Simplified Calculation of the Exponential Integral

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Calculations of the negative exponential integral function [1]

$$(1) \qquad -\operatorname{Ei}(-x) = \int_x^\infty \frac{e^{-t}}{t}\,dt$$

are usually made using one method for $x < 3$, another for $3 < x < 20$, and a third for $x > 20$. The intermediate region is of considerable interest for many applications, and it is here that the function is most difficult to compute. The Ruedenberg [2] and Kotani [3] methods are valid for any argument, but difficulties in application limit their usefulness to the intermediate region where less complicated methods fail.

The authors wish to present a simple method which is useful over the entire range of arguments. The method is based on a Taylor's expansion of the slowly varying *exponential product function*, $-e^x \operatorname{Ei}(-x)$. An interpolation formula is obtained by which this function may be computed from a table value, $-e^{x_0}\operatorname{Ei}(-x_0)$; then $-\operatorname{Ei}(-x)$ is obtained upon dividing by the exponential. The accuracy of the result is limited only by the accuracy of the table value and that of the computed exponential. The positive exponential integral

$$\operatorname{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t}\,dt$$

may be computed by analogous means.

The formulae for the negative exponential product function are developed as follows:

Let

$$(2) \qquad g(x) = -e^x \operatorname{Ei}(-x),$$

then

$$\begin{aligned} g'(x) &= g(x) - x^{-1} \\ g''(x) &= g'(x) + x^{-2} \\ g'''(x) &= g''(x) - 2!x^{-3} \\ g^{iv}(x) &= g'''(x) + 3!x^{-4}. \end{aligned}$$

One may show by induction that

$$(3) \qquad g^{(n)}(x) = g^{(n-1)}(x) + (-1)^n(n-1)!x^{-n}.$$

The Taylor's expansion, then, is

$$(4) \qquad \begin{aligned} -e^x \operatorname{Ei}(-x) = {} & g(x_0) + g'(x_0)(x-x_0) + (1/2!)g''(x_0)(x-x_0)^2 \\ & + (1/3!)g'''(x_0)(x-x_0)^3 + \cdots + (1/n!)g^{(n)}(x_0)(x-x_0)^n + R_{n+1} \end{aligned}$$

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where

$$R_{n+1} = \frac{1}{(n+1)!} g^{(n+1)}(x_1)(x - x_0)^{n+1}$$

with $x < x_1 < x_0$.

The preceding equations may be combined to produce the simple formula

$$-e^{x} \operatorname{Ei}(-x) = T_0 + \sum_{n=1}^{\infty} T_n \tag{5}$$

where

$$T_0 = g(x_0) = -e^{x_0} \operatorname{Ei}(-x_0) \tag{6}$$

and the T's are related by the recursion relation

$$T_n = \frac{x - x_0}{n}\left[T_{n-1} + \frac{(x - x_0)^{n-1}}{(-x_0)^n}\right] \quad (n \geq 1). \tag{7}$$

For the positive exponential product function we have

$$e^{-x} \operatorname{Ei}(x) = U_0 + \sum_{n=1}^{\infty} U_n \tag{8}$$

where

$$U_0 = e^{-x_0} \operatorname{Ei}(x_0) \tag{9}$$

and

$$U_n = -\frac{x - x_0}{n}\left[U_{n-1} + \frac{(x - x_0)^{n-1}}{(-x_0)^n}\right] \quad (n \geq 1). \tag{10}$$

Equations (5) through (10) are very convenient for computing the negative or positive exponential product functions where a table value is known. For example, with $\left|\frac{x - x_0}{x_0}\right| \leq \frac{1}{50}$, convergence to 16 figures is obtained with 12 or fewer terms; convergence to 8 figures requires 6 or fewer terms. The spacing of the arguments in the accompanying table is such that this condition is fulfilled over most of the range $2.5 \leq x \leq 80$.

To prevent loss of figures, $x - x_0$ should be negative for computing $-e^{x} \operatorname{Ei}(-x)$ and positive for computing $e^{-x} \operatorname{Ei}(x)$. The accompanying table of $-e^{x} \operatorname{Ei}(-x)$ was computed starting with the value for $x = 80$, where $-e^{x} \operatorname{Ei}(-x)$ was computed using the asymptotic expansion; each computed value became the T_0 for the next lower argument. The table of $e^{-x} \operatorname{Ei}(x)$ was computed going up from the value of the function for $x = 0.2$, where $\operatorname{Ei}(x)$ was computed using the usual MacLaurins series, then multiplied by e^{-x} to obtain the product function. All calculations were made on an IBM 650 computer using double precision, floating point arithmetic.

EXPONENTIAL INTEGRALS AND EXPONENTIAL PRODUCT FUNCTIONS

x	e^{-x}Ei(x)	Ei(x)	$-e^{x}$Ei(-x)	-Ei(-x)
0.20	-0.6728006649831373(0)	-0.8217605879024003(0)	0.1493348746932240(1)	0.1222650544183893(1)
0.25	-0.4225331193688149(0)	-0.5425432646619137(0)	0.1340885444831393(1)	0.1044282634443738(1)
0.30	-0.2242223687152438(0)	-0.3026685392658259(0)	0.1222535605080586(1)	0.9056766516758467(0)
0.35	-0.6302307596780976(-1)	-0.8943400191844643(-1)	0.1127045349862318(1)	0.7942154346208358(0)
0.40	0.7022622616783961(-1)	0.1047652186193248(0)	0.1047828008456006(1)	0.7023801188656624(0)
0.45	0.1816318255906013(0)	0.2848554053465651(0)	0.9807147233583903(0)	0.6253313163232692(0)
0.50	0.2754982985512703(0)	0.4542199048631736(0)	0.9229106324837305(0)	0.5597735947761608(0)
0.55	0.3549918279209135(0)	0.6152906570621864(0)	0.8724573131594157(0)	0.5033640813923938(0)
0.60	0.4225198103287018(0)	0.7698812899373594(0)	0.8279334352735088(0)	0.4543795031894021(0)
0.65	0.4799618229587596(0)	0.9193864682454431(0)	0.7882775692635576(0)	0.4115169759494795(0)
0.70	0.5288172627521610(0)	0.1064907194624291(1)	0.7526780200295871(0)	0.3737688432335091(0)
0.75	0.5703036403055990(0)	0.1207332816001222(1)	0.7205015065870451(0)	0.3403408129112300(0)
0.80	0.6054242952633644(0)	0.1347396548212326(1)	0.6912453978028315(0)	0.3105965785455430(0)
0.85	0.6350164235265754(0)	0.1485714176225254(1)	0.6645047874381068(0)	0.2840192685024615(0)
0.90	0.6597860062607939(0)	0.1622811713696867(1)	0.6399492266392997(0)	0.2601839393259996(0)
0.95	0.6803337743392053(0)	0.1759145611867690(1)	0.6173059209526070(0)	0.2387375236537346(0)
1.00	0.6971748832350661(0)	0.1895117816355937(1)	0.5963473623231941(0)	0.2193839343955203(0)
1.05	0.7107540774523513(0)	0.2031087184099664(1)	0.5768820704052510(0)	0.2018728132201966(0)
1.10	0.7214575567019491(0)	0.2167378279563403(1)	0.5587475561702364(0)	0.1859909045360402(0)
1.15	0.7296223877856724(0)	0.2304288251855629(1)	0.5418049014535823(0)	0.1715553536299986(0)
1.20	0.7355440612554694(0)	0.2442092285192652(1)	0.5259345318947846(0)	0.1584084368514626(0)
1.25	0.7394826255791207(0)	0.2581047974355476(1)	0.5110328836740476(0)	0.1464133725259102(0)
1.30	0.7416677163903875(0)	0.2721398880232024(1)	0.4970097482616443(0)	0.1354509578491291(0)
1.35	0.7423027172881568(0)	0.2863377453173074(1)	0.4837861375242837(0)	0.1254168443886644(0)
1.40	0.7415682306020866(0)	0.3007207464150646(1)	0.4712925524860814(0)	0.1162193125713579(0)
1.45	0.7396249943951793(0)	0.3153106049387744(1)	0.4594675683131685(0)	0.1077774398689766(0)
1.50	0.7366163509600129(0)	0.3301285449129798(1)	0.4482566692915830(0)	0.1000195824066327(0)
1.55	0.7326703489608585(0)	0.3451954502797438(1)	0.4376112831118401(0)	0.9288210816420927(-1)
1.60	0.7279015439636994(0)	0.3605319949019469(1)	0.4274879752994281(0)	0.8630833369753978(-1)
1.65	0.7224125488366733(0)	0.3761587568694135(1)	0.4178477732671577(0)	0.8024762667334322(-1)
1.70	0.7162953753046061(0)	0.3920963201354904(1)	0.4086555960005607(0)	0.7465464440125304(-1)
1.75	0.7096326000236425(0)	0.4083653659076957(1)	0.3998797703779726(0)	0.6948868460463879(-1)
1.80	0.7024983823417449(0)	0.4249867557487934(1)	0.3914916189703562(0)	0.6471312936386886(-1)
1.85	0.6949593560157989(0)	0.4419816079804075(1)	0.3834651071497194(0)	0.6029496682837346(-1)
1.90	0.6870754132617678(0)	0.4593713686953585(1)	0.3757765396688848(0)	0.5620437817453485(-1)
1.95	0.6789003963937472(0)	0.4771778784589879(1)	0.3684042987138854(0)	0.5241437956799866(-1)
2.00	0.6704827097900733(0)	0.4954234356001890(1)	0.3613286168882226(0)	0.4890051070806112(-1)
2.05	0.6618658628802201(0)	0.5141308568493948(1)	0.3545313797517170(0)	0.4564056299118849(-1)
2.10	0.6530889531759691(0)	0.5333235359655902(1)	0.3479959534707185(0)	0.4261434150851507(-1)
2.15	0.6441870969978426(0)	0.5530255009015579(1)	0.3417070338905328(0)	0.3980345604801850(-1)
2.20	0.6351918144137759(0)	0.5732614699814381(1)	0.3356505139529960(0)	0.3719113705193257(-1)
2.25	0.6261313739651271(0)	0.5940569075037338(1)	0.3298133668814699(0)	0.3476207311944460(-1)
2.30	0.6170311019689313(0)	0.6154380791333508(1)	0.3241835429648885(0)	0.3250226717162159(-1)
2.35	0.6079136605259716(0)	0.6374321074059444(1)	0.3187498781096143(0)	0.3039890883162248(-1)
2.40	0.5987992978087817(0)	0.6600670276350539(1)	0.3135020126067469(0)	0.2844026093579567(-1)
2.45	0.5897060737337514(0)	0.6833718444858624(1)	0.3084303187941878(0)	0.2661555839389467(-1)
2.50	0.5806500637222667(0)	0.7073765894578601(1)	0.3035258364859841(0)	0.2491491787026973(-1)
2.55	0.5716455429153259(0)	0.7321123795013333(1)	0.2987802152032416(0)	0.2332925697125217(-1)
2.60	0.5627051529145893(0)	0.7576114769787598(1)	0.2941856623768447(0)	0.2185022180408219(-1)
2.65	0.5538400528724073(0)	0.7839073511713031(1)	0.2897348968068591(0)	0.2047012192534298(-1)
2.70	0.5450600565375012(0)	0.8110347415222037(1)	0.2854211067605018(0)	0.1918187182658638(-1)
2.75	0.5363737566762730(0)	0.8390297228026057(1)	0.2812379121729163(0)	0.1797893821545441(-1)
2.80	0.5277886381277471(0)	0.8679297723809165(1)	0.2771793304851186(0)	0.1685529244521605(-1)
2.85	0.5193111806092019(0)	0.8977738397739503(1)	0.2732397457133851(0)	0.1580536752680431(-1)
2.90	0.5109469522665601(0)	0.9286024186566349(1)	0.2694138803956638(0)	0.1482401922726118(-1)

2.95	0.5027006948559761(0)	0.9604576215068175(1)	0.2656967701046732(0)	0.1390649081872684(-1)
3.00	0.4945764013486412(0)	0.9933832570625417(1)	0.2620837402553185(0)	0.1304838109419704(-1)
3.05	0.4865773866677791(0)	0.1027424910770988(2)	0.2585703849668428(0)	0.1224561531143291(-1)
3.10	0.4787063521935835(0)	0.1062630028410981(2)	0.2551525477685169(0)	0.1149441876565585(-1)
3.15	0.4709654446071541(0)	0.1099048003074495(2)	0.2518263039623042(0)	0.1079129272585048(-1)
3.20	0.4633563095871836(0)	0.1136730265696873(2)	0.2485879444773646(0)	0.1013299249934911(-1)
3.25	0.4558801408222987(0)	0.1175730379330493(2)	0.2454339610699440(0)	0.9516507415488778(-2)
3.30	0.4485377247567375(0)	0.1216104137362286(2)	0.2423610327385172(0)	0.8939042542032138(-2)
3.35	0.4413294814467530(0)	0.1257909665881873(2)	0.2393660132383432(0)	0.8398001968161591(-2)
3.40	0.4342555018691712(0)	0.1301207530414045(2)	0.2364459195921340(0)	0.7890973505565841(-2)
3.45	0.4273155819913683(0)	0.1346060847236905(2)	0.2335979215045578(0)	0.7415714674756149(-2)
3.50	0.4205092538831298(0)	0.1392535399515234(2)	0.2308193315980103(0)	0.6970139857548393(-2)
3.55	0.4138358141250072(0)	0.1440699758487413(2)	0.2281075963956555(0)	0.6552308508859890(-2)
3.60	0.4072943497445684(0)	0.1490625409953709(2)	0.2254602879853118(0)	0.6160414331797309(-2)
3.65	0.4008837618910396(0)	0.1542386886323746(2)	0.2228750963044691(0)	0.5792775329626956(-2)
3.70	0.3946027874400052(0)	0.1596061904491687(2)	0.2203498219926746(0)	0.5447824656770463(-2)
3.75	0.3884500187028375(0)	0.1651731509818928(2)	0.2178823697628117(0)	0.5124102198586810(-2)
3.80	0.3824239214001773(0)	0.1709480226516090(2)	0.2154707422475070(0)	0.4820246816497640(-2)
3.85	0.3765228510448905(0)	0.1769396214728705(2)	0.2131130342810929(0)	0.4534989201084495(-2)
3.90	0.3707450678673489(0)	0.1831571434644303(2)	0.2108074275813037(0)	0.4267145281218571(-2)
3.95	0.3650887504044733(0)	0.1896101817952653(2)	0.2085521857982297(0)	0.4015610142149997(-2)
4.00	0.3595520078636207(0)	0.1963087447005622(2)	0.2063456499010558(0)	0.3779352409848906(-2)
4.05	0.3541328913629877(0)	0.2032632742038675(2)	0.2041862338758001(0)	0.3557409062811431(-2)
4.10	0.3488294041416436(0)	0.2104846656832318(2)	0.2020724207096828(0)	0.3348880636069710(-2)
4.15	0.3436395108245115(0)	0.2179842883208902(2)	0.2000027586399285(0)	0.3152926785319663(-2)
4.20	0.3385611458205194(0)	0.2257740064778208(2)	0.1979758576467600(0)	0.2968762181941948(-2)
4.25	0.3335922209256595(0)	0.2338662020364075(2)	0.1959903861721022(0)	0.2795652712273958(-2)
4.30	0.3287306321967914(0)	0.2422737977564123(2)	0.1940450680471100(0)	0.2632911956823758(-2)
4.35	0.3239742661566163(0)	0.2510102816915399(2)	0.1921386796130670(0)	0.2479897927226724(-2)
4.40	0.3193210053853184(0)	0.2600897327160515(2)	0.1902700470215048(0)	0.2336010040655829(-2)
4.45	0.3147687335498526(0)	0.2695268472131682(2)	0.1884380437005717(0)	0.2200686313127430(-2)
4.50	0.3103153399177213(0)	0.2793369669793958(2)	0.1866415679757465(0)	0.2073400754714614(-2)
4.55	0.3059587233982997(0)	0.2895361084014081(2)	0.1848796408339623(0)	0.1953660951105750(-2)
4.60	0.3016967961512960(0)	0.3001409929647570(2)	0.1831512038210900(0)	0.1841005817241503(-2)
4.65	0.2975274867987521(0)	0.3111690791564253(2)	0.1814553170635290(0)	0.1735003509942023(-2)
4.70	0.2934487432740670(0)	0.3226385958261276(2)	0.1797910574053859(0)	0.1635249487508885(-2)
4.75	0.2894585353388472(0)	0.3345685770742831(2)	0.1781575366533868(0)	0.1541364705263895(-2)
4.80	0.2855548567959244(0)	0.3469788987377533(2)	0.1765538999222755(0)	0.1452993936878324(-2)
4.85	0.2817357274246173(0)	0.3598903165477514(2)	0.1749793240740063(0)	0.1369804212159651(-2)
4.90	0.2779991946622346(0)	0.3733245060378098(2)	0.1734330162445464(0)	0.1291483362705985(-2)
4.95	0.2743433350539052(0)	0.3873041042833270(2)	0.1719142124525677(0)	0.1217738667517481(-2)
5.00	0.2707662554910572(0)	0.4018527535580318(2)	0.1704221762847322(0)	0.1148295591275326(-2)
5.10	0.2638410218985874(0)	0.4327570763576145(2)	0.1675155916170776(0)	0.1021300107861738(-2)
5.20	0.2572089914235682(0)	0.4662485050579675(2)	0.1647078767047833(0)	0.9086216124486596(-3)
5.30	0.2508561971769137(0)	0.5025573030483023(2)	0.1619940141687699(0)	0.8086083340807593(-3)
5.40	0.2447692200166432(0)	0.5419347580097915(2)	0.1593693236120981(0)	0.7198044498634734(-3)
5.50	0.2389351937712944(0)	0.5846551424982394(2)	0.1568294335801275(0)	0.6409260498657626(-3)
5.60	0.2333418047996327(0)	0.6310178597429926(2)	0.1543702562925014(0)	0.5708401696482117(-3)
5.70	0.2279772869743245(0)	0.6813497923753799(2)	0.1519879648307450(0)	0.5085464802750316(-3)
5.80	0.2228304130069319(0)	0.7360078735067486(2)	0.1496789725060241(0)	0.4531612834935893(-3)
5.90	0.2178904828866256(0)	0.7953819014488757(2)	0.1474399141665211(0)	0.4039035089431294(-3)
6.00	0.2131473100815936(0)	0.8598976214243920(2)	0.1452676292338869(0)	0.3600824521626587(-3)
6.10	0.2085912060470571(0)	0.9300200998696368(2)	0.1431591462840558(0)	0.3210870279496547(-3)
6.20	0.2042129634943658(0)	0.1006257419406609(3)	0.1411116690100267(0)	0.2863763442277715(-3)
6.30	0.2000038387995428(0)	0.1089164725275854(3)	0.1391225634235193(0)	0.2554714278071156(-3)
6.40	0.1959555338649284(0)	0.1179348657001819(3)	0.1371893461691771(0)	0.2279479559293729(-3)
6.50	0.1920601776925695(0)	0.1277472202332260(3)	0.1353096738395544(0)	0.2034298668393982(-3)
6.60	0.1883103078813109(0)	0.1384260014082264(3)	0.1334813331918336(0)	0.1815837392834511(-3)
6.70	0.1846988522199469(0)	0.1500504234452441(3)	0.1317022321783076(0)	0.1621138451969451(-3)

6.80	0.1812191105152683(0)	0.1627070875714314(3)	0.1299703917123769(0)	0.1447577922448996(-3)
6.90	0.1778647367655084(0)	0.1764906810901577(3)	0.1282839381003267(0)	0.1292826835984703(-3)
7.00	0.1746297217657902(0)	0.1915047433355014(3)	0.1266410960766328(0)	0.1154817316103382(-3)
7.10	0.1715083762120765(0)	0.2078625049687226(3)	0.1250401823871386(0)	0.1031712700936947(-3)
7.20	0.1684953143532593(0)	0.2256878077010638(3)	0.1234795998702534(0)	0.9218811688716205(-4)
7.30	0.1655854382269262(0)	0.2451161122163741(3)	0.1219578319914612(0)	0.8238724444880836(-4)
7.40	0.1627739225026122(0)	0.2662956028236372(3)	0.1204734377909765(0)	0.7363972149250618(-4)
7.50	0.1600561999466174(0)	0.2893883982001446(3)	0.1190250472084111(0)	0.6583089326708023(-4)
7.60	0.1574279475144716(0)	0.3145718784980836(3)	0.1176113567519010(0)	0.5885877207538390(-4)
7.70	0.1548850730705739(0)	0.3420401400887054(3)	0.1162311254823222(0)	0.5263261311592179(-4)
7.80	0.1524237027292334(0)	0.3720055903178158(3)	0.1148831712860667(0)	0.4707165377202777(-4)
7.90	0.1500401688070737(0)	0.4047006958537438(3)	0.1135663674123762(0)	0.4210399713154200(-4)
8.00	0.1477309983734010(0)	0.4403798995348383(3)	0.1122796392534993(0)	0.3766562284392490(-4)
8.10	0.1454929023825139(0)	0.4793217220795804(3)	0.1110219613479563(0)	0.3369951045538391(-4)
8.20	0.1433227653699474(0)	0.5218310666214330(3)	0.1097923545890153(0)	0.3015486214352037(-4)
8.30	0.1412176356931865(0)	0.5682417457866691(3)	0.1085898836221073(0)	0.2698641333378787(-4)
8.40	0.1391747162963691(0)	0.6189192529621442(3)	0.1074136544163729(0)	0.2415382104688129(-4)
8.50	0.1371913559778528(0)	0.6742638015183372(3)	0.1062628119968480(0)	0.2162112104348337(-4)
8.60	0.1352650411391756(0)	0.7347136580797110(3)	0.1051365383249826(0)	0.1935624583178277(-4)
8.70	0.1333933879938504(0)	0.8007487984901243(3)	0.1040340503162546(0)	0.1733059664226153(-4)
8.80	0.1315741352145440(0)	0.8728949179282845(3)	0.1029545979846070(0)	0.1551866314943838(-4)
8.90	0.1298051369974674(0)	0.9517278297119495(3)	0.1018974627043105(0)	0.1389768554948170(-4)
9.00	0.1280843565232139(0)	0.1037878290717090(4)	0.1008619555806409(0)	0.1244735417800627(-4)
9.10	0.1264098597937903(0)	0.1132037295059577(4)	0.9984741592148153(-1)	0.1114954242452099(-4)
9.20	0.1247798098261796(0)	0.1234961881775222(4)	0.9885320980260409(-1)	0.9988069188131810(-5)
9.30	0.1231924611834212(0)	0.1347481506725405(4)	0.9787872871997763(-1)	0.8948487551284568(-5)
9.40	0.1216461548248895(0)	0.1470505033890186(4)	0.9692338832298657(-1)	0.8017896729601667(-5)
9.50	0.1201393132581710(0)	0.1605028406632339(4)	0.9598662722292924(-1)	0.7184774692384830(-5)
9.60	0.1186704359756800(0)	0.1752143065472738(4)	0.9506790587161234(-1)	0.6438829145515731(-5)
9.70	0.1172380951598980(0)	0.1913045185462769(4)	0.9416670550526139(-1)	0.5770864831009909(-5)
9.80	0.1158409316418655(0)	0.2089045813431421(4)	0.9328252714933810(-1)	0.5172665329506271(-5)
9.90	0.1144776510982969(0)	0.2281581993287537(4)	0.9241489068019260(-1)	0.4636887757134862(-5)
10.00	0.1131470204734108(0)	0.2492228976241878(4)	0.9156333393978808(-1)	0.4156968929685324(-5)
10.20	0.1105790630932119(0)	0.2974929110318170(4)	0.8990669587339479(-1)	0.3341860537450944(-5)
10.40	0.1081282973538746(0)	0.3550055375863218(4)	0.8830924437781962(-1)	0.2687469579012960(-5)
10.60	0.1057867456051234(0)	0.4245733837203044(4)	0.8676784941570828(-1)	0.2161908580423391(-5)
10.80	0.1035471325769754(0)	0.5075963394298464(4)	0.8527959866698414(-1)	0.1739661463910517(-5)
11.00	0.1014028126361853(0)	0.6071406374098612(4)	0.8384177886034595(-1)	0.1400300304247442(-5)
11.20	0.9934770531270865(-1)	0.7265341584654344(4)	0.8245185899881711(-1)	0.1127462885929706(-5)
11.40	0.9737623811909750(-1)	0.8697813403189956(4)	0.8110747525831423(-1)	0.9060375098752769(-6)
11.60	0.9548329579931656(-1)	0.1041701743937324(5)	0.7980641736726444(-1)	0.7315126235039412(-6)
11.80	0.9366417524513860(-1)	0.1248097173811727(5)	0.7854661630012471(-1)	0.5894576310576063(-6)
12.00	0.9191454540389659(-1)	0.1495953266639753(5)	0.7732613313891923(-1)	0.4751081824672494(-6)
12.20	0.9023041162183965(-1)	0.1793682693358488(5)	0.7614314897517273(-1)	0.3830347307052797(-6)
12.40	0.8860808379858205(-1)	0.2151418606972272(5)	0.7499595574034253(-1)	0.3088774964230226(-6)
12.60	0.8704414807105058(-1)	0.2581368766955510(5)	0.7388294786642593(-1)	0.2491344257486993(-6)
12.80	0.8553544145078415(-1)	0.3098242945367525(5)	0.7280261469016544(-1)	0.2009914618092013(-6)
13.00	0.8407902916722514(-1)	0.3719768849068904(5)	0.7175353352446240(-1)	0.1621866218801433(-6)
13.20	0.8267218437250664(-1)	0.4467314972049650(5)	0.7073436332946521(-1)	0.1309010974877020(-6)
13.40	0.8131236994086161(-1)	0.5366642637434736(5)	0.6974383892351446(-1)	0.1056719669032301(-6)
13.60	0.7999722212377197(-1)	0.6448814140122464(5)	0.6878076568086320(-1)	0.8532220142276633(-7)
13.80	0.7872453585069002(-1)	0.7751289527606312(5)	0.6784401466898389(-1)	0.6890451641697467(-7)
14.00	0.7749225149209301(-1)	0.9319251363396537(5)	0.6693251818343963(-1)	0.5565631111145182(-7)
14.20	0.7629844291821436(-1)	0.1120720505289322(6)	0.6604526564283448(-1)	0.4496349363543785(-7)
14.40	0.7514130671741855(-1)	0.1348091227623865(6)	0.6518129981035035(-1)	0.3633142877075455(-7)
14.60	0.7401915244119631(-1)	0.1621972713070857(6)	0.6433971331189784(-1)	0.2936159782022293(-7)
14.80	0.7293039376885267(-1)	0.1951941917756394(6)	0.6351964542401636(-1)	0.2373284118082841(-7)
15.00	0.7187354049241071(-1)	0.2349558524907683(6)	0.6272027910740924(-1)	0.1918627892147867(-7)
15.20	0.7084719123528884(-1)	0.2828777324759337(6)	0.6194083826443568(-1)	0.1551318435489763(-7)

15.40	0.6985002682813623(-1)	0.3406446704365455(6)	0.6118058520104437(-1)	0.1254522935050609(- 7)
15.60	0.6888080427400526(-1)	0.4102911290368417(6)	0.6043881827555632(-1)	0.1014663520014089(- 7)
15.80	0.6793835124275987(-1)	0.4942740579967479(6)	0.5971486971841596(-1)	0.8207854989978216(- 8)
16.00	0.6702156104139909(-1)	0.5955609986708370(6)	0.5900810360855644(-1)	0.6640487249441043(- 8)
16.20	0.6612938801293519(-1)	0.7177366287113441(6)	0.5831791399338751(-1)	0.5373179810396308(- 8)
16.40	0.6526084332170970(-1)	0.8651316186170387(6)	0.5764372314063416(-1)	0.4348330246413268(- 8)
16.60	0.6441499108764715(-1)	0.1042978487220513(7)	0.5698497991134497(-1)	0.3519427311392819(- 8)
16.80	0.6359094483601590(-1)	0.1257600130511370(7)	0.5634115824436879(-1)	0.2848908328641833(- 8)
17.00	0.6278786423285517(-1)	0.1516637894042517(7)	0.5571175574347673(-1)	0.2306431989821655(- 8)
17.20	0.6200495207939739(-1)	0.1829327507651212(7)	0.5509629235909709(-1)	0.1867485703456464(- 8)
17.40	0.6124145154161804(-1)	0.2206832955834809(7)	0.5449430915734226(-1)	0.1512262441400273(- 8)
17.60	0.6049664359352493(-1)	0.2662650482689691(7)	0.5390536716964754(-1)	0.1224754734668012(- 8)
17.80	0.5976984465499713(-1)	0.3213097505446692(7)	0.5332904631692088(-1)	0.9920236807498873(- 9)
18.00	0.5906040440693245(-1)	0.3877904330597444(7)	0.5276494440262507(-1)	0.8036090344828678(- 9)
18.20	0.5836770376819408(-1)	0.4680930346916436(7)	0.5221267616968677(-1)	0.6510530577803806(- 9)
18.40	0.5769115302038587(-1)	0.5651030950391941(7)	0.5167187241655524(-1)	0.5275161160153598(- 9)
18.60	0.5703019006785616(-1)	0.6823107006991331(7)	0.5114217916812135(-1)	0.4274662841215528(- 9)
18.80	0.5638427882155116(-1)	0.8239375386198908(7)	0.5062325689756015(-1)	0.3464286669773704(- 9)
19.00	0.5575290769642903(-1)	0.9950907251046845(7)	0.5011477979547992(-1)	0.2807829097060795(- 9)
19.20	0.5513558821311919(-1)	0.1201949067167750(8)	0.4961643508305196(-1)	0.2275996053466976(- 9)
19.40	0.5453185369538304(-1)	0.1451988610649781(8)	0.4912792236605991(-1)	0.1845081053098080(- 9)
19.60	0.5394125805571199(-1)	0.1754255781301086(8)	0.4864895302704905(-1)	0.1495896868258273(- 9)
19.80	0.5336337466209903(-1)	0.2119698184951045(8)	0.4817924965297609(-1)	0.1212911993048645(- 9)
20.00	0.5279779527964813(-1)	0.2561565266405659(8)	0.4771854549596084(-1)	0.9835525290649882(-10)
20.50	0.5143515241690600(-1)	0.4114309041706761(8)	0.4660449650238127(-1)	0.5826274488896287(-10)
21.00	0.5014133864682561(-1)	0.6612718635548492(8)	0.4554136165450517(-1)	0.3453201267146756(-10)
21.50	0.4891124541721118(-1)	0.1063506337502374(9)	0.4452572364680038(-1)	0.2047762688262039(-10)
22.00	0.4774025998569045(-1)	0.1711446713003637(9)	0.4355446469261788(-1)	0.1214937895620437(-10)
22.50	0.4662420634887792(-1)	0.2755734002959935(9)	0.4262473437505852(-1)	0.7211669954118324(-11)
23.00	0.4555929447721889(-1)	0.4439663698302712(9)	0.4173392155600062(-1)	0.4282684795665672(-11)
23.50	0.4454207651365593(-1)	0.7156337008899201(9)	0.4087962975740886(-1)	0.2544407277410374(-11)
24.00	0.4356940883854058(-1)	0.1154115391849183(10)	0.4005965552384032(-1)	0.1512305893999706(-11)
24.50	0.4263841909814872(-1)	0.1862155313713956(10)	0.3927196935293355(-1)	0.8992239668453620(-12)
25.00	0.4174647745066453(-1)	0.3005950906525549(10)	0.3851469884490402(-1)	0.5348899755340217(-12)
25.50	0.4089117140987806(-1)	0.4854436678520199(10)	0.3778611377527881(-1)	0.3182899598082895(-12)
26.00	0.4007028376945511(-1)	0.7842940991898187(10)	0.3708461283934969(-1)	0.1894685885674978(-12)
26.50	0.3928177317438785(-1)	0.1267636845388481(11)	0.3640871185375504(-1)	0.1128240131086243(-12)
27.00	0.3852375697492631(-1)	0.2049649711988081(11)	0.3575703323153753(-1)	0.6720637435262040(-13)
27.50	0.3779449605489522(-1)	0.3315330364332410(11)	0.3512829657302925(-1)	0.4004597190457930(-13)
28.00	0.3709238137315206(-1)	0.5364511859231470(11)	0.3452131023684736(-1)	0.2386941511933733(-13)
28.50	0.3641592199588499(-1)	0.8683284776434690(11)	0.3393496377383885(-1)	0.1423163037419925(-13)
29.00	0.3576373442994828(-1)	0.1405991957584069(12)	0.3336822112256107(-1)	0.8487759778353563(-14)
29.50	0.3513453309465803(-1)	0.2277306064537837(12)	0.3282011447828864(-1)	0.5063524030714399(-14)
30.00	0.3452712179236185(-1)	0.3689732094072742(12)	0.3228973875898013(-1)	0.3021552010688812(-14)
30.50	0.3394038605740911(-1)	0.5979962712214315(12)	0.3177624660143239(-1)	0.1803519735842456(-14)
31.00	0.3337328627949772(-1)	0.9694555759683940(12)	0.3127884382925677(-1)	0.1076767038616238(-14)
31.50	0.3282485151125753(-1)	0.1572095592474019(13)	0.3079678534154509(-1)	0.6430269957499400(-15)
32.00	0.3229417388175753(-1)	0.2550043566357787(13)	0.3032937137733325(-1)	0.3840961801225067(-15)
32.50	0.3178040354772841(-1)	0.4137424382005289(13)	0.2987594411636582(-1)	0.2294832419214651(-15)
33.00	0.3128274412294852(-1)	0.6714640184076497(13)	0.2943588458134151(-1)	0.1371384348448747(-15)
33.50	0.3080044853367856(-1)	0.1089989174729090(14)	0.2900860981088176(-1)	0.8197129056219255(-16)
34.00	0.3033281525443544(-1)	0.1769803724411627(14)	0.2859357027600140(-1)	0.4900676118392788(-16)
34.50	0.2987918488392727(-1)	0.2874275355830471(14)	0.2819024751594533(-1)	0.2930483385121722(-16)
35.00	0.2943893702575893(-1)	0.4669055014466160(14)	0.2779815197195296(-1)	0.1752705938994737(-16)
35.50	0.2901148744267182(-1)	0.7586196776515777(14)	0.2741682099987429(-1)	0.1048486852619120(-16)
36.00	0.2859628545669380(-1)	0.1232852079912098(15)	0.2704581704463538(-1)	0.6273339009762242(-17)
36.50	0.2819281157072319(-1)	0.2003950446679199(15)	0.2668472596137432(-1)	0.3754171926217522(-17)
37.00	0.2780057528982065(-1)	0.3257988998672264(15)	0.2633315546967471(-1)	0.2247020697588571(-17)
37.50	0.2741911312288861(-1)	0.5297811170425472(15)	0.2599073372874117(-1)	0.1345164720542498(-17)

38.00	0.2704798674752830(-1)	0.8616388199965787(15)	0.2565710802261442(-1)	0.8054106914290750(-18)
38.50	0.2668678132271802(-1)	0.1401631180584185(16)	0.2533194354563262(-1)	0.4823152104821002(-18)
39.00	0.2633510393558843(-1)	0.2280446200301903(16)	0.2501492227933002(-1)	0.2888779301522701(-18)
39.50	0.2599258217000951(-1)	0.3710918879133971(16)	0.2470574195283762(-1)	0.1730477137053579(-18)
40.00	0.2565886278597515(-1)	0.6039718263611241(16)	0.2440411507962858(-1)	0.1036773261451657(-18)
40.50	0.2533361049989646(-1)	0.9831586535606510(16)	0.2410976806414384(-1)	0.6212501625936037(-19)
41.00	0.2501650685691183(-1)	0.1600664914324504(17)	0.2382244037245185(-1)	0.3723166776459978(-19)
41.50	0.2470724918720716(-1)	0.2606425970774542(17)	0.2354188376164872(-1)	0.2231619830976716(-19)
42.00	0.2440554963912712(-1)	0.4244796092136851(17)	0.2326786156319983(-1)	0.1337790881001177(-19)
42.50	0.2411113428255924(-1)	0.6914059660093667(17)	0.2300014801586681(-1)	0.8020753140591072(-20)
43.00	0.2382374227669796(-1)	0.1126348290166967(18)	0.2273852764426145(-1)	0.4809496556950018(-20)
43.50	0.2354312509685454(-1)	0.1835160585140183(18)	0.2248279467942528(-1)	0.2884299346610979(-20)
44.00	0.2326904581547836(-1)	0.2990444718632337(18)	0.2223275251815504(-1)	0.1729959874281648(-20)
44.50	0.2300127843300251(-1)	0.4873673371763902(18)	0.2198821321808334(-1)	0.1037732683120525(-20)
45.00	0.2273960725452828(-1)	0.7943916035704454(18)	0.2174899702578527(-1)	0.6225690809462383(-21)
45.50	0.2248382630872347(-1)	0.1294998150712611(19)	0.2151493193541692(-1)	0.3735433849181410(-21)
46.00	0.2223373880563390(-1)	0.2111342388647824(19)	0.2128585327560503(-1)	0.2241531759744300(-21)
46.50	0.2198915663039922(-1)	0.3442722209946245(19)	0.2106160332249963(-1)	0.1345234573407572(-21)
47.00	0.2174989987012736(-1)	0.5614329680810343(19)	0.2084203093707619(-1)	0.8074197842725814(-22)
47.50	0.2151579637141969(-1)	0.9156833466011563(19)	0.2062699122493194(-1)	0.4846720698787642(-22)
48.00	0.2128668132625326(-1)	0.1493630213112993(20)	0.2041634521696516(-1)	0.2909664190405842(-22)
48.50	0.2106239688412107(-1)	0.2436633239310381(20)	0.2020995956945640(-1)	0.1746960447705315(-22)
49.00	0.2084279178850680(-1)	0.3975442747903745(20)	0.2000770628218984(-1)	0.1048981164236802(-22)
49.50	0.2062772103593020(-1)	0.6486764087887549(20)	0.1980946243336086(-1)	0.6299351408664804(-23)
50.00	0.2041704555594399(-1)	0.1058563689713169(21)	0.1961510993011487(-1)	0.3783264029550459(-23)
51.00	0.2000835201197715(-1)	0.2819875254895520(21)	0.1923762933791553(-1)	0.1365001019107898(-23)
52.00	0.1961570627560364(-1)	0.7514792768154392(21)	0.1887440774812071(-1)	0.4926747149729344(-24)
53.00	0.1923818093493666(-1)	0.2003417881855170(22)	0.1852465196867329(-1)	0.1778863059762939(-24)
54.00	0.1887491871954997(-1)	0.5343023851440743(22)	0.1818762658359069(-1)	0.6425012932971142(-25)
55.00	0.1852512598949623(-1)	0.1425468664988751(23)	0.1786264878519243(-1)	0.2321396656266892(-25)
56.00	0.1818806693669695(-1)	0.3804324292226218(23)	0.1754908375138907(-1)	0.8390028962572610(-26)
57.00	0.1786305840936042(-1)	0.1015643484474065(24)	0.1724634050210516(-1)	0.3033272919545714(-26)
58.00	0.1754946528267050(-1)	0.2712338189315405(24)	0.1695386817786948(-1)	0.1096955100092402(-26)
59.00	0.1724669630961408(-1)	0.7245699996696615(24)	0.1667115269121852(-1)	0.3968178474512270(-27)
60.00	0.1695420039481329(-1)	0.1936182213929277(25)	0.1639771370804653(-1)	0.1435867565681257(-27)
61.00	0.1667146324187138(-1)	0.5175318892687175(25)	0.1613310192157824(-1)	0.5197021115800324(-28)
62.00	0.1639800433125279(-1)	0.1383722092026783(26)	0.1587689658639005(-1)	0.1881515229760728(-28)
63.00	0.1613337419128045(-1)	0.3700646202538074(26)	0.1562870328398534(-1)	0.6813505112488122(-29)
64.00	0.1587715192959883(-1)	0.9899640925974456(26)	0.1538815189494344(-1)	0.2467968559452694(-29)
65.00	0.1562894299654450(-1)	0.2648932738634548(27)	0.1515489475569561(-1)	0.8941525120697856(-30)
66.00	0.1538837715539018(-1)	0.7089712555436261(27)	0.1492860498060671(-1)	0.3240286570846400(-30)
67.00	0.1515510663747108(-1)	0.1897969740295516(28)	0.1470897493231828(-1)	0.1174497563867716(-30)
68.00	0.1492880446283458(-1)	0.5082177148694186(28)	0.1449571482528862(-1)	0.4258090299574639(-31)
69.00	0.1470916290933767(-1)	0.1361153829291226(29)	0.1428855144918970(-1)	0.1544076991753565(-31)
70.00	0.1449589211510103(-1)	0.3646352759579736(29)	0.1408722700032681(-1)	0.5600306285813438(-32)
71.00	0.1428871880095736(-1)	0.9770156145547613(29)	0.1389149801056327(-1)	0.2031612451633665(-32)
72.00	0.1408738510104050(-1)	0.2618382465224057(30)	0.1370113436438754(-1)	0.7371465348386786(-33)
73.00	0.1389164749098190(-1)	0.7018607129747796(30)	0.1351591839577345(-1)	0.2675151499435887(-33)
74.00	0.1370127580433766(-1)	0.1881709898766298(31)	0.1333564405737618(-1)	0.9710069409656658(-34)
75.00	0.1351605232888535(-1)	0.5045869418512775(31)	0.1316011615539307(-1)	0.3525117355552349(-34)
76.00	0.1333577097532315(-1)	0.1353314556155587(32)	0.1298914964411187(-1)	0.1279970898341254(-34)
77.00	0.1316023651169158(-1)	0.3630268948220829(32)	0.1282256897478357(-1)	0.4648361947902859(-35)
78.00	0.1298926385753334(-1)	0.9739891688771744(32)	0.1266020749400019(-1)	0.1688384028133141(-35)
79.00	0.1282267743242176(-1)	0.2613622063250456(33)	0.1250190688724092(-1)	0.6133553950256061(-36)
80.00	0.1266031055403288(-1)	0.7014600004904800(33)	0.1234751666367861(-1)	0.2228543258688473(-36)

In using this table each tabulated entry is to be multiplied by the power of 10 given in the parentheses located just to the right of the last significant digit listed.

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1. See Frank E. Harris, "Tables of the exponential integral Ei(x)," *MTAC*, v. 11, 1957, p. 9–16 for a summary of these methods. The authors are grateful to Dr. Harris for the use of his material prior to publication.

2. K. Ruedenberg, C. C. J. Roothaan, & W. Jaunzemis, "Study of two-center integrals useful in calculations of molecular structure. III. A unified treatment of the hybrid, coulomb, and one-electron integrals," *Jn. Chem. Phys.*, v. 24, 1956, p. 201–20.

3. M. Kotani, A. Amemiya, E. Ishiguro, & T. Kimura, *Table of Molecular Integrals*, Maruzen Co., Ltd., Tokyo, 1955.