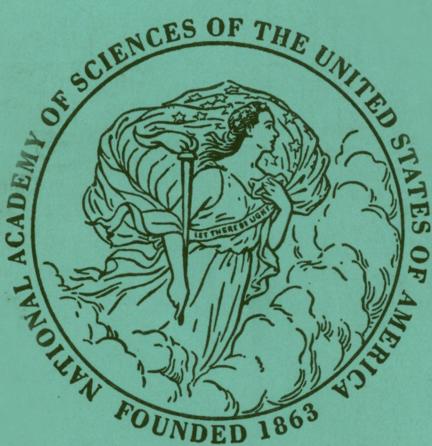


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# Mathematical Tables *and other* Aids to Computation



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# Mathematical Tables *and other* Aids to Computation

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A Quarterly Journal

Edited by

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C. V. L. SMITH

C. C. CRAIG

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