we may expect our lemma for $n = k = 7$ (case a)) to be about three times as fast as sequential coding. (In one program coded by the author, the optimizing of a payroll calculation resulted in a reduction in computing time sufficient to cause calculations to take place at full reading and punching speeds.) A desirable property of the suggested optimizing code is that a single routine may be written for one case of the lemma and arbitrary n and modulus; assignment of particular values to these parameters may be done by the insertion of a single card in the optimizing deck. Therefore a sequential program may easily be optimized for several different values of n and modulus in order to test the best mode of optimizing for the particular routine. If a program contains mostly add, subtract, store, and branch commands, computing speed may be increased by using $n < 7$. Given n, one should select a case of the lemma and k to maximize (modulus) ≤ 50.

Although the lemma is simple enough to use as a manual technique for concomitantly coding and optimizing, we do not recommend the procedure, because testing out a sequential routine has important time saving advantages and because the automatic method for optimizing is so easy and fast.

A routine using case a) of the lemma has been written by the author for M.I.T.'s Statistical Services; the program occupies less than 100 locations and will optimize a one/card deck at full punching speed of 100 cards/minute.

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1. B. GORDON & A. J. DALTON, "Optimizing program," Equitable Life Assurance Society and IBM, New York, 1955 (mimeographed).
2. BARRY GORDON, "An optimizing program for the IBM 650," *J. Assn. for Comp. Machinery*, v. 3, 1956, p. 3–5.
3. S. POLEY & G. L. MITCHELL, "S. O. A. P., IBM 650 symbolic optimal assembly program, programmer's guide," IBM, New York, 1955.
4. E. F. SHEPHERD, "An automatic method of optimum programming," *IBM Technical Newsletter No. 10*, New York, 1955, p. 95–104.
5. IBM, "Type 650 manual of operation, first revision," New York, 1955, p. 70–89.
6. D. W. SWEENEY, "A note on optimum programming and the IBM type 650 operation code usage," *IBM Technical Newsletter No. 10*, New York, 1955, p. 105–107.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

49.—A. V. LEBEDEV & R. M. FEDOROVA, *Spravočhnik po Matematičheskim Tablit͡sam* (*Guide to Mathematical Tables*), Moscow, 1956, xlvi + 549 p., 26 cm. Price 29.20 rubles.

In this *Guide to Mathematical Tables* the authors have extended the largest available index to mathematical tables, Fletcher, Miller, and Rosenhead [1], to include tables published in books up through 1952 or later and tables published in journals through 1953. These dates seem more recent than the closing dates for Schütte's index [2].

The preparation of this index was necessitated by the great activity in table making and related parts of numerical analysis in USSR. The *Guide* was prepared as a convenience both to scholars and to practical workers. Thus, it is to serve a somewhat wider class of users than Fletcher, Miller, and Rosenhead [1] or

Schütte [2]. On the whole, [1] was directed to the fairly advanced computer, and references to elementary tables other than the most basic were largely omitted. For users outside USSR, the present *Guide* will serve approximately the same purpose, but four- and five-digit tables most easily available in USSR were listed and some items not conveniently available in USSR (or elsewhere) are omitted. The present work seems to share with Fletcher, Miller, and Rosenhead [1] the decision to eliminate tables based on physical measurements. Schütte [2] directed his effort toward the listing of practically useful and available tables.

There are presumably no entries in Fletcher, Miller, and Rosenhead [1] which are accidentally excluded from the present *Guide*, for the authors acknowledge having used [1], and also point out that many of their references are from other indexes or references and are not from a direct examination of the table reported. They also acknowledge help received by consulting H. T. Davis [3] and *MTAC*. However, many references in [1] are not found here—five of the twenty-five references to J. R. Airy noted in [1] are not found here. Presumably this is in accord with restrictions imposed purposely by the present authors.

A detailed table of contents extending over 42 pages makes reference to the book somewhat easier than reference to earlier indexes. However, the casual user, who has not memorized the classification system, will lose much time leafing through this table of contents seeking proper chapter headings. There is a name index, but no index of functions tabulated.

Approximate translation of the chapter headings follows:

Chapter 1.	Powers, and rational and algebraic functions
Chapter 2.	Trigonometric functions and various quantities connected with circles and spheres
Chapter 3.	Exponential and hyperbolic functions
Chapter 4.	Common and natural logarithms
Chapter 5.	Factorials, Eulerian integrals, and related functions
Chapter 6.	Sine, cosine, exponential, and logarithmic integrals and related functions
Chapter 7.	Probability integrals and related functions
Chapter 8.	Elliptic integrals and elliptic functions
Chapter 9.	Legendre polynomials and Legendre functions
Chapter 10.	Cylindrical functions
Chapter 11.	Certain special functions and integrals
Chapter 12.	Solutions of certain equations
Chapter 13.	Sums and quantities connected with finite differences
Chapter 14.	Mathematical constants
Chapter 15.	Prime numbers, factors, products, quotients, and fractions.

The user must be willing to thumb through several pages of the table of contents to find the table he seeks. This requirement might have been alleviated by an index of functions listed, such as the one in [1], and the reviewer would certainly be happier with such an index. However, this omission may amount to a negligible time loss. Fresnel integrals, for example, are listed in Chapter 7. The user might seek them here in any one of several ways: adequate knowledge of special functions and the arrangement of the *Guide* might lead him to look here, he might leaf through the eighteen pages required before he finds them in the table of contents, he might look for Fresnel in the name index and find a reference, or he might seek in the name index the name of the author of some known table of Fresnel integrals.

Bibliographic references are grouped together in the back of the book according to the chapter. Thus, under Fresnel in the index one finds a reference to the bibliography of Chapter 7. Under the chapter groupings, the bibliographic references are arranged alphabetically according to author or title (whichever seemed more appropriate to the authors) with Cyrillic characters preceding Latin. Page references are usually given to papers in journals, but not to collections of tables, such as Jahnke-Emde. Principal libraries in USSR owning the tables listed are noted in the bibliography.

Typography in this book, at least so far as formulas and important listings are concerned, is excellent. A few letters are printed poorly in the verbal portions, but on the whole the book is easy to read, easy to use, and appealing.

Utility of indexes of this kind must be obvious.

Throughout the book notation is seemingly consistent with that of [1]. Perhaps one of the great services that [1] and this book may provide (along with the Bateman Project volumes [4]) will be a general consistency in notation among various users of special functions. It is unfortunate that this consistency has not developed spectacularly during the ten years that [1] has been available. The reviewer still hopes that the large efforts of the USSR table makers, the Royal Society table markers, the NBS table makers, and other large groups, will lead to some unification of terminology. At least the three groups mentioned are largely consistent. However, we have recently seen Harvard University tables of the "Error Integrals" and the Cambridge University Press edition of "Fresnel Integrals" listing functions which do not conform with the suggestions of these scholarly indexes.

This *Guide* has an important place on the reviewer's bookshelf.

C. B. T.

1. A. Fletcher, J. C. P. Miller, & L. Rosenhead, *An Index of Mathematical Tables*, Scientific Computing Service Limited, London, 1946.
2. Karl Schütte, *Index Mathematischer Tafelwerke und Tabellen*, R. Oldenbourg, München, 1955.
3. Harold T. Davis & Vera Fisher, *A Bibliography of Mathematical Tables*, Northwestern University, Evanston, Illinois, 1949.
4. Arthur Erdélyi, Wilhelm Magnus, Fritz Oberhettinger, & Francesco G. Tricomi, *Higher Transcendental Functions*, McGraw-Hill Book Co., Inc., New York, 1953.

50[F].—D. H. Lehmer, "On the diophantine equation $x^3 + y^3 + z^3 = 1$," London Math. Soc., *Jn.*, v. 31, 1956, p. 275–280.

The author makes a study of the diophantine equation (1), $x^3 + y^3 + z^3 = 1$, and finds a sequence of explicit parametric solutions of which (2), $x = 9t^4$, $y = -9t^4 + 3t$, $z = -9t^3 + 1$, is the simplest. It is shown how some of these solutions give rise to an infinity of others. Several numerical instances are given, of which the following are typical:

$$x = 3753, \qquad y = -2676, \qquad z = -3230,$$
$$x = 3753, \qquad y = -5262, \qquad z = 4528,$$
$$x = -98196\ 140, \quad y = 78978\ 818, \quad z = 76869\ 289.$$

The study was prompted by a table of Miller and Woollett [1] in which of 21 non-trivial solutions of (1) with $|x|, |y|, |z| \leq 3164$, only 8 are covered by (2). Unfortunately the solutions obtained by the author all fall outside the range of this table, and so there are still solutions of (1) unaccounted for by any parametric formula.

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1. J. C. P. MILLER & M. F. C. WOOLLETT, "Solutions of the Diophantine equation $x^3 + y^3 + z^3 = k$," London Math. Soc., *Jn.*, v. 30, 1955, p. 101–110.

51[F].—H. J. GODWIN, "Real quartic fields with small discriminant," London Math. Soc., *Jn.*, v. 31, 1956, p. 478–485.

The author lists all totally real quartic fields of discriminants less than 11,664 giving full details (for simplicity) in the case of discriminants less than 2,000. There are 45 such fields with quadratic sub-field, starting with $R(7 + 2\cdot 5^{\frac{1}{2}})^{\frac{1}{2}}$ of discriminant 725, and 19 such fields without quadratic sub-field starting with one generated by $x^4 - 8x^3 + 20x^2 - 17x + 3$ of discriminant 1,957. The author conjectures the first two different fields of like discriminant are those generated by $x^4 - 10x^3 + 30x^2 - 32x + 10$ and $R(17 + 4\cdot 2^{\frac{1}{2}})^{\frac{1}{2}}$ of discriminant 16,448. The tables corroborate B. N. Delone and D. K. Faddeev [1], who produced a similar table with discriminants up to 8,112. The computations were apparently done by hand.

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1. B. N. DELONE & D. K. FADDEEV, "Teoriîa irratsional'nosteĭ tret'eĭ stepeni" (Theory of cubic irrationalities), Akad. Nauk SSSR, Mat. Inst. Steklova, *Trudy*, v. 11, 1940.

52[F, L].—D. H. LEHMER, "On the roots of the Riemann zeta-function," *Acta Mathematica*, v. 95, 1956, p. 291–298.

The author gives the results of numerical computations undertaken on SWAC relating to the behavior of the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, on the critical line $\sigma = 1/2$, $t > 0$. The first 10,000 zeros of $\zeta(s)$ were calculated, and it was found that these are all simple and have real part 1/2. Thus the Riemann hypothesis is true at least for $t \leq 9878.910$.

As an auxiliary tabulation, the number of failures of Gram's law among the first n Gram intervals for $n = 1000(1000)10{,}000$ was recorded. Gram's law fails for about 814 instances below 10,000.

The author adds in proof that a few more hours of machine time were used to examine the next 15,000 roots. The behavior of $\zeta(s)$ becomes steadily "worse" but all roots have real part 1/2.

The total computing time amounted only to several hours.

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53[H, X].—W. L. WILSON, JR., *Tables of Inverses to Laplacian Operators over Triangular Grids*, two typewritten leaves $8\frac{1}{2} \times 11$ inches and one transparency 27×48 inches deposited in UMT FILE.

Tables to 6D of the inverses of matrices L_n, $n = 2(1)7$, where L_n is a matrix of $n(n + 1)/2$ rows and columns which is an analogue of six times the Laplacian operator over a regular triangular grid of $n(n + 1)/2$ points.

The Laplacian operator replaces a vector over the grid by itself minus one-sixth the sum of its components at the six or fewer adjacent points in the grid.

These may be interesting in connection with [1].

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Calculation on SWAC was supported by the Office of Naval Research.

1. I. J. GOOD, "On the numerical solution of integral equations." [*MTAC*, this issue, p. 81–83.]

54[I, X].—NBS Applied Mathematics Series, No. 35, *Tables of Lagrangian Coefficients for Sexagesimal Interpolation*, U. S. Govt. Printing Office, Washington, D. C., 1954, ix + 157 p., 26 cm. Price $2.00.

This work tabulates Lagrangian interpolation coefficients for use with tables with increments of one degree, one hour, or convenient fractions or multiples of these increments; entries are to 8D (with accuracy estimated at about 2 in the last decimal). In each case entries are for range $\theta = 0(1'')60'$.

These tables form a companion to the decimal tables [1], in which importance of such works is estimated. No comparable tables for sexagesimal arguments are known to the reviewer.

Sixty randomly chosen arguments were tested in each table using SWAC (through the support of the Office of Naval Research), and the estimate of accuracy to within 2 in the last decimal is consistent with the SWAC results.

Format is the standard style of the AMS, and the tables are, for the most part, easy to read and to use. Some faulty printing has been discovered, and a clarification sheet issued by the National Bureau of Standards for inclusion with the volumes lists the following values difficult to read in some copies:

Page	*Minutes*	*Seconds*	*Coefficients*
23	43	8	A_1: 0.6178 4506
134	6	13	A_0: 0.9525 3516
		14	0.9523 7455
		15	0.9522 1377
		16	0.9520 5282.

C. B. T.

1. NYMTP, A. N. LOWAN, technical director, *Tables of Lagrangian Interpolation Coefficients*, New York, Columbia University Press, 1944. [RMT 162, *MTAC*, v. 1, p. 314–315.]

55[I, X].—H. E. SALZER, "Coefficients for complex osculatory interpolation over a Cartesian grid," *J. Math. Phys.*, v. 35, 1956, p. 152–163.

The Hermite interpolation formula for a function f, given together with its derivative, at $z_1, \cdots, z_n$, is

$$f(z) \doteqdot \sum \{L_k^{(n)}(z)\}^2\{1 - 2L_k^{(n)\prime}(z_k)(z - z_k)\}f(z_k) + \sum \{L_k^{(n)}(z)\}^2(z - z_k)f'(z_k)$$

where the summation is for $k = 1$ to $k = n$ and where

$$L_k^{(n)}(z) = \prod_{j=1}^{n} (z - z_j)/(z_k - z_j);$$

the error is a multiple of $f^{(2n)}$. This is very handy when f and f' are conveniently available, e.g., in tables where J_0 and J_1 are given.

In practice, we assume the z_k are (some of) the vertices of the square in the grid containing z. In the case $n = 2$, we take z_0 as the south-west vertex, $z_1 = z_0 + h$; if $n = 3$ we add $z_2 = z_0 + ih$ and if $n = 4$ we also take $z_3 = z_0 + (1 + i)h$. We have $z = z_0 + (p + iq)h$ where $0 \leq p; q \leq 1$. The basic formula can be written

$$f(z_0 + (p + iq)h) \doteqdot \sum \{A_k^{(n)}(p + iq)f(z_k) + hB_k^{(n)}(p + iq)f'(z_k)\}$$

where the $A_k^{(n)}$, $B_k^{(n)}$ are polynomials of degree $2n - 1$ in $p + iq$.

The tables give values of the polynomials $A_k^{(n)}, B_k^{(n)}$ for $p = 0(.1)1, q = 0(.1)1$, for $n = 2, 3, 4$. For $n = 2$, values are given to 3D; for $n = 3$, values are given to 5D; for $n = 4$, values are given to 8D; all values are exact.

The construction and checking of the tables is described.

J. T.

56[L].—O. EMERSLEBEN, "Anwendungen zahlentheoretischer Abschätzungen bei numerischen Rechnungen," *Z. angew. Math. Mech.*, v. 33, 1953, p. 1–3.

O. EMERSLEBEN, "Über Summen Epsteinscher Zetafunktionen regelmässig Verteilter 'unterer' Parameter," *Math. Nachr.*, v. 13, 1955, p. 59–72.

O. EMERSLEBEN, "Über Zwei Epsteinsche Zetafunktionen 4. und 8. Ordnung," *Ber. Math. Tagung*, 1953, p. 233–250.

O. EMERSLEBEN, "Über eine doppelperiodische Parallelströmung zäher Flüssigkeiten," *Z. angew. Math. Mech.*, v. 35, 1955, p. 156–160.

The author lists a large number of properties of the Epstein zeta-function in these papers, defined by

$$(p)\, Z\left|\begin{matrix} 0 \\ \frac{1}{2} \end{matrix}\right|(s) = \sum_{n=1}^{\infty} (-1)^n \frac{r_p(n)}{n^{s/2}},$$

where $r_p(n)$ is the number of representations of n as a sum of p squares, as well as some related functions. It is shown how a judicious use of the number-theoretic properties of the function can be used in numerical evaluation. Thus, the author gives to 6 decimals

$$(p)\, Z\left|\begin{matrix} 0 \\ \frac{1}{2} \end{matrix}\right|(p), \quad \text{for} \quad p = 1, 2, 3, 4, 6, 8.$$

In addition, the author gives exact values or 6 or more decimal place values for the Epstein zeta-functions

$$Z\begin{vmatrix}0 & 0\\ 0 & \tfrac{1}{2}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0\\ \tfrac{1}{4} & \tfrac{1}{4}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0\\ \tfrac{1}{3} & \tfrac{1}{3}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0\\ 0 & \tfrac{1}{3}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0\\ \tfrac{1}{6} & \tfrac{1}{6}\end{vmatrix}(2),$$

$$Z\begin{vmatrix}0 & 0\\ \tfrac{1}{6} & \tfrac{1}{2}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0\\ \tfrac{1}{6} & \tfrac{1}{3}\end{vmatrix}(2),\quad Z\begin{vmatrix}0 & 0 & 0\\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2}\end{vmatrix}(1),\quad Z\begin{vmatrix}0 & 0 & 0\\ \tfrac{1}{4} & \tfrac{1}{4} & \tfrac{1}{4}\end{vmatrix}(1),\quad Z\begin{vmatrix}0\\ \tfrac{1}{2}\end{vmatrix}(1),$$

as well as for some other functions which are combinations of these.

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57[L].—T. H. Southard, "Approximation and table of the Weierstrass $\wp$ function in the equianharmonic case for real argument." [*MTAC*, this issue, p. 99–100.]

A sparse table giving $f(z; 0, 1) = \wp(z; 0, 1) - 1/z^2$, $z = 0(.1).8(.05)1.55$, 7D, with modified second differences. Interpolation accurate to 6D using Everett's formula is claimed possible.

The paper also contains an approximation of $H(y) = x^2\,\wp(x; 0, 1)$ where $y = x^6$, $0 \leq x \leq 1.53$, 7S. This approximation is a cubic polynomial in y.

Early tabulation to 5D is in [1] for the $\wp$ function.

C. B. T.

1. Eugene Jahnke & Fritz Emde, *Tables of Functions with Formulae and Curves*, Dover Publications, New York, 1945, p. 102–104.

58[L].—F. H. Hollander & C. B. Tompkins, "What you should know about digital computers," *Chem. Eng. Prog.*, v. 52, 1956, p. 451–454.

T. H. Southard's approximation (but not the table) to $\wp(x; 0, 1)$. (See Review **57** above.)

C. B. T.

59[L, P, S].—G. Belford, L. Jackson Laslett, & J. N. Snyder, *Table Pertaining to Solutions of a Hill Equation*, 1956, 420 p., two $8\frac{1}{2}'' \times 11''$ multilithed copies in notebook binders deposited in the UMT file.

Solutions of a Hill equation of the form

$$d^2y/dt^2 + (A + B\cos 2t + C\cos 4t + D\cos 6t)Y = 0,$$

have been tabulated for various values of the coefficients A, B, C, and D, as described by Belford, Laslett, and Snyder in this issue of *MTAC* [1].

The introduction to the table gives a derivation of the equations used to evaluate the tabulated quantities, and a detailed discussion of the method of estimating the truncation error.

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1. G. Belford, L. Jackson Laslett, & J. N. Snyder, "Table pertaining to solutions of a Hill equation." [*MTAC*, this issue, p. 79–81.]

60[M].—D. BIERENS DE HAAN, *Nouvelles Tables d'Intégrales Définies*, Hafner Publishing Co., New York, 1957 (edition of 1867—corrected), xiv + 716 p., 24 cm. Price $12.50.

This is an unaltered reproduction of G. E. Stechert's 1939 edition of the same tables, which in turn was a "corrected" facsimile edition of the original 1867 edition of Bierens de Haan's classical work, augmented by an English translation, due to J. F. Ritt, of the preface. No indication of the nature and extent of the corrections made can be found in either the 1939 or the present edition.

Bierens de Haan's gigantic work hardly needs recommendation. For the uninitiated reader we mention that it is probably the most extensive published collection of *definite* integrals involving *elementary* functions; there are no indefinite integrals, no definite integrals which can be derived from elementary indefinite integrals, and no integrals involving higher transcendental functions.

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61[P].—ERNST GLOWATZKI, *Sechsstellige Tafel der Cauer-Parameter*, Abh. Bayer. Akad. Wiss. Math.-Nat. Kl. (N.F.), v. 67, 1955, 37 p.

These tables are of use in the design of electrical networks, see W. Cauer [1]. They give, for $\theta = 0(1°)90°$ and for $m = 1(1)n$, $n = 1(1)12$, the values to 6D of $a_m = (\sin\theta)^{\frac{1}{2}} \operatorname{sn}(mK/n; \sin\theta)$, to 6S of Δ and to 3D of $-\ln\Delta$. Here, if n is odd, $\Delta = a_n^{-1}\Pi_\nu a^2_{2\nu-1}$, where ν runs from 1 to $\frac{1}{2}(n+1)$, while if n is even $\Delta = \Pi_\nu a^2_{2\nu-1}$, where ν runs from 1 to $\frac{1}{2}n$. When $n = 1, 2, 3, 5, 6, 9, 10$, the computations were carried out by hand to 12D, using the tables of G. W. and R. M. Spenceley [2]. When $n = 4, 7, 8, 11, 12$, the computations were carried out on the Göttingen automatic computer G1, using the series expansions, and working to 9D. No details of the checking are given. The tables are clearly printed.

The parameters a_m are the zeros of the Zolotareff functions of degree n, based on the interval I: $(-k^{\frac{1}{2}}, k^{\frac{1}{2}})$, where $\theta < k = \sin\theta < 1$. The Zolotareff function is that rational function of x, with numerator and denominator of degree n, with value unity at $x = 1$, which gives best approximation (in the sense of minimal deviation) to zero in the interval I and to ∞ in the complement of the interval $(-k^{-\frac{1}{2}}, k^{-\frac{1}{2}})$. The maximal deviation from zero in I is Δ. For an account of these functions see H. Piloty [3].

J. T.

This review was prepared by John Todd for *Mathematical Reviews*.

1. W. CAUER, *Theorie der linearen Wechselstromschaltungen*, Akademie Verlagsgesellschaft, Leipzig, 1941.
2. G. W. & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*, Smithsonian miscellaneous collections, v. 109 (Publication 3863), The Smithsonian Institution, Washington, D. C., 1947.
3. H. PILOTY, "Zolotareffsche rationale Funktionen," *Z. angew. Math. Mech.*, v. 34, 1954, p. 175–189.

62[P].—T. SASAKI, *The Effect of Earth and Sea on the Propagation of Radio Waves*, Numerical Computation Bureau, Report No. 6, Tokyo, 1952. 14 mimeographed pages of theory, 25 cm., and 6 tables of functions, 25 × 34 cm.

The mathematical approach is similar to that of Sommerfeld [1]. The sea is treated as finitely conductive as well as the land. Diffraction of radio waves in the air by the boundary of the land and the sea is calculated. The attenuation of radio waves in the air above the land and above the sea is estimated. The dipole is assumed to be vertical, and is placed at the surface of the land and of the sea. The results are plotted in terms of the electric field as a function of the distance from the land-sea boundary. Results are given for the case where the receiver is located on the surface of the earth and at higher altitudes.

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1. A. SOMMERFELD, "Über die Ausbreitung der Wellen in der drahtlosen Telegraphie," *Ann. Physik*, v. 28, 1909, p. 685.

63[K, L, P].—J. HALCOMB LANING, JR., & RICHARD H. BATTIN, *Random Processes in Automatic Control*, McGraw-Hill Series in Control Systems Engineering, McGraw-Hill Book Co., Inc., New York, 1956, ix + 434 p., 23 cm. Price $10.00.

This book deals with the theory of random signals and noise as applied to automatic control system analysis and synthesis.

Appendix B contains two tables of a functional relationship discussed earlier in the book. These two tables relate the "correlation" functions, respectively, of the input and output of two types of signal amplitude limiters. The input is supposed to be a stationary Gaussian stochastic process with zero mean and normalized correlation function, $\rho(t)$. The limiters are supposed symmetric. The authors denote by x the ratio of the limiter threshold to the root mean square amplitude of the input. In the cases considered, for each value of t, the normalized output correlation functions depend only on the values of ρ and x. Table I gives the value of the normalized correlation function for the output of an "ideal" limiter for $\rho = .05(.05)1.00$ and $x = .2(.2)2.0$. Table II gives the corresponding relationship for the output of an "approximate" limiter for $\rho = .1(.1)1.0$, and $x = .6(.2)2.0$.

Appendix D contains some properties of the polynomials in ρ^{-ct} which are orthonormal functions with uniform weight on $(0, \infty)$. Tables are given of the unnormalized (i.e., multiplied by $c^{-1}/2$) functional values of the first five such functions (i.e., those having degree 1, 2, 3, 4, and 5), for $ct = 0(.01)1(.02)5(.1)9.9$.

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64[S].—J. L. WOLFSON & H. S. GELLMAN, *Five Figure Tables for the Conversion of Electron Momentum to Electron Kinetic Energy from 100 to 20000 Gauss-Cm*, Atomic Energy of Canada, Ltd., Chalk River, Ontario, 1954 (reprinted July, 1956), A.E.C.L. Report No. 327, i + 43 p., 27 cm. Price $1.00.

The tables give the kinetic energy T in kev of an electron of momentum $B\rho$ in gauss-cm, according to the formula

$$T = 510.969\left[\sqrt{\left(\frac{B\rho}{1704.42}\right)^2 + 1} - 1\right]$$

for $B\rho = 100(1)2000(10)10000(100)20000$, 5S. Tables of proportional parts allow linear interpolation with an accuracy of 0.1 to 0.01%.

Similar tables may also be found in Kai Sieghahn [1], and in NBS Applied Mathematics Series, No. 13 [2].

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1. Kai Sieghahn, *Beta- and Gamma-Ray Spectroscopy*, North Holland Publishing Co., Amsterdam, 1955, p. 926–931.
2. NBS Applied Mathematics Series, No. 13, *Tables for the Analysis of Beta Spectra*, U. S. Gov. Printing Office, Washington, D. C., 1952, p. 15–16. [RMT 1117, *MTAC*, v. 7, 1953, p. 180–181.]

65[S, T].—O. Emersleben, "Der Wert der Madelung Konstanten des Steinsalzgitters," Wissenschaftliche Zeitschrift der Universität Greifswald, v. 3, 1953–54, *Math. naturw. Reihe*, p. 607–617.

This is a useful survey of the historical development of our knowledge of the accurate value of Madelung's Constant (i.e., the constant A when writing the electrostatic energy per ion pair in an infinitely extended NaCl type lattice of alternating charges $\pm e$ in the form Ae^2/a, where a is the distance between nearest ions (=half the cubic lattice constant), and the ions are considered as point charges). The value first calculated by Madelung in 1918 is given in (1)—correcting a trivial numerical error—as 1.746(6). Ewald, employing a method of evaluating the lattice sum $\sum_n \pm e^2/r_n$ by Fourier (or Theta-) transformation, obtained 1.747 in 1921; and Emersleben, in considering the more general lattice sums $\sum_n \pm e^2/r_n^s$ as functions of the exponent s and by following in the complex s-plane the analytical continuation from the region of real $s > 3$, where convergence is absolute, to smaller real s values and eventually to $s = 1$, obtained in his dissertation with Born, 1922, a new approach to the mathematical convergence problem. The numerical value then given to 5 decimals was later (1950) extended by him to 14 decimal places, and independently by Y. Sakamoto (1953) in Japan to 16 decimal places: $A = 1.74756\ 45946\ 33182\ 163 \pm 9$. The paper under review reveals very little about the details of the calculations, tables and machines used, or number of terms required, except indicating that the calculation followed essentially Ewald's method and that special tables (with 15 and more decimal places) were calculated for the Error Integral with argument $\sqrt{n\pi}$, n integer [1].

An interesting paragraph deals with the relation of the Madelung Constant and the Epstein Zeta-functions in p dimensions (see Review **56**, p. 109),

$$(p)Z\left|\begin{matrix}0\\k\end{matrix}\right|(s) = \sum_{n=1}^{\infty} e^{2\pi i k n} r_p(n) n^{-s/2},$$

where $r_p(n)$ is the number of lattice vectors of length $\sqrt{n} = (n_1^2 + n_2^2 + \cdots + n_p^2)^{\frac{1}{2}}$ from the origin of a p-dimensional cubic lattice. It had been shown by Emersleben in 1950 that in one dimension $-A = {}^{(1)}Z\left|\begin{matrix}0\\\frac{1}{2}\end{matrix}\right|(1) = 2\ln 2$ and in two dimensions $-A = {}^{(2)}Z\left|\begin{matrix}0\\\frac{1}{2}\end{matrix}\right|(1)$; furthermore that for $p = 1, 2, 4, 8$, it is possible to express

$(p)Z\begin{vmatrix}0\\ \frac{1}{2}\end{vmatrix}(p)$ in a closed form $(= -\pi \ln 2$ for $p = 2)$ and that there exist functional equations between the $(p)Z\begin{vmatrix}0\\ k\end{vmatrix}(s)$ for $k = \frac{1}{2}$ and $k = 0$ when $p = 1, 2, 4, 8$, such as

$$^{(2)}Z\begin{vmatrix}0\\ \frac{1}{2}\end{vmatrix}(s) = (2^{1-s/2} - 1)\cdot {}^{(2)}Z\begin{vmatrix}0\\ 0\end{vmatrix}(s).$$

This last relation can conveniently be used for the calculation of the Madelung constant for a square net. These relations go back to relations between the numbers $r_p(n)$, i.e., the number of lattice points at distances $\sqrt{n}$ from the origin. These relations are only known for $p = 2, 4, 8$, namely, if u denotes an odd value of n, and l any integer

$$r_2(2^l u) = r_2(u); \quad r_4(2^l U) = 3r_4(u); \quad r_8(2^l u) = 1/7(8.2^{3l} - 15)r_8(u).$$

The search for further relations of this kind, which may lead to further relations between the Zetas and be used in calculating the Madelung constants is being continued by the author [2].

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1. Otto Emersleben, "Numerische Werte des Fehlerintegrals für $\sqrt{n\pi}$," *Z. angew. Math. Mech.*, v. 31, 1951, p. 393–394.
2. Otto Emersleben, "Über Summen Epsteinscher Zetafunktionen regelmässig Verteilter 'unterer' Parameter," *Math. Nachr.*, v. 13, 1955, p. 59–72.

66[S, T].—O. Emersleben, "Mit welcher Genauigkeit benötigt man die Madelung-konstante eines Kristalls beispielweise vom Steinsalzgitter?" *Z. für Physik. Chemie*, v. 204, 1955, p. 43–55.

In this paper the author stresses that the knowledge of an accurate value of Madelung's constant A is of practical importance for a discussion of the Coulomb part of the surface energy of finite crystals, for the application of Born's cycle (where all energy terms should add up to zero), and in determining which of several crystal structures is the more stable (e.g., NaCl or CsCl type). In these applications the uncertainty in the values of some of the other physical constants, such as electron charge, Avogadro number, or lattice constant, cancel away, so that it becomes important to know A correctly to about 10^{-4} of its value.

The paper contains a plot of Born's ground potential which may be found useful for calculations of energies in lattices.

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67[L, X].—A. Erdélyi, W. Magnus, F. Oberhettinger, & F. G. Tricomi, *Higher Transcendental Functions*. Based, in part, on notes left by H. Bateman. McGraw-Hill, New York, Toronto, London, v. 1, 1953, xxvi + 302 p., 23 cm. Price $6.50; v. 2, 1953, xvii + 396 p., 23 cm. Price $7.50; v. 3, 1955, xvii + 292 p., 23 cm. Price $6.50.

The late Harry Bateman left at his death in 1946 extremely voluminous materials for a planned *Guide to the Functions* of very wide scope. Consideration was given to the possibility of using this material for the benefit of the mathematical world. The Bateman manuscript project has been supported by the Office of Naval Research and the California Institute of Technology, of which Bateman was such an eminent member. In the event, the four distinguished mathematicians principally engaged on the project, with Erdélyi as director, found it necessary, if a useful work was to be produced in a reasonable time, to narrow the scope of their efforts. They also, understandably, found it easier to compile their own account and to make only limited use of Bateman's notes. What we have, therefore, is a joint work produced to the memory of Bateman, rather than an execution of Bateman's design. The whole work consists of the three volumes now under review, together with two volumes of *Tables of Integral Transforms* reviewed separately [Rev. **107**, *MTAC*, v. 10, 1956, p. 252–254].

It is clear that the volumes on *Higher Transcendental Functions* are of the first importance. Their general scope may be indicated by the chapter headings.

Vol.	*Chap.*	
I	I	The gamma function
	II	The hypergeometric function
	III	Legendre functions
	IV	The generalized hypergeometric series
	V	Further generalizations of the hypergeometric function
	VI	Confluent hypergeometric functions
II	VII	Bessel functions
	VIII	Functions of the parabolic cylinder and of the paraboloid of revolution
	IX	The incomplete gamma functions and related functions
	X	Orthogonal polynomials
	XI	Spherical and hyperspherical harmonic polynomials
	XII	Orthogonal polynomials in several variables
	XIII	Elliptic functions and integrals
III	XIV	Automorphic functions
	XV	Lamé functions
	XVI	Mathieu functions, spherical and ellipsoidal wave functions
	XVII	An introduction to the functions of number theory
	XVIII	Miscellaneous functions
	XIX	Generating functions

A glance at this list is enough to show that the field covered, despite intentional limitation, is still unusually wide. As a handbook or compendium, in which proofs are normally omitted or indicated in outline, the work is distinguished not only by the breadth of its conception but also by the depth of its treatment. It discusses topics and contains results which are not touched upon at all in any other general compendium of comparable size. The references for each chapter are gathered together at the end of the chapter; completeness is rightly disclaimed-but the select references provided will be found highly useful. Any applied mathe, matician accustomed to work with Whittaker and Watson's *Modern Analysis* at hand may be encouraged to add *Higher Transcendental Functions* to his apparatus.

Prospective users may ask, Is this a completely reliable work of reference, in the sense that an applied mathematician may extract any formula from it and use it with confidence? There is no doubt that some members of the mathematical public expect an affirmative answer, but anyone with experience in such enterprises is aware that the answer is likely to be negative, and so it is in this instance. It is no surprise to the reviewer to find a few errata given on loose sheets; he would have been astonished to find matters otherwise, and has no doubt that further errata will come to light. All past experience shows that the removal of all error from such copious and varied collections of results requires a prodigious effort, larger even than that which has already, to our great benefit, been lavished on the work. Trivial misprints are fairly easily found. Creditably reliable though the work is, the reviewer would nevertheless advise using it as a source of inspiration about the *kind* of results which exist, rather than as a collection of results guaranteed in every detail.

It seems a pity that a work of such importance is not printed from type, but the varityping is good of its kind.

The reviewer may perhaps claim licence to put on record that his first name is wrongly spelled (v. 2, p. 382, for Allan read Alan).

A. F.

68[X].—NBS Applied Mathematics Series, No. 15, *Problems for the Numerical Analysis of the Future*, U. S. Gov. Printing Office, Washington, D. C., 1951, iv + 21 p., 26.0 cm. Price $0.20.

When asked to review a set of papers written over eight years ago on *Problems for the Numerical Analysis of the Future*, one is sorely tempted to seize the unfair advantage afforded by the passage of years and discuss just how far the writers' prophecies have been fulfilled. Better feelings prevail, however, when the amount of work in which one would be involved by such a display of ill-nature is realised; this reviewer is going to give a purely factual account of the contents of the pamphlet.

The first paper, by D. R. Hartree, is entitled "Some unsolved problems in numerical analysis." Following a brief introduction designed to illustrate the difference between formal analytical and practical numerical methods, he gives examples of problems to which the answer was not known. These include the elimination of approximately known roots of polynomial equations, the solution of systems of simultaneous nonlinear algebraic equations, in which he considers the possibility of using a steepest descent method to minimise the sum of the squares of the residuals, and two problems concerning relaxation methods. Further sections concern characteristic value problems in ordinary differential equations, and the motion of a continuous distribution of charge under mutual forces between its parts. Finally, he considers the psychological problems that may arise in numerical analysis and the use of computers.

The contribution by S. Lefschetz is entitled "Numerical calculations in nonlinear mechanics." He discusses what is known of the solutions of van der Pol's equation, and points out how valuable would be a numerical investigation for a whole range of values of μ, the parameter occurring in the dissipative term. The necessity for numerical work is even more pronounced in the case of van der Pol's equation with a forcing term, in which the chief interest lies in the existence of subharmonic resonance oscillations. He ends with the hope that the advent of electronic machines will enable nonlinear problems to be tackled directly, without the customary process of linearization.

Bernard Friedman, in "Wave propagation in hydrodynamics and electrodynamics," considers examples of the propagation of waves in which the difficulty of obtaining a solution is due to (1) an inhomogeneous medium, (2) complicated boundary conditions, (3) nonlinear equations, and (4) free boundaries. The respective examples cited by him are (1) the propagation of electromagnetic waves through the atmosphere, (2) tides in the ocean, (3) flood waves in rivers, and (4) the impact of a sphere on a fluid.

The fourth and final paper concerns the then relatively new subject of "Linear programing," and is written by George B. Dantzig, a pioneer in this field. He discusses the problem in very general terms, illustrating it with references to the housewife buying food most economically, the Air Force requirements to use men and equipment in the most efficient manner, and the automobile industry's production problems. A brief description of the "simplex" technique is included, and he concludes with the observation that the computers then available were not fast enough to solve many of these problems with existing techniques.

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69[X].—ALBERT A. BENNETT, WILLIAM E. MILNE, & HARRY BATEMAN, *Numerical Integration of Differential Equations*, Dover Publications, Inc., New York, 1956, 108 p., 20 cm. Price $1.35.

This is an unaltered reproduction of a monograph which appeared in 1931 as Bulletin 92 of the Committee of the Division of Physical Sciences of the National Research Council. Judging from the many references which have been made to the original edition, it has served well its purpose of easing the conscience of pure mathematicians who neglected the numerical aspects of differential equations. The book contains indeed much valuable information on topics which are more or less intimately connected with numerical computation; however, there is relatively little material on the subject of the title as it is understood today.

Chapter I, "The interpolation polynomial" (40 p., including a bibliography of 13 p.), by A. Bennett, is an extremely scholarly account of the various classical representations of the Lagrangian interpolation polynomial, its integral and its first derivative. The subject of chapter II, "Successive approximations" (20 p.), by the same author, is given a much wider scope than would be necessary for the numerical solution of differential equations. There is a wealth of historical detail on such matters as the computation of $\sqrt{2}$ by Indian mathematicians, and the method of Taylor's expansion is traced to its origins. From the practical point of of view the most useful part of the book is chapter III, "Step-by-step methods of integration" (17 p., including a bibliography of 3 p.), by W. E. Milne, a miniature version of the chapters 2, 4, 5, and 6 of Milne's more recent textbook on the subject [1]. In chapter IV, "Methods for partial differential equations" (16 p.), by the late H. Bateman, difference methods are mentioned; naturally, nothing is reported here on the problems which arise in their actual use (degree of convergence, stability, iterative methods for solving partial difference equations), because work in these lines was almost nonexistent in 1931. There are some remarks, interspersed with many references, on variational methods, and more than a third of the chapter is taken up by a discussion of the extension of solutions of (ordinary) differential equations to non-integral values of a parameter, notably by E. T. Whittaker's cardinal series.

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1. W. E. MILNE, *Numerical Solution of Differential Equations*, John Wiley & Sons, Inc., New York, 1953.

70[X].—L. COUFFIGNAL, *Résolution Numérique des Systèmes d'Équations Linéaires*, Gauthier-Villars, Paris, v. 2, 1956, iii + 180 p., 21 cm. Price $5.86.

As the title implies, the author, in this volume, is concerned with the specific problem of finding the numerical solution to a system of linear algebraic equations.

Chapter 1 contains the definitions and notation. The cracovians of Banachiewicz [1] are used rather than the more familiar matrices. These tableaux use column-by-column multiplication instead of row-by-column multiplication and the author has reversed the usual meaning of the subscripts of an element a_{ij}, i.e., the i denotes the column and the j denotes the row containing a_{ij}.

In the second chapter he introduces the concepts of the determinant of a tableau, the rank of a tableau, and the *réduit* of a tableau. This latter term denotes a tableau obtained by a transformation which eliminates the nth row and mth column while replacing a_{ij} by its transform, a'_{ij}, where

$$a'_{ij} = a_{ij} - a_{in}a_{mj}/a_{mn}.$$

In the third chapter the author gives the formal solution of a system of linear equations. His algorithm involves the construction of *réduits* of a tableau, and the formulas are easily programmed for hand or machine computation.

In chapter 4 numerical solutions are discussed, and a numerical analysis of the effects of errors in the coefficients and constants of the system is included. There is a discussion of round-off errors and methods to reduce these to a minimum. A Relaxation method of Southwell [2] is described briefly. The final section of the chapter is concerned with systems of equations which are not normal.

The fifth and final chapter makes up approximately half of the book. It contains a detailed discussion of computational procedures which includes copies of computation forms useful for the hand computer and the computer using a desk calculator. Sample problems are worked out in detail, and the step-by-step procedures used are presented in the form of lists of computation steps. These lists play the same role as the flow charts which some programmers construct prior to coding a problem for solution using an automatic computing machine.

The author claims that the column-by-column multiplication employed using cracovians justifies his choice of notation. His viewpoint is undoubtedly influenced by the fact that the book is written as a manual for hand calculation. Automatic calculation is mentioned only briefly (page 90), and the discussion is restricted to electronic analog machines.

The book is well written and will serve as a good reference for those interested in manual methods for solving linear equations. The printing is easily legible and entirely satisfactory, although there are a few minor misprints, e.g., in the definition of cracovian multiplication on page 12, the left member of equation (16) should be $\|b_{nq}\|$ rather than $\|b_{np}\|$, and several page numbers listed in the table of contents are incorrect.

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1. T. BANACHIEWICZ, "Résolution d'un système d'équations linéaires algébriques par division," *Enseignement Math.*, v. 39, 1942–1950, p. 34–45.
2. R. V. SOUTHWELL, *Relaxation Methods in Engineering Science*, Clarendon Press, Oxford, 1940.

71[L, X].—J. CRANK, *The Mathematics of Diffusion*, Oxford University Press, New York, 1956, vi + 347 p., 23.5 cm. Price $8.00.

The author gives a very extensive and detailed discussion of solutions of the diffusion equation. The results are presented in closed form and graphically. The major portion of the book is concerned with analytical solutions. However one of the thirteen chapters is devoted to numerical methods for dealing with a parabolic partial differential equation.

The first six chapters of the book are devoted to the derivation of the diffusion equation and its solution, in case the diffusion coefficient is constant. Separate chapters are devoted to the discussion of problems with plane, cylindrical and spherical symmetry, respectively. Both steady state and time dependent solutions satisfying various boundary and initial conditions are built up from the elementary solutions of the diffusion equation.

Chapter VII contains a discussion of a number of problems in which diffusion occurs in two distinct regions separated by a moving boundary or interface across which there is either a discontinuity in concentration or the gradient in concentration, the dependent variable of the problem. The problems treated are variants of a single mathematical problem which was treated generally by P. V. Danckwerts [1].

Chapter VIII treats problems in which one substance may be absorbed by another through which it can diffuse and with which it can also react chemically. This chapter is concerned with two dependent functions of position and time, the concentration c, and the amount of immobilized solute s which are linked by a pair of partial differential equations one of which is a parabolic one. Methods for dealing with special equations of this sort are discussed.

Chapter IX discusses solutions of nonlinear parabolic differential equations which may be reduced to ordinary differential equations by requiring the independent variables to occur only in particular combinations. Various methods of successive approximation to the resulting nonlinear ordinary differential equation are given.

Chapter X, entitled Finite-Difference Methods, is concerned with methods for obtaining numerical solutions to parabolic partial differential equations. The author discusses the equation

$$\partial c/\partial T = \partial^2 c/\partial X^2$$

as an illustrative example and in terms of it gives various "recipes" for obtaining numerical solutions. Although the critical ratio $\delta T/(\delta X)^2$ where δT and δX represent the mesh sizes used in T and X respectively is restricted in each of the recipes to be less than one the author never describes adequately the role that this quantity plays in the convergence of the numerical solution to the analytic one and the stability of the numerical algorithm. Moreover the possibility of obtaining stable and convergent algorithms with $\delta T/(\delta X)^2 > 1$ in cases where suitable boundary conditions are given is not even mentioned in spite of the fact that implicit algorithms are given.

This inadequacy in the discussion of numerical methods may be related to the fact that the author's discussion of the direct use of high-speed digital machines in the solution of parabolic differential equations is restricted to two sentences although a page is devoted to King's work on the application of the Monte Carlo method to diffusion problems and five pages are devoted to a discussion of the use of differential analyzers and other analog devices for the obtaining of numerical solutions to diffusion problems.

Thus while this chapter contains some useful information concerning the obtaining of numerical solutions to diffusion problems, it cannot be said to be a modern or definitive discussion of the numerical analysis involved or even of the computing instruments available.

The contents of Chapters XI and XII are described by their titles which are The Definition and Measurement of Diffusion Coefficients and Some Calculated Results for Variable Diffusion Coefficients respectively.

Chapter XIII is concerned with the problem of the diffusion of one substance through the pores of a solid body which may absorb and immobilize some of the diffusing substance with the evolution or absorption of heat. The mathematical problem involved is one in which there are two dependent variables C, the concentration, and T, the temperature, which satisfy a pair of coupled parabolic partial differential equations.

The book contains twenty short numerical tables of various mathematical functions which are needed in the evaluation of expressions given in the text. Included in these is a table of the error function and associated functions and tables giving the roots of various transcendental equations. The methods of computing these tables are not discussed.

A. H. T.

1. P. V. DANCKWERTS, "Unsteady-state diffusion or heat-conduction with moving boundary," Faraday Soc., *Trans.*, v. 46, 1950, p. 701–712.

72[M, S, X].—MARCEL VAN LAETHEM, *Une Méthode Nouvelle et Générale de Calcul des Intégrales Généralisiées*, Publications Universitaires de Louvain, Louvain (Belgique), 1956, viii + 180 p., 23 cm. Price 250 Fr.b. ($5.00).

This book is concerned with defining, evaluating the integral,

$$J = \int_a^b y\,dx$$

where $y(x)$ has a singularity, a pole, at $x = b$ and applying the results of the evaluation to various problems. The author considers the quantity

$$\chi(x) = \int_b^x y\,dx$$

and evaluates x as a power series in χ. This series is then inverted to give χ as a function of x. The coefficients of the various power series are given in detail as are estimates for the remainders.

The main application is to the problem of determining the trajectories of ions in a cylindrical condensor. The results of the method used by the author for the evaluation of integrals of the form of J, are compared to the results obtained by a method used by O. Godart which involves a change of variable and numerical evaluation of the resulting integral.

A. H. T.

73[Z].—*The Proceedings of the Institution of Electrical Engineers*, v. 103, Part B, Supplement No. 1–3, 1956 (Convention on Digital Computer Techniques), British Institution of Electrical Engineers, London. Price one pound.

A well organized picture of the state of digital computer techniques in Great Britain was presented in London during April 1956. The full proceedings of the "Convention of Digital Computer Techniques" has now been published by the British Institution of Electrical Engineers, Savoy Place, London W C 2, as Part B, Supplement Numbers 1, 2, 3 of its Proceedings.

The portion of the conference concerned with applications of digital computers was particularly strong in its examples of engineering applications. Eight papers described the methods and results of machine solutions of problems in proton synchrotron design, load flow in power distribution systems, power systems engineering switching circuits, transformer design, synchronons motor calculations, electric traction, the design of linear and nonlinear control systems, and the solution of a nonlinear heat conduction problem concerned with the freezing of fish. In most cases the authors were cognizant of the tutorial value of these papers beyond the time of presentation and went to greater than average pains to describe the preparation of the problems for the digital computer.

A relative scarcity of papers on business and industrial control applications was compensated for by extensive discussion, particularly concerning the industrial control area.

Formal problems in numerical analysis were dealt with by a few papers concerned with the solution of linear elliptic partial differential equations, calculation of characteristic roots and vectors of a real symmetric matrix, predictor-corrector methods for numerical integration, and the general programming strategy developed at the University of Manchester MARK I computing facility.

The description of modern digital computer systems was separated into categories of "Commercially Available Computers" and "Experimental Computers."

The commercially available machines presented were the "DEUCE" (English Electric Co.); "MERCURY" and "PEGASUS" (Ferranti); "400" Series (Elliott Bros.); "HEC" and a punched card calculator series (British Tabulating Machine

Co., Ltd.); PCC (Powers-Samas); and the "650," "704," and "705" (IBM). Of particular interest are the characteristics of the "DEUCE" and the "MERCURY."

The "DEUCE" is based on the development of the "PILOT ACE" at the National Physical Laboratory. Confidence derived from the high reliability and productivity achieved with the prototype machine led to an increased level of speed and complexity of organization using the same acoustic delay line and circuit components. A graded sequence of acoustic delay lines containing 1, 2, 4, or 32 words are incorporated with a highway busbar system to provide very flexible organization of information transfers between the delay lines. These fast access memories are backed up by 8192 words of magnetic drum storage. Optimal programming techniques are facilitated by the flexible organization and instruction code structure.

The "MERCURY" is based on the development of the University of Manchester MARK II. "MERCURY" is a serial machine using a digit repetition rate of 1 Mc/s. A random access magnetic core memory is provided compatibly by parallel transfers of 10 bits with a 10 microsecond cycle. The random access memory is, in turn, backed up by four magnetic drums each having a capacity of 163,840 bits. Seven B-Registers and efficient floating point and fast multiplication organization lead to high operating speeds and a powerful instruction code. The description of "MERCURY" restricts itself primarily to logical differences between it and the MARK II. Full design details are covered in other papers describing the experimental Manchester machine.

The experimental machines discussed were MARK II (University of Manchester); an acoustic delay line accounting machine (British Tabulating Machine Co.); "NICHOLAS" (Elliott Bros.); "EDSAC II" (Cambridge University); "IMP" (Imperial College, University of London); "ACE" (National Physical Laboratory); "BESM" (USSR Academy of Sciences). As indicated earlier the Manchester group has done a substantial amount of new work and provides an excellent 21-page exposition of the MARK II organization. A concise description of the EDSAC II organization gives a little more insight into the microprogramming approach of the Cambridge group.

In the same category of experimental machines, there were two papers presented in a session on "logical design." The first of these provides an extensive discussion of the thinking leading to the organizational form of the projected "ICCE II" at Imperial College. This is a very valuable paper insofar as it describes approaches often left unmentioned in expositions occurring after the fact of machine construction. The large number of variables and inconclusiveness of decisions reached however serve to emphasize the need for a studied characteristic set of problems which might be used as a coarse yardstick for relative evaluation of machine organizations.

The second paper discussed an experimental machine called "TACT" being developed at the National Physical Laboratory. The main feature of this machine is a step in the direction of automatic optimal programming for a machine with a large main cyclic memory. A small random access memory is provided and numbers are stored with a "tag," i.e., an additional set of bits carried with the number. When a three-address instruction is to be performed, it first inquires whether any of the required numbers are in the immediate access memory cells. If so, the operation may be performed immediately; if not, it transfers one of the immediate access quantities to the lower speed memory and brings the necessary quantity to the immediate access cell. In computations with high repetition rate of operation on a few quantities this type of organization would lead to increased efficiency.

Most of the latter half of the convention concerned itself with component researches.

The session on rapid access storage described the magnetic core memory developed for Ferranti's "MERCURY," magnetic core logical circuitry, multistable vacuum tube circuits, and the design of the CRT memory for The Manchester MARK II.

In addition to two papers describing magnetic tape storage devices at Cambridge and Ferranti, there was a presentation of work on variable reluctance reading heads being done at Manchester University. The latter is an important area of investigation since it provides one avenue towards the alleviation of the bottleneck between the high-speed computer and terminal data presentation.

A number of papers on the transistor as a computing element describe the design of and experience with two small computers at the Atomic Energy Research Establishment and at Manchester University. In addition to these there are two good papers presenting studies of circuit techniques using transistors and magnetic cores at Manchester.

Two digital analog conversion schemes were discussed. The first utilized a binary number to time interval to voltage output conversion achieving an accuracy of one part in two inches. The second used a binary coded disc to digitalize a rotating shaft position to an accuracy of the order of one minute of arc.

Several points of special interest emerged in the session on the computer in a nonarithmetic role including game playing, pattern recognition, and mechanical translation. A. D. Booth mentioned a promising avenue towards mechanical recognition evaluated at Birkbeck College. A voltage time picture of sound is sampled and the number representing the waveform, its time derivative, and its time integral are established as the criteria for recognition. Booth reports an accuracy of better than 0.1% in recognizing the spoken digits 0 to 9. For character recognition he generates numbers representing the first intersection of the character with each *raster* line. A predetermined set of numbers for the characters to be recognized is then compared with the output of the circuit. A minimum of the sum of the absolute values of the differences is sought. If there is ambiguity in the result for a chosen discrimination interval, the system goes back and generates a second set of numbers based on another criterion and compares with a second precalculated set to remove the ambiguity. No quantitative results were reported. Another paper describes work at IBM in the U. S. on character recognition, where the problem is attacked by providing a digital computer with the quantized representation of the entire character field. With this set of information the analysts are then in a position to test various scanning and testing procedures using the logic of the machine. Such studies led to the development of the particular system described in which the character is scanned serially with a fine spot; the light signals are quantized into black and white cells; the binary data from each vertical scan line are coded in three decimal digits representing the number, position, and size of the black areas. A large distribution of character samples was taken from inked ribbons used for 150% of nominal life with identical samples taken at equal intervals during that period. The criteria for recognition were then established by a "learning" process of testing and modifying the criteria.

Concerning mechanical translation, A. F. Parker-Rhodes of the Cambridge Language Research Unit presented the type of computer operations needed for further advances and I. S. Muchin of the Academy of Sciences of the USSR described in detail an experiment in machine translation carried out on BESM.

The last session of the convention covered miscellaneous circuit techniques and components of computers. These included several papers on the use of electromagnetic and acoustic delay lines. One interesting paper from the Manchester University group described a multi-input analog adder for use in a fast binary multiplier. Another paper provided a good exposition of the design of a parallel arithmetic unit at Imperial College.

As a whole, the publication of the convention proceedings represents a contribution to the digital computer field not only by providing the results of British ingenuity, but by the effort made in many papers to describe their work without assuming an intimate knowledge of it by the reader. This effort strongly affects progress.

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74[Z].—S. A. LEBEDEV, *Élektronnye Vychislitel'nye Mashiny i Obrabotka Informatsii* (*Electronic Computing Machines and Information Processes*), Akad. Nauk SSSR, Moscow, 1956, 47 p., 20 cm. Price 65 kopeks.

This pamphlet is one of the series of educational booklets issued by the Academy for popular consumption. Its author is the Academician in charge of BESM (this name is made up of the initials of the four Russian words standing for High-speed Electronic Computing Machine) and principal compiler of the Academy's Index of Mathematical Tables, Moscow, 1956 (Rev. **49**, p. 104–106, this issue). The booklet is couched in delightfully lucid language and is profusely illustrated with diagrams, photographs, and sample programs. The modest price, equivalent to about 15 cents, makes it available to anyone having an interest in the subject, and its high quality guarantees complete satisfaction to each purchaser.

The Introduction points out the benefits of high-speed computation and emphasizes, in particular, the achievements made possible through the use of BESM not only in numerical computation but also in the initial—and quite spectacular—attempts to translate scientific English into Russian.

The first chapter is devoted to a careful explanation of how electronic machines function. In the second, we find an excellent discussion of the binary versus the decimal systems with a complete explanation of the engineering principles which are used to represent the former system. This is followed by some very timely observations on the pros and cons of floating radix point representation of numbers.

The last Chapter deals with a detailed description of BESM. Since the features of this machine are already familiar to the readers of this magazine, there is no need to repeat the facts here [1].

The pamphlet is enriched by an excellent appendix, which can easily serve as a textbook for tyros in coding. It outlines the basic principles of program-preparation, and illustrates them by actual examples. It touches upon the use of subroutines and the choice of suitable checks to insure the accuracy of the computations.

An altogether charming booklet for 65 kopecks!

IDA RHODES

National Bureau of Standards
Washington, D. C.

1. S. A. LEBEDEV, "The high-speed electronic calculating machine of the Academy of Sciences of the U.S.S.R.," *J. of the Assn. for Computing Machinery*, v. 3, 1956, p. 129–133.

TABLE ERRATA

252. A. A. ABRAMOV, *Tablitsy* ln $\Gamma[z]$ *v Kompleknoĭ Oblasti* (*Tables of* ln $\Gamma[z]$ *in a Complex Region*), Akad. Nauk SSSR, Moscow, 1953.

Published in the back of A. D. Smirnov's book, *Tablitsy funktsy Éĭri i Spetsial'nykh Vyrozhdennykh Gipergeometricheskikh Funktsy* (*Tables of Airy Functions and Special Degeneration of Hypergeometric Functions*) (Akad. Nauk SSSR, Moscow, 1955) are lists of errors discovered in the tables of the Akad. Nauk series issued previously. Included in these lists of errors are the following in Abramov's tables:

Page	*Line*	*Column*	*As printed*	*Should be*
14	1 from top	6 from right	−060	−160
26	18 from bottom	3 from left	302	102
54	19 from top	6 from right	89	79
61	17 from bottom	3 from left	76	70
82	24 from top	2 from left	397633	297633
82	25 from top	2 from left	202955	302955
104	11 from top	4 from right	49	46
131	8 from top	5 from left	22	32
141	10 from top	4 from left	673267	073267
143	1 from top	1 from right	80,12817	0,128178
190	4 from top	3 from left	29	19
217	5 from bottom	2 from right	22	32
218	2 from top	2 from right	0	10
273	15 from bottom	2 from right	37	27
286	8 from bottom	5 from left	0	9
319	17 from top	4 from right	45	25
321	8 from top	6 from right	35	25

253. AKADEMIIA NAUK SSSR. Institut Tochnoĭ mekhanikĭ i vychislitel'noĭ tekhniki. Matematicheskie Tablitsy. *Desiatiéna Tablitsy logarifmov kompleksnykh chisel i perekhoda ot dekartovykh koordinat k poliarnym. Tablitsy funktsiĭ* [*Ten place tables of logarithms of complex numbers and of the transformation from cartesian to polar coordinates. Tables of functions*] ln x, arctg x, $\frac{1}{2}$ ln $(1 + x^2)$, $(1 + x^2)^{\frac{1}{2}}$. Moscow, 1952. [See RMT **1206**, *MTAC*, v. 8, 1954, p. 149.]

Published in the back of A. D. Smirnov's book, *Tablitsy funktsy Éĭri i Spetsial'nykh Vyrozhdennykh Gipergeometricheskikh Funktsy* (*Tables of Airy Func-