09449	20575 02720	11776 66490
31	29 60100 00000	90 09678 70530	106 27448 83200	60 94424 08575
32	2 35462 50000	7 18162 79535	8 48719 87200	4 87553 92686
33	5 27800 00000	16 12843 71885	19 09362 52416	10 98606 36200
34	3 71109 37500	11 36002 96719	13 47010 56000	7 76189 27750
35	42073 20000	1 28996 67393	1 53183 84640	88390 53405
36	15732 50000	48307 39641	57443 94240	33188 78290
37	110 75680 00000	340 55121 91752	405 47804 77440	234 54768 17376
38	2 10141 25000	6 46960 33017	7 71221 74976	4 46607 36890
39	162 31600 00000	500 31598 86648	597 07490 30400	346 12071 08975
40	60868 50000	1 87827 19263	2 24385 63840	1 30202 14830
41	232 47840 00000	718 12596 64887	858 73970 38080	498 75081 74880
42	20540 78125	63512 51391	76018 48320	44189 30054
43	326 26230 00000	1009 73689 02621	1209 60610 46784	703 71461 10995
44	23 98987 50000	74 30964 12747	89 09135 87200	51 87085 58550
45	29 97592 00000	92 92752 52542	111 49875 60960	64 96436 60120
46	32 78616 25000	101 71796 68323	122 13515 05920	71 21093 96670
47	609 64540 00000	1892 78696 17566	2274 28873 01120	1326 89717 57951
48	44 11907 50000	137 07292 96629	164 80909 88544	96 21535 22330
49	116 35800 00000	361 74885 94863	435 21933 31200	254 23175 77800
50	11 70852 37500	36 42374 38257	43 84741 78560	25 62783 46170
55	72 03784 00000	224 70962 31828	271 21051 23840	158 90962 19655
60	90190 10000	2 81945 39373	3 41001 83040	2 00203 30330
65	2221 01488 00000	6955 56538 78194	8426 95127 85920	4955 69306 17040
70	4 05619 50000	12 72174 09462	15 43510 42560	9 08966 25820
75	5728 75996 00000	17990 21499 30594	21853 97089 07520	12884 96683 37390
80	54 65473 50000	171 82003 02777	208 94140 82560	123 31650 52770
85	2596 32172 80000	8169 81995 99016	9943 97965 51680	5874 12798 03360
90	10 88321 93750	34 27430 51133	41 75079 01440	24 68240 72530
95	53325 11184 00000	1 68057 75451 50504	2 04862 57586 99520	1 21195 43569 16940
100	289 70259 50000	913 60782 27063	1114 39267 43040	659 67523 74030
110	426 47618 65000	1346 41741 20579	1644 09078 57920	974 26207 50590
120	1854 50345 50000	5860 08155 50893	7161 98637 15840	4247 76149 86730
130	246 25783 87500	778 73957 60319	952 45103 92320	565 30709 21190
140	1403 31064 50000	4440 50315 83341	5434 43528 90880	3227 48153 82810
150	2525 53336 75000	7995 91816 89387	9790 93084 56960	5817 86105 19470
160	4540 42488 90000	14381 92813 69761	17618 76032 71680	10474 04799 54110
170	33154 45489 12500	1 05061 18754 89836	1 28759 13219 27680	76575 96983 55960
180	1578 07226 50000	5002 48741 89213	6133 07876 96640	3648 78220 39130
190	66264 52127 75000	2 10126 72999 12417	2 57699 37361 30560	1 53363 04256 60970
200	11388 63953 50000	36124 21246 46949	44315 47415 85920	26380 70362 73690
250	7 23664 40627 25000	22 97934 88571 27124	28 22047 17200 17920	16 81746 04905 78440
300	11 09639 82405 50000	35 26089 65656 34607	43 33395 84199 06560	25 84226 93115 84670
350	8314 03097 25000	26432 75819 78739	32500 91815 73120	19391 64584 02590
400	64 44977 33995 50000	204 98191 74911 47797	252 13382 69862 29760	150 49140 16471 56570
450	352 28055 64733 00000	1120 75082 63074 81512	1378 95553 32454 80960	823 29572 38353 04720
500	125 38767 78511 50000	399 00262 63551 63831	491 04016 57646 28480	293 23935 52389 40110
600	42 15203 66228 50000	134 18049 83957 52039	165 18859 10014 77120	98 68113 06717 90590
700	275 89915 32841 50000	878 47158 42992 69631	1081 74412 43622 60480	646 37486 41980 26110
800	4333 64236 10980 50000	13800 98178 29522 42307	16997 54339 11945 62560	10158 39760 93632 11670
900	20 44825 96356 50000	65 12908 25497 38921	80 22554 64819 91680	47 95270 76172 09010
1000	4173 69687 28899 50000	13295 01253 33637 91153	16378 55590 39777 38240	9790 95570 52294 74930

$$A_m(n) \equiv C_m(n)/D(n)$$

$C_6(n)$	$-C_8(n)$	$C_4(n)$	$D(n)$	n
9 79776	1 09375	4096	17 71561	11
4 08240	46875	1792	1 86624	12
251 94240	29 53125	1 14688	48 26809	13
6 56100	78125	3072	67228	14
119 75040	14 43750	57344	7 59375	15
242 49456	29 53125	1 18272	10 48576	16
7642 25280	938 43750	37 84704	241 37569	17
291 89160	36 09375	1 46432	7 08588	18
24249 45600	3016 40625	123 00288	470 45881	19
250 19280	31 28125	1 28128	4 00000	20
9073 65888	1139 53125	46 85824	122 52303	21
3044 96010	383 90625	15 84128	35 43122	22
1 45202 80320	18370 62500	760 38144	1480 35889	23
2197 69200	278 90625	11 57632	19 90656	24
59962 29120	7630 87500	317 52192	488 28125	25
13054 29048	1665 46875	69 45792	96 53618	26
1 90634 08320	24376 40625	1018 71616	1291 40163	27
6881 17680	881 71875	36 91776	43 02592	28
10 23865 92000	1 31441 40625	5513 05216	5948 23321	29
3356 75340	431 68125	18 13504	18 22500	30
17 40140 96256	2 24142 18750	9430 22080	8875 03681	31
1 39434 37200	17986 71875	757 78560	671 08864	32
3 14653 24800	40645 31250	1714 58560	1434 96441	33
2 22614 73000	28792 96875	1216 05120	965 50276	34
25383 19680	3286 96875	138 97728	105 04375	35
9542 20176	1237 03125	52 35776	37 79136	36
67 51048 55040	8 76103 59375	37117 62432	25657 26409	37
1 28682 09585	16715 78125	708 84352	470 45881	38
99 82611 07200	12 97931 25000	55087 26784	35187 43761	39
37586 65680	4891 21875	207 76448	128 00000	40
144 10366 24896	18 76778 75000	79781 56032	47501 04241	41
12778 00920	1665 46875	70 85056	40 84101	42
203 64693 14880	26 56247 34375	1 13077 49376	63213 63049	43
15 02180 31600	1 96071 09375	8352 31488	4535 19616	44
18 82672 24320	2 45896 21875	10481 33632	5535 84375	45
20 65056 11676	2 69886 09375	11510 75328	5921 43556	46
385 02943 31520	50 35017 81250	2 14867 39456	1 07792 15329	47
27 93569 40720	3 65521 40625	15606 92224	7644 11904	48
73 85691 45600	9 66895 31250	41305 53856	19773 26743	49
7 44917 56920	97571 03125	4170 27072	1953 12500	50
46 29945 48480	6 07819 27500	26035 32288	11072 25625	55
58443 18480	7686 65625	329 83808	129 60000	60
1448 97504 99840	190 86846 62500	8 20246 28224	3 01675 56250	65
2 66126 95395	35101 68750	1510 37952	525 21875	70
3776 79708 80640	498 70901 43750	21 48204 29824	7 11914 06250	75
36 18202 54320	4 78228 93125	20619 21792	6553 60000	80
1725 00930 20160	228 19233 93750	9 84679 05536	3 01719 61250	85
7 25382 11880	96028 40625	4146 75968	1230 18750	90
35641 78576 35840	4721 49427 75000	204 01836 19584	58 80735 12500	95
194 11799 75280	25 73016 46875	1 11245 79648	31250 00000	100
286 98566 76540	38 07823 09375	1 64796 83584	44289 02500	110
1252 31690 15280	166 30058 15625	7 20314 72128	1 86624 00000	120
166 78169 60040	22 16320 54875	96064 01024	24134 04500	130
952 77671 51760	126 68776 65625	5 49437 39136	1 34456 00000	140
1718 37290 32470	228 60431 34375	9 91946 47552	2 37304 68750	150
3095 03664 38160	411 93370 96875	17 88228 87936	4 19430 40000	160
22636 89142 54560	3014 04135 37500	130 89219 01056	30 17196 12500	170
1079 00963 20080	143 71729 55625	6 24342 77248	1 41717 60000	180
45366 41990 62620	6044 39890 03125	262 66401 20832	58 80735 12500	190
7805 89094 59440	1040 30841 90625	45 21987 15904	10 00000 00000	200
4 98146 85057 61440	66458 97608 62500	2891 83356 88704	610 35156 25000	250
7 66004 75168 47920	1 02265 53463 21875	4452 95462 36672	911 25000 00000	300
5750 83455 86340	768 14416 59375	33 46373 43744	6 69921 87500	350
44 64661 54117 18320	5 96568 31390 40625	25998 64798 46912	5120 00000 00000	400
244 31894 33391 90720	32 65522 01225 25000	1 42352 93948 39552	27679 21875 00000	450
87 04077 55242 93360	11 63635 64672 46875	50737 51712 78976	9765 62500 00000	500
29 30103 02575 41840	3 91854 54213 46875	17091 66070 20544	3240 00000 00000	600
191 97238 00716 45360	25 67946 25583 71875	1 12034 08376 11776	21008 75000 00000	700
3017 57739 70941 79920	403 72375 76966 34375	17 61680 06252 95872	3 27680 00000 00000	800
14 24648 08957 91760	1 90631 47076 53125	8319 52049 17616	1537 73437 50000	900
2909 16581 42133 49680	389 31783 01659 28125	16 99247 76781 56288	3 12500 00000 00000	1000

Use of the above values of S_m in (1) for $n = 42$ gave $S_{42} = 3.1229\ 14703$, from which $42 \times S_{42} = 131.16241\ 75$, agreeing with the true value of $131.16244\ 63$ to within 3 units in the 8th significant figure. But the same formula (1) for the very next or 11th zero was so accurate as to give an answer of $33.77582\ 019$, which agrees with the true value of $33.77582\ 021$ to 2 units in the 10th significant figure.

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The work reported was done while the author worked at the Diamond Ordnance Fuze Laboratories, Washington, D. C.

1. H. E. SALZER, "A simple method for summing certain slowly convergent series," *J. Math. Phys.*, v. 33, 1955, p. 356-359.
2. H. E. SALZER & R. ZUCKER, "Table of the zeros and weight factors of the first fifteen Laguerre polynomials," *Amer. Math. Soc., Bull.*, v. 55, 1949, p. 1004-1012.
3. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Mathematical Tables, v. VI, *Bessel Functions*, Part I, Cambridge Univ. Press, 1950, p. 171.
4. R. B. HORGAN, "Table of coefficients for the partial summation of series" [see note below, this issue].

TECHNICAL NOTES AND SHORT PAPERS

Table of Coefficients for the Partial Summation of Series

This note supplies a table of the coefficients $A_m(n)$ as described by Herbert E. Salzer [1]. The computation of the table was undertaken at the suggestion of the referee of Dr. Salzer's paper and the Chairman of the Editorial Committee. Responsibility for its correctness does not fall upon Dr. Salzer.

For $m = 4(1)10$ and $n = 11(1)50(5)100(10)200(50)500(100)1000$, the table lists

$$(1) \quad A_m(n) = \frac{m^6 \prod_{j=4}^{10} (n - j)}{n^6(n - m) \prod' (m - j)}$$

to fifteen decimals. ($j = m$ is absent from \prod' .)

The computation was performed on the SWAC machine by means of double precision routines. The seven values of

$$(2) \quad \frac{m^6}{\prod' (m - j)}$$

were stored as constants in the computing routine. For each n , the seven values

of $\frac{\prod_{j=4}^{10} (n - j)}{n^6(n - m)}$ were obtained and then multiplied by (2), resulting in (1).

The results were converted to 25-decimal numbers and punched on cards by SWAC. They were listed on an International Business Machines Corporation