REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

49 [F, Z].—MARTIN DAVIS, *Computability and Unsolvability*, McGraw-Hill Book Company, Inc., New York, 1948, xxv + 210 p., 23 cm. Price $7.50.

The following is a quotation from the author's preface.

"This book is an introduction to the theory of computability and noncomputability, usually referred to as the theory of recursive functions. This subject is concerned with the existence of purely mechanical procedures for solving problems. Although the theory is a branch of pure mathematics, it is, because of its relevance to certain philosophical questions and to the theory of digital computers, of potential interest to nonmathematicians. The existence of absolutely unsolvable problems and the Gödel incompleteness theorem are among the results in the theory of computability which have philosophical significance. The existence of universal Turing machines, another result of the theory, confirms the belief of those working with digital computers that it is possible to construct a single 'all-purpose' digital computer on which can be programmed (subject of course to limitations of time and memory capacity) any problem that could be programmed for any conceivable deterministic digital computer. This assertion is sometimes heard in the strengthened form: anything that can be made completely precise can be programmed for an all-purpose digital computer. However, in this form, the assertion is false. In fact, one of the basic results of the theory of computability (namely, the existence of nonrecursively enumerable sets) may be interpreted as asserting the possibility of programming a given computer in such a way that it is impossible to program a computer (either a copy of the given computer or another machine) so as to determine whether or not a given item will be a part of the output of the given computer. Another result (the unsolvability of the halting problem) may be interpreted as implying the impossibility of constructing a program for determining whether or not an arbitrary given program is free of 'loops.'

"Because it was my aim to make the theory of computability accessible to persons of diverse backgrounds and interests, I have been careful (particularly in the first seven chapters) to assume no special mathematical training on the reader's part."

The author succeeds admirably in presenting a readable, motivated, yet not verbose, exposition of the theory of recursive functions. The not mathematically mature reader will, however, find the reading slow, but this is probably intrinsic to the subject matter. The notion of a Turing machine is taken as basic, and the reviewer is gratified to note that the author sharply distinguishes what a Turing machine is from how it behaves. Up to the appearance of this book hardly any two articles on Turing machines employed exactly the same concepts of a Turing machine and functions computed by such a machine. This made it necessary to check that results established for one concept held for another. Since the concepts (mentioned above) employed by the author seem highly satisfactory it is to be hoped that the tendency to introduce even more variants will be attenuated.

While the book appears suitable for the classroom, this usefulness would have been greatly enhanced by the inclusion of exercises which, in part, would indicate alternative lines of development. The reader familiar with programming digital computers will note that construction of Turing machines is an activity similar to programming. One should be wary, however, in interpreting results concerning Turing machines as results concerning digital computers, for this implies, in particular, a willingness to endow the (idealized) digital computer with (potentially) infinite memory. For some purposes, at least, it is more appropriate to take as a mathematical model of a digital computer the notion of a finite (memory) automaton. The significance of this change in mathematical model is indicated by the fact that the analogue of the holding problem for finite automata is recursively solvable. This may be interpreted as implying the possibility of constructing a program for a Universal Turing Machine for determining whether or not an arbitrarily given program for a digital computer is free of "loops."

The chapter headings are: Computable Functions, Operations on Computable Functions, Recursive Functions, Turing Machines Self-Applied, Unsolvable Decision Problems, Combinatorial Problems, Diophantine Equations, Mathematical Logic, The Kleene Hierarchy, Computable Functionals, and The Classification of Unsolvable Decision Problems.

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50 [G, H, W, X, Z].—Mathematisches Labor der Technischen Hochschule Wien, *MTW Mitteilungen*, Vol. 5, 1958, 380 p., 24 cm. Price DM 15.

Of the 380 pages in this volume, two hundred are devoted to contributed articles, the remainder to various progress reports, reviews, news, etc. The relative proportions of these features are about the same as in earlier volumes [1]. The "literature reports" (references to new publications, arranged by subject matter) seem to have been discontinued after March, 1958.

Of the contributed articles, some are accounts of new research results, some are expository surveys, and some are reports on meetings or talks. The following are likely to be of interest to the computer mathematician. Most of the articles are in German; the titles below are translated.

W. Knödel: A Transportation Problem. The usual algorithm for solving transportation problems is applied to the Austrian sugar industry, using standard punched-card machines.

W. Spindelberger: Determination of complex roots of algebraic equations on small electronic computers.

W. Gröbner: Computation of powers of matrices. The mth power of an $n \times n$ matrix A, where m is considerably larger than n, is expressed as a polynomial $p_m(A)$ in A of degree at most n; more precisely, of degree at most d, where $d < m$ is the degree of the minimal polynomial of A. The coefficients of p_m can be found by recurrence with respect to m, if the coefficients of the minimal polynomial are known.

H. Rechberger and H. Sequenz: Basic problems in simulation of transients in reactors. A discussion of analog and mixed digital-analog systems for this problem.

K. Zuse: "Feldrechenmaschine." This is the name given by the author to a proposed logical design for a digital computer with single commands causing arithmetic operations on "fields of data," such as vectors or matrices.

E. Egervary: Remarks on the transportation problem. A theorem of the author has been used by H. W. Kuhn and M. Flood to develop an algorithm for the Assignment Problem of linear programming. In the present paper the author modifies the method of Kuhn and Flood for transportation problems with integer coefficients; these can be considered as special cases of assignment problems.

F. G. TRICOMI: Active problems in the theory of ordinary differential equations. This is a survey of the recent literature.

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1. *MTAC*, v. 13, 1959, p. 67–68 (Review 10).

51 [G, S].—B. J. SEARS & M. G. RADTKE, *Algebraic Tables of Clebsch-Gordan Coefficients*, Report AECL No. 746, 1954, II + 10 p., 27 cm. Available from Scientific Document Distribution Office, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada. Price $.50.

Prior to the development of quantum mechanics, the Clebsch-Gordan series played an important role in the theory of invariants of binary forms and elsewhere in group theory. In quantum mechanics, the coefficients in these series give the matrix components needed for the coupling of angular-momentum vectors. If two angular momenta having quantum numbers j_1 and j_2 and components m_1 and m_2 (all integral or half-integral) are coupled to give resultants j and m, the matrix component $(j_1\, j_2\, m_1\, m_2 \mid j\, m)$ is a Clebsch-Gordan coefficient.

The algebraic formula for these coefficients involves a complex sum; hence, to facilitate computation of the numerical values needed by theoretical physicists studying atomic and nuclear structure, algebraic tables were devised. The present volume contains six tables in which the values of j_1 are successively fixed at $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, and 3. The rows of the tables correspond to the permitted values of j, from $j_2 + j_1$ to $j_2 - j_1$ $(j_2 \geqq j_1)$, while the columns correspond to the values of m_1 from $+j_1$ to $-j_1$. The entries are comparatively simple algebraic expressions in j and m.

Only the table for $j_1 = \frac{5}{2}$ represents a new calculation from the definitions. The first four tables are rearrangements of those appearing in Condon and Shortley, *Theory of Atomic Spectra* (1935), while the last was computed by Falkoff, Collady, and Sells in 1952. The rearrangement by the present authors involves expressing the entries in terms of j and m instead of j_2 and m (actually j_1 and m in previous tables where j_2 was fixed), and leads to slightly simpler algebraic expressions but no essential reduction in time required for numerical substitution.

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52 [G, X].—FRANZ E. HOHN, *Elementary Matrix Algebra*, The Macmillan Company, New York, 1958, xi + 305 p., 24 cm. Price $10.00.

For several years the reviewer has taught beginning graduate courses in the computational methods of linear algebra, for which it is essential that the student have a strong knowledge of concrete linear algebra, of finite-dimensional linear analysis, and of the analytic geometry of hyperplanes and quadric surfaces in n dimensions. Even reasonably good students who have studied Birkhoff and MacLane [1] seem astonishingly unable to use the basic theorems of linear algebra in the concrete analysis of actual computations. As an example, students need a working familiarity with the Jordan canonical form of a linear transformation and its relation to the biorthogonal system of eigenvectors and principal vectors of the transformation. Without this knowledge it is quite impossible to understand, for example, the inner working of the Liebmann process or successive overrelaxation. Another important need is a familiarity with the principal vector norms of numerical analysis—the pth-power norms for $p = 1, 2, \infty$—and the associated matrix norms.

The reviewer is often asked to recommend suitable textbooks for remedial work in this area. Such books as Perlis [2] seem a little too elementary, and skip the analysis and geometry in favor of the algebra. Perhaps the best reference has been the first chapter of Faddeeva [3], until now available only in Russian or in an informal translation. And Faddeeva, good as it is in algebra and analysis, lacks the geometric point of view and has no exercises.

The reviewer therefore welcomed the arrival of Hohn's book as a new textbook in this field, not written by a pure algebraist (the author is an associate professor of mathematics at Illinois), and directed to future applied mathematicians who are at the junior-senior level in a university. The book is now reviewed from the standpoint of numerical analysis only, although it is directed to a much wider audience.

The reviewer's conclusion is that Hohn has written an unusually good exposition of the material covered, and that the exercises are especially good. Students will find it very readable, and the author presupposes only college algebra. On the other hand, this is basically another book on algebra, with very little analysis or geometry. Moreover, the algebra, while adequate on many topics (like Hermitian forms), seems deplorably lacking in any mention of the canonical forms under similarity. For the needs of numerical analysis as the reviewer sees them, the book would have to be supplemented by other sources of linear algebra and analysis.

There are nine chapters and three appendices. Chapter 1 introduces the reader to matrix notation. Chapter 2 defines a determinant (as the sum of $n!$ terms), and gives basic properties. Chapter 3 deals with solving linear systems and inverting matrices. Chapter 4 covers rank and equivalence of matrices, with a proof that the rank of AB cannot exceed the rank of A or B. The concept of a *number field* (subfield of the complex numbers) is used in most further work.

Linear dependence is the topic of Chapter 5, and the fundamental existence theorem for a linear algebraic system is stated here in terms of rank. Chapter 6 is devoted to vector spaces and linear transformations. We get to Sylvester's law on the nullity of AB in terms of the nullities of A and B. The Jordan canonical form is apparently nowhere even alluded to, nor is any other canonical form under similarity, and this disappoints the reviewer very much. In Chapter 7 we meet some analysis during the development of unitary and orthogonal transformations. We again meet the important existence theorem for a linear algebraic system $AX = B$, this time in terms of the orthogonality of B to the nullspace of A^* (called the *tranjugate* of A), but the nature of the general solution is not mentioned.

In Chapter 8 the characteristic equation is discussed, with the Cayley-Hamilton theorem, and we do learn about diagonalizing a Hermitian matrix. In Chapter 9 are discussed quadratic and Hermitian forms, definiteness, and even Cochran's theorem on representing a semidefinite form as a sum of squares.

The appendices are devoted to the $\sum$ and $\prod$ notations for sums and products, to the algebra of complex numbers, and to isomorphism as a concept. There follows an excellent 7-page bibliography of books on linear algebra and its applications in pure mathematics, numerical analysis, economics, psychology, electrical engineering, chemistry, etc.

The author consistently denotes matrices and column vectors by capital letters, but frequently uses component equations. He thus happily seems to make the notation the reader's ally instead of an enemy.

One should mention that a little desk-machine numerical analysis is contained in the book. First, there is an elimination method for evaluating a determinant (called the "sweep-out" process), with a numerical example of order 3 and a reference to Dwyer [4] for more details. The other exposition is of Gaussian elimination for solving a linear algebraic system, and an abridged form thereof, with two more numerical examples of order 3. For the student interested in round-off errors, there is a reference to von Neumann and Goldstine [5], but not to its sequel [6]. There is no mention of automatic computers as a tool for solving linear systems!

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REFERENCES

1. Garrett Birkhoff & Saunders MacLane, *A Survey of Modern Algebra*, The Macmillan Co., New York, 1941.
2. Sam Perlis, *Theory of Matrices*, Addison-Wesley Press, Cambridge, Mass., 1952.
3. V. N. Faddeeva, *Vychislitel'nye Metody Lineĭnoĭ Algebry*, Gostekhizdat, Moscow and Leningrad, 1950. Translated by Curtis D. Benster, under the title *Computational Methods of Linear Algebra*, Dover Publications, New York, 1959.
4. Paul S. Dwyer, *Linear Computations*, John Wiley & Sons, New York, 1951.
5. John von Neumann & H. H. Goldstine, "Numerical inverting of matrices of high order," Amer. Math. Soc., *Bull.*, vol. 53 (1947), p. 1021–1099.
6. Herman H. Goldstine & John von Neumann, "Numerical inverting of matrices of high order, II," Amer. Math. Soc., *Proc.*, vol. 2 (1951), p. 188–202.

53 [H, I, X].—W. J. Cunningham, *Introduction to Nonlinear Analysis*, Electrical and Electronic Engineering Series, McGraw-Hill Book Co., Inc., New York, 1958, ix + 349 p., 23 cm. Price $9.50.

This book is the well planned and well written outgrowth of a graduate course in electrical engineering. The subject matter is the practical analysis of nonlinear ordinary differential equations. A list of the chapter headings follows.

1. Introduction
2. Numerical Methods
3. Graphical Methods
4. Equations with Known Exact Solutions
5. Analysis of Singular Points
6. Analytical Methods
7. Forced Oscillating Systems
8. Systems described by Differential-difference Equations
9. Linear Differential Equations with Varying Coefficients
10. Stability of Nonlinear Systems

The 47 pages of the second and third chapters give a short and practical account of methods for desk calculation and for graphical solution. The remaining chapters frequently refer to these methods. The book contains many excellent figures. The exposition is bolstered with carefully worked out examples for each new idea, as is appropriate for a good textbook. The homework problems are given in 16 pages at the end of the book and are arranged according to the appropriate chapter. A bibliography, with comments on relevance to the book, is given in seven pages. A five-page index completes the book.

E. I.

54 [H, S].—A. Jeffrey, "Tables of characteristic roots and functions arising in neutron transport problems," 15 p. Deposited in the UMT File.

Application of the spherical harmonic method to neutron transport problems [1], [2] requires the knowledge of the roots ν of a certain determinantal equation (equation (1)), and of the functions $G_n(\nu)$ of those roots (given in equations (2) and (3) below).

$$(1)\qquad \begin{vmatrix} 1-c & \nu & 0 & 0 & \cdots & 0 & 0 \\ \nu & 3 & 2\nu & 0 & \cdots & 0 & 0 \\ 0 & 2\nu & 5 & 3\nu & \cdots & 0 & 0 \\ 0 & 0 & 3\nu & 7 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & (N-1)\nu & 0 \\ 0 & 0 & 0 & 0 & \cdots & (2N-1) & N\nu \\ 0 & 0 & 0 & 0 & \cdots & N\nu & (2N+1) \end{vmatrix} = 0$$

$$(2)\qquad (n+1)G_{n+1}(\nu) + \frac{2n+1}{\nu} G_n(\nu) + nG_{n-1}(\nu) = 0, n = 1, 2, 3, \cdots, N$$

$$(3)\qquad G_0(\nu) = 1, \qquad G_1(\nu) = \frac{(c-1)}{\nu}$$

Tables have been constructed listing the positive roots ν of equation (1) for $N =$ 9, 11, 13 and for c in the range 0(0.05)0.9(0.01)1.0. An abbreviated tabulation of the functions $G_n(\nu_i)$ of equations (2) and (3) is also given for argument c in the range 0(0.1)0.8(0.05)0.95, 0.99, 1.0. These 7D data have been deposited in the *MTAC* repository for Unpublished Mathematical Tables.

H. P.

1. J. C. Mark, *The Spherical Harmonics Method*, Parts I and II, Atomic Energy of Canada Limited, Reports CRT-340 and CRT-338.
2. B. Davison, *Neutron Transport Theory*, Oxford University Press, 1957.

55 [I].—H. E. Salzer, *Tables of Osculatory Interpolation Coefficients*, National Bureau of Standards Applied Math. Series, No. 56, U. S. Government Printing Office, Washington, D.C., 1959, xi + 25 p., 26 cm., Price $.30.

Let f(x) be a function defined over a region X, with at least $2n$ bounded derivatives in X. Let $f_i = f(x_i)$; $f_i' = [df(x)/dx]_{x=x_i}$. The well-known Lagrange-Hermite formula for osculatory interpolation has the form

$$f(x) = f(x_0 + ph) = \sum_{i=-[(n-1)/2]}^{[n/2]} \{A_i^n(p)f_i + hB_i^n(p)f_i'\} + R_{2n}$$

where

$$R_{2n} = \frac{f^{(2n)}(w)h^{2n}}{(2n)!} \prod_{j=-[(n-1)/2]}^{j=[n/2]} (p-j)^2.$$

In the above $[s]$ denotes the greatest integer in s, and w is an inner point of X.

An inspection of the form of the remainder shows that for comparable values of the derivative of order $2n$, the formula given above is more accurate than the corresponding ordinary Lagrangian formula of degree $2n$ (which involves the same amount of work). The above formula is therefore more advantageous in cases where $f(x)$ is required, and where both $f(x)$ and $f'(x)$ are known or easily obtainable at points x_i. The author tabulates the coefficients A_i and B_i over the following range: $n = 2, 3$, $p = -[(n - 1)/2]$ (.01) $[(n/2)]$, exact; $n = 4, 5$, to 9D, same range and interval.

The table should prove to be very useful in application. The author gives the inexact values to within 1.5 units in the last place, but adds: "This accuracy is not absolutely guaranteed, owing to the large amount of extra work that would be necessary to check it completely." Considering the author's past performance, a table issued under his name has indeed a high probability of being accurate. It is, nevertheless, a sobering fact that rigorous methods of checking tables are nowadays considered too expensive to be worth while!

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56 [I, X].—L. Fox, *The Numerical Solution of Two-Point Boundary Problems in Ordinary Differential Equations*, Clarendon Press, Oxford, 1957, xi + 371 p., 24 cm. Price $9.60.

This volume is the first in a series of monographs on numerical analysis planned by E. T. Goodwin and the late D. R. Hartree. In it, the numerical solution of one-dimensional boundary problems by finite difference methods receives the most comprehensive treatment which has as yet appeared in print. The emphasis is on purely numerical methods of universal applicability; "semi-analytical" techniques such as the methods of Ritz and Galerkin are not treated.

The author modestly disclaims all pretensions to rigorous or highbrow mathematics. According to his own words, the book is intended for the practical computer and for the student seeking a career in computation. Yet the book can hardly be dismissed as mathematically trivial; it would take a competent analyst to prove, say, the convergence of some of the algorithms proposed in the book, or to establish rigorous error bounds. Students of numerical analysis looking for topics for Ph.D. dissertations will find ample raw material here.

The recurrent theme of the author's treatment of boundary problems is the systematic use of the *difference correction*. This is mathematically (although not numerically) equivalent to improving a crude first approximation to a solution by taking into account the error terms in the finite difference approximation to the differential operator. As the author puts his basic philosophy (p. 35): "Most writers neglect the difference correction, using more accurate formulae for the derivatives, or a small enough interval, so that the error is either negligible or kept within reasonable bounds. In this book we shall work with a reasonably large interval and take full account of the difference correction. A large interval involves a correspondingly small number of algebraic equations and a proportionately small amount of labour: the inclusion of the difference correction yields high accuracy at this large interval." He also says in the preface: "Throughout I have taken the view that truncation errors in finite difference equations should not be tolerated." While it would be easy to take issue with these principles, if taken too literally, it cannot be denied that the author makes a strong case for a reasonable interpretation of them.

After introductory chapters of a general nature on the algebra of finite difference operators and on the solution of algebraic equations, the author considers second-order problems in Chapter 4. There follows, somewhat surprisingly, a chapter on the solution of initial-value problems by boundary-value techniques. Further chapters concern equations of higher than second order, eigenvalue problems, accuracy and precision, and miscellaneous methods. The discussion proceeds almost wholly by examples, all fully worked (on a desk computer). Many of the examples illustrate special difficulties and points of interest, such as the treatment of singularities, changes of the basic interval, and boundary conditions at infinity. Some of the famous tricks due to classical British numerical analysts, such as L. F. Richardson's deferred approach to the limit and A. C. Aitken's δ^2-method, are discussed in detail.

The author writes a terse, but at the same time fluent and highly readable English. The Clarendon Press has with this volume once more demonstrated its tradition of superb mathematical printing.

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57 [K].—FORMAN S. ACTON, *Analysis of Straight-Line Data*, John Wiley & Sons, Inc., New York, 1959, xiii + 267 p., 24 cm. Price $9.00.

As the title states, this is a book fitting a straight line to a set of data. Every conceivable set of a priori situations is assumed, and then the techniques for making the fit are fully and clearly discussed based on these assumptions. All the necessary side problems are considered and illustrated by examples.

The author's preface states very well why the book was written, and I would like to quote a few lines.

"This book was written from the conviction that we need more detailed expositions of classical and modern statistical techniques than are now possible in a general text surveying the entire field. Accordingly, I have selected the topics most pertinent to the engineer or physical scientist when he deals with data containing one or more lines entrapped within his experimental variability. He may seek to reveal the line, or he may desire to remove it in order better to examine the residual fluctuations. In either event, he must encounter the philosophy and arithmetic of the analysis of variance—a philosophy that all too often has been shoved off as a stepchild because its mathematical theory is rather tedious and pedestrian.

"In particular, I wanted to set down a strong plea for the use of these analytical techniques by the experimentalist himself, rather than by a professional statistician.

The structure of the experiment determines the proper method for analyzing its data, and the experimenter himself knows that structure best. Since his statistical background is apt to be shaky, he needs to be guided—at times in considerable detail—and he usually prefers concrete examples from which general principles may be inferred. I have therefore tried to introduce each subject with data from a real experiment—examining them at first broadly, and then again with an increase in detail that occasionally they do not deserve. When I have felt that they will not mislead, I have sometimes relied on explanations that are heuristic, but in general I have tried to emphasize the essential questions that are posed by the mathematical models our experimenter may choose in analyzing his data. It is with the influence of the model on the nature of the extractable information that I have been chiefly concerned."

The author also states that he did not intend the book for "classroom use," but rather as a detailed reference book. It is quite clear that Professor Acton has done a very excellent job and carried out fully the goal he set for himself.

After a brief introduction in Chapter 1, the author discusses in Chapters 2 and 3 the standard straight-line-fit problem in every way possible. Some needed information about the bivariate normal is presented in Chapter 4. Chapter 5 then considers the non-standard straight line fit when both variables are subject to error. Chapter 6 takes up the straight line fit in the analysis of variance and presents (for the first time in a text, to my knowledge) information of great value when "doing" an analysis of variance. Following Chapter 7, which is devoted to non-linear fit and orthogonal polynomial fit, the last three chapters round out the text by treating questions which usually come up in connection with least-squares problems, namely, transformations, rejection of extreme values, and cumulative frequency fitting.

Having this book available will really be a great boon for any statistician who is in daily contact with non-statistical scientific people. The nonstatistician would profit from exposure to it. It is an excellent book.

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58 [K, P].—STEPHEN H. CRANDALL, Editor, *Notes for the M. I. T. Special Summer Program on Random Vibration*, The Technology Press of the Massachusetts Institute of Technology, Cambridge, Massachusetts, 1958, viii + 418 p., 196 Fig., 22 cm. Price $10.00.

The first four chapters of these notes cover mechanical vibrations with deterministic excitation (S. H. Crandall), the mathematical description of random processes (W. M. Siebert), stochastic processes of mechanical origin (H. Poritsky), and statistical properties of response to random vibrations. The remaining eight chapters cover instrumentation for random vibration analysis (T. F. Rona), simulation equipment (K. J. Metzgar & D. E. Priest), and a very wide variety of discussions of special fields in which random vibration analysis is important: for example, structural damping (T. H. H. Pian), the fatigue of metals (F. A. McClintock), the response of structures to random pressures and to jet noise in particular (A. Powell), the estimation of sound-induced missile vibrations (I. Dyer), and mechanical design for random loading (R. M. Mains).

The treatment of the basic material in the early chapters is good, but rather abbreviated, especially if the students taking the course are not already reasonably familiar with the leading concepts and the mathematical methods that have been developed for the study of random vibrations. A person without such a background would probably not get much out of this brief and rapid treatment, and one well equipped would presumably find it superfluous. The chapters on applications cover a wide field, and in each case would appear to be of interest only to specialists. It seems to me that a far sounder course would have resulted if no attempt had been made to cover detailed applications, and if instead the course had been devoted to a more careful and detailed presentation of the basic theoretical material, followed by the chapters on instrumentation and on testing devices.

C. V. L. S.

59 [L].—Louis Robin, *Fonctions Spheriques de Legendre et Fonctions Spheroidales*, Gauthier-Villars, Paris. Tome 1, 1957, xxxvi + 201 p.; Tome 2, 1958, vii + 384 p., 24 cm. Price $18.50.

The mathematical community will be grateful to Dr. Robin for providing an up-to-date and virtually encyclopedic work on Legendre functions, which is certain to take its place among the most important books on its subject, and in many respects is likely to become the standard work of reference in this field.

The book is written primarily from the point of view of applied mathematicians, having in mind the numerous classical applications of Legendre functions and spherical harmonics as well as some modern applications, such as in antenna theory. It gives a wealth of formulae and results. These are clearly stated, with the conditions of validity scrupulously described in each case. The majority of the results are proved in detail; in cases where proofs have been omitted or severely curtailed, references to the literature are included. Errors found in the literature have been corrected, so that some of the results appear here correctly for the first time, and others are new or due to the author, who has been engaged in research on Legendre functions for a number of years.

The contents of the first two volumes of this treatise will now be summarized and briefly commented upon.

Topics considered in Chapter I include the separation of the wave equation in curvilinear coordinates, Legendre's differential equation, Legendre polynomials and their properties, and Legendre functions of the second kind and their properties.

Chapter II deals with the associated Legendre functions P_n^m and Q_n^m for integral values of m and non-negative integral values of n. A special feature of this chapter is the careful consideration of P_n^m for negative integers m, including $m < -n$.

Chapter III treats spherical harmonics of (positive, negative, or zero) integral degree. Included therein is an account of Maxwell's generation of spherical harmonics and transformation formulae to new coordinate axes, as well as the addition theorem for Legendre polynomials and expansions in spherical surface harmonics. Appendices contain explicit expansions and formulae for Legendre functions and associated Legendre functions with integral m and n.

The second volume begins with Chapter IV, which occupies 211 pages. It is devoted to an investigation of the associated Legendre function $P_n^m(z)$ for general (complex) values of m, n, and z. Corresponding to such values, this function is defined by a double loop integral. The connection with the hypergeometric equation is established, and numerous hypergeometric series are given for P_n^m and Q_n^m. The analytic nature of the Legendre functions as functions of the three complex variables m, n, z is carefully described, and expansions are given also in the exceptional (logarithmic) cases as well as for the derivatives of the functions with respect to m and n. Chapter V is concerned with approximations, asymptotic expansions, and inequalities. This is the largest published collection of material of this kind, and some of it is new or has not previously appeared in a book. Chapter VI relates to expansions in Legendre polynomials, associated Legendre functions, and spherical surface harmonics. Both the (pointwise) convergence and various kinds of summability of these expansions are investigated, and some examples of such expansions are given. Expansions involving $P_{n_k}^m(\cos\theta)$ are also discussed when the n_k are roots of a transcendental equation. The Gibbs phenomenon is described.

A forthcoming third volume will contain four chapters on addition theorems, zeros, applications, and related functions, including spheroidal functions, and an appendix on numerical tables of Legendre functions.

The book is very lucidly written and can be recommended both for study and as a work of reference. References to the literature are given throughout the text at appropriate places, but there is no systematic bibliography in the two volumes under review.

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60 [P, S, T, X, Z].—L. H. HERBACH, Editor, *Symposium on Digital Computing in the Chemical and Petrochemical Industries*, New York University-International Business Machines Corporation Symposium, New York, 1958, vi + 189 p., 28 cm. Price $7.00.

This report contains a series of papers given at the Symposium on Digital Computing in the Chemical and Petrochemical Industries held at New York University in May 1958. This was the second symposium in this series. The first symposium was held in early 1957 and was devoted to the application of computers in the aircraft industry. A list of papers follows.

1. "A Numerical Solution to the Miscible Displacement Equation" by D. U. von Rosenberg.
2. "Thermodynamic Properties of Neon" by J. B. Butt, J. C. Leutwyler, B. F. Dodge & R. W. Southworth.
3. "Numerical Experiments in Chemical Reactor Design" by L. Nemerever.
4. "Design of a Distributed Control System" by O. Bilous.
5. "Supervision of an Operating Plant by a Digital Computer" by W. D. Mohr.
6. "Multicomponent Ion Exchange Column Calculations" by J. Dranoff & L. Lapidus.

7. "Multicomponent Distillation Calculation on the IBM 704" by J. Greenstadt, Y. Bard, & B. Morse.
8. "Source-Sink Method for Oil-well Fracture Problem" by M. Friedman, E. Mehr, & C. Harris.
9. "Solution of Diffusion Equations for Photochemical Initiation by Light that is Intermittent in Space" by R. M. Noyes.
10. "Molecular Orbital Calculations" by J. Auffray.
11. "The Role of the Digital Computer in the Structure Determination of Complex Organic Molecules by X-ray Analysis" by G. A. Jeffry.
12. "The Use of Digital Computers in X-ray Crystallography" by W. G. Sly.
13. "The Engineer, the Computer, and the General Purpose Program" by E. P. Bartkus & W. M. Carlson.
14. "The Use of Linear Programming Techniques in Crude Oil Production" by A. S. Lee & J. Aronofsky.
15. "Product Allocation by Linear Programming" by K. H. Shaffir & T. Zang.
16. "Library Programs for Chemical Engineering Applications" by G. Trimble.

H. P.

61 [**P, X**].—Ruel V. Churchill, *Operational Mathematics*, McGraw-Hill Book Co., Inc., New York, 1958, ix + 337 p., 23 cm. Price $7.00.

The first edition of this book, published in 1944, was entitled *Modern Operational Mathematics in Engineering*. With the exception of the last chapter, which is concerned with Fourier transforms, both editions are devoted to an exposition of the theory of the Laplace transformation and its applications to physical problems, at about the junior-senior level of difficulty. The present edition is an enlargement by about ten per cent of the old, with many sections extensively rewritten and new problems added. Considerably rewritten are the portions of the book concerned with fundamental properties of analytic functions and with Sturm-Liouville systems. The arrangement of chapters is changed not at all, and of subject matter within chapters, very little from the earlier edition. The exposition is sound and lucid, and the book should have wide appeal.

Tables are essentially as in the earlier edition, namely, a table of Laplace transforms containing 125 entries, and a short table of finite Fourier sine and cosine transforms.

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62 [S].—J. P. Schiffer, *Tables of Charged Particles Penetrabilities*, ANL-5739, Argonne National Laboratory, Lemont, Ill., 1957, 59 p., 28 cm. Price $.40. Office of Technical Services, Washington 25, D. C.

Charged particles penetrabilities defined as $A_L^2\ (x, g) = F_L^2\ (x, g) + G_L^2\ (x, g)$ where F_L and G_L are the regular and irregular Coulomb functions, have been calculated for $L = 0$ to 4, $x \equiv \rho/2\eta = 0.05(.01)0.70$, $g \equiv \sqrt{2\rho\eta} = 1.6(.1)7.0$. The range covered corresponds roughly for protons on nuclei to $z \simeq 15\ (\backsim 1)75$ and bombarding energies up to 0.7 of the Coulomb barrier height.

The calculations were carried out on an IBM 650, neglecting $F_L{}^2$ and using the approximate formulae of Feshbach, Shapiro and Weisskopf [1] for G_0 and G_0'.

The tables are reproduced from the machine listings in floating decimal format with all quantities given to four figures. The accuracy was checked where possible against the entries of Bloch, et al [2]; regions of agreement to better than two per cent and better than five per cent with the latter reference are indicated. Other regions are admittedly meaningless.

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1. H. FESHBACH, I. SHAPIRO & V. F. WEISSKOPF, *Tables of Penetrabilities for Charged Particle Reactions*, New York Operations Office of AEC 3077, 1953.
2. I. BLOCH, ET AL., "Coulomb functions for reactions of protons and alpha-particles with the lighter nuclei," *Revs. Mod. Phys.*, v. 23, 1951, p. 147–182.

63 [S, X].—B. NOBLE, *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*, Pergamon Press, Inc., New York, 1959, 246 p., 21 cm. Price $10.00.

This excellent book is a definite contribution to modern applied mathematics. It brings together, in a well-integrated manner, results which are widely dispersed in the scientific literature.

Professor Noble has isolated the fundamental idea behind the Wiener-Hopf technique. He recognizes that it deals with certain factorization problems for functions of a complex variable and that it is only indirectly related to the Wiener-Hopf integral equation with which it is usually associated.

The classical Wiener-Hopf procedure starts from the Wiener-Hopf integral equation:

$$\lambda h(x) + f(x) = \int_0^\infty k(x-t)h(t)\,dt\,, \qquad 0 < x < \infty\,. \tag{1}$$

In the applications which are considered in Noble's book, λ vanishes. Define $h(x)$ and $f(x)$ to be 0 for negative x, and let $e(x)$ be the value of the integral in (1) for negative x. Introduction of Fourier transforms reduces equation (1) to:

$$F_+(\alpha) + E_-(\alpha) = K(\alpha)\,H_+(\alpha), \tag{2}$$

where F_+ and H_+ are analytic in an upper half plane, E_- is analytic in a lower half plane, K is analytic in a strip. $K(\alpha)$ can be factored as K_+/K_- and F_+K_- can be split as $C_+ + C_-$. This yields

$$E_-(\alpha)K_-(\alpha) + C_-(\alpha) = H_+(\alpha)K_+(\alpha) - C_+(\alpha), \tag{3}$$

where E_- and H_+ are unknown. The remarkable fact is that both can be determined from the single equation (3). The left and right sides of (3) usually have a common strip of analyticity, so that either side is the analytic continuation of the other. Both sides are therefore equal to the same entire function $M(\alpha)$, which often turns out to be a constant or a polynomial. $M(\alpha)$ can be determined from the asymptotic behavior of the functions appearing in equation (3). Some of these asymptotic behaviors are postulated on physical grounds, the others being found from equation (1).

Once $M(\alpha)$ has been determined, H_+ and E_- are computed from (3), and inversion gives $h(x)$.

A typical physical application occurs in the diffraction of waves by a half-plane. The boundary value problem can be reduced to an integral equation of type (1), where $k(x - t)$ is the Green's function for the partial differential equation, $h(x)$ is the current on the screen, and $f(x)$ is the incoming wave on the screen.

Noble points out that the functional equation (2) can be obtained directly from the boundary-value problem without reference to the integral equation. The advantages of this point of view are the following:

a. $K(\alpha)$ can be obtained with ease by transforming the partial differential equation directly without introducing the Green's function k explicitly.

b. We are led to possible further applications of the Wiener-Hopf technique to problems which cannot be formulated as an equation of type (1). The generalized two-part boundary-value problems of S. Karp fall in this category. The Wiener-Hopf technique can be extended to cope with these problems, the appropriate transform being a Mellin, Hankel, or Kantorovich-Lebedev transform rather than the ordinary Fourier transform used for problem (1).

There are many other interesting features of Noble's book. A comparison of the Wiener-Hopf technique with Titchmarsch's dual integral equation approach is given. One also finds a careful discussion of the relationship between the Wiener-Hopf problem and the Hilbert problem. An extensive collection of problems (taken from acoustics, electromagnetic theory, fluid flow, elasticity, and neutron diffusion) is worked out by the author. He has also included a wealth of non-trivial exercises, many of which are likely starting points for further research.

There are a number of topics omitted by Professor Noble, which I would have liked to see included. These omissions do not detract appreciably from the over-all excellence of the volume, but they might easily be remedied in the next edition:

a. The work on approximate methods in connection with Wiener-Hopf problems. The powerful techniques developed by Harold Levine receive only cursory mention.

b. The attempt to develop a two-dimensional Wiener-Hopf technique. J. Radlow has made a start toward the solution of the quarter-plane problem. This leads to difficult questions in the theory of functions of two complex variables.

c. There is a fairly extensive literature in probability theory on the Wiener-Hopf method. The interest is with solutions of integral equations of type (1) which are required to be cumulative probability distributions. Significant results have been obtained in this direction by F. Pollaczek, W. L. Smith, and F. Spitzer.

In conclusion, Professor Noble is to be thanked for his valuable analysis of an important tool of mathematical physics.

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64 [W, X].—K. J. ARROW, L. HURWICZ, & H. UZAWA, *Studies in Linear and Non-Linear Programming*, Stanford Mathematical Studies in the Social Sciences, II, Stanford University Press, Stanford, California, 1958, iv + 229 p., 25 cm. Price $7.50.

The general mathematical programming problem can be defined as: Determine the n-vector x which maximizes $f(x)$ subject to the conditions

$$g_i(x) \geqq 0, \qquad i = 1, 2, \ldots m$$
$$x \geqq 0.$$

Linear programming is the special case where $f(x)$ and the constraints $g_i(x)$ are linear.

Since the development of the Simplex Method, linear programming has had many applications to management problems. Also, as a representation of the economic problem of optimum allocation of scarce resources, linear programming (with its duality theorem) has become an important concept in economic analysis.

Refinements to linear programming have been extensive in recent years, but developments in the more general aspects of mathematical programming have not been nearly as rapid. Computational procedures have been developed for the case where the functional $f(x)$ is quadratic and where $f(x)$ is convex and separable for each x_j. All of these procedures have used the Simplex Method, or some variant, as the basic computational technique. Of more general nature is the basic paper by H. Kuhn and A. W. Tucker entitled "Non-linear Programming," which appeared in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics*, 1951, p. 481–492. In this paper, Kuhn and Tucker give conditions under which the mathematical programming problem is equivalent to finding the saddle point of the Lagrangian function, $\varphi(x, u) = f(x) + u \cdot g(x)$.

The Kuhn-Tucker theorem on non-linear programming provides the basis of this book, which is in three parts. Part I, entitled Existence Theorems, is mainly concerned with extensions of the Kuhn-Tucker theorem. Part II, entitled Gradient Method, is devoted to extensive studies of the gradient method applied to the saddle point of the Lagrangian function. Of special interest is the development of a modified gradient method which converges for all concave programming problems, (that is, where $f(x)$ and $g(x)$ are concave functions of x) without restriction. Examples of this modified gradient method applied to two small linear programming problems are presented in detail. Part III, entitled Methods of Linear and Quadratic Programming, discusses the solution of three specific problems whose special structures are used to develop methods of solution. Also, a method for solving linear programs based on a theorem on extreme points of convex polyhedral cones is presented.

This book, which is a collection of related papers published for the first time, is the second in the Stanford Mathematical Studies in the Social Sciences series. Like the first, *Studies in the Mathematical Theory of Inventory and Production*, by Arrow, Karlin, and Scarf, it is a valuable addition to the literature. It should be of considerable interest to mathematical economists and management scientists concerned with generalizations of linear programming.

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65 [W, X].—A. W. TUCKER & R. D. LUCE, *Contributions to the Theory of Games*, Annals of Mathematics Studies, v. 4, Princeton University Press, New Jersey, 1959, 453 p., 24 cm. Price $6.00.

This is the fourth in a series of books presenting papers relating to the theory of games; the editors claim that it is the last. The whole series has been a most valuable collection of papers; if nothing else, they indicate the mathematical, and hence the numerical, difficulty which is to be encountered in attempts to treat competitive situations quantitatively.

The present volume is perhaps further removed from computational method than any of the earlier ones. It still contributes to the description of the quantitative difficulties occurring in the description of competitive situations.

One of the most valuable contributions of this volume is a bibliography by Dorothea M. Thompson and Gerald L. Thompson; this is a list of about a thousand of the most readily available contributions to game theory.

A second valuable contribution is the introduction, presumably written by the editors, outlining what is in the volume and what the general significance of each paper is. This introduction constitutes what would be the most valuable review possible for this volume, but it is unfortunately too long to reproduce here.

A third interesting item is a translation into English by Mrs. Sonya Bargmann of von Neumann's famous early paper "Zur Theorie der Gesellschaftsspiele," which appeared in 1928.

While the present volume has only these fairly remote offerings to make to computation, the fact that so much present day computing is concerned with economic problems makes the material worth noting here. In addition, the reviewer would like to add his applause to that already bestowed on the editors of this series of volumes for their contributions in publishing research works in a single field grouped into a single volume with a good editorial introduction to guide the reader. The time the editors have expended on this task is certainly great, but to the reader the contribution is most valuable. An expansion of this idea would be most valuable if the editorial time could be made available. Specifically, it would seem appropriate to have journal editors relay some of the information they receive in the form of referees' reports to aid the cursory readers of their journals.

C. B. T.

66 [X].—GLENN JAMES & ROBERT C. JAMES, Editors, *Mathematics Dictionary*, 2nd Edition, D. Van Nostrand, Princeton, 1959, 546 p., 24 cm. Price $15.00.

The new edition of this well known reference is about 25 percent longer than the 1949 edition. The new material includes terms from "modern algebra, number theory, topology, vector spaces, the theory of games and linear and dynamic programming, numerical analysis, and computing machines." Entirely new are the four foreign language dictionaries of mathematical terms in the appendix: French—19 pages, German—16 pages, Russian—22 pages, and Spanish—15 pages.

The dictionary contains a wealth of information and makes interesting browsing. While the book is a "must" for mathematical libraries, it is not difficult to find many minor defects. Since the editors, in their preface, invite comments, here are some of mine.

Sins of omission and commission: Where are "Class Number," "Continuum Hypothesis", "Diophantine Approximation", "Dirichlet Series", and "Distribution, (L. Schwartz)"? On the other hand must we have "Commercial Bank", "Horsepower", "Mariner's Compass", etc.? Sins of misplaced emphasis: After "Möbius function" one would expect something about the Möbius inversion formula, but instead we find the more sophisticated, but less fundamental, relation to the Riemann Hypothesis. Improper definitions: "Unique Factorization Theorem" and "Factorial Series" are not properly defined. Improper spelling: "Fresnal", "Marriot". While "Tchebycheff" and even "Tschebyscheff" are not "wrong," the more modern "Chebyshev" could at least be given as a cross-reference. Typography: "Joukowski Transformation" is listed as a subheading of "Jordan." Non sequitur: "Fibonacci Sequence" has a mysterious reference to "Farey Sequence", but this should cause the reader little inconvenience since it is only one page away.

The appendices contain elementary tables of logarithms, trigonometric functions, compound interest, etc. There is also a table of "Denominate Numbers" (2 pints = 1 quart, 7 days = 1 week, 20 grains = 1 scruple, 2352 pounds = 1 Cornish mining ton, 24 sheets = 1 quire, 500 sheets = 1 long ream, etc.). British readers susceptible to acute nostalgia should be forewarned of the item which states that the British monetary pound is normally equal to 5 dollars. The editors seem especially fond of the fact that a gallon equals 231 cubic inches, since this is repeated three times. With due respect to them, could not the space be better used in the tables of integrals which follows? The type size there is almost unbelievably small.

The foreign-language dictionaries and the hundreds of new terms such as "Haar measure", "Tychonoff space", etc., add considerably to the value of this useful reference book.

D. S.

67 [Z].—A. D. Booth & K. H. V. Booth, *Automatic Digital Calculators*, Second Edition, Academic Press Inc., New York, 1956, 261 p., 21 cm. Price $6.00.

This book is a non-mathematical introduction to the principles and applications of computers employing tubes and other electronic devices. It is a very general coverage of the entire digital computing field.

The first three chapters of the book are devoted to the evolution of calculating machines to the modern general-purpose electronic digital computer.

The next four chapters discuss the overall design of a computing system: the memory, control, arithmetic unit, and input-output units. Each unit is described in terms of serial, parallel, and decimal operation, along with the peculiarities of each type of operation and number system being used.

Four chapters are devoted to a general discussion of circuits and hardware which make up the logic of the different systems. The different types of storage, such as magnetic core, magnetic drum, magnetic disc, magnetic wire, mercury delay lines, magnetostrictive delay lines, and other types of storage, are described in terms of general operating principles.

The remaining five chapters are devoted to coding for a digital computer and digital computer applications. The definition of a code and discussion of its form and contents are described.

The book gives the reader a very good introduction to the general principles of coding, operation and application of high-speed electronic computers and computer components. Some previous background in mathematics, or a previous knowledge of computer circuitry, is required for a thorough understanding of the contents. In clear and concise terms, the authors cover the subjects in seventeen well written chapters. The book is written primarily for new workers in the field. Engineers in other fields who wish to become acquainted with the subject of computers will find the text both interesting and informative. The book lays a firm foundation for subsequent reading in the digital computer field. It is not intended, however, that the book be of use to engineers who are well informed in computing techniques and equipment. The excellent illustrations of circuitry serve well as an aid to comprehension.

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68 (Z).—WATTS S. HUMPHREY, JR., *Switching Circuits with Computer Applications*, McGraw-Hill Book Company, Inc., New York, 1958, viii + 264 p., 23 cm. Price $8.50.

After an introductory chapter mostly devoted to the binary number system then follows a fairly long chapter on Boolean algebra: the presentation is intuitive and practical, the basic ideas and results, for example, being set forth by means of Venn diagrams. This is immediately applied in Chapter 3 to the study of relay contact networks. Chapter 4 is devoted to codes; included are discussions of the various kinds of undetected errors using binary-coded decimal and excess-three codes, of modulo-nine checking, of parity checking, and of error-correcting codes. Chapter 5 is on minimizing aids, and is done almost entirely in terms of the Karnaugh map. Chapter 6, on circuit logic, covers two- and three-level diode logic and transistor logic. Chapter 7, on Boolean matrices, deals with connection matrices of networks, their modification and manipulation. This lays the foundation for the treatment, in Chapter 8, of bilateral networks, both symmetrical and antisymmetrical, and their reduction. Chapter 9, on cascaded networks, deals with complex networks built up by cascading simpler ones, and considers clocked and unclocked pulse logic and level logic. Finally, Chapter 10 is devoted to sequential circuits, defined as circuits containing memory (or time-delay) elements.

The discussion throughout is practical and didactic rather than theoretical. Many worked examples are included in the text; these are very instructive, in most cases are easy to follow, and supplement and elucidate the occasionally rather sketchy presentation of ideas. This should make the book particularly valuable for self-study, though it is also very suitable as a textbook. The emphasis throughout is on practical methods of minimizing switching networks, and no single method or procedure is presented as a cure-all. The very sensible conclusion of the introduction to the chapter on Boolean matrices could well be taken as a maxim: "It is, however, no different from the other switching-circuit design techniques: it is an aid to the designer and does not replace skill and experience."

C. V. L. S.