Consequently, by taking a positive sign for q', the new cubic is in the form

$$x^3 - 3px - (4p^3 - 27q^2)^{\frac{1}{2}} = 0. \tag{14}$$

From this cubic, values of $(e_2 - e_3)$ and then e_2 and e_3 are computed to $16D$ as shown in Table 2.

It is mentioned that the values of the function, for $0 \leq a < 1$ or in general for ω_2/ω_1 purely imaginary, can be computed from the tabulated values with the aid of the following relation [1]

$$\wp(\lambda z \mid \lambda\omega_1, \lambda\omega_2) = \lambda^{-2}\wp(z \mid \omega_1, \omega_2) \tag{15}$$

where λ is a constant, real or complex.

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A Note on the Nonexistence of Certain Projective Planes of Order Nine

By Raymond B. Killgrove

1. Introduction. Every finite projective plane may be coordinatized in at least one way [1]. In this process some line is chosen to be the line at infinity, and the points not on this line are represented by an ordered pair of elements. The elements x and y for any point (x, y) on a given line of the plane satisfy the equation $y = x \cdot m \circ b$, where m and b are specific elements for the given line. This ternary operation on x, m, and b includes an additive loop in a special case.

A sequence of SWAC computer routines has been written to search for all planes having a specific additive loop in an appropriate ternary ring. Using these routines, a complete search had been made previously using the elementary Abelian group for the additive loop [2]. Now a complete search has been made using the

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cyclic group for the additive loop. No planes were found. This entire note parallels the paper [2].

2. The lines consistent with the cyclic additive pencil. It is sufficient to find the affine planes of order nine where the lines parallel to the line $y = x$ are determined by the cyclic group. In particular these lines are of the form $y = x + c$ where y may be computed for each x and a given c by addition modulo nine.

We represent any possible line for a geometry as a permutation on nine marks in the following way: a number a and its image b under the permutation define a point (a, b) of the line. All of these lines can interesect in at most one point with the lines of the cyclic groups. We can also restrict ourselves to the pencil of lines going through the origin (0, 0). Only 225 lines were found which satisfy these criteria.

By applying the permutations defined by the additive loop, the full list of 2,025 lines consistent with the cyclic group may be found. Having the lines $x = c$, $y = c$, in addition to $y = x + c$, one needs to find sets of 63 lines from the 2,025 which satisfy the affine plane axioms.

To save time one considers the automorphisms which preserve the 27 known lines, and in particular the subgroup of automorphisms which fix (0, 0). The cyclic group of order nine has such an automorphic subgroup of order 6. When this subgroup is applied to the 31 lines of the 225 containing the point (1, 2), one obtains 25 conjugate classes of lines. The conjugate classes are placed in some preference order. In this case the order chosen was 4, 16, 8, 9, 10, 11, 13, 15, 18, 19, 20, 23, 24, 25, 2, 3, 6, 7, 17, 21, 12, 14, 1, 22, 5. All but the last five classes mentioned in this order have six members. Conjugate classes 12 and 14 have three members, 1 and 22 have two members, and 5 has only one member.

3. Construction of pencils through (0, 0). Using a card sorter one finds the lines of the reduced set consistent with each line through (1, 2). Conjugate classes of lines preceding the current sorting line in preference order are removed also. This set of sorted cards is given the computer, which in turn does further sorting and the printing out of complete Latin squares. On the average each conjugate class yielded eight Latin squares. These Latin squares represent possible pencils of lines through (0, 0).

Next we use a set of 225 transformed lines which are obtained by applying one of the non-identity permutations to the original 225 lines. The computer then lists the transformed lines consistent with a given Latin square. In the previous elementary Abelian group case [2], whenever the list of transformed lines associated with some Latin square could themselves form a second Latin square, one usually finds a geometry. In this cyclic group case a second consistent Latin square was never found. Hence the search ended here, as there are no possible planes in this situation.

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