where

$$\begin{cases} \operatorname{tg} \theta = \alpha_{ki}/\alpha_{ik} \\ \operatorname{tg} 2\varphi = 2\sqrt{\alpha^2_{ik} + \alpha^2_{ki}}/(\alpha_{ii} - \alpha_{kk}) \end{cases}$$

Then we have for $r < i$ or $r > k$: for $i < r < k$:

$$\begin{cases} \alpha'_{ri} = x_1 + z_1 \\ \alpha'_{ir} = x_2 + z_2 \\ \alpha'_{rk} = x_3 - z_3 \\ \alpha'_{kr} = x_4 - z_4 \end{cases} \qquad \begin{cases} \alpha'_{ri} = x_1 - z_2 \\ \alpha'_{ir} = x_2 + z_1 \\ \alpha'_{rk} = x_3 - z_4 \\ \alpha'_{kr} = x_4 + z_3 \end{cases}$$

and further $\alpha'_{ii} = \frac{1}{2}(\alpha_{ii} + \alpha_{kk} + R)$, $\alpha'_{kk} = \frac{1}{2}(\alpha_{ii} + \alpha_{kk} - R)$, $\alpha'_{ik} = \alpha'_{ki} = 0$. Here $R^2 = (\alpha_{ii} - \alpha_{kk})^2 + 4(\alpha^2_{ik} + \alpha^2_{ki})$. When both r and s differ from i and k we have $\alpha'_{rs} = \alpha_{rs}$. The indices i and k can be chosen conveniently, e.g. to fulfill the condition $|\alpha_{ik}| + |\alpha_{ki}| = \max$.

The whole procedure is now repeated until all eigenvalues have been established with sufficient accuracy in the main diagonal.

This method has been successfully tested on SMIL, the electronic computer of Lund University, up to $n = 15$. The matrices had all elements in the main diagonal $= 1$, all upper-diagonal elements $= 1 - i$ and all lower-diagonal elements $= 1 + i$. The eigenvalues of these matrices can also be computed directly: $\lambda_k^{(n)} = \cot(\pi(4k + 1)/4n)$; $k = 0, 1, \cdots (n - 1)$. An accuracy of about 8 decimal digits was easily obtained; further it turned out that the time consumed was about 4 times longer than for the corresponding diagonalization of real symmetric matrices.

Department of Numerical Analysis
Sölvegatan 14
Lund, Sweden

1. A. S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Co., Inc., New York, 1953.
2. E. G. Kogbetliantz, "Solution of linear equations by diagonalization of coefficients matrix," *Quart. Appl. Math.*, v. 13, 1955, p. 123–132.
3. J. Greenstadt, "A method for finding roots of arbitrary matrices," *MTAC*, v. 9, 1955, 47–52.
4. M. Lotkin, "Characteristic values of arbitrary matrices," *Quart. Appl. Math.*, v. 14, 1956, p. 267–275.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

86[C].—NBS Applied Mathematics Series, No. 53, *Table of Natural Logarithms for Arguments Between Five and Ten to Sixteen Decimal Places*, U. S. Government Printing Office, Washington, D. C., 1958, xiii + 506 p., 26 cm. Price $4.00.

This book tabulates $\log_e x$, $x = 5(.0001)10$, 16D, $\log_e x$, $x = 2(1)10$, 40D, and $\log_e(1 + x)$ and $-\log_e(1 - x)$, $x = 10^{-n}(10^{-n})10^{-n+1}$, $n = 1(1)13$, 25D. In addition there are sixteen constants given to 20D.

With AMS 31 (see *MTAC*, v. 8, Rev. **1167**, 1954, p. 76) these give $\log_e x$ for $x = 0(.0001)10$, 16D and enough auxiliary material to make the computation of any natural logarithm to accuracy compatible with the tables reasonably convenient. (Actually, either volume suffices without intolerable inconveniences.) To this end there is a careful discussion of interpolation methods in an introduction written by Arnold N. Lowan. There are no tabulated differences or other aids to interpolation; linear interpolation yields 9D accuracy.

The volume is a reprint of one volume of the NYWPA "Table of Natural Logarithms," which appeared in four volumes. Two of the volumes (containing logarithms of the integers between 1 and 1 00000) evidently will not be reprinted, and the third is AMS 31 mentioned above.

Three trivial misprints and one worse misprint in the earlier printing are noted. The trivial misprints all involve arguments. The important misprint corrects the value of $\log_e 9.6061 = 2.26239\ 83133\ 487638$; the originally printed value was erroneous in the seventh through the twelfth significant digits.

The use of these valuable tables since their first printing in 1941 confirms their excellence and accuracy. The format is the same as that of AMS 31, which in this reviewer's eyes is perfectly adequate. The printing is easily legible, and the book is generally up to the high standards of this valuable series published by the National Bureau of Standards in scholarship and utility.

C. B. T.

87[D, E, L, S, X].—PHILIP M. MORSE & HERMAN FESHBACH, *Methods of Theoretical Physics*, Parts I & II, McGraw-Hill Book Company, Inc., New York, 1953. 23 cm., xxii × 1978 p. Price $30.00.

This remarkable two-volume work is already well known to most readers of *MTAC*, but a review seems to be in order as a means of indexing much of its content which relates to computation and numerical analysis.

The volumes are written by physicists for physicists, and they apply analysis to problems of physics without attempting to study the complete mathematical formulation or background of the methods used. However, the material presented certainly is as good a collection as is easily available of partial differential equations of mathematical physics. The treatment is largely by separation of variables and by transform methods.

There is no quarrel with the mathematical attainments of these authors, nor with their motivation (which, after all, is toward getting the answers to physical problems and hence entirely in keeping with the objectives of *MTAC*). The reviewer, however, has never fully accepted the development of the usual equation of the vibrating string through the use of infinitesimals, although it must be equivalent in the final analysis to a derivation using tension and inertial forces (the last integrated) over all portions of the string. Morse and Feshbach use the infinitesimal approach, whereas the integral approach is found, for example, in [1]. The reviewer also tends to view with some alarm the use of Dirac delta functions, introduced in Morse and Feshbach (p. 123) with a warning that "differentiation should be attempted only with considerable caution." This is still a minor criticism; the function gives valuable hints when properly used, the authors seem not to misuse it, and the argument here may be almost pedantic.

A more constructive approach here would be to turn to parts of the book which are likely to be of help to readers of *MTAC* in connection with numerical analysis and computation. These include methods of solution, discussion of some of the more important special functions, and some tables.

Chapter 4 contains sections on gamma and elliptic functions, asymptotic series and the method of steepest descent, conformal mapping including the Schwarts-Christoffel transformation, and Fourier transforms (along with some mention of Laplace and Mellin transforms). There is a table of properties of and relations between gamma functions, elliptic functions and theta functions.

Chapter 5 contains a table of separable coordinates in three dimensions and a general table of second order differential equations and their solutions.

Chapter 6 contains a table of useful eigenfunctions and their properties (polynomials of Gegenbauer, Laguerre and Hermite).

Chapter 7 contains a short table of Green's functions.

Chapter 8 contains a section on applications of transforms to the solution of integral equations and a short table of integral equations and their solutions.

Chapter 9 is devoted to approximate methods—perturbation and variational methods. It includes a tabulation of methods.

Chapter 10, devoted to solutions of Laplace's and Poisson's equations, lists information about and expansions of trigonometric and hyperbolic functions, Bessel functions and Legendre functions.

Chapter 11, on wave equations, takes up cylindrical Bessel functions, Weber functions, Mathieu functions, spherical Bessel functions. It also includes a three page table of Laplace transforms.

Chapter 12, devoted to diffusion and wave mechanics, gets to Jacobi polynomials and semi-cylindrical functions.

Chapter 13 includes spherical vector harmonics.

Finally there is a set of tables. These include the following: for $x = 0(.2)8$, $\sin x$, $\cos x$, $\tanh x$, and e^{-x} all 4D, and $\tan x$, $\sinh x$, $\cosh x$, and e^{x} all 5S; for $x = 0(.05)2$, $\sin \pi x$, $\cos \pi x$, $\tanh \pi x$, and $e^{-\pi x}$, all 4D, and $\tan \pi x$, $\sinh \pi x$, $\cosh \pi x$, and $e^{\pi x}$ all 5S; the functions α, θ, χ and $|\zeta|$, 4D, and φ (in degrees) 2D, where $\tanh [\pi(\alpha - i\beta)] = \theta - i\chi = |\zeta| e^{(-i\varphi)}$, $\tanh \pi\alpha = 0(.05).95$, $\beta = 0(.05).45(.025).5$; the inverse of this function, giving α and β, 4D, for $\theta = 0(.2)4$ and $\chi = 0(.2)1.6(.4)2$; for $x = 0(.1)4(.2)12$, in χ, $\sinh^{-1} \chi$ and $\cosh^{-1} \chi$ (where real), 4D; spherical harmonics, $P_\alpha^\beta(x)$, $\alpha = 0(1)3$, $\beta = 0(1)\alpha$, $x = 0(.05)1$, 4D or 5S; Legendre functions, $P_\alpha^\beta(x)$ and $Q_\alpha^\beta(x)$, $\alpha = 0(1)2$, $\beta = 0(1)\alpha$, and $(\alpha,\beta) = (\pm\frac{1}{2}, 0)$, $(\pm\frac{1}{2}, 1)$, $(-\frac{1}{2}, 2)$, $(\frac{2}{3}, 0)$, $x = 1(.2)3(.5)8$, 4D or 5S, $P_\alpha^\beta(ix)$ and $Q_\alpha^\beta(ix)$, $x = 0(.2)2(.5)7.5$, 5D or 4 to 5S; Bessel functions $J_\alpha(x)$, $N_\alpha(x)$, $I_\alpha(x) = i^{-\alpha} J_\alpha(ix)$, $j_\alpha(x) = \sqrt{\pi/2x}\, J_{\alpha+\frac{1}{2}}(x)$ and $n_\alpha(x) = \sqrt{\pi/2x}\, N_{\alpha+\frac{1}{2}}(x)$, $x = 0(.1).2(.2)8$; (Here $N_\alpha(x)$ denotes a Bessel function of the second kind and is equivalent to $Y_\alpha(x)$ of Fletcher, Miller and Rosenhead [2]); Legendre functions $P_\alpha (\cos \nu)$, $\alpha = -1(1)9$, $\theta = 0(5)90°$, 4D; auxiliary functions $C_\alpha(x)$, $C_\alpha'(x)$, $\delta_\alpha(x)$ and $\delta_\alpha'(x)$ such that $J_\alpha(z) = C_\alpha(z) \sin [\delta_\alpha(z)]$, $dJ_\alpha(z)/dz = - C_\alpha'(z) \sin [\delta_\alpha'(z)]$, $N_\alpha(z) = -C_\alpha(z) \cos [\delta_\alpha(z)]$, $dN_\alpha(z)/dz = C_\alpha'(z) \cos [\delta_\alpha'(z)]$, δ and δ' in degrees, $\alpha = 0$, 1 and $x = 0(.1)1(.2)5$, $\alpha = 2$, 3 and $x = .1(.1).2(.2)5$, $\alpha = 4, 5$ and $\chi = .4(.2)5$, $\alpha = 6, 7$ and $\chi = 1(.2)5$, $\alpha = 8, 9$ and $x = 1.6(.2)5$, 4D or 5S and angles 2D; auxiliary functions $D_\alpha(x)$, $D_\alpha'(x)$, $\delta_\alpha(x)$ and $\delta_\alpha'(x)$ related analogously to $j_\alpha(x)$ and $n_\alpha(x)$, but this δ_α, δ_α' is different from that of the preceding table, $\alpha = 0$, 1 and $x = .1(.1)1(.2)5$, $\alpha = 2$, 3 and

$x = .1(.1).2(.2)5$, $\alpha = 4, 5$ and $x = .6(.2)5$, $\alpha = 6, 7$ and $x = 1(.2)5$, $\alpha = 8, 9$ and $x = 2(.2)5$, amplitudes 4D or 5S andangles (in degrees) 2D; periodic Mathieu functions, even and odd, $Se_\alpha\ (h, \cos x)$, $\alpha = 0(1)4$, and $So_\alpha\ (h, \cos x)$, $\alpha = 1(1)4$, $h^2 = 0(1)9$, $\chi = 0(10)90°$, 4D; quantities associated with periodic Mathieu functions, $M_\alpha{}^e(h) = \int_0^{2\pi} [Se_\alpha\ (h, \cos \chi)]^2 dx$, Je_α, $C_\alpha{}^e$, $\delta_\alpha{}^e$ such that $Je_\alpha + iNe_\alpha = -C_\alpha{}^e ie^{i\delta_\alpha{}^e}$ and analogous functions for odd functions, $M_\alpha{}^0$, $Jo_\alpha{}'$, $C_\alpha{}^0$, $\delta_\alpha{}^0$ with $Jo_\alpha{}' + No_\alpha{}' = iC_\alpha{}^0 e^{i\delta_\alpha{}^0}$, $h^2 = 0(1)9$, $\alpha = 1(1)4$ and $\alpha = 0$ for even functions, δ in degrees 2D, other quantities mostly 4D.

On the whole, this is an impressive collection of information, both in volume and in range of topics covered, and it represents an enormous effort on the part of its highly competent authors. References are adequate, although no attempt was made to include exhaustive bibliographies. Misprints detected by the rather cursory reading of the reviewer were trivial. Numerous and well chosen problems are included with each chapter. Reference is facilitated by summaries at the ends of the chapters, but the broad scope of the book precludes complete reference details in the fields covered. While the work is intended as a text book for a course the authors have presented at the Massachusetts Institute of Technology for several years, many people will find that it is more useful as a guide to be used in their courses rather than a text book to be followed faithfully; in this role of guide the book gives a valuable list of equations, solutions and significance. The nature of the information available in the book is described in the words of the authors as follows: "The present treatise, therefore, is primarily concerned with an exposition of the mathematical tools which have proved most useful in the study of the many field constructs in physics, together with a series of examples, showing how the tools are used to solve various physical problems. Only enough of the underlying physics is given to make the examples understandable.

"This is not to say that the work is a text on mathematics, however. The physicist, using mathematics as a tool, can also use his physical knowledge to supplement equations in a way in which pure mathematicians dare not (and should not) proceed. He can freely use the construct of the point charge, for example; the mathematician must struggle to clarify the analytic vagaries of the Dirac delta function. The physicist often starts with the solution of the partial differential equation already described and measured; the mathematician often must develop a very specialized network of theorems and lemmas to show exactly when a given equation has a unique solution. The derivations given in the present work will, we hope, be understandable and satisfactory to physicists and engineers, for whom the work is written; they will not often seem rigorous to the mathematician."

The Table of Contents follows:

Vol. Chap.

I 1. Types of Fields

1.1 Scalar Fields

1.2 Vector Fields

1.3 Curvilinear Coordinates

1.4 The Differential Operator

1.5 Vector and Tensor Formalism

1.6 Dyadics and Other Vector Operators
1.7 The Lorentz Transformation, Four-Vectors, Spinors

2\. 2.1 The Flexible String
2.2 Waves in an Elastic Medium
2.3 Motion of Fluids
2.4 Diffusion and Other Percolative Fluid Motion
2.5 The Electromagnetic Field
2.6 Quantum Mechanics

3\. Fields and the Variational Principle
3.1 The Variational Integral and the Euler Equations
3.2 Hamilton's Principle and Classical Dynamics
3.3 Scalar Fields
3.4 Vector Fields

4\. Functions of a Complex Variable
4.1 Complex Numbers and Variables
4.2 Analytic Functions
4.3 Derivatives of Analytic Functions, Taylor and Laurent Series
4.4 Multivalues Functions
4.5 Calculus of Residues; Gamma and Elliptic Functions
4.6 Asymptotic Series: Method of Steepest Descent
4.7 Conformal Mapping
4.8 Fourier Integrals

5\. Ordinary Differential Equations
5.1 Deparable Coordinates
5.2 General Properties, Series Solutions
5.3 Integral Representations

6\. Boundary Conditions and Eigenfunctions
6.1 Types of Equations and of Boundary Conditions
6.2 Difference Equations and Boundary Conditions
6.3 Eigenfunctions and Their Use

7\. Green's Functions
7.1 Source Points and Boundary Points
7.2 Green's Functions for Steady Waves
7.3 Green's Function for the Scalar Wave Equation
7.4 Green's Function for Diffusion
7.5 Green's Function in Abstract Vector Form

8\. Integral Equations
8.1 Integral Equations of Physics, Their Classification
8.2 General Properties of Integral Equations
8.3 Solution of Fredholm Equations of the First Kind
8.4 Solution of Integral Equations of the Second Kind
8.5 Fourier Transforms and Integral Equations

II 9. Approximate Methods
9.1 Perturbation Methods
9.2 Boundary Perturbations
9.3 Perturbation Methods for Scattering and Diffraction
9.4 Variational Methods

10. Solutions of Laplace's and Poisson's Equations
 10.1 Solutions in Two Dimensions
 10.2 Complex Variables and the Two-dimensional Laplace Equation
 10.3 Solution for Three Dimensions
11. The Wave Equation
 11.1 Wave Motion on One Space Dimension
 11.2 Waves in Two Dimensions
 11.3 Waves in Three Space Dimensions
 11.4 Integral and Variational Techniques
12. Diffusion, Wave Mechanics
 12.1 Solutions of the Diffusion Equation
 12.2 Distribution Functions for Diffusion Problems
 12.3 Solutions of Schroedinger's Equation
13. Vector Fields
 13.1 Vector Boundary Conditions, Eigenfunctions and Green's Functions
 13.2 Static and Steady-state Solutions
 13.3 Vector Wave Solutions

C. B. T.

1. S. L. SOBALEV, *Uravneniia matematicheskoi fiziki* (*Equations of Mathematical Physics*), Moskva, 1950.
2. A. FLETCHER, J. C. P. MILLER, & L. ROSENHEAD, *An Index of Mathematical Tables*, Scientific Computing Service, 23 Bedford Square, London, W. C. 1, 1946. [See *MTAC*, v. 2, 1946–47, RMT **233**, p. 13–18.]

88[F].—R. J. PORTER, *Irregular Negative Determinants of Exponent* $3n$ with their critical classes. Part III, from $-D = 1\ 00000$ to $1\ 50000$. 215 typewritten pages, 25.5×10 cm., deposited in *UMT* file.

This table of numbers occurring in the theory of quadratic forms completes the listing of "critical classes" to $-D = 1\ 50000$. The list of determinants is in three parts [*UMT* **155**, *MTAC*, v. 7, p. 34; *UMT* **185**, *MTAC*, v. 8, p. 96; Review **62**, *MTAC*, v. 9, p. 126]. The determinants are serially numbered and listed together with their critical classes in the present table [Review **84**, *MTAC*, v. 9, p. 126; Review 113, *MTAC*, v. 11, p. 275]. The lists were compiled with the aid of a *UMT* list of groups and series [Review **3**, *MTAC*, v. 9, p. 26] by the same author. The 58 values of $-D < 1\ 50000$ which have exponent of irregularity 9 are listed and discussed in [1]. The 14 values of $-D < 1\ 50000$ which have exponent of irregularity 6 are

17561	70244	91299	1 23539	1 28739
55555	80755	1 12723	1 25443	1 29355
67899	91083	1 19195	1 26755	

The other 5764 values of $-D < 1\ 50000$ have exponent of irregularity 3.

J. L. SELFRIDGE

University of California
Los Angeles, California

1. R. J. PORTER, "On irregular negative determinant of exponent $9n$," *MTAC*, v. 10, 1956, p. 22–25.

89[F].—C. L. BAKER & F. J. GRUENBERGER, *Primes in the Thousandth Million*, 1958, 3 p. $8\frac{1}{2}'' \times 11'' + 39$ $17'' \times 22''$ blue line ozalid prints deposited in the UMT file.

A list of 47,957 primes found between, and including, 999,000,011 and 999,999,937 together with the first eighteen 10-digit primes.

This table came into being as a by-product of another table of primes: the 5,761,456 eight-digit primes, which will eventually be published.

A description of the method of construction of the tables on the IBM model 704 computer is included.

C. B. T.

90[F].—O. P. GUPTA, "Partitions into exactly *k* distinct primes," Panjab Univ., *Res. Bull.*, No. 107, p. 283–290, May 1957, Hoshiarpur, India.

The function tabulated is $R_k(n)$ the number of partitions of n into precisely k distinct primes > 1, for all possible values of k and for $n = 1(1)300$, together with the total number $R(n)$ of partitions of n into distinct primes. The function $R_k(n)$ was determined recursively by means of the auxiliary function $R_k(n, p_1)$ defined as the number of partitions of n into k distinct primes of which the smallest is p_1, so that

$$R_k(n, p_1) = \sum_{p>p_1} R_{k-1}(n - p_1, p)$$

and

$$R_k(n) = \sum_{p\geq 2} R_k(n, p).$$

Sample values are

$$R_2(300) = 21, \quad R_8(300) = 13504, \quad R_{13}(300) = 87, \quad R(300) = 53040.$$

D. H. LEHMER

Department of Mathematics
University of California
Berkeley, California

91[F].—M. S. CHEEMA & H. GUPTA, *Tables of Partitions of Gaussian Integers.* Natl. Inst. of Sciences, India, Mathematical Tables, v. 1, xii + 67 p., 1956. 30 × 24 cm. Price 15 Rps. Paper Bound.

The number $B(n, m)$ of partitions of the Gaussian integer $n + im$ into non-zero Gaussian integers of the first quadrant is tabulated for $m, n = 0(1)50$ in the first ten pages of the volume, even though $B(n, m) = B(m, n)$. In other words $B(n, m)$ is the number of partitions of the bipartite number (n, m) into bipartite summands (r, s) where $0 \leq r$, $0 \leq s$, $r + s > 0$. On p. 65 the number of unrestricted partitions

$$p(n) = B(n, 0)$$

is tabulated along with its sum function

$$p_1(n) = \sum_{k=0}^{n} p(k)$$

for $n = 0(1)50$. The rest of the volume is devoted to the auxiliary function $B_k(n, m)$ generated by

$$\left(\sum_{n,m=0}^{\infty} B_k(n, m)x^n y^m \right) \prod_{r=0}^{\infty} \prod_{t\geq k} (1 - x^r y^t) \prod_{s=1}^{\infty} (1 - x^2) \equiv 1$$

which was used recursively to compute $B(n, m) = B_1(n, m)$. The printing is beautifully done.

D. H. LEHMER

Department of Mathematics
University of California
Berkeley, California

92[F].—HARVEY COHN, "Some bi-quadratic class numbers," [*MTAC*, this issue, p. 213–217.]

This is a tabulation of 446 sets (a, b, R, H) as is described in the attached note.

93[F, K].—RICHARD T. BURCH, *Approximate Values of Stirling Numbers of the Second Kind for the First Hundred Degrees*, Department of Defense, 134 p., multigraphed, 27 cm. Deposited in the UMT File.

The Stirling Numbers, $u(N, A)$, of the second kind are defined by the polynomial identity

$$x^N = \sum_{A=1}^{N} u(N, A), x^{(A)}, \quad \text{where} \quad x^{(A)} = \prod_{k=0}^{A-1} (x - k).$$

The recursion $u(N + 1, A) = A\,U(N, A) + u(N, A - 1)$ is used to calculate the numbers to four significant figures for N (the degree): $1(1)100$ and $A:1(1)N$. A sum $U(N) = \sum_{k=1}^{N} U(N, k)$ and a sum check U' recursively computed from $U(N + 1) = 1 + \sum_{j=1}^{N} \binom{N}{j} U(j)$ are included. Values are expressed in 'floating' form: $1344 + 12$ standing for $1344 \cdot 10^{12}$. The number $u(N, A)$ is the number of (ordered) selections of N items from A categories, at least one item being selected from each category.

J. D. SWIFT

University of California
Los Angeles, California

94[G].—K. YAMAMOTO, "Structure Polynomial of Latin Rectangles and Its Application to a Combinatorial Problem," *Memoirs of the Faculty of Science*, Kyushu Univ., S. A., v. 10, No. 1, 13 p., 1956. Fukuoka, Japan.

The problem mentioned in the title is due to J. Touchard and asks for the number $N_n^{(k)}$ of permutations on

$$1, 2, 3, \cdots, n$$

which are discordant with the first k cyclic permutations

$$\begin{array}{c}(1, 2, 3, \cdots, n)\\(2, 3, 4, \cdots, 1)\\\cdots\cdots\cdots\cdots\\(k, k+1, \cdots, k-1)\end{array}$$

This generalizes the famous problème des rencontres ($k = 1$) and the problème des ménages ($k = 2$). There is given (p. 13) a table of N_n in the case $k = 3$ for $n = 3(1)20$. Two auxiliary functions

$$N_n^* = V_n + 2, \quad N_n' = \{N_{n+1} + (-1)^n N^*_{n+1}\}/n$$

where

$$V_n = a^n + b^n \quad (a + b = 1, ab = -1)$$

in Lucas' companion to the Fibonacci series, are given for $n = 3(1)20$.

D. H. Lehmer

Department of Mathematics
University of California
Berkeley, California

95[G, P].—Lorenzo Lunelli, "Numerazione delle maglie in una rete completa," Instituto Lombardo di Scienze e Lettere, *Rendiconti*, v. 91, 1957, p. 903–911.

[G, P].—Emanuele Biondi, "Numerazione delle maglie in una rata qualsiasi," Instituto Lombardo di Scienze e Lettere, *Rendiconti*, v. 91, 1957, p. 912–926.

[X, Z].—Lorenzo Lunelli, "Determinazione delle maglie in une rete mediante una calcolatrice elettronica," Instituto Lombardo di Scienze e Lettere, *Rendiconti*, v. 91, 1957, p. 927–935.

These papers are concerned with the calculation of the number of polygonal subnets contained in networks of lines connecting nodes (as in electrical networks). In particular, the first paper derives a formula for the number of polygonal subnets contained in a "complete net" (wherein each pair of nodes is connected by one and only one line). The formula, for n nodes, is

$$M_c(n) = \sum_{q=3}^{n} \frac{(q-1)!}{2} \binom{n}{q}.$$

The derivation is straightforward.

The second paper derives in a somewhat different fashion a formula for the number of polygonal subnets contained in a *general* net. The procedure described considers the "complementary net" (the additional lines required to produce from the given net a complete net with the same number of nodes). Each set of ϕ lines contained in the complementary net is then classified into one of three types: 1) they form a closed polygon of ϕ lines. (In this case there is only one polygonal subnet which passes through all ϕ lines and the notation is used

$M_\phi{}^0(n) = 1$); 2) they form a number, γ, of separate connected sequences of lines (excluding those of the first case) such that at each node at most two lines meet. In these situations, the number of polygonal subnets passing through all ϕ lines is given by

$$M_\phi{}^\gamma(n) = \sum_{q=\gamma+\phi}^{n} 2^{\gamma-1} \binom{n-\gamma-\phi}{q-\gamma-\phi} (q-\phi-1)!, \quad \gamma+\phi \geqq 3;$$

3) all other situations (involving more than two lines meeting at some node). These must have zero polygonal subnets passing through all ϕ lines.

Let the complementary net of the given net consist of L lines. Let it have, for each ϕ, $m_\phi{}^\gamma$ sets of ϕ lines forming γ separate connected sequences. Finally, let $M_c(n)$ be the number of polygonal subnets in the complete net of n nodes. Then, the number of polygonal subnets in the given net, as derived in the paper, is given by the formula

$$M = M_c(n) + \sum_{j=1}^{L} (-1)^j \sum_{i=0}^{j} m_j{}^i M_j{}^i(n).$$

The paper then derives the formula for the number of polygonal subnets in a complete net by induction on n. Specifically, by the above formula,

$$M_c(n-1) = M_c(n) - (n-1)M_1{}^0(n) + \binom{n-1}{2} M_2{}^1(n).$$

It is then shown that $M_c(n) = M_c(n-1) + \frac{1}{2}(n-1)M_1{}^0(n)$. The formula for $M_c(n)$, as given in the first paper, is then derived by induction on n.

The third paper is concerned with the use of the CRC-102A at the Polytechnic Institute of Milan for determination of the number of polygonal subnets in a network. The procedure uses a representation of the set of lines in a given subnet as a vector with ones and zeros for its components—a one for a given component if the corresponding line in the original net is part of the subnet and a zero if the corresponding line is not part of the subnet. The original net itself is then represented by a matrix (the Poincaré matrix) with a row for each node showing by such a vector the subnet of lines meeting at the node. The columns of this matrix then indicate the endpoint nodes for each line. If the Poincaré matrix is multiplied onto the vector representing a given subnet, a product vector is obtained. The given subnet then is a polygonal subnet if and only if the product vector consists entirely of twos and zeros. The computer program examines in this manner each of the possible subnets of the given net and tabulates the polygonal subnets. The program includes an output conversion routine to represent the make-up of each polygonal subnet in convenient form for the human operator. The program is deliberately unsophisticated, and therefore uses no criteria other than the make-up of the product vector for determining the polygonal subnets.

R. M. Hayes

Business Systems Department
Magnavox Research Laboratory
Los Angeles, California

96[I, X, Z].—National Physical Laboratory, *Modern Computing Methods*, Notes on Applied Science No. 16, Published by National Physical Laboratory, Obtainable from Her Majesty's Stationery Office, London, England, 1957, vi + 128 p., 24 cm. Price 10s 6d. net.

This is a pamphlet which recounts the experience of professional computers attacking extensive problems on an automatic digital computer of considerable power. It is scholarly in that full use was made of literature known to these competent people, but it is not pedantic and it does not contribute particularly to the advanced research literature on numerical analysis.

However, the usefulness of this book to people who are meeting extensive computations for the first time is great, particularly because the larger and more detailed books in numerical analysis seem to be only mildly influenced by the development of large machines. The reviewer is consistent in recommending it to each person starting to work in his own group, and any careful reader of the pamphlet is likely to profit. So far as the reviewer knows the material in the book is not conveniently available in any other place, and considering the size of the pamphlet the choice of material presented is excellent.

Despite this approbation, it should be noted that this material is dated—many contributions to extensive computation have been made since the preparation of this material. Thus it should be augmented if it is to be used as a guide to a course in modern computation. Problems for students would also be required if it were to be used as a text. Numerous examples are worked out in the text, however, and these illustrate the methods and make reading easier. Occasionally the authors or compositors have been careless in displaying the results of computation. For example the publisher has noted that in Table 2 on page 49 a proper conclusion is drawn from incorrectly copied inputs to the table—a type of computing error which must be unusual among the distinguished group contributing to this book. On page 99 the right column of numbers in an example illustrating Euler's transformation is spaced artistically but not conveniently for following the example (which is easy to follow anyhow). Those of us who contend that time and effort spent on care and neatness in computing (and, from the reviewer's point of view, even more importantly in coding for automatic machines) are particularly good investments hope that these slight faults will be eliminated in later editions.

The table of contents follow:

Chapters

1. Linear Equations and Matrices (1)
2. Linear Equations and Matrices (2)
3. Roots of Polynomial Equations
4. Latent Roots of Matrices
5. Finite-difference Methods
6. Ordinary Differential Equations (1)
7. Ordinary Differential Equations (2)
8. Hyperbolic Partial Differential Equations
9. Elliptic and Parabolic Partial Differential Equations

10. Relaxation Methods
11. Tabulation of Mathematical Functions
12. Computation of Mathematical Functions

C. B. T.

97[K].—W. J. DIXON & F. J. MASSEY, JR., *Introduction to Statistical Analysis*, McGraw-Hill Book Co., New York, 2nd ed., 1957, xii + 488 p., 24 cm. Price $6.00.

The first edition of this excellent textbook contained an unusually extensive and useful set of tables. An important feature of the second edition is the addition of 21 more tables. Besides this, 12 of the tables of the first edition have been extended in the second edition; five have been modified, one was reduced and one omitted. The present review will list the new tables and indicate the other changes.

The new tables are:

A-4. Cumulative areas under the normal frequency curve to 4D for the standardized deviate = −3.25(.05)3.25.

A-8b (1) gives the .1, .5, 1, 2.5, 5, 10, 90, 95, 97.5, 99, 99.5, 99.9 percentiles of the standardized range to 2D for samples of N = 2(1)20 from normal.

A-8b (4). The variance and the efficiency to 3D or 3S of four estimates of the mean, the median, the midrange, the average of the best two ordered observations, and the average of all but two extreme observations in samples of N = 2(1)20 from normal.

A-8b (5). Standardized expected values of the order statistics to 3D in samples of N = 2(1)20 from normal.

A-8b (6). The best linear estimates of σ and their efficiencies to 3D for samples N = 2(1)10 from normal.

A-10b. Percentile values covering the range .005 to .125 to 3D for the number of occurrence in samples of N from the binomial with $p = \frac{1}{2}$ for $N = 3(1)100$.

A-12a. A chart showing the power curves for one- and two-sided tests for means and differences of means from normal with σ known for the significance levels .01 and .05 for two-sided tests and .005 and .025 for one-sided tests and for values of the test criterion in the range 0 to 4.5.

A-12b. For the same sampling situation as in A-12a, but with σ known or unknown, values of the test criterion to 2D or more required to make the power of the test .1(.1).9, .95, .975, .99, .995 for degrees of freedom 4(1)10, 12, 16, 24, 36, ∞ and for significance levels .005, .0125, .025, .05 for one-sided tests and .01, .025, .05, .1 for two-sided tests.

A-12c. For the same sampling situation as in A-12b, the sample sizes needed in the case of one mean and the equal sample sizes needed in the case of two means to attain the power .5(.1).9, .95, .99 for test criterion = .1, .2(.2)2, 3 for the significance levels, .005, .125, .025, .05 for one-sided tests.

A-19. Distribution of the signed rank statistic T. For samples of $N = 1(1)20$, the 100_α and 100 $(1 - \alpha)$ percentile values of T for α's in the range .005–.125.

A-20. Distribution of the rank T'. For all pairs of sample sizes from (1, 1) to (10, 10), the cumulative probability distribution of T' is given to 3D.

A-28. Values of $p(1 - p)$, $\sqrt{p(1 - p)}$, $1 - p^2$, $1 - (1 - p)^2$, 2 arcsin $\sqrt{p}$ and 2 arcsin $\sqrt{1 - p}$ to 4 and 5D for $p = .01(.01)1$.

A-29a. Values of $\binom{N}{X} p^x(1 - p)^{N-x}$ to 4D for $N = 2(1)10$ and $p = .01$, .05(.05).5 and $\frac{1}{3}$.

A-29b. All values of $\binom{N}{X}$ for $N = 1(1)20$.

A-29c. Values of $N!$ to 5S for $N = 1(1)100$.

A-30a. 95, 97.5, 99, 99.5, and 99.95 percentiles of the distribution of the coefficient of correlation to 3D in samples of $N = 5(1)20(2)30(10)60$, 80, 100, 250, 500, 1000, ∞ from an uncorrelated normal bivariate universe.

A-30b. Values of $\frac{1}{2} \ln \frac{1 + r}{1 - r}$ to 5D for $r = -.99(.01).99$.

A-30c. Selected values of Σd_i^2, used in the rank correlation coefficient, that will be equalled or exceeded with probabilities $\leq .175$ in samples of $N = 4(1)10$.

A-30d. The probabilities to 3D that the quadrant sum S, for the corner test for association, will be equalled or exceeded in samples of $2N$, for $N = 3(1)5$, 7, ∞ and $S = 8(1)23$.

A-31. Reciprocals of N to 5D for $N = 1(.01)1.1(.1)9.9$.

A-33. 4D values of $\sqrt{N}$ for $N = 1(.01)(10)(.1)99.9$.

Tables in the new edition which have been significantly extended beyond their counterparts in the first edition are:

A-5. Percentiles of the t distribution. 60, 70, 80 and 90 percentile values have been added and 99.75 percentile values deleted.

A-6b. Percentiles of the χ^2/df distributions. .05, .1, 20(10)80, 99.9 and 99.95 percentile values have been added and the table now gives values for 1(1)20(2)30(5)60(10)100(20)200(50)500, 750, 1000, 5000 and ∞ d.f.

A-7c. This is a much extended table of percentiles of the F distribution. For the degrees of freedom, $\nu_1 = 1(1)12$, 15, 20, 24, 30(10)60, 100, 120, 200, 500, ∞ and $\nu_2 = 1(1)12$, 15, 20, 24, 30, 40, 60, 120, ∞, the values of F to 3S or 3 or more D are given for which the cumulative probabilities are .0005, .001, .005, .01, .025, .05, .1, .25, .5, .75, .9, .95, .975, .99, .995, .999, and .9995.

A-8b(1). This now gives for samples of 2(1)20 from normal, unbiased estimates of σ as a multiple of the sample range to 3D, the variance of the estimate to 3S and its efficiency to 3D.

A-8b(3). Modified linear estimates of σ in samples of N from normal are now given for $N = 2(1)20$ with the coefficients to 4D together with their variances to 3S and the efficiencies to 3D.

A-8c(1). Upper (and lower) 95, 97.5, 99, 99.5, 99.9 and 99.95 percentiles of the ratio of the deviation of the sample mean from the population to the range in samples of N from normal for $N = 2(1)20$ are given to 3D for the first four percentiles and to 2D for the last two.

A-8c(2). In the new tables the same set of percentiles is given as in A-8c(1) and the same D for the difference of two sample means divided by the average of the ranges for sample of sizes $N_1 = N_2 = 2(1)20$.

A-8d. For the ratio of the ranges in samples of N_1 and N_2 from normal (substitute F ratio), .5, 1, 2.5, 5, 95, 97.5, 99 and 99.5 percentiles are now given to 2 or 3S again for N_1, $N_2 = 2(1)10$.

A-8e. Criteria for testing for extreme mean. Table 8e of the first edition is extended to give critical values at the .5%, 2%, 10%, 20% and 30% levels as well as the 1% and 5% levels.

A-11. This differs in form from Table 11 of the first edition. It now gives the complete distribution to 3D of the total number of runs in samples of (N_1, N_2) for N_1, $N_2 = 2(1)10$. For $N_1 = N_2 = 11(1)20(5)100$, the table gives the number of runs that occur with probabilities $\leqq .005$, .01, .025, .05, .95, .975, .99, .995, the mean number of runs to the nearest integer, and the variance and the standard deviation to 2D.

A-13. Table 13 of the first edition is replaced by charts for the power of the analysis of variance test. If for K samples of n each from $N(\mu_i, \sigma^2)$; $i = 1, \cdots, K$; we designate by φ^2 the average squared deviation of the μ_i's from their mean divided by σ^2/n, the charts enable one to read the power of the test with significance levels .01 and .05 for the appropriate range of values of φ for the degrees of freedom, $n = 1(1)8$ and $n_2 = 6(1)10, 12, 15, 20, 30, 60, \infty$.

A-14. The values of $\ln (1 - \beta)/\alpha$ to 3D are now given for $\alpha, \beta = .001, .005, .01, (.01).05(.05).25$.

Table 15. The values of the cumulative Poisson distribution to 3D are now given for $Np = .05(.05)1(.1)2(.2)4$.

A-16. Tolerance factors for normal distributions. This is the same as table 16 of the first edition with the exception that the tabulation is for sample sizes, $N = 2(1)27, 30(5)100(10)200, 250, 300(100)1000$, instead of for $N = 2(1)$-$102(2)180(5)300(10)400(25)750(50)1000, \infty$.

A-18. Percentiles of the distribution of the ratio of the range to the standard deviation (as defined in this text) in samples of K from normal. Upper 95 and 99 percentiles are given for $\nu = 1(1)9$ degrees of freedom and upper and lower values for $\nu = 10(1)20, 24, 30, 40, 60, 120, \infty$, all to 3S for $K = 2(1)20$.

Table 19. Determination of the second sample size, of the first edition is omitted from the second edition.

This extensive and useful set of tables in itself very considerably enhances the value of the second edition as compared to the first.

C. C. C.

98[K].—B. L. van der Waerden, *Mathematische Statistik* Springer-Verlag, Berlin, Germany, 1957, ix + 360 p., 23 cm. Price DM 49.60.

This is an example of a book written in the careful style of van der Waerden. It includes at the beginning a chart of the type used in his famous *Algebra* indicate earlier chapters which are essential to an understanding of later ones. At the end it includes an index of examples classified according to the field in which they were generated, and there is a short English-German glossary.

The purpose of this review is to announce tables in the book, and not particularly to discuss material covered. However, it might be noted that the author describes the central question of statistics as that of how far quantities estimated from random samples may vary from the corresponding ideal values (translation approximate only). To attack this question he studies the axiomatic development of probability and its applications to statistics, including studies of confidence limits and confidence bands. He then turns to the theory of estimation, and then to the testing of hypotheses.

Tables include the normal distribution (area) $\Phi(t)$, $t = -3.9(.1)-3(.01)$-$3(.1)3.9$, 4D, the inverse function for the normal distribution $\Psi(x)$, $x = 0(.001).99$, 2D; g and g^2, the (one sided) .05, .1, .5, 1, 2.5 and 5% points for the normal distribution and the χ^2 distribution with one degree of freedom, 2D; exact and asymptotic 1 and 5% points for Smirnoff's test, $n = 5, 8, 10(10)50$, 4D and the ratio of the values 3D; exact and asymptotic 1 and 5% points for Kolmogorov's test for $n = 5(5)30(10)80$ and the 5% values for $n = 90, 100$, 4D with ratios 3D; .1, 1, and 5% points of χ^2 with f degrees of freedom, $f = 1(1)100$, 3S with suitable formula for larger f; (one sided) .05, .5, 1, and 2.5% points for Student's test with f degrees of freedom, $f = 1(1)30(10)60(20)100, 200, 500, \infty$, 4S; 5% points for F-test with f_1 and f_2 degrees of freedom in numerator and denominator respectively, $f_1 = 1(1)20(2)30(10)60(20)100$, $f_2 = 1(1)30(2)50(10)100(25)150, 200, 300, 500, 1000$, 3D; the 1% points with f_1 as above and $f_2 = 2(1)30(2)50(5)60$-$(10)100(25)150, 200, 300, 500, 1000$, and the .1% points for $f_1 = 1(1)10(5)20, 30, 50, 100$ and $f_2 = 2(1)20(2)30(10)60(20)100, 200, 500, \infty$, 3S; (one sided) .5, 1 and 2.5% points for the sign test, $n = 5(1)100$, integral; tables for Wilcoxon's test of the number of inversions in a series with g marks x and h marks y with $g = 2$ and $h = 5(1)10$, and $g = 3(1)10$, $h = g(1)10$, the probabilities not exceeding 5% are listed 3S; tables for the X-test, (one sided) .5, 1, and 2.5% points for $X = \sum_{r \epsilon x} \left(\frac{r}{n+1}\right)$, and corresponding values of X for $n = 6(1)50$ and $g - h = 0(1)5$, 3S, where g and h are as in Wilcoxon's test above, $n = g + h$, r is the set of rank numbers of x in the sequence of x and y, and ψ is the inverse normal distribution function; also an auxiliary table of $Q = \frac{1}{n} \sum_{r=1}^{n} \psi^2 \left(\frac{r}{n+1}\right)$, $n = 1(1)150$, 3D; (one sided) .1, 1, 2, and 5% points for the correlation coefficient, $f = n - 2 = 1(1)20(5)50(10)100$, 3D. Many of these are reprinted from other sources, which are acknowledged.

C. B. T.

99[K].—H. A. DAVID, "The ranking of variances in normal populations," Amer. Stat. Assn., *Jn.*, v. 51, 1956, p. 621–626.

This paper considers two procedures for ranking, or grouping, the variances σ_t^2 $(t = 1, \cdots, k)$ of k normal populations by use of mean square estimates s_t^2 each based on ν degrees of freedom.

Let $X_1 < X_2 < \cdots < X_k$ be an ordered sample of k independent observations from a unit normal distribution. Table I gives, to 3S and for $k = 2(1)12$, values G such that the expected number of differences, $X_{t+1} - X_t$, which exceed G is $\alpha = .05$ and $\alpha = .01$. The title of Table I refers to these as "Critical 100α% levels . . ." and the text material points out that they are not percentage points. If $X_t = \log s_t^2$ Table I can be used to rank the variances σ_t^2.

Table II gives, to 3S, for $k = 2(1)12$, for $\nu = 2, 4(1)10, 12, 15, 20, 30, 60, \infty$, and for $\alpha = .05$, $\alpha = .01$, values of $R\alpha(\nu)$ such that the expected number of ratios s^2_{t+1}/s_t^2 (when the s_t^2 have been ordered by size) which exceed $R\alpha(\nu)$ is α. Again the title refers to "Critical 100α% levels. . . ." The title has a misprint, the division bar being printed as *m*.

Table III (titled "Significant Maximum F-Ratios in a 1% Level Multiple F_{max}-Ratio Test. (Normal Variation Assumed)") gives, to 3S, for $\alpha = .01$, for $k = 2(1)12$, and for $\nu = 2(1)10, 12, 15, 20, 30, 60, \infty$, values $C\alpha(\nu, k)$ which are the upper $100\{1 - (1 - \alpha)^{k-1}\}\%$ points of the maximum ratio of the s_t^2.

F. J. MASSEY, JR.

University of California
Los Angeles 24, California

100[K].—D. G. CHAPMAN, "Estimating the parameters of a truncated gamma distribution," *Ann. Math. Stat.*, v. 27, 1956, p. 498–506.

To facilitate computation of maximum likelihood estimates of parameters of the gamma or Type III distribution with density function

$$f(x) = \frac{a^b}{\Gamma(b)} e^{-a(x-c)} (x - c)^{b-1}; \quad x \geq c,$$
$$= 0; \qquad x < c,$$

in the case $c = 0$, a table of the function

$$b = \gamma^{-1}[\ln \bar{x} - \overline{\ln x}],$$

where $\bar{x}$ designates the sample mean and $\overline{\ln x}$ designates the mean of natural logarithms of the sample observations, is given for $[\ln \bar{x} - \overline{\ln x}] = .01(.001).99$, $.1(.005).495, .50(.01)1$. For values of the argument from .01 — .049, b is given to 4D. Elsewhere it is given to 3S. The author states that a complete tabulation of $\gamma(b)$ with its first and second differences, correct to one figure in the fifth decimal, for $b = .01(.01)2, 2(.02)5, 5(.1)20, 20(1)100$, is available in mimeographed form from the Laboratory of Statistical Research, University of Washington.

A new procedure bearing certain resemblances to a minimum chi square method is suggested for estimating parameters of a truncated gamma distribution. The asymptotic variance-covariance matrix of these estimates is determined.

The author points out that his method is applicable to a number of other truncated distributions whether truncation is in the tails or the center of the distribution.

A. C. COHEN, JR.

The University of Georgia
Athens, Georgia

101[K].—R. J. BUEHLER, "Confidence intervals for the product of two binomial parameters," Amer. Stat. Assn., *Jn.*, v. 52, 1957, p. 482–493.

The author considers the problem in which two binomial populations have been sampled and that k_1 failures are observed in a sample of n_1 from the first population and k_2 failures are observed in a sample of n_2 from the second and one is interested in finding the number $C_{n_1n_2}(k_1, k_2; \alpha)$ having the property that

$$\text{Prob}\,\{0 \leq P_1P_2 \leq C_{n_1n_2}(k_1, k_2; \alpha)\} \geq \alpha \quad \text{for all} \quad 0 \leq P_1, P_2 \leq 1$$

where P_1 and P_2 are the probabilities associated with the respective statistically independent populations. That is, one is interested in obtaining an upper confidence interval in the sense of Neyman for the product of two probabilities of statistically independent binomial populations.

The author proposes that the above problem be solved by taking

$$C_{n_1n_2}(k_1, k_2; \alpha) = C_{n_1}(k_1, \sqrt{\alpha}) \cdot C_{n_2}(k_2, \sqrt{\alpha})$$

where $C_{n_1}(k_1, \sqrt{\alpha})$ and $C_{n_2}(k_2, \sqrt{\alpha})$ are such that

$$\text{Prob}\,\{0 \leq P_i \leq C_{n_i}(k_i; \sqrt{\alpha})\} \geq \sqrt{\alpha} \quad \text{for} \quad 0 \leq P_i \leq 1, i = 1, 2.$$

Approximate solution to the above inequalities are obtained under the assumption that n is large and C is small by using Poisson terms for the binomial terms. These approximations are tabulated.

Table 1 gives values of $nC_n(k; \alpha)$ to 3S for $k = 0(1)6, 8, 10, 20, 30$ and $\alpha = .90$, .95 and .99.

To obtain the confidence limit $C_{n_1n_2}(k_1, k_2; \alpha)$ which are tabulated in Table 3, the author resolves the lack of uniqueness problem associated with the solution of the inequality by imposing a generalized condition of (1) Regularity: If $k_1 < k_2$ then $C_n(k_1; \alpha) < C_n(k_2, \alpha)$ and (2) Shortness: $C_n(k; \alpha)$ should be as small as possible. The 3S values for $n_1n_2C_{n_1n_2}(k_1, k_2; \alpha)$ are tabulated for $k_1 = 0(1)5$ and $k_2 = 0(1)29$ for $\alpha = .90$ and .95. The symmetry allows the interchange of k_1 and k_2.

The final Table 5 gives unsymmetrical values for the confidence limits $n_1n_2C_{n_1n_2}(k_1, k_2; 0.9)$ to 1D for $k_1, k_2 = 0(1)4$ which are shortened by use of an auxiliary random element in a fashion similar to that suggested by Stevens [1] and Eudey [2].

C. F. KOSSACK

Purdue University
Lafayette, Indiana

1. W. L. STEVENS, "Fiducial limits of the parameter of a discontinuous distribution," *Biometrika*, v. 37, 1950, p. 117–129.
2. M. W. EUDEY, "On the treatment of discontinuous random variables," *Stat. Lab. Technical Report* No. 13, Univ. of Calif., Berkeley, 1949.

102[K].—JOSEPH BERKSON, "Tables for the maximum likelihood estimation of the logistic function," *Biometrics*, v. 13, 1957, p. 28–34.

Table 2 in this paper gives to 5D the function $p = 1/[1 + e^{-l}]$ and (directly beneath the value of p) the function $w = p(1 - p)$, for argument $l = 0(.01)4.99$. The text states that the tables are correct to ± 1 in the fifth place and that linear interpolation in the table will also give values correct to ± 1 in the fifth place.

Suppose $P(x) = 1/[1 + e^{-(\alpha+\beta x)}]$. Then $P(x)$ is said to be a logistic function of x. If, for each of a set of values $x_1, \cdots, x_k$, there are performed n_i independent binomial trials with probability of success $P_i = P(x_i)$, the results may be used to estimate the parameters α and β in a number of ways. The table of values of the functions p and w facilitates the calculation of the Maximum Likelihood estimates of α and β. The text explains the method and gives a numerical example (table 1).

PAUL MEIER

University of Chicago
Chicago, Illinois

103[K].—A. J. DUNCAN, "Charts of the 10 per cent and 50 per cent points of the operating characteristic curves for fixed effects analysis of variance F tests, $\alpha = 0.01$ and 0.05," Amer. Stat. Assn., *Jn.*, v. 52, 1957, p. 345–349.

The symbols and expressions α, β (ordinate of OC curve), f_1, f_2, OC(1-power) have meanings usual in testing hypotheses and (fixed effect) variance analysis. Duncan gives four (very small) charts for ϕ (defined below)

(1) $\alpha = .05, \beta = .10, f_1 = 1(1)8, f_2 = 6(1)10, 12, 15, 20, 30, 40, 60, \infty$

(2) $\alpha = .05, \beta = .50$, f_1 as above, f_2 as above

(3) $\alpha = .01, \beta = .10$, f_1 as above, f_2 as above

(4) $\alpha = .01, \beta = .50$, f_1 as above, f_2 as above

where Φ is defined for (a) one-way analysis, (b) two-way analysis, (c) Latin Squares. For example, for a Latin Square,

$$\phi^2 = \sum_k \delta_k^2/\sigma^2$$

for testing treatment effects, the basic linear model being

$$X_{ij} = \mu + \tau_i + \theta_j + \delta_k + \epsilon_{ijk}.$$

These charts (described by the author as "essentially a special condensation of the Pearson and Hartley charts") should be examined along with similar (power) tables and charts by Fox, [1], Lehmer [2], Pearson and Hartley [3], and the pathbreaking paper by Tang [4].

HAROLD FREEMAN

Massachusetts Institute of Technology
Cambridge, Massachusetts

1. MARTIN FOX, "Charts of the power of the F-test," *Ann. Math. Stat.*, v. 27, 1956, p. 484–497. [*MTAC*, v. 11, Rev. 28.]
2. EMMA LEHMER, "Inverse tables of probabilities of errors of the second kind," *Ann. Math. Stat.*, v. 15, 1944, p. 388–398.
3. E. S. PEARSON & H. O. HARTLEY, "Charts of the power function for analysis of variance tests derived from the noncentral F-distribution," *Biometrika*, v. 38, 1951, p. 112–130. [*MTAC*, v. 8, 1954, *RMT* 1183, p. 84.]
4. P. C. TANG, "The power function of the analysis of variance test with tables and illustrations of their use," *Stat. Res. Memoirs*, v. 2, 1938, p. 126–149 + tables.

104[K].—BRUNO DE FINETTI, "Una legge riguardante l'estinzione nei processi di eliminazione," *Giornale dell' Instituto Italiano degli Attuari*, v. 15, 1953, p. 94–99.

A population, initially of n elements, is subjected to successive trials with elimination probability q (for each individual, each trial) until complete extinction. The author gives a recurrence formula and an exact series formula for $P_m{}^n$ = the probability that the number of survivors eliminated at the last step (before extinction) is m, $m = 1, 2, \cdots, n$. An approximate formula is derived, $P_m = q^m/m|\log(1 - q)|$, which, [1], is not the limit of $P_m{}^n$ as n becomes infinite. As is shown in [1], $P_m{}^n$ approaches a periodic function of $\log n$ for whose values P_m is an average for large n.

A table of $P_m{}^n$ is given for the cases through $n = 10$ for $q = \frac{1}{2}$. Exact expressions are indicated and values (quotients) are given up to maximum 6S. There is an error of $+1$ in the least significant digit for $P_6{}^6$. Round-off is uniform over n for fixed m but (starting at $n = 3$) works to the disadvantage of $\sum_m P_m{}^n = 1$. It is of interest to note that for the case of $q = \frac{1}{2}: P_3^3 = P_2^3; P_9^{10} < P_8^{10} < P_7^{10} < P_{10}^{10}$.

W. H. MARLOW

The George Washington University
Logistics Research Project
Washington, D. C.

1. GIUSEPPE OTTAVIANI, "A proposito della legge di estinzione nei processi di eliminazione," *Giornale dell' Instituto Italiano degli Attuari*, v. 15, 1953, p. 100–114.

105[K, W, Z].—ALISON DOIG, "A bibliography on the theory of queues," *Biometrika*, v. 44, 1957, p. 490–514.

An extensive and valuable bibliography on the theory of queues including related topics and papers devoted to computational aspects of the subject.

C. B. T.

106[L].—K. A. KARPOV & S. N. RAZUMOVSKI, *Tablit͡sy integral'nogo logarifma* (*Tables of the logarithmic integral*). Akad. Nauk SSSR, Vych͡islitel'ny T͡sentr, *Matematich͡eskie Tablit͡sy* (Acad. Sci. USSR, Computational Center, *Mathematical Tables*). Izdatel'stro Akad. Nauk SSSR (Press of the Academy of Sciences of the USSR). Moscow, 1956. 320 p., 26.7 cm. Price 33.30 rubles.

The main table (pages 11–311) lists values of the logarithmic integral

$$\mathrm{li}\, x = \int_0^x dt/\ln t \quad (x \geq 0),$$

understood as a principal value when $x > 1$, to 7S for

$$x = 0(.0001)2.5(.001)20(.01)200(.1)500(1)10{,}000(10)24{,}990.$$

There are no differences except in the first five pages ($x \leq .5$), where first and second differences are provided.

Pages 312–315 contain an auxiliary table of $\text{li}\, x - \ln|1 - x|$ to 6D without differences for $x = .95(.0001)1.0499$. Page 316 tabulates $\frac{1}{2}t(1 - t)$ to 4D without differences for $t = 0(.001)1$.

The main table was computed on the high-speed electronic calculating machine (BÉSM) of the Academy of Sciences. It was found expedient to use, chiefly, numerical quadrature by Simpson's rule, reserving the formula $\text{li}\, x = \text{Ei}\,(\ln x)$ for the computation of pivotal and checking values. The error is stated not to exceed 0.6 final units. A correction slip pasted in gives the following correction on p. 186:

$$\text{li}\, 25.82, \quad \textit{for} \quad 0.1175643 \cdot 10^2 \quad \textit{read} \quad 0.1176643 \cdot 10^2.$$

Although the range $0 \leq x \leq 24{,}990$ does not include the high values (up to at least $x = 10^8$) of interest in number theory, and although extra figures may be found elsewhere for a few arguments, this is nevertheless by far the largest table of the logarithmic integral available. It will be noticed that throughout the interval $.0100 \leq x \leq 24{,}990$ there is always at least a three-figure argument, and that the main table runs to 300 pages. None of its predecessors comprises more than a page or two. In these circumstances one cannot seriously dissent from the claim that this is the first table of the logarithmic integral which is anything like complete. It can hardly fail to be found of use.

On pages 318–319 are given corrections to four other tables published in the same series in 1954 and 1955.

A. F.

107[L].—E. N. Dekanosidze, *Tablit͡sy t͡silindricheskikh funkt͡siĭ ot dvukh peremennykh* (*Tables of cylinder functions of two variables*). Akad. Nauk SSSR, Vychislitel'ny T͡sentr, *Matematicheskie Tablit͡sy* (Acad. Sci. USSR, Computational Center, *Mathematical Tables*). Izdatel'stvo Akad. Nauk SSSR (Press of the Academy of Sciences of the USSR), Moscow, 1956. 494 p. + 2 leaves of diagrams, 26.7 cms. Price 50 rubles.

These tables were computed at the Institute of Precise Mechanics and Computational Technique on the initiative of Professor P. I. Kuznet͡sov. The functions tabulated are Lommel functions of two variables, $U_\nu(w, z)$, $V_\nu(w, z)$, which are related to Bessel functions and were first introduced in connection with the study of diffraction. Apart from the use of ν and w instead of n and y, the functions are as described in *MTAC*, v. 1, 1944, p. 258 and FMR *Index*, p. 301.

The tables (pages 11–492) are for $\nu = 1$ and 2, the four functions U_1, U_2, V_1, V_2 being listed in parallel columns. All values are to 6D and there are no differences. There is a separate table for each of the following values of w:

$$w = .5(.02)1.2(.05)4(.1)10.$$

For $w \leq 6.2$, the second variable takes the values of $z = w(.01)W$, where W is always fairly close to $4\sqrt{w}$; for $w \geq 6.3$, on the other hand, $z = w(.01)10$. The resulting region of tabulation in the (w, z) plane is illustrated in Figure 3.

Various formulas are given for extending the scope of the tables. For example, computation for suffixes ν which are integers greater than 2 is possible if the present tables are supplemented by tables of Bessel functions J of integral order; negative values of w and z are covered by symmetries; while a further valuable resource is that, for integral ν, the U and V functions are interchanged if w is replaced by z^2/w. Asymptotic expansions for large w and z are given. Some numerical examples are worked out.

The computation of the tables is briefly sketched in mathematical terms, but nothing is said about the machine or machines used. The errors are stated not to exceed unity in the sixth decinal place. The mere four pages of the Introduction contain so many formulas that they will be largely intelligible to mathematicians without a knowledge of Russian.

A. F.

108[L, S].—Kurt Alder & Aage Winther, *Tables of the Classical Orbital Integral in Coulomb Excitation*, Mat. Fys. Medd. Dan. Vid. Selsk., Bind 31, No. 1, 1956, 74 p., 9 cm. Price 3.00 kr.

This paper contains a tabulation of certain functions which arise in the classical description of Coulomb excitation. The functions tabulated here are the orbital integrals $I_{\lambda\mu}(\nu, \zeta)$, defined by

$$I_{\lambda\mu}(\nu, \zeta) = \int_{-\infty}^{+\infty} \{e^{i\zeta[\epsilon \sinh w + w]}\} \frac{[\cosh w + \epsilon + i\sqrt{\epsilon^2 - 1} \sinh w]^{\mu} dw}{[\epsilon \cosh w + 1]^{\lambda+\mu}},$$

the differential cross-section functions $\dfrac{df_{E\lambda}(\nu, \zeta)}{d\Omega}$, and $\dfrac{df_{M\lambda}(\nu, \zeta)}{d\Omega}$ given by

$$\frac{df_{E\lambda}(\nu, \zeta)}{d\Omega} = \frac{4\pi^2}{(2\lambda + 1)^3} \sum_{\mu} \left| Y_{\lambda\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 |I_{\lambda\mu}(\nu, \zeta)|^2 \sin^{-4} \nu/2,$$

$$\frac{df_{M\lambda}(\nu, \zeta)}{d\Omega} = \frac{4\pi^2}{(2\lambda + 1)^2} \sum_{\mu} \frac{(\lambda + 1)^2 - \mu^2}{\lambda^2(2\lambda + 3)} \left| Y_{\lambda+1,\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \cdot |I_{\lambda+1,\mu}(\nu, \zeta)|^2 \cdot \cot^2 \nu/2 \cdot \sin^{-4} \nu/2;$$

and the total cross-section functions $f_{E\lambda}(\nu, \zeta)$ and $f_{M\lambda}(\nu, \zeta)$ given by

$$f_{E\lambda}(\zeta) = 2\pi \int_0^{\pi} \frac{df_{E\lambda}(\nu, \zeta)}{d\Omega} \sin \nu d\nu$$

$$f_{M\lambda}(\zeta) = 2\pi \int_0^{\pi} \frac{df_{M\lambda}(\nu, \zeta)}{d\Omega} \sin \nu d\nu.$$

The tables of the orbital integrals were obtained by numerical integration using the high-speed electronic computer BESK at Stockholm.

The orbital integrals were calculated for the following values of the parameters:

$$\nu = 10°(10°)180°$$

and

$$\zeta = 0.0(0.1)1.0(0.2)2.0, 4.0 \quad \text{for } \lambda = 1 \text{ and } 2$$
$$\zeta = 0.2(0.2)1.0(0.5)2.0, 4.0 \quad \text{for } \lambda = 3 \text{ and } 4,$$
$$\mu = -\lambda, -\lambda + 2, \cdots \lambda.$$

The tables are said to be accurate to the number of decimals given, which is six, except for small $\nu(10°)$.

The differential and total cross section functions are tabulated for the following parameter values:

$$\frac{df_{E\lambda}(\nu, \zeta)}{d\Omega} \text{ and } f_{E\lambda}(\zeta) \quad \text{for } \lambda = 1, 2, 3, \text{ and } 4$$

$$\frac{df_{M\lambda}(\nu, \zeta)}{d\Omega} \text{ and } f_{M\lambda}(\zeta) \quad \text{for } \lambda = 1 \text{ and } 2.$$

In the introduction to the tables properties of the orbital integrals are described. A review of the Coulomb excitation process wherein these integrals are discussed is given in the following publication: K. Alder, A. Bohr, T. Huus, B. Mottelson and A. Winther, *Rev. of Mod. Phys.* v. 28, 1956, p. 432.

Copies of these tables may be obtained from

Secretary
Det Kongelige Danske Videnskabernes Selskab
Dantes plads 5
Copenhagen V, Denmark,

at a cost of 10.00 kr.

LLOYD D. FOSDICK

University of Illinois
Urbana, Illinois

109[L, V].—JACK N. NIELSEN, *Tables of Characteristic Functions for Solving Boundary-Values Problems of the Wave Equation with Application to Supersonic Interference*, Report NACA TN 3873, 1957, 245 p., diagrams and Tables, 26 cm.

The present Technical Note deals with the problem of calculation of the pressure distribution on quasi-cylindrical bodies of revolution including the different possible cases of interference. The principles of the referring theory had been outlined in detail in several previous reports of Jack N. Nielsen and other authors (see References). This theory contains the so-called W_m-functions, which had been tabulated for different values of the quantities x (streamwise distance) and 1 (radial distance of unity corresponding to the body surface), W_0 occurring in the formula for pressure coefficient at the body surface, the higher order W-function being needed in connection with a calculation of interference pressure fields.

Nielsen stated some disadvantages of these $W_m(r, 1)$-functions, if the external pressure field (and not only the pressure at the body surface) was to be calculated. In this case singularities occurred and the results obtained as differences of two large numbers became inaccurate. Therefore a new method was developed based on a more general set of functions $W_m(x, r)$, r being the radial distance from body center line. Nielsen found that with this new function it is as easy to compute the pressures away from the body as those on the body.

The new $W_m(x, r)$ functions are defined as the inverse Laplace transformation of a function containing modified Bessel functions of the second kind:

$$W_m(x, r) = L^{-1}\left[e^{s(r-1)} \frac{K_m(sr)}{K_m'(s)} + \frac{1}{\sqrt{r}}\right].$$

They are the solution of the partial differential equation

$$\frac{\partial 2 W_m}{\partial r^2} + \frac{1}{r}\frac{\partial W_m}{\partial r} - \frac{\partial 2 W_m}{\partial \eta^2} - \frac{m^2}{r^2} W_m = 0$$

with the boundary condition

$$W_m = \frac{(1/8) - (m^2/r)}{r^{\frac{3}{2}}} + \frac{(3/8) + (m^2/r)}{r^{\frac{1}{2}}} \quad \text{at} \quad \eta = x - r + 1 = 0$$

$$\frac{\partial W_m}{\partial r} = 0 \quad \text{at} \quad r = 1.$$

The author discusses the different methods of evaluating this formulae particularly adapted to automatic computation. He found that the following 3 methods if used in a combined manner are to be considered as the most suitable one:

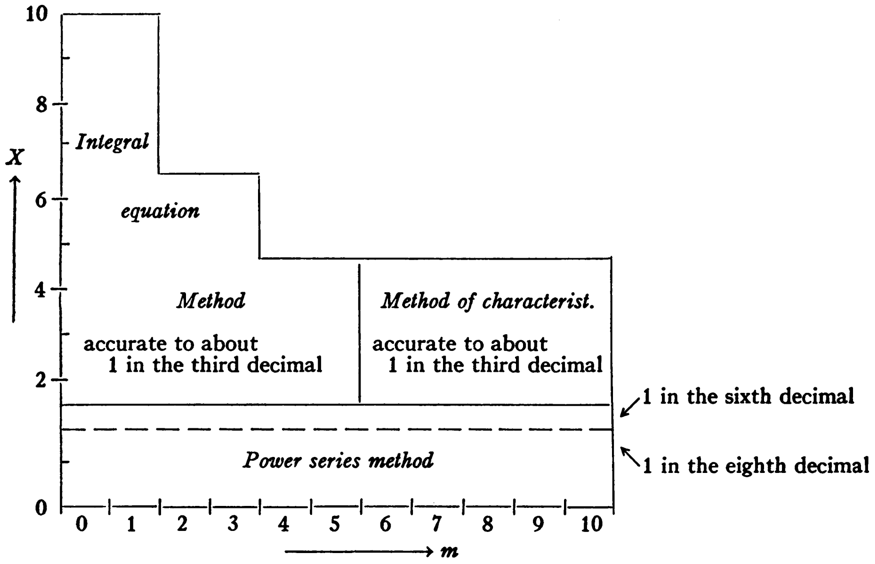
1. Power series in x with coefficients depending on m and r
2. Integral-equation methods
3. Characteristics using finite difference equations

Thus the calculated tables of $W_m(x, r)$ cover a range of the order m from 1 to 10. The range of x is less for the higher order functions than the lower order functions because they become smaller faster. The actual ranges of the parameter x and r are

m	x	r (all values for each value m and r)
0 1	0 to 10	1, 1.1, 1.25, 1.5
2 3	0 to 2	2.0, 3.0, 4.0
4 5 6 7 8 9 10	0 to 5	6.0, 8.0, 10.0

On the whole 69,000 values are contained in the prescribed tables.

The author has intensively investigated the question of the accuracy and came to the following results;



As to the physical significance of the $W_m(x, r)$ function, the author explains that these functions are to be considered as cylindrical pressure waves associated with a step in the radius of a streamwise quasi-cylindrical body.

The author finally interprets the application of the $W_m(x, r)$ functions to some aerodynamic examples which partly have hitherto not appeared in the literature, as

1) Calculation of the pressure field due to an indented body
2) Flow field of an infinite cylinder impulsively moved in axially direction in a viscous incompressible field
3) Wave drag of corrugated bodies
4) Arbitrary quasi-cylindrical body alone
5) Body-body or shock-body interference
6) Wing-body interference
7) Wortex-panel interference
8) Non-aerodynamic applications

The reviewed Technical Note represents without doubt a very remarkable contribution for a general solution of boundary value problems of the second kind involving the wave equation in three dimensions with approximately circular cylindrical boundaries or involving the unsteady heat-conduction equation in two space dimensions with nearly circular boundaries.

Reference is made on some publications which deal with the same or with similar problems; listed below are some of the most important.

ALFRED WALZ

National Bureau of Standards
Washington, D. C.

1. JACK N. NIELSEN, "Quasi-cylindrical theory of wing-body interference at supersonic speeds and comparison with experiment," NACA Report 1252, 1955.

2. JACK N. NIELSEN, "General theory of wave drag reduction for combinations employing quasi-cylindrical bodies with an application to swept-wing and body combinations," NACA TN 3722, 1956.

3. M. J. LIGHTHILL, "Supersonic flow past bodies of revolution," R. & M., No. 2003, British A. R. C., January 3, 1945.
4. D. G. RANDALL, "Supersonic flow past quasi-cylindrical bodies of almost circular cross-section," British R. A. E. TN Aero 2404, November, 1955.
5. G. K. BATCHELOR, "The skin friction on infinite cylinders moving parallel to their length," British A. R. C. 15,105, Fluid Motion Subcommittee F. M. 1772, Aug. 6, 1952.

110[P, Z].—W. W. DOBROWOLSKI, *Theorie der Mechanismen zur Konstruktion ebener Kurven*, Akademie-Verlag, Berlin, 1957, viii + 134 p., 24 cm. Price DM 19, bound.

This work was part of an extensive symposium on mechanisms and machines and appeared in Trudy Seminara Teorii po Mashin i Mekhanizmov 9, no. 36 (1950), Akademiya Nauk SSSR.

The constructions of projective geometry are implemented in mechanical devices to draw curves. A point constrained to a line is replaced by a pin constrained in a straight slot. A line passing through a fixed point is replaced by a slotted bar pivoted about a fixed pin. The transfer of angular rotation from one line to another is accomplished by locking the opposite sides of a parallelogram linkage to the rotating lines. The demonstrations are quite detailed and elementary with very little use of algebra. Some of the mechanisms have more theoretical than practical value.

A conic section can be drawn by a mechanism which simulates Pascal's theorem. If five points A, B, C, D, E of the conic are given, the variable sixth point M traces the conic. Fixed slots through BE and EC are used to guide pins which move, also, in three rotating slots which are hinged at H (the intersection of AB and CD) and points A and D. If the pin L lies on the triplet of lines (BE, HL, DL) and the pin K on (EC, HL, AK), then the pin M will be the intersection of AK and DL. Variations of the mechanism are shown corresponding to special cases of the theorem, namely, when any five elements are given (points and tangents).

Third and fourth degree curves are generated by compounding the simpler quadratic linkages, or by the use of four-bar and crosshead mechanisms. Mechanisms for quadratic transformations and Cremona transformations are described. Some higher degree curves, like the Lissajous figures, can be made by a combination of two functions of the same parameter. An inversor mechanism can convert a curve into another of higher degree.

MICHAEL GOLDBERG

Bureau of Ordnance
Navy Department
Washington, D. C.

111[S, X, Z].—S. FLUGGE, Editor, *Handbuch der Physik* (*Encyclopedia of Physics*), Band 1, Mathematische Methoden I (Mathematical Methods 1), Springer-Verlag, Berlin, 1956, vii + 364 p., 24.5 cm. Price DM 72.

[S, X, Z].—S. FLUGGE, Editor, *Handbuch der Physik* (*Encyclopedia of Physics*), Band 2, Mathematische Methoden II, Springer-Verlag, Berlin, 1955, vii + 520 p., 24.5 cm. Price DM 88.

The two volumes contain as complete a compendium of useful mathematical methods and theorems as one is ever likely to compile at any one time as part of an encyclopedia of physics. It would be impracticable to give more than a brief indication of the contents and depth of the separate sections prepared by various authors.

The first three sections of Volume I were written by Josef Lense and cover (a) calculus, Lebesgue integration, complex variable theory, ordinary differential equations—89 pp.; (b) partial differential equations culminating in the classification of the semi-linear second order equation in two independent variables as elliptic, parabolic or hyperbolic and the Riemann method for solving the linear hyperbolic equation—30 pp.; and (c) elliptic functions and integrals—27 pp.

The next section on the special functions of mathematical physics was written by J. Meixner and gives a condensed but thorough development of the properties of all of the commonly used functions and relates them to the study of partial differential equations by the method of separation of variables—71 pp.

The final section of Volume I, by F. Schlögl, gives a detailed account of the techniques for studying the boundary value and eigenvalue problems for second order partial differential equations and includes the related aspects of the theory of integral equations and the calculus of variations (use is made of the Dirac delta function in studying the Green's function). Most of the material relates to elliptic equations and less space is devoted to the parabolic and hyperbolic equations—135 pp.

Volume II begins with a thorough treatment of selected topics in modern algebra by G. Falk which contains a discussion of linear algebra, the theory of group representations and the representation theory of algebras, with some brief indication of the application to mechanics—116 pp.

The second section is by H. Tietz and covers vectors, analytic geometry, projective geometry, differential geometry of curves and surfaces, Riemann geometry, tensor analysis, theory of spinors, contact and canonical transformations for a Hamiltonian system of equations—81 pp.

The next section by I. N. Sneddon is the only one written in English and is devoted to a detailed study of many integral transforms, their properties and their application to typical physical problems. It is supplemented by a discussion of Lebesgue integration, a development of the theory of linear operators and functionals in Hilbert and Banach spaces, and a short description of the theory of distributions (of L. Schwartz)—51 pp.

The section on numerical and graphical methods is written by L. Collatz. He discusses the operation of the slide rule, nomography, smoothing of data, approximations of functions, location of roots of polynomials and more general functions, solution of linear and non-linear systems of algebraic equations by direct and iterative methods, determination of eigenvalues of matrices; finite differences, interpolation and numerical integration; numerical methods for ordinary differential equations, initial value problems and mixed initial-boundary value problems for partial differential equations, methods of Ritz and Trefftz; integral and functional equations. The descriptions of methods are accompanied by adequately worked out examples—122 pp.

The last section of 28 pages is by H. Bückner and describes the operation and logical structure of analogue and digital computers and the coding of problems.

The above studies are of high quality and should serve as a valuable guide to the branches of mathematics that are most usable today in classical and modern

Physics. The treatments are generally self contained, but references to the mathematical literature are given to make it possible to keep the size of these volumes manageable. Mention is made of the fact that the subject of Probability Theory is not contained in these books. Otherwise the two books are a well-rounded and remarkably well coordinated set of chapters in what might be called Applied Mathematics in spite of its high purity.

E. I.

112[V].—J. J. STOKES, *Water Waves*, Interscience Publishers, New York, 1957, xxviii + 567 p., 23 cm. Price $12.00.

This book gives a lucid, well written, and connected account of the theory of wave motion in incompressible fluids with a free surface and subjected to gravitational and other forces. The author pays special attention to some of the more recent work in the field to which he has made significant contributions during and since World War II. He has taken considerable pains to supply background material in hydrodynamics, some of the mathematical tools needed and an extensive bibliography.

There are four main parts of the book: Part I, comprising Chapters 1 and 2, presents the basic hydrodynamic theory for non-viscous incompressible fluids. It also contains the derivation of the equations of the two principle approximate theories which form the basis for the discussion in the remainder of the book. The first of these is a consequence of the assumption that the wave amplitudes are small. The second comes from the assumption that the depth of the liquid is small. In both cases the approximate theories are derived from formal expansions with respect to a small dimensionless parameter and methods are given for determining the higher order terms. The derivation of the shallow water theory follows that first given by K. O. Friedrichs.

Part II, which consists of Chapters 3 to 9 inclusive, describes the linear theory obtained when it is assumed that the wave amplitudes are small. The main mathematical tool used is potential theory. This part is divided into three subsections: (A) dealing with wave motions that are simple harmonic oscillations in time; (B) dealing with unsteady or transient motions that arise from initial disturbances starting from rest; and (C) dealing with waves on a running stream.

At the end of subdivision A, the author outlines other methods for dealing with boundary problems which differ from those he discussed at some length earlier. In particular, the Wiener-Hopf method is briefly discussed, and the author "hazards the opinion that problems solvable by the Wiener-Hopf technique will, in general, prove to be solvable more easily by other methods. . . ." This reviewer is among those who do not share this opinion.

Part III, consisting of Chapters 10 and 11, deals with shallow water theory, the approximation resulting from the assumption that the depth of the liquid is small. This assumption leads to a system of non-linear partial differential analogous to the equations describing the motion of compressible gases.

The first part of Chapter 10 contains a detailed discussion of the theory of characteristics based on the discussion given by Courant and Friedrichs in their book, *Supersonic Flow and Shock Waves*. Simple waves and propagating discontinuities are discussed. Chapter 10 contains a detailed discussion of a variety of concrete problems including conditions for the occurrence of a bore and a hydraulic jump, the motion resulting from the breaking of a dam, and the motion of frontal discontinuities in the atmosphere. It concludes with a discussion of some applications of the linearized version of the shallow water theory.

Chapter 11, entitled "Mathematical Hydraulics," deals with flows and wave motions in rivers and other open channels with rough sides. The fundamental equations derived in Chapter 10 are modified to include the force of resistance due to the rough sides which is taken to be proportional to the square of the velocity. This chapter contains an illustration of the length to which the author has gone to present portions of the book in such a way that they can be read, to a large extent, independently of the rest of the book. It contains a brief discussion of the method of characteristics which was given in Chapter 10; Figure 11.4.1, page 473, is identical with Figure 10.2.2, page 298.

A numerical method for calculating solutions of the differential equations is presented briefly. The method is based on the method of characteristics, but is applied in the x, t plane. The reader is referred to the paper of Courant, Isaacson and Rees for a discussion of the convergence of the method. No mention of stability questions is made.

The result of calculations for the prediction of an actual flood in the Ohio and at its junction with the Mississippi are reported on. A comparison of the predicted with the observed flood is also made. The author states "there is no doubt that this method of dealing with flood waves in rivers is entirely feasible since it gives accurate results without the necessity for unduly large amounts of expensive computing time on a machine such as the UNIVAC." The author also argues that such computations are cheaper and at least as good and as feasible as those made with the "analogue computers" previously used, namely scale models of long rivers or river systems.

Part IV, consisting of Chapter 12, discusses a few analytical solutions of the equations describing exact non-linear theory. This chapter deals with problems whose solutions can be expressed in power series. The book ends with an exposition of Levi-Civita's theory of progressive waves of finite amplitude in water of infinite depth which satisfy exactly the non-linear free surface conditions.

This stimulating book which contains a selection of material chosen because it interests the author provides an excellent treatment of many topics in the theory of incompressible flow. It will be useful to both students and research workers in the field.

A. H. T.

113[W, Z].—ANDREW D. BOOTH, L. BRANDWOOD, & J. P. CLEAVE, *Mechanical Resolution of Linguistic Problems*, Academic Press, Inc., New York, 1958, vii + 306 p., 21.5 cm. Price $9.80.

Modern electronic computers have been operating for more than ten years. Despite extensive experience with them and a host of innovations, little is known about their limitations.

It has been shown that some computers can play checkers, and some have played chess after a fashion, and some have even been forced reluctantly to learn a little. Each of these demonstrations throws some light on the ability of the computer, and each of them up to now has resolved the question in favor of ability rather than of limitation. The exploration of these limits is an outstanding scientific problem.

Another demonstration which in addition promises to be productive of useful results is that of translation from one language to another. The first of several demonstrations was made in 1953 by International Business Machines and Professor Dostert of Georgetown University. Others have followed. But it still remains to be shown that the machines can turn out useful translations.

Professor Booth has been thinking of this problem since 1947, and he and his colleagues have put into this somewhat uneven book a resume of their work and ideas.

There is a chapter on APEXC, the machine they have at the University of London. This is a modest machine, inadequate for the large task of translating languages, and the fact that they have successfully used it to demonstrate many of the steps of translating is a tribute to the ingenuity of the authors.

The basic processes of making concordances and using dictionaries are described. Catalogues of sentence structures can be made in the same way.

There is a chapter on the analysis of the style of Plato, a problem of interest to linguists, historians, and philosophers. Plato's style changed as he grew older, with one striking discontinuity at about age sixty. This change in style enables scholars to divide the works into two groups, those earlier and those later than the change. Refinements in these studies of style lead to extensive burdensome drudgery at which a computer should excel. This is an excellently written chapter on a highly technical subject.

Getting back to translating words rather than counting them, there is a discussion of the mechanics of breaking words into stems and endings in order to have short dictionaries. Compound words, such as abound in technical German, can also be broken. If they are not broken, the size of the dictionary gets out of all bounds. Words with multiple translations, such as English "but" into German "aber" or "sondern," or "know" into "kennen" or "wissen," must be handled with regard to the context. The two words on either side are frequently enough for this purpose. The comparison of the expenses of word splitting and full dictionaries is confused.

The methods by which the computer could do the work of stem recognition, identification of parts of speech, and acting on the dictionary information about syntax are described in terms of analogous operations with punched cards, which are apparently thought to be more picturesque than the arcane operations of a computer. The most time-consuming operation will be that of consulting the dictionary. Considerable thought has been given to this, with the conclusion that the best way will be a modification of the bracketing method, in which the middle entry of the dictionary is consulted first, thereby establishing which half contains the word, then this operation is repeated on the proper half, and so on. In the introduction Booth says that this method was first given a practical form in 1955, but the reviewer is sure that versions were in use before that time. Some estimates of numbers of steps needed are made based on Zipf's law (attributed to Estoup). Some methods of compressing words to fit in small boxes are discussed, but nothing about the origins of these ideas is said, nor is any evaluation of their usefulness made. The reviewer is under the impression that these are Russian ideas, and that they are probably not very useful. Ways in which the machine could resolve ambiguities by determining the subject matter are given. For instance a word which could be either "resistance" or "opposition" would be referred back to an earlier frequency count. This count of key words would determine that the subject was electricity, in which case the first would be used, or politics, whence the choice might be the latter. Very polished translations might be made by having a large collection of literary quotations, from which phrases of just the right flavor could be selected; to succeed this would require large storage and sophisticated programming, and a fast machine.

Braille II offers a fascinating opportunity to use the principles of mechanical translation against a simpler problem. Braille II uses a system of shorthand with a moderately complex set of rules, such as "do not use a contraction to bridge the components of compound words," or, some pairs of words can be treated as one word "where the sense permits." These rules can be reasonably well met with a dictionary of a thousand entries. Unfortunately—from the viewpoint of a computer enthusiast—this transcription is now being done by volunteers gratis, leaving no opportunity for the expensive machine.

Translation from French to English has been planned in some detail for APEXC. The small size of the memory permits only 250 French words in the vocabulary besides rules enabling the machine to recognize masculine from feminine, singular from plural, and most verb forms. Words are split into stems and endings. Not all the letters of a word need to be used; for example in *recevrai* only *rec* is needed to identify the stem and only *rai* the ending. If the vocabulary were more extensive more complication would be needed. There are plans for 46 verb endings. The authors assert that grammar and ambiguous endings are the only sources of real difficulty. No mention is made of how strange words will be handled. The procedures seem well thought-out, and sound as though they will work. We can look forward to seeing some examples of its output.

The chapter on translating German into English is quite in contrast to that on French. It is 162 pages long, more than half the book. It is not based on a machine experiment but only on an armchair study. Again in contrast with the chapter on French it looks toward the difficulties of the future rather than the opportunities of the present. The study is based on Oswald and Fletcher's "Proposals for the Mechanized Resolution of German Syntax Patterns" in *Modern Language Forum* XXXVI, 1951. Most of the examples of German used to illustrate the study are from a critique of Thomas Mann's novels and tend to emphasize the difficulties to be encountered rather than the infrequency of these unusual forms. One concludes that this untutored machine is to translate Mann at his deepest immediately or it will be considered a failure. Oswald and Fletcher have listed 82 syntactical units which can be used by a machine to analyze sentences. A second part of the study is the examination of a particular piece of text (7000 words) dealing with the specific subject of electron optics. Counts were made of the vocabulary and the syntactical rules which would need to be invoked. The number of words needed was 960. Each case of multimeaning, where in different contexts different English words are required, is taken up. Most of them are prepositions or adverbs. The variations in meaning are illustrated by *auf dem Tisch,* *auf dem Tanz*, and *auf dem Lande*, in which *auf* is respectively "on," "at," and "in."

There is a resume of an article by I. S. Mukhin, *Proc I. E. E.* **103** (B), 463–472, 1956, telling of a machine program for translating English into Russian. Translation in this direction is more difficult than in the opposite, for the machine must synthesize the endings of the Russian words from the word order of the English. Each Russian word in the dictionary has 17 associated grammatical facts. After all these facts have been marshalled the Russian sentence is constructed. It may even happen that a word with its 17 grammatical properties is suppressed. If an English word is capable of more than one Russian equivalent, depending on context, this fact is recorded in the dictionary from which the machine (which is BESM) then has recourse to a second and more elaborate dictionary to resolve the ambiguity. Idioms are handled the same way.

A very short chapter discusses the proposal of a meta-language or intermediate language understood only by the machine, the object being to simplify translating among many languages. With such a meta-language translation from any one of N languages to any other could be done by $2N$ programs. Without it $N(N - 1)$ might be necessary. The authors point out that if instead of a meta-language one of the N languages is selected to play a central role, then the number of programs is reduced to $2(N - 1)$, smaller yet.

Last there is some discussion of a possible computer designed especially for translating languages. Armchair daydreaming about computers with peculiar properties is not without its rewards. It commonly brings out the fact that the modern computer is nearly optimal, and that abandoning any of its main properties for other advantages is rarely attractive. In this case the authors suppose that since the dictionary will be changed only rarely the ability to selectively alter the memory can be abandoned (perhaps only in part, although this point is not clear) in favor of a very large store. Using glass discs very large dictionaries (several million words) with small access times are economically feasible. It appears later in the discussion that the authors would like to be able to compile concordances and word counts; this would require a selectively alterable store. With a serial or serial-parallel machine words of variable length are feasible and at the same time certain economies become possible. The authors also recommend replacing multiplication and division by other instructions. One such replacement would be a transfer of control conditioned on a comparison of two words. Another would be to locate and copy in the accumulator an entry from the dictionary. An interesting suggestion would enable the machine to insert an additional word into a list of words which was already monotonic non-decreasing. This would have the new word in the register; it would then iterate the operation of copying the word at x into $x + 1$ and then comparing it with the word in the register. If the word in the register is smaller continue with address x decreased by one; if it is large or equal copy from the register into $x + 1$ and go to the next operation.

Despite its uneven quality this book is an interesting exposition. It has a lively approach and promises exciting advances for the future. With enough effort machines can not only translate but can translate adequately, or even better.

H. H. Campaigne

National Security Agency
Research and Development
Fort George G. Meade, Maryland

114[X].—R. A. BUCKINGHAM, *Numerical Methods*, G. P. Putnam's Sons, New York, 1957, xii + 597 p., 21.5 cm. Price $15.00.

This book, very comprehensive and very well written, is a combination of a reference book and a text book on standard numerical methods. The emphasis is on techniques and procedures, the proofs are as a rule omitted. The author gives a very detailed discussion of most of the methods available with abundant references to the literature and each important technique is followed by an example with all the necessary comments. Every chapter contains a collection of problems. Here is the list of topics: An introduction to computation; Lagrangian methods of interpolation, differentiation and integration; Newton's methods of divided differences; Differences at constant interval; Symbolic methods; Solution of ordinary differential equations; Polynomial and other algebraic equations; Fitting of data by the method of least squates; Direct solutions of simulaneous linear equations; Matrices and determinantal equations; Indirect methods for linear equations; Ordinary differential and Fredholm integral equations; Functions of two variables; and Partial differential equations. The book ends with useful appendices on: Relations between powers and factorials; Summary of difference formulas with remainder; Lagrangian formulas for differentations and integration; Orthogonal polynomials for curve fitting; Index on useful formulas in text.

The reviewer noticed the following omissions: a very incomplete discussion on ill conditioned matrices and an absence of warning that normal systems of equations have a tendency to be ill-conditioned and are almost certainly ill-conditioned for systems of order six or greater. Other omissions, like the Runge-Kutta method, are mentioned by the author in the preface.

This book, in the reviewer's opinion, is probably the best on the subject and is highly recommended not only for mathematicians but for all scientists with any interest in computing.

T. LESER

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

115[X].—N. ARONSZAJN, A. DOUGLIS, & C. B. MORREY, JR., Editors, *Transactions of the Symposium on Partial Differential Equations*, Interscience Publishers, New York, 1956, vi + 334 p., 26 cm. Price $6.50.

The Program of the Berkeley Symposium on Partial Differential Equations, June 20–July 1, 1955, had two parts. One consisted of twenty hours of expository lectures on linear elliptic equations, linear hyperbolic equations, non-linear partial differential equations, and approximations to solutions of differential equations. The other was composed of thirty-two 45-minute lectures on a wide variety of topics. Twenty-eight of these papers, first published in *Communications on Pure and Applied Mathematics*, v. IX, No. 3, 1956, have been reissued in book form. A program of such length, with participants drawn from the leading centers of research in partial differential equations in the United States, not to mention contributions by a number of visiting scholars from abroad, would be expected to give a comprehensive picture of recent American acitivity in this field of mathematics. The book contains material to suit all interests, from casual beginners' to specialized experts', from proofs to conjectures. The subjects range from specific, concrete, but delightful results obtained by simple, subtle devices from very special properties of the particular problems considered, to broad general conclusions derived by very abstract techniques.

To discuss all twenty-eight papers adequately in a short review would be impossible. Readers interested in numerical analysis and applied mathematics would find material of fairly immediate usefulness and value in at least half of the papers. The present discussion will arbitrarily be restricted to these, and even they will be described essentially by titles. Three papers are explicitly devoted to numerical analysis. John Todd has given elegant systematic stability analyses for several methods for approximating parabolic and hyperbolic differential equations. H. F. Weinberger and G. E. Forsythe have determined bounds for the eigenvalues of vibrating membrane problems in terms of eigenvalues for corresponding difference equations. L. E. Payne's new isoperimetric inequalities represent another approach to the basic problem of the two preceding papers. Many other papers deal with techniques and results useful for solving problems in applied mathematics. Noteworthy among these are A. E. Heins' excellent account of the scope and limitations of the method of Wiener and Hopf and J. J. Stoker's discussion of the determination of appropriate radiation conditions for steady state problems by passage for the limit in related nonsteady wave problems. Special representations of harmonic functions by S. Bergman, polyharmonic functions by A. Huber, and of singular and regular solutions of Cauchy problems for Euler-Poisson-Darboux equations by J. B. Diaz typify results that find frequent application. Information such as that about the nature of discontinuities of solutions of quasi-linear equations discussed by Y. W. Chen, about singularities of generalized axisymmetric potentials considered by Erdelyi, and several papers of estimates, growth, and local behavior of solutions has obvious importance for both pure and applied mathematics. In this connection, L. Bers' very lucid survey of local properties of solutions of elliptic partial differential equations deserves especial mention.

John H. Giese

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

116[X].—F. B. Hildebrand, *Methods of Applied Mathematics*, Prentice-Hall, Inc., New York, 1956, xi + 523 p., 21 cm. Price $8.50.

"The principal aim of this volume is to place at the disposal of the engineer or physicist the basis of an intelligent working knowledge of a number of facts and techniques relevant to four fields of mathematics which usually are not treated in courses of the 'Advanced Calculus' type, but which are useful in varied fields of application. The text includes the result of a series of revisions of material originally prepared in mimeographed form for use at the Massachusetts Institute of Technology." The four fields are: Matrices, determinants and linear equations, Calculus of variations, Difference equations, and Integral equation; these are covered in chapters of about equal length. Each chapter is followed by an impressive collection of examples, about 100 to a chapter, and a list of references.

Readers who are mainly concerned with numerical analysis will find much useful material in this book. Some is not where it might be expected, e.g., the extremal properties of characteristic numbers of a matrix is discussed in the examples.

The appearance of the book suffers from a very narrow margin.

JOHN TODD

California Institute of Technology
Pasadena, California

117[Z].—A. GIET, *ABACS or Nomograms*, Philosophical Library, New York, 1956, ix + 225 p., 22 cm. Price $12.00.

This is a translation of a book recently published in France and still available in a French edition [1]. Without sophistication it tells how to construct the simplest nomograms—Cartesian nomograms (essentially contours sketched on a plane with cartesian or other coordinates), alignment charts, and a few others. There is no real mathematical treatment, and the work can be followed by anyone with knowledge of graphs similar to that obtainable in our secondary schools.

The presentation is mainly by working out examples, each of which is stated as a formula pertaining to some shop or engineering problem.

The utilitarian nature of the book is illustrated by an appendix consisting of several sections. One concerns the advantages of Cartesian nomograms, and is largely concerned with their retention of accuracy even when the material on which they are drawn is stretched, deformed, or torn; this permits photography without worry about shrinking of paper during development of the print, for example. A second section of the appendix concerns advantages of alignment charts—accuracy, ease of reading, ease of interpolation, and so on. Another section lists several common forms of relations ($f_1 + f_2 + f_3 = f_1f_2f_3$ is one type listed, where the functions f_i are to be taken into account by suitable choice of scales) with recommended types of charts.

This is a useful text for those who want to construct nomograms, which may be the cheapest, the most convenient and the most widely used computational aids. Hence it has some considerable importance.

As noted above, it has no theory or bibliographic references. Readers requiring more theory would probably be happier with any of several books listed in the bibliography below, say [9], [10], [7] and possible [1]. Some bibliographical references are found in all these and in [5]. A few helpful tables are included in [3]. Otherwise, the present book is similar in content to those listed below, and if it is chosen by a reader this choice must be based on arrangment of material, wealth of examples (and many of the others listed below use the method of exposition by worked examples), or simplicity of exposition.

While the reviewer does not object to the utilitarian nature of the book, he would have been more favorably impressed if a few examples of the greatest attainments in the art of nomography has been cited; the common General Radio reactance charts relating inductance capacity, reactance and frequency for sinusoidal voltages [11] and similar charts have saved electrical engineers enough time to pay for all the world's research on nomographs, and the nomograms by Rybner [12] are examples of a professional nomographer of a high level of competence attacking a problem of seeming forbidding extent. It seems unfortunate that such examples are not made available to the student reading Giet's book.

An abstract of the table of contents follows; there is no index.

I. Relations Between Two Variables

1. Diagrams
2. Scales

II. Cartesian ABACS

3. Relations Involving Three Variables
4. Superimposed Cartesian ABACS for Three Variables
5. Superimposed Cartesian ABACS for Four Variables
6. Superimposed Cartesian ABACS for Relations Involving n Variables ($n > 4$)

III. Alignment Charts

7. Parallel Coordinates
8. Graphical Representation of a Relation of the First Degree Between Three Variables
9. Standard Forms of Alignment Chart for Relations Between Three Variables
10. Charts for Four Variables
11. Charts for Three Variables Requiring the Use of Auxiliary Variables
12. Combined Charts for Several Equations Involving Common Variables

IV. Alignment Charts Not Based on Parallel Coordinates

13. Charts with Concurrent Scales
14. Circular ABACS
15. Double-Alignment Charts for a Relation Between Four Variables of the Form $f_1f_2 = f_3f_4$

V. Relations Between n Variables ($n > 4$)

16. Relations of the Form $f_1 + f_2 + f_3 + \cdots + f_n = 0$
17. Relations of the Form $f_1 \cdot f_2 \cdot f_3 \cdots f_{n-1} = f_n$

Appendix

Choice of Methods: Choice of Type of ABAC

C. B. T.

1. W. J. ALLCOCK & J. R. JONES, *The Nomogram*, Pitman, London, 1946.
2. S. BRODETSKY, *A First Course in Nomography*, Bell, London, 1920.
3. D. S. DAIRS, *Empirical Equation and Nomography*, McGraw-Hill, New York, 1942.
4. MAURICE KRAITCHIK, *Alignment Charts*, van Nostrand, New York, 1944.
5. A. S. LEVENS, *Nomography*, John Wiley and Son, New York, 1948.
6. M. LIEN, *Nomografi*, Karlebo, Stockholm, 1945.
7. J. LIPKA, *Graphical and Mechanical Computation*, John Wiley and Son, New York, 1918.
8. C. O. MACKEY, *Graphical Solutions*, John Wiley and Son, New York, 2nd edition, 1944.
9. W. MEYER ZUR CAPELLEN, *Leitfaden der Nomographic*, Springer, Berlin, 1953.
10. M. W. PENTKOWSKI, *Nomographic*, Akademie, Berlin, 1953.
11. F. E. TERMAN, *Radio Engineers' Handbook*, McGraw-Hill, New York, 1943.
12. J. RYBNER, *Nomograms of Complex Hyperbolic Functions*, Gjellerups, Copenhagen, 2nd edition, 1955. [*MTAC*, v. 11, 1957, Rev. 80, p. 207.]

118[Z].—T. E. IVALL, Editor, *Electronic Computers*, Philosophical Library, Inc., New York, viii + 167 p., 22 cm. Price $10.00.

This book is a non-mathematical introduction to the principles and applications of computers employing tubes and other electronic devices. The treatment has been made very general in order to provide a broad background of the entire field of computing.

The first chapter of the book discusses some of the outstanding contributions made to the computing field over the past three hundred years.

The second chapter discusses the general principles of computing. Comparisons are made between pencil and paper, desk calculator, and electronic computer methods, in doing a particular type of mathematical operation.

The next three chapters are devoted to analogue computers. Typical circuits of analogue computers are explained in general terms. Descriptions are given on how the circuits operate, how they are constructed, and combined to fulfill the requirements for a particular application. The types of inputs required, scaling, types of outputs and some of the applications for which an analogue computer are best suited are described.

The remaining six chapters are devoted to digital computers. The basic arithmetic operations are discussed along with some of the different types of circuits required to perform these operations. Storage systems such as magnetic drums and discs, magnetic tape, magnetic core arrays, and cathode ray tubes are discussed and comparisons are made between the relative speed of the different systems.

The applications of digital computers to data processing, machine tool control, production planning are discussed in general terms.

The book gives the reader a painless introduction to the principles of operation and application of high-speed electronic computers and computer components. A background in mathematics or a previous knowledge of computer circuitry is not required for a thorough understanding of the contents. In clear and concise terms, the author covers the subject in eleven well-written chapters. The book is written primarily for electronic technicians and students. Engineers in other fields who wish to become acquainted with the subject of computers will find the text both interesting and informative. The book lays a firm foundation for subsequent detailed reading in the digital computer field. Persons in the sales, managerial, administrative, and executive fields would find it quite helpful. It is not intended, however, that the book be of use to engineers who are well informed in computing techniques and equipment. The excellent illustrations of components and circuitry serve well as an aid to comprehension.

CHESTER H. WALLIN

Computing Laboratory, BRL
Aberdeen Proving Ground, Maryland

119[Z].—R. K. LIVESLEY, *Digital Computers*, Cambridge University Press, London, 1957, viii + 53 p., 21 cm. Price $1.75.

This slim volume of 53 pages is published in the Cambridge engineering series. It amounts to a brief introduction to digital computers for engineers who are prospective users of machine. The author starts by analyzing a simple calculation (the solution of three simultaneous linear algebraic equations in three unknowns) to show what arithmetical and logical steps are involved. He then describes a simple computer which could carry out this calculation. Next a brief and sketchy treatment is given of the physical equipment needed for memory and input-output units; no attempt is made to show how control and information processing functions are actually carried out. There follows a chapter on programming, stressing the utility of subroutines and showing how they can be organized into complete programs. The book is concluded by a chapter giving a brief sketch of typical engineering problems requiring extensive computations.

The writing is clear, unpretentious, and easy to follow. The book could be understood, with a little additional explanation, by an interested reader of limited scientific background.

C. V. L. S.

120[Z].—G. A. Montgomerie, *Digital Calculating Machines*, Blackie and Son Limited, Glasgow, Scotland, 1956, vii + 262 p., 22 cm. Price 30s. net.

This is an elementary practical guide without pretentions of scholarship and without efforts to romanticize computers. The material covered includes all common types of computers, but much is sparse, particularly with regard to larger computers. Illustrations of common machines are included, but nothing like complete coverage of machines available (particularly those easily available outside but not within the United Kingdom) is attempted.

Additional references are given at the end of each chapter; the author's view that reference material for most topics is not useful or is hard to obtain is not entirely shared by the reviewer, but, on the other hand, it seems to the reviewer that by the time a reader has advanced to a point where he needs such references they may become known through other contacts. In any case the bibliographic references are not complete.

In short, what is put together is an elementary work giving the first principles of operation of the various types of machines discussed. It is a practical work, although it does not group and exhibit recipes for particular calculations. Square roots are taken by repeated subtraction on page 95, for example, with a suggestion that one of the methods described later in the book be used for most work. On page 117, square roots are again mentioned and logarithms are suggested. Finally, on page 141 the common iteration, $x_n = \frac{1}{2}(x_{n-1} + a/x_{n-1})$ is introduced as the best way to find $a^{\frac{1}{2}}$, but never is there given a rule for continuing by this method an extraction started by successive subtraction, which is easily brought about on some machines. Still, such methods are usually (and properly) brought to users' attention by the manufacturers of the various machines, and description in a small book such as the present with no reference to a laboratory instrument would entail difficult exposition.

There is an index of processes treated, on pages 143–144, and this has some of the uses of a set of recipes.

Analysis of strong and weak points of machines and their applications like that in [1] is missing almost entirely. The Mechanical principles are treated, but not deeply, or to the extent some are described in [2].

Thus, what is put together is a sound but not advanced text, neither deep nor encyclopedic in nature, but containing material not easily available elsewhere in English. It is mainly useful in connection with manually operated machines, here mainly presenting a description for those who have not yet used such machines to any great extent. It describes many but not all types of machines. There is undoubtedly a set of readers who will find such a text more useful than [2], [3], [4], [5] or [6], for example.

The table of contents follows:

Chapters

I. Introduction
II. Key-responsive Adding Machines (Group I, Type 1)
III. Key-set Adding Machines (Group I, Type 2)
IV. Four-rule Machines, Lever-set, Hand-operated (Group I, Type 3)
V. Four-rule Machines, Key-set and Electrically-operated (Group I, Type 3, cont.)
VI. Four-rule Automatic Machines (Group I, Type 4)
VII. Computing with Four-rule Machines
VIII. Punched-card Machines (Group II)
IX. Special-purpose Machines (Group III)
X. Universal Sequence-controlled Machines (Group IV)
XI. Electronic Sequence-controlled Machines (Group IV cont.)
XII. Programming of Sequence-controlled Machines

C. B. T.

1. L. J. Comrie, "The application of commercial calculating machines to scientific computing," *MTAC*, v. 2, 1946, p. 149–159.
2. F. J. Murray, *The Theory of Mathematical Machines*, King's Crown, New York, 1947 and (revised) 1948.
3. F. A. Willers, *Mathematische Maschines and Instrumente*, Akademie, Berlin, 1951.
4. F. A. Willers, *Mathematisch Instrumente*, Gruyter, Berlin, 1926.
5. W. Meyer zur Capellen, *Mathematische Instrumente*, Akademie, Leipzig, 1949.
6. E. Hersburgh, *Modern Instruments and Methods of Calculation*, Bell, London, undated.

121[Z].—Edmund C. Berkeley & Lawrence Wainwright, *Computers, Their Operation and Applications*, Reinhold Publishing Corporation, New York, 1956, ix + 366 p., 22 cm. Price $8.00.

The authors admit in their preface that it is a hopeless task to gather all the information about computing machinery. Their approach is to "do the best one can to report information already learned—incomplete as it is—indicating areas where other information is needed." The appeal is to the general reader, and it is truly unfortunate that the book is so disorganized as to be confusing to the novice in the field.

One assumes from the first chapters that the approach is for the non-expert, yet the section on analog equipment (written by the second author) does not hesitate to use derivatives and other mathematical operators with the assumption of previous knowledge on the reader's part.

The peculiar mixture of the general and the specific which is the hallmark of the first author leads him to make some misleading and some inaccurate statements. A few quotations will perhaps suffice to show what is meant.

On the operation of overflow he states on page 41: "Another useful operation is *overflow*, in a counter; this is the production of a number in the counter which is beyond its capacity, giving rise to a carry signal that can be employed for useful purposes. This would happen for example in a six-place counter holding 999784 if 4000 were added."

Unfortunately, he says nothing more about the usefulness of overflow, nor what purposes the carry signal could be employed for. In addition, the quotation serves to mar the distinction between adders and counters, which he does not clarify elsewhere.

On accuracy he states on page 58: "An automatic digital computer is surprisingly accurate. One of the earliest machines, the Harvard IBM Mark I, represented numbers of 23 decimal digits. . . . " This, by the author's own definitions in the glossary appended to the book, is not accuracy, but precision. A few sentences further on he repeats the error: "Accuracy to such a high degree is not often useful." Again it is precision which is meant. Finally to conclude this exhibition, and make "confusion worse confounded," we have: "One machine, the IBM Electronic Selective Sequence Calculator, . . . calculated the motion of the moon with new and remarkable *accuracy*, locating the moon *precisely* in the heavens at six-hour intervals for four centuries." (Italics are the reviewer's.)

In Section IV he deals with types of automatic computing machines that are not digital or analog computers. In drawing this artificial distinction he only succeeds in completely confusing the reader. He lists, among other non-computers on page 162, an astronomical telescope aiming equipment to adjust the direction of a telescope to follow a particular star field. This is as clear an example of an analog computer as one would wish, and the author himself says so on page 293: "And automatic analog computers are responsible for keeping a telescope pointed at a heavenly body while the astronomer watches it through the eyepiece."

It would have been far better to avoid such artificial distinctions if he was not able to be consistent in his definitions. Such loose thinking could be demonstrated by many more references to the text.

Section III, written by Lawrence Wainwright, is, by contrast, "a good deed in a naughty world." It is a clear elementary exposition of the functions and elements of analog computers, with examples of various types. The book would be worthwhile if it were all on a par with this section.

Section VII, on applications, spends a good deal of space on arguments as to the usefulness of computers in fields where they have long made their mark. The actual descriptions of applications in some fields are very terse. In mathematics, the whole field of numerical analysis is ignored. Computers, according to this author, have determined whether numbers are prime, and calculated the values of π and e to 2000 decimal places. The basic importance of being able to ask the computer the right questions so as to get the right answers is nowhere stressed. Even the fact that computers have opened a whole new realm of unsolved problems is glossed over.

The book concludes with a section of references, a roster of organizations and computing services, and a glossary. The index reveals no mention of the name of von Neumann, an omission which the author should find it hard to justify.

I have rarely read a book which could make me angry by the mere style of the author; this one does. I have tried to approach it with the objectivity of a person new to the subject, as a general reader might. Even this did not help; the fuzzy thinking obscured all the difficult parts, and the "cute" analogies which compare a computer to a large railroad yard with switching stations are of no avail in the serious business of understanding what a computer really is.

F. HOLLANDER

University of California
Los Angeles, California

122[Z].—GEORGE F. FORBES, *Digital Differential Analyzers, An Applications Manual for Digital and Bush Type Differential Analyzers*, Third Edition, Published by the author, Pacoima, California, 1956, xii + 154 + 19 p. of Appendices and 9 p. of index sheets, bound, soft cover. Price $7.50.

[Z].—GEORGE F. FORBES, *Digital Differential Analyzers, An Applications Manual for Digital and Bush Type Differential Analyzers*, Fourth Edition, Published by the author, Pacoima, California, 1957, iii + iv + 154 + 22 p. of Appendices and 8 p. Index sheets. Price $5.00.

This book is strictly as described in the title, an applications manual. A very brief introduction to the principles of digital differential analyzers is given in Chapters I and II, not sufficient to explain their operation fully to one unacquainted with devices of this type. However, this is not a serious defect, since the detailed information on the use of these machines contained in the remaining twenty-seven chapters will only be of interest to those who really understand them anyway. One's general impression is that the author presents this material carefully and precisely, and that it must be very useful to the audience to whom it is addressed.

The Third and Fourth Editions are essentially identical. The author states that some few corrections have been made in the Fourth, while the only additions are those of a few extra Exercises and a few extra items in the Bibliography.

C. V. L. S.

123[Z].—O. J. DAHL, *Autocoding for the Ferranti Mercury Computer (MAC)*, NDRE Report No. 24, Oslo, Norway, 1957, iii + 92 + 11 p. of appendices.

This is the description of the external features of an algebraic compiling system similar to FORTRAN written for the Ferranti Mercury Computer. In addition to the floating-point representation of numbers which is built into the machine, the system allows arithmetic operations on complex numbers, double-length floating-point numbers, and short integers. Programs are written as sequential lists of directives, or instructions, which may themselves be elaborate algebraic formulas composed of pre-defined variables and constants. For example, the directive to compute

$$x/2(1 + x^2)^{\frac{1}{2}} + \tfrac{1}{2}\ln\,(x + (1 + x^2)^{\frac{1}{2}})$$

is programmed as

$$(X/2)Q(1 + X^*2) + (/2)\mathrm{IEXP}(X + Q(1 + X^*2)) \rightarrow Y$$

where X denotes a single variable previously defined, Q the square root, IEXP the inverse exponential, * the power, + addition, and / division. Functions available directly in the symbolism are exponential, sin, cos, tan, square root and their inverses as well as more complicated functions such as integral and derivative. Other functions can be incorporated as subroutines.

The built-in B-registers are not referred to directly; a postfix method of indexing sets of variables is used. For example, the matrix element a_{ij} is written as An12 where index nl is used to count rows and n2 to count columns. For all programs the extents of fields of variables must be defined in a set of field foundations preceding the instructions.

The Mercury high-speed store consists of 16 pages of 32 floating-point numbers each. This is backed up by a magnetic drum from which a page at a time may be transferred. Large matrices, for example, must be stored on the drum and transferred as needed to the high-speed store. Matrices are frequently filled out to multiples of 32 by adding zero elements. A field in the H. S. store is referred to by a symbol such as FA, on the drum as FMA. Operations on an entire field in the H. S. store are possible without counting. For example, the directive 0→FA sets the whole field FA to zero. Facilities are also available for constructing compound logical statements. In the report, programs are shown for various matrix and vector operations, the solution of linear equations by the Jordan-elimination method, and there is a discussion of indexing for non-rectangular fields.

The normal method of looping, using counters and conditional jump instructions, can be used but a more sophisticated method, using operators, is also described. These operators cause the sequence of directives within their range of influence to be executed systematically for all possible values of an index stated in the operator.

The system is said to prepare programs comparable to manually coded ones. Storage allocation of program and data is still the responsibility of the programmer if these cannot all be contained in the H. S. store. No indication of experience with the system is given nor is there very much information about the mechanism of compilation. It appears to be a comprehensive scheme admirably suited to the needs of a seasoned programmer. The more elementary sections of the report would undoubtedly permit even the beginner to use the system.

J. N. P. Hume

University of Toronto
Toronto, Canada

124[Z].—O. J. Dahl, *Multiple Index Countings on the Ferranti Mercury Computer*, NDRE Report No. 23, Norwegian Defence Research Est., Oslo, Norway, 1957, iv + 43 + 10 p. of diagrams.

This report describes a systematic method of handling multiple countings in programs for problems such as linear algebra or tensor calculus. The form of counting depends on how a set, or field, of data is stored in the machine. The dimensionality of a field stored completely in the high-speed store is the number of indices describing an element of the field, e.g., 1 for a vector, 2 for a matrix. The range of an index is known as an extent of the field. In general an extent may be a function of other indices; fields whose extents are constant are rectangular fields. The H. S. store of a computer is essentially a one-dimensioned field and the problem as stated is "to store an n-dimensioned field in a linear one." For example, matrices can be stored as sets of linear subfields, end-to-end, e.g., row by row. Use of the secondary store, the magnetic drum, can increase the number of indices. For example, if a single column of the matrix occupies several sectors of the drum, countings must proceed both on sectors and elements within sectors. Various methods of partitioning matrices are discussed. Flow diagrams and Mercury Computer Programs are given for a variety of partitioning methods. In fitting a field into the machine storage, when it cannot be divided into an integral number of similar subfields, zeros may be added to complete subfields. It is found best to complete at least all H.S. store subfields with zeros allowing for incompleteness on the drum only. Countings may avoid operations on the zeros but the gain in time for large matrices appears to be offset by the higher degree of complexity in the innermost countings.

J. N. P. Hume

University of Toronto
Toronto, Canada

125[Z].—Lorenzo Lunelli, "Un comando di computo di cicli per una calcolatrice elettronica aritmetica a tre indirizzi," *La Ricerca Scientifica*, v. 27, 1957, p. 3381–3394.

This paper describes a "cycle computing instruction" added to the CRC-102 A digital computer located at the Centro di Calcoli Numerici of the Milan Polytechnical School. Essentially, the command combines counter modification (by subtraction of binary one from specified sections of the contents of a counter cell—any desired cell of the memory) with branch point on zero results in any section so specified. The feature of interest in the command is the implicit capability of combining several separate counts in a single cell with branching possible on each such count. The command is mechanized as follows: Let the command be represented by $Cm_1m_2m_3$. Then, m_1 is the counter cell; (m_2) specifies the division of (m_1) into separate counters by locating contiguous binary ones in the corresponding bit positions of (m_2) (individual counters are therefore separated by at least one bit); the command subtracts one from the least significant bit of each of those sections of (m_1) for which (m_2) has such a contiguous sequence of binary ones and replaces (m_1) by the result; m_3 specifies the location of the next command if none of the sections so specified becomes all zero; otherwise the next command is taken from the address following that of the command being executed. The use of this command is particularly effective for address modification in a repetitive subroutine. The paper includes illustrations of this type of operation.

An appendix to the paper describes a proposed command with somewhat similar purpose but different mechanization. The proposed command is similar in function to the familiar "B-box" except that it includes the modification of the B-box with the actual augmentation of the command, rather than with the branching decision. The proposed mechanization is as follows: Let the command be $Cm_1m_2m_3$; then m_1 is the location of the command to be executed; m_2 is the

"B-box," and its contents modify (m_1); (m_3) is the modifier of (m_2); the command results in the execution of the command stored in m_1 as modified by (m_2) and in the replacement of the original contents of m_2 by the result of $(m_2) + (m_3)$.

No time estimates are given for either of the two commands. However, the first one appears to be fairly cheap, requiring perhaps one or two word times more than a normal subtract command. On the other hand, the second, proposed, command requires four memory look-ups in addition to those of the command being modified and executed as part of the operation. Under many circumstances this would be very costly in time. These commands are quite representative of some of the complex built-in sub-routines which can fairly easily be added to a micro-programmed machine such as the CRC-102A. Such built-in sub-routines are much faster than the corresponding programmed sub-routine. On the other hand, it is characteristic of the micro-programmed approach that the resulting built-in sub-routines are slower than they would be if designed otherwise; the advantage of micro-programming is the relatively efficient use of equipment.

R. M. Hayes

Business Systems Department
Magnavox Research Laboratory
Los Angeles, California

TABLE ERRATA

265. Edwin P. Adams, *Smithsonian Mathematical Formulae and Tables of Elliptic Functions*, Second reprint, The Smithsonian Institute, Washington, D. C., 1947. See also *MTAC*, v. 1, p. 191, 325; v. 2, p. 46, 352–3; v. 3, p. 314, 423; v. 6, p. 236.

P. 122, formula **6.42,** no. 4, the sign before the $\sum$ should be $-$.

P. 127, formula **6.475,** no. 2, the formula is correctly printed in this reprint. See *MTAC*, v. 2, p. 46.

P. 139, formula **6.821,** in the numerator of the fraction in the second summation (involving sines), *for* 1, *read* n.

P. 139, formula **6.822,** the formula should read

$$e^{cx} = \frac{2c}{\pi}\left\{\frac{e^{c\pi}-1}{2c^2} - \sum_{n=1}^{\infty}\frac{1-(-1)^n e^{c\pi}}{c^2+n^2}\cos nx\right\} \quad [0 \leqq x \leqq \pi]$$

E. G. H. Comfort

Ripon College
Ripon, Wisconsin

266. F. J. Massey, jr., "Distribution table for the deviation between two sample cumulatives," *Annals of Math. Stat.*, v. 23, 1952, p. 435–441.

The entire table was recomputed using the IBM 701 electronic computer at the University of California's Computer Center, Berkeley, California. In addition to the following 46 errors there are approximately twenty other entries in which the last digit has been rounded incorrectly. The 16 corrections marked with an