

Tables of Values of the Modified Mathieu Functions

By E. T. Kirkpatrick

1. **Introduction.** Mathieu functions are encountered in physical problems involving elliptical boundaries. When the method of separation of variables is applied to the wave equation or the diffusion equation expressed in elliptical coordinates, there result [1, p. 170] the Mathieu and modified Mathieu equations,

$$(1) \quad \frac{d^2 y}{du^2} + (a - 2q \cos 2u)y = 0$$

$$(2) \quad \frac{d^2 y}{du^2} - (a - 2q \cosh 2u)y = 0.$$

The notation used is that of Goldstein, Ince and McLachlan. Solutions to the modified Mathieu equation (2) can be assumed to be of the form

$$(3a) \quad y = Ce_{2n}(u, q) = \sum_{r=0}^{\infty} A_{2r}^{(2n)} \cosh 2ru$$

$$(3b) \quad y = Ce_{2n+1}(u, q) = \sum_{r=0}^{\infty} A_{2r+1}^{(2n+1)} \cosh (2r + 1)u$$

$$(3c) \quad y = Se_{2n+1}(u, q) = \sum_{r=0}^{\infty} B_{2r+1}^{(2n+1)} \sinh (2r + 1)u$$

$$(3d) \quad y = Se_{2n+2}(u, q) = \sum_{r=0}^{\infty} B_{2r+2}^{(2n+2)} \sinh (2r + 2)u.$$

When the above infinite hyperbolic series are substituted into equation (2), recurrence relationships may be derived for allowable values of the characteristic numbers a_{2n+p} ($p = 0$ or 1) for the even functions, or b_{2n+s} ($s = 1$ or 2) for the odd functions for a given value of q . Recurrence equations also give the allowable values of the Fourier coefficients $A_{2r+p}^{(2n+p)}$ and $B_{2r+s}^{(2n+s)}$

The formulas for both the characteristic numbers and the Fourier coefficients associated with each type of solution [1, p. 29, 37] are given below:

For $y = Ce_{2n}(u, q)$

$$(4a) \quad a_{2n} = \frac{2q^2}{a - 2^2} - \frac{q^2}{a - 4^2} - \frac{q^2}{a - 6^2} - \dots - \frac{q^2}{a - (2r)^2} - \dots$$

$$(4b) \quad v_{2r-2} = \frac{q}{a - (2r)^2 - qv_{2r}}, \quad r \geq 2$$

$$(4c) \quad v_0 = \frac{2q}{a - (2r)^2 - qv_2}, \quad r = 1$$

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where

$$(4d) \quad v_{2r-2} = \frac{A_{2r}^{(2n)}}{A_{2r-2}^{(2n)}}.$$

For $y = Ce_{2n+1}(u, q)$

$$(5a) \quad a_{2n+1} = 1 + q + \frac{q^2}{a - 3^2} - \frac{q^2}{a - 5^2} - \dots - \frac{q^2}{a - (2r + 1)^2} - \dots$$

$$(5b) \quad v_{2r-1} = \frac{q}{a - (2r + 1)^2 - qv_{2r+1}}, \quad r \geq 2$$

$$(5c) \quad v_1 = \frac{a - 1 - q}{q}, \quad r = 1.$$

For $y_{2n+1} = Se_{2n+1}(u, q)$

$$(6a) \quad b_{2n+1} = 1 - q + \frac{q^2}{a - 3^2} - \frac{q^2}{a - 5^2} - \dots - \frac{q^2}{a - (2r + 1)^2} - \dots$$

$$(6b) \quad v_{2r+1} = \frac{q}{a - (2r + 1)^2 - qv_{2r+1}}, \quad r \geq 2$$

$$(6c) \quad v_1 = \frac{a - 1 + q}{q}, \quad r = 1.$$

For $y_{2n+2} = Se_{2n+2}(u, q)$

$$(7a) \quad b_{2n+2} = 4 + \frac{q^2}{a - 4^2} - \frac{q^2}{a - 6^2} - \dots - \frac{q^2}{a - (2r)^2} - \dots$$

$$(7b) \quad v_{2r-2} = \frac{q}{a - (2r)^2 - qv_{2r}}, \quad r \geq 3$$

$$(7c) \quad v_2 = \frac{a - 4}{q}, \quad r = 2.$$

As a consequence [1, p. 24] of the Goldstein-Ince normalization

$$(8) \quad \int_0^{2\pi} y^2 du = \pi, \text{ then for } y = Ce_{2n}(u, q), \quad 1 = 2 [A_0^{(2n)}]^2 + \sum_{r=1}^{\infty} [A_{2r}^{(2n)}]^2$$

and for the remaining three types,

$$(9) \quad 1 = \sum_{r=0}^{\infty} [A_{2r+1}^{(2n+1)}]^2 = \sum_{r=0}^{\infty} [B_{2r+1}^{(2n+1)}]^2 = \sum_{r=0}^{\infty} [B_{2r+2}^{(2n+2)}]^2.$$

By dividing equations (8) and (9) through by $[A_0^{(2n)}]^2$ there are obtained equations of the form

$$(10) \quad \left[\frac{1}{A_0^{(2n)}} \right]^2 = 2 + \left[\frac{A_2^{(2n)}}{A_0^{(2n)}} \right]^2 + \left[\frac{A_4^{(2n)}}{A_0^{(2n)}} \right]^2 + \left[\frac{A_6^{(2n)}}{A_0^{(2n)}} \right]^2 + \dots$$

$$= 2 + v_0^2 + (v_2 \cdot v_0)^2 + (v_4 \cdot v_2 \cdot v_0)^2 + \dots$$

2. Computation of the Modified Mathieu Functions. For a given value of q , the characteristic number is found by using a trial, error and interpolation method

using the appropriate transcendental continued fraction of the form of equation (4a). The computation was generally started with the 21st term equal to zero, and then the 20th, 19th, etc., terms were computed in turn.

The v_{2r} are all known from equations of the type (4b) and (4c) above and therefore A_0 can be evaluated from equation (10). Equation (4d) will then give the value of the remaining A_{2r} . The value of the modified Mathieu function can then be computed by summing the appropriate hyperbolic series.

In [2], E. L. Ince has given tables for the characteristic numbers, Fourier coefficients and values of the Mathieu functions satisfying equation (1). The method used by Ince was to apply the above formulas; the same method was used by the author to evaluate the modified Mathieu functions satisfying equation (2). The numerical work was accomplished by using an IBM 650 Digital Computer and the Wolontis Interpretive System of coding. The characteristic numbers and the Fourier coefficients are the same for both the ordinary and the modified functions. However, the problem of summing the hyperbolic series is much more difficult than that of summing the trigonometric series. Whereas the $\cos 2ru$ term is bounded by plus or minus one, the $\sinh 2ru$ and $\cosh 2ru$ approach infinity as r tends to infinity. Therefore many more terms of the hyperbolic series must be considered before it is possible to generate a product $A_{2r}^{(2n)} \cosh 2ru$ which will be negligibly small.

As more terms are taken, difficulties in computation are encountered. The v_{2r} can be made as small as desired by choosing r sufficiently large. In most cases after the characteristic number had been found by using the appropriate transcendental continued fraction, the computation of the Fourier coefficients was started by letting $v_{40} = 0$, and then compute the $v_{38}, v_{36}, \dots, v_0$, in turn. As these values were computed, they were stored and used to form the products $v_0^2, (v_2 \cdot v_0)^2$, etc., for equation (10). It is to be noted that the smallest number which can be computed using the Wolontis Interpretive System is 10^{-50} . At first consideration this may not appear to be a serious limitation. However, in order to obtain, for example, fifteen coefficients up to $A_{28}^{(2n)}$, from the relations $A_{2r+2} = v_{2r} A_{2r}$, the v_{28} must be available. For it to be available the product of $(v_{26} \cdot v_{24} \cdot \dots \cdot v_0)^2$ must be greater than 10^{-50} , or the unsquared value must be greater than 10^{-25} . Except for v_0, v_2 and v_4 which may sometimes be greater than one, all the remaining values are less than one. In fact, for convergence, $v_{2r} \rightarrow 0$ as $r \rightarrow \infty$. If, for example, the "average" size of the v_{2r} is of the order of 10^{-2} , only twelve v_{2r} can be multiplied together before their product becomes less than 10^{-25} . Therefore, for this example, the maximum number of coefficients that can be generated will be twelve or thirteen. A machine error stop was avoided by counting the products as they were formed and at the same time checking their magnitude. The maximum number of coefficients generated was twenty; that is, up to and including coefficient $A_{38}^{(2n)}$. The number of coefficients generated will depend on the values of q and order $2n$ being considered.

3. Description of Computer Programs. A computer program was developed for the IBM 650 Digital Computer which will compute the characteristic numbers and Fourier coefficients and will sum the appropriate series in order to find the value of either the ordinary or the modified Mathieu function. A complete description of this program titled "A Program for the Computation of Mathieu and Modified Mathieu Functions" is available from the author and will be published separately.

In order to ensure five figure accuracy for parameters in the ranges $0 \leq q \leq 20$, $0 \leq n \leq 3$ and $0 \leq u \leq 1.0$, the program can use a maximum of twenty terms. For parameters outside the given ranges, the accuracy is unknown. However, the user can estimate the accuracy by noting the magnitude of the last term added to the series.

4. Discussion of Tables. Tables 1-12 are provided herein for values of the four types of modified Mathieu function $Ce_{2n}(u, q)$, $Ce_{2n+1}(u, q)$, $Se_{2n+1}(u, q)$ and $Se_{2n+2}(u, q)$, for the ranges $q = 1(1)10(2)20$, $u = 0.1(0.1)1.0$ and $n = 0, 1, 2$. The Fourier coefficients computed in order to obtain these values were compared with existing tables [1] for over 150 values, and no discrepancy was found in the first seven digits of any coefficient generated. Also, the magnitude of the last term added in the series was noted to ensure the accuracy of the last position published.

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Mechanical Engineering Department
 University of Toledo
 Toledo, Ohio

1. H. W. McLACHLAN, *Theory and Application of Mathieu Functions*, Oxford University Press, London, 1947.
 2. E. L. INCE, "Tables of the elliptical cylinder functions," Roy. Soc., Edinburgh, *Proc.*, v. 52, 1932, p. 355-423.

TABLE 1
 Values of Modified Mathieu Function $Ce_0(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | .38010 | .36588 | .34209 | .30863 | .26554 |
| 2 | .19704 | .18047 | .15340 | .11679 | .07224 |
| 3 | .10989 | .09505 | .07142 | .04076 | .00576 |
| 4 | .06564 | .05329 | .03412 | .01034 | -.01491 |
| 5 | .04130 | .03127 | .01612 | -.00179 | -.01932 |
| 6 | .02702 | .01895 | .00709 | -.00623 | -.01807 |
| 7 | .01822 | .01174 | .00249 | -.00734 | -.01515 |
| 8 | .01259 | .00738 | .00017 | -.00705 | -.01206 |
| 9 | .00887 | .00468 | -.00094 | -.00624 | -.00931 |
| 10 | .00636 | .00298 | -.00142 | -.00529 | -.00705 |
| 12 | .00341 | .00118 | -.00153 | -.00355 | -.00388 |
| 14 | .00190 | .00042 | -.00126 | -.00229 | -.00203 |
| 16 | .00110 | .00010 | -.00095 | -.00144 | -.00101 |
| 18 | .00065 | -.00002 | -.00069 | -.00089 | -.00045 |
| 20 | .00040 | -.00007 | -.00049 | -.00054 | -.00016 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | .21308 | .15200 | .08378 | .01097 | -.06241 |
| 2 | .02222 | -.02961 | -.07835 | -.11775 | -.14074 |
| 3 | -.02984 | -.06129 | -.08309 | -.08980 | -.07753 |

TABLE 1 (Continued)

| | | | | | |
|----|---------|---------|---------|---------|---------|
| 4 | -.03764 | -.05332 | -.05766 | -.04792 | -.02464 |
| 5 | -.03275 | -.03843 | -.03382 | -.01887 | .00267 |
| 6 | -.02529 | -.02528 | -.01712 | -.00270 | .01269 |
| 7 | -.01841 | -.01551 | -.00677 | .00478 | .01379 |
| 8 | -.01290 | -.00880 | -.00094 | .00728 | .01122 |
| 9 | -.00876 | -.00443 | .00199 | .00722 | .00773 |
| 10 | -.00576 | -.00172 | .00319 | .00606 | .00457 |
| 12 | -.00219 | .00073 | .00314 | .00319 | .00054 |
| 14 | -.00055 | .00127 | .00211 | .00113 | -.00092 |
| 16 | .00011 | .00112 | .00116 | .00007 | -.00106 |
| 18 | .00032 | .00081 | .00051 | -.00033 | -.00074 |
| 20 | .00034 | .00052 | .00014 | -.00039 | -.00038 |

TABLE 2

Values of Modified Mathieu Function $Ce_1(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | .85596 | .85372 | .84884 | .83954 | .82334 |
| 2 | .67799 | .66081 | .63071 | .58567 | .52324 |
| 3 | .50011 | .47318 | .42746 | .36215 | .27703 |
| 4 | .35310 | .32216 | .27089 | .20034 | .11323 |
| 5 | .24612 | .21525 | .16523 | .09889 | .02144 |
| 6 | .17252 | .14384 | .09841 | .04040 | -.02332 |
| 7 | .12249 | .09685 | .05717 | .00848 | -.04153 |
| 8 | .08824 | .06581 | .03192 | -.00795 | -.04594 |
| 9 | .06445 | .04509 | .01652 | -.01561 | -.04369 |
| 10 | .04769 | .03109 | .00721 | -.01840 | -.03863 |
| 12 | .02697 | .01493 | -.00151 | -.01739 | -.02697 |
| 14 | .01582 | .00713 | -.00408 | -.01366 | -.01732 |
| 16 | .00955 | .00329 | -.00433 | -.00994 | -.01054 |
| 18 | .00591 | .00138 | -.00379 | -.00696 | -.00612 |
| 20 | .00373 | .00044 | -.00306 | -.00475 | -.00337 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | .79697 | .75652 | .69758 | .61571 | .50717 |
| 2 | .44103 | .33753 | .21334 | .07268 | -.07467 |
| 3 | .17349 | .05584 | -.06712 | -.18088 | -.26516 |
| 4 | .01512 | -.08442 | -.17102 | -.22623 | -.23096 |
| 5 | -.05858 | -.12880 | -.17365 | -.17745 | -.13069 |
| 6 | -.08277 | -.12515 | -.13706 | -.10917 | -.04351 |
| 7 | -.08263 | -.10328 | -.09387 | -.05217 | .01053 |
| 8 | -.07238 | -.07784 | -.05696 | -.01349 | .03573 |
| 9 | -.05914 | -.05494 | -.02954 | .00899 | .04194 |
| 10 | -.04621 | -.03648 | -.01097 | .01974 | .03785 |
| 12 | -.02564 | -.01254 | .00704 | .02185 | .02025 |
| 14 | -.01264 | -.00112 | .01088 | .01463 | .00571 |
| 16 | -.00526 | .00325 | .00923 | .00728 | -.00172 |
| 18 | -.00142 | .00421 | .00625 | .00231 | -.00401 |
| 20 | .00037 | .00374 | .00359 | -.00032 | -.00369 |

TABLE 3

Values of Modified Mathieu Function $Ce_2(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 1.0988 | 1.1373 | 1.2008 | 1.2883 | 1.3974 |
| 2 | 1.0549 | 1.0724 | 1.0988 | 1.1297 | 1.1585 |

TABLE 3 (Continued)

| | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|
| 3 | 0.96379 | 0.96299 | 0.95773 | 0.94205 | 0.90754 |
| 4 | 0.85071 | 0.83405 | 0.80203 | 0.74858 | 0.66614 |
| 5 | 0.72581 | 0.69617 | 0.64300 | 0.56150 | 0.44693 |
| 6 | 0.59925 | 0.56021 | 0.49256 | 0.39375 | 0.26316 |
| 7 | 0.48147 | 0.43689 | 0.36159 | 0.25581 | 0.12340 |
| 8 | 0.37974 | 0.33303 | 0.25593 | 0.15146 | 0.02752 |
| 9 | 0.29657 | 0.25032 | 0.17560 | 0.07791 | -0.03165 |
| 10 | 0.23086 | 0.18674 | 0.11699 | 0.02902 | -0.06385 |
| 12 | 0.14063 | 0.10314 | 0.04636 | -0.01995 | -0.08086 |
| 14 | 0.08716 | 0.05686 | 0.01298 | -0.03411 | -0.07028 |
| 16 | 0.05511 | 0.03123 | -0.00175 | -0.03393 | -0.05319 |
| 18 | 0.03549 | 0.01694 | -0.00746 | -0.02879 | -0.03736 |
| 20 | 0.02324 | 0.00892 | -0.00894 | -0.02268 | -0.02491 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 1.5243 | 1.6624 | 1.8011 | 1.9253 | 2.0134 |
| 2 | 1.1753 | 1.1676 | 1.1197 | 1.0143 | 0.83594 |
| 3 | 0.84378 | 0.73947 | 0.58490 | 0.37624 | 0.12180 |
| 4 | 0.54718 | 0.38688 | 0.18749 | -0.03624 | -0.25175 |
| 5 | 0.29722 | 0.11677 | -0.07877 | -0.25853 | -0.37664 |
| 6 | 0.10531 | -0.06602 | -0.22436 | -0.33044 | -0.34055 |
| 7 | -0.02480 | -0.16773 | -0.27310 | -0.30268 | -0.22806 |
| 8 | -0.10013 | -0.20621 | -0.25818 | -0.22648 | -0.10465 |
| 9 | -0.13427 | -0.20357 | -0.21119 | -0.14039 | -0.00653 |
| 10 | -0.14150 | -0.17901 | -0.15482 | -0.06588 | 0.05530 |
| 12 | -0.11743 | -0.11198 | -0.05854 | 0.02440 | 0.08984 |
| 14 | -0.08057 | -0.05594 | -0.00279 | 0.05087 | 0.06479 |
| 16 | -0.04921 | -0.02021 | 0.02043 | 0.04520 | 0.02999 |
| 18 | -0.02702 | -0.00098 | 0.02511 | 0.02973 | 0.00504 |
| 20 | -0.01284 | 0.00753 | 0.02147 | 0.01512 | -0.00745 |

TABLE 4

Values of Modified Mathieu Function $Ce_3(u, q)$

| | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|
| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
| 1 | 1.1052 | 1.2213 | 1.4222 | 1.7189 | 2.1258 |
| 2 | 1.1586 | 1.2504 | 1.4052 | 1.6250 | 1.9097 |
| 3 | 1.1812 | 1.2486 | 1.3585 | 1.5061 | 1.6816 |
| 4 | 1.1654 | 1.2097 | 1.2780 | 1.3609 | 1.4428 |
| 5 | 1.1197 | 1.1428 | 1.1740 | 1.2010 | 1.2055 |
| 6 | 1.0538 | 1.0580 | 1.0563 | 1.0356 | 0.97750 |
| 7 | 0.97392 | 0.96113 | 0.93084 | 0.87011 | 0.76270 |
| 8 | 0.88402 | 0.85635 | 0.80166 | 0.70851 | 0.56459 |
| 9 | 0.78762 | 0.74742 | 0.67297 | 0.55533 | 0.38747 |
| 10 | 0.68880 | 0.63876 | 0.54956 | 0.41562 | 0.23594 |
| 12 | 0.50147 | 0.44023 | 0.33678 | 0.19314 | 0.02033 |
| 14 | 0.34898 | 0.28634 | 0.18502 | 0.05393 | -0.08713 |
| 16 | 0.23815 | 0.18014 | 0.09012 | -0.01830 | -0.12097 |
| 18 | 0.16205 | 0.11131 | 0.03578 | -0.04848 | -0.11669 |
| 20 | 0.11083 | 0.06792 | 0.00673 | -0.05603 | -0.09738 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 2.6602 | 3.3395 | 4.1775 | 5.1777 | 6.3233 |
| 2 | 2.2548 | 2.6466 | 3.0569 | 3.4362 | 3.7080 |
| 3 | 1.8671 | 2.0337 | 2.1383 | 2.1234 | 1.9238 |
| 4 | 1.4998 | 1.4989 | 1.3999 | 1.1630 | 0.76444 |

TABLE 4 (Continued)

| | | | | | |
|----|----------|----------|----------|----------|----------|
| 5 | 1.1629 | 1.0441 | 0.82247 | 0.48677 | 0.06084 |
| 6 | 0.86034 | 0.66443 | 0.38210 | 0.03295 | -0.32004 |
| 7 | 0.59306 | 0.35396 | 0.05843 | -0.24680 | -0.47537 |
| 8 | 0.36249 | 0.10894 | -0.16367 | -0.38955 | -0.47787 |
| 9 | 0.17129 | -0.07282 | -0.29711 | -0.42722 | -0.38634 |
| 10 | 0.02180 | -0.19459 | -0.35593 | -0.39057 | -0.25026 |
| 12 | -0.15498 | -0.28704 | -0.31889 | -0.21098 | 0.01050 |
| 14 | -0.20395 | -0.25165 | -0.19309 | -0.03416 | 0.14269 |
| 16 | -0.18368 | -0.17205 | -0.07499 | 0.06593 | 0.15185 |
| 18 | -0.13945 | -0.09636 | 0.00146 | 0.09533 | 0.10267 |
| 20 | -0.09483 | -0.04108 | 0.03835 | 0.08428 | 0.04596 |

TABLE 5

Values of Modified Mathieu Function $Ce_4(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 1.1086 | 1.3389 | 1.7569 | 2.4173 | 3.4036 |
| 2 | 1.1402 | 1.3447 | 1.7094 | 2.2701 | 3.0767 |
| 3 | 1.1750 | 1.3539 | 1.6668 | 2.1337 | 2.7777 |
| 4 | 1.2090 | 1.3622 | 1.6246 | 2.0030 | 2.4995 |
| 5 | 1.2357 | 1.3633 | 1.5764 | 1.8712 | 2.2341 |
| 6 | 1.2485 | 1.3508 | 1.5161 | 1.7329 | 1.9764 |
| 7 | 1.2436 | 1.3212 | 1.4415 | 1.5868 | 1.7257 |
| 8 | 1.2209 | 1.2753 | 1.3539 | 1.4351 | 1.4843 |
| 9 | 1.1830 | 1.2159 | 1.2564 | 1.2812 | 1.2549 |
| 10 | 1.1330 | 1.1460 | 1.1523 | 1.1277 | 1.0391 |
| 12 | 1.0056 | 0.98400 | 0.93253 | 0.82970 | 0.65183 |
| 14 | 0.85343 | 0.80451 | 0.71006 | 0.55534 | 0.33119 |
| 16 | 0.69134 | 0.62346 | 0.50214 | 0.32192 | 0.08994 |
| 18 | 0.53665 | 0.45884 | 0.32677 | 0.14485 | -0.06528 |
| 20 | 0.40354 | 0.32374 | 0.19407 | 0.02753 | -0.14399 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 4.8340 | 6.8683 | 9.7122 | 13.6152 | 18.8542 |
| 2 | 4.1905 | 5.6763 | 7.5866 | 9.9321 | 12.6332 |
| 3 | 3.6170 | 4.6510 | 5.8388 | 7.0688 | 8.1249 |
| 4 | 3.1011 | 3.7660 | 4.4043 | 4.8617 | 4.9166 |
| 5 | 2.6316 | 2.9989 | 3.2303 | 3.1799 | 2.6894 |
| 6 | 2.2011 | 2.3346 | 2.2773 | 1.9215 | 1.1991 |
| 7 | 1.8078 | 1.7636 | 1.5147 | 1.0045 | 0.25483 |
| 8 | 1.4518 | 1.2788 | 0.91563 | 0.35942 | -0.29441 |
| 9 | 1.1330 | 0.87224 | 0.45490 | -0.07303 | -0.56620 |
| 10 | 0.84996 | 0.53553 | 0.10988 | -0.34146 | -0.64899 |
| 12 | 0.38468 | 0.04324 | -0.30499 | -0.53216 | -0.49268 |
| 14 | 0.04888 | -0.24184 | -0.44407 | -0.44014 | -0.17824 |
| 16 | -0.15973 | -0.35575 | -0.40144 | -0.23144 | 0.09096 |
| 18 | -0.25421 | -0.34593 | -0.26929 | -0.02886 | 0.22516 |
| 20 | -0.26510 | -0.26980 | -0.12543 | 0.10202 | 0.23050 |

TABLE 6

Values of Modified Mathieu Function $Ce_5(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 1.1412 | 1.5282 | 2.2710 | 3.5374 | 5.6077 |
| 2 | 1.1563 | 1.5151 | 2.1935 | 3.3245 | 5.1202 |
| 3 | 1.1731 | 1.5042 | 2.1202 | 3.1239 | 4.6683 |

TABLE 6 (Continued)

| | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|
| 4 | 1.1921 | 1.4957 | 2.0516 | 2.9353 | 4.2504 |
| 5 | 1.2132 | 1.4897 | 1.9873 | 2.7581 | 3.8641 |
| 6 | 1.2359 | 1.4855 | 1.9267 | 2.5910 | 3.5062 |
| 7 | 1.2590 | 1.4818 | 1.8680 | 2.4318 | 3.1731 |
| 8 | 1.2802 | 1.4763 | 1.8091 | 2.2778 | 2.8609 |
| 9 | 1.2970 | 1.4665 | 1.7472 | 2.1263 | 2.5659 |
| 10 | 1.3064 | 1.4497 | 1.6800 | 1.9752 | 2.2856 |
| 12 | 1.2955 | 1.3884 | 1.5250 | 1.6710 | 1.7661 |
| 14 | 1.2437 | 1.2906 | 1.3455 | 1.3692 | 1.3036 |
| 16 | 1.1582 | 1.1651 | 1.1512 | 1.0788 | 0.90042 |
| 18 | 1.0480 | 1.0205 | 0.95063 | 0.80629 | 0.55675 |
| 20 | 0.92029 | 0.86472 | 0.75164 | 0.55842 | 0.27427 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 8.9280 | 14.1878 | 22.4276 | 35.1766 | 54.6063 |
| 2 | 7.8948 | 12.0901 | 18.2886 | 27.1920 | 39.5195 |
| 3 | 6.9605 | 10.2507 | 14.7938 | 20.7544 | 28.0285 |
| 4 | 6.1173 | 8.6426 | 11.8561 | 15.6014 | 19.3790 |
| 5 | 5.3573 | 7.2398 | 9.3977 | 11.5090 | 12.9574 |
| 6 | 4.6719 | 6.0181 | 7.3491 | 8.2873 | 8.2676 |
| 7 | 4.0527 | 4.9552 | 5.6499 | 5.7767 | 4.9121 |
| 8 | 3.4918 | 4.0311 | 4.2479 | 3.8446 | 2.5754 |
| 9 | 2.9824 | 3.2293 | 3.0994 | 2.3820 | 1.0082 |
| 10 | 2.5195 | 2.5361 | 2.1678 | 1.2989 | 0.01418 |
| 12 | 1.7215 | 1.4339 | 0.83522 | -0.01644 | -0.84049 |
| 14 | 1.0801 | 0.65007 | 0.04279 | -0.56769 | -0.86016 |
| 16 | 0.57767 | 0.11871 | -0.37198 | -0.67437 | -0.54741 |
| 18 | 0.19727 | -0.21384 | -0.52797 | -0.54412 | -0.17196 |
| 20 | -0.07276 | -0.38751 | -0.51169 | -0.31371 | 0.12777 |

TABLE 7

Values of Modified Mathieu Function $Se_1(u, q)$

| | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|
| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
| 1 | .06840 | .13532 | .19914 | .25799 | .30961 |
| 2 | .04709 | .09159 | .13086 | .16208 | .18239 |
| 3 | .03295 | .06296 | .08715 | .10274 | .10727 |
| 4 | .02347 | .04404 | .05895 | .06582 | .06293 |
| 5 | .01700 | .03131 | .04047 | .04256 | .03661 |
| 6 | .01251 | .02260 | .02815 | .02771 | .02090 |
| 7 | .00934 | .01654 | .01981 | .01811 | .01152 |
| 8 | .00705 | .01224 | .01408 | .01184 | .00593 |
| 9 | .00538 | .00916 | .01009 | .00772 | .00264 |
| 10 | .00415 | .00691 | .00728 | .00499 | .00075 |
| 12 | .00253 | .00404 | .00385 | .00197 | -.00083 |
| 14 | .00159 | .00242 | .00207 | .00065 | -.00111 |
| 16 | .00102 | .00149 | .00111 | .00009 | -.00098 |
| 18 | .00067 | .00093 | .00060 | -.00012 | -.00075 |
| 20 | .00044 | .00059 | .00032 | -.00018 | -.00053 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | .35130 | .37984 | .39160 | .38277 | .34981 |
| 2 | .18892 | .17923 | .15176 | .10677 | .04736 |
| 3 | .09899 | .07736 | .04382 | .00256 | -.03898 |
| 4 | .04974 | .02742 | -.00056 | -.02823 | -.04767 |
| 5 | .02305 | .00415 | -.01572 | -.03061 | -.03443 |

TABLE 7 (Continued)

| | | | | | |
|----|---------|---------|---------|---------|---------|
| 6 | .00883 | -.00566 | -.01822 | -.02392 | -.01929 |
| 7 | .00153 | -.00889 | -.01587 | -.01590 | -.00797 |
| 8 | -.00197 | -.00906 | -.01215 | -.00920 | -.00104 |
| 9 | -.00341 | -.00795 | -.00856 | -.00440 | .00249 |
| 10 | -.00377 | -.00644 | -.00559 | -.00130 | .00381 |
| 12 | -.00317 | -.00366 | -.00177 | .00138 | .00329 |
| 14 | -.00219 | -.00178 | -.00004 | .00169 | .00172 |
| 16 | -.00136 | -.00070 | .00054 | .00123 | .00053 |
| 18 | -.00079 | -.00015 | .00061 | .00069 | -.00007 |
| 20 | -.00042 | .00008 | .00049 | .00030 | -.00028 |

TABLE 8

Values of Modified Mathieu Function $Se_2(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 0.16828 | 0.33969 | 0.51700 | 0.70220 | 0.89595 |
| 2 | 0.13834 | 0.27606 | 0.41188 | 0.54311 | 0.66499 |
| 3 | 0.11228 | 0.22132 | 0.32313 | 0.41239 | 0.48190 |
| 4 | 0.09035 | 0.17576 | 0.25071 | 0.30867 | 0.34203 |
| 5 | 0.07234 | 0.13880 | 0.19312 | 0.22858 | 0.23834 |
| 6 | 0.05782 | 0.10933 | 0.14814 | 0.16794 | 0.16323 |
| 7 | 0.04622 | 0.08610 | 0.11343 | 0.12265 | 0.10981 |
| 8 | 0.03702 | 0.06788 | 0.08682 | 0.08914 | 0.07235 |
| 9 | 0.02973 | 0.05364 | 0.06650 | 0.06449 | 0.04641 |
| 10 | 0.02396 | 0.04251 | 0.05100 | 0.04642 | 0.02867 |
| 12 | 0.01573 | 0.02696 | 0.03013 | 0.02361 | 0.00874 |
| 14 | 0.01048 | 0.01733 | 0.01791 | 0.01154 | 0.00030 |
| 16 | 0.00709 | 0.01128 | 0.01068 | 0.00523 | -0.00276 |
| 18 | 0.00486 | 0.00743 | 0.00638 | 0.00201 | -0.00345 |
| 20 | 0.00337 | 0.00495 | 0.00380 | 0.00043 | -0.00317 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 1.0969 | 1.3008 | 1.4992 | 1.6785 | 1.8187 |
| 2 | 0.76997 | 0.84717 | 0.88237 | 0.85901 | 0.76084 |
| 3 | 0.52245 | 0.52335 | 0.47414 | 0.36785 | 0.20632 |
| 4 | 0.34263 | 0.30332 | 0.22081 | 0.09992 | -0.04168 |
| 5 | 0.21658 | 0.16068 | 0.07444 | -0.02837 | -0.12215 |
| 6 | 0.13090 | 0.07253 | -0.00276 | -0.07653 | -0.12279 |
| 7 | 0.07428 | 0.02093 | -0.03795 | -0.08311 | -0.09329 |
| 8 | 0.03790 | -0.00711 | -0.04926 | -0.07112 | -0.05861 |
| 9 | 0.01528 | -0.02059 | -0.04809 | -0.05321 | -0.02935 |
| 10 | 0.00180 | -0.02546 | -0.04128 | -0.03567 | -0.00851 |
| 12 | -0.00948 | -0.02312 | -0.02421 | -0.01017 | 0.01098 |
| 14 | -0.01095 | -0.01606 | -0.01094 | 0.00207 | 0.01293 |
| 16 | -0.00912 | -0.00960 | -0.00310 | 0.00583 | 0.00876 |
| 18 | -0.00664 | -0.00501 | 0.00069 | 0.00557 | 0.00415 |
| 20 | -0.00446 | -0.00214 | 0.00207 | 0.00396 | 0.00098 |

TABLE 9

Values of Modified Mathieu Function $Se_3(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 0.28177 | 0.58335 | 0.92520 | 1.3289 | 1.8174 |
| 2 | 0.25585 | 0.52460 | 0.81849 | 1.1481 | 1.5208 |
| 3 | 0.22907 | 0.46514 | 0.71366 | 0.97678 | 1.2509 |
| 4 | 0.20280 | 0.40767 | 0.61469 | 0.81981 | 1.0124 |

TABLE 9 (Continued)

| | | | | | |
|----------|-------------|-------------|-------------|-------------|-------------|
| 5 | 0.17782 | 0.35376 | 0.52376 | 0.67953 | 0.80655 |
| 6 | 0.15465 | 0.30434 | 0.44202 | 0.55677 | 0.63263 |
| 7 | 0.13358 | 0.25990 | 0.36990 | 0.45135 | 0.48858 |
| 8 | 0.11473 | 0.22060 | 0.30732 | 0.36236 | 0.37149 |
| 9 | 0.09810 | 0.18633 | 0.25380 | 0.28836 | 0.27798 |
| 10 | 0.08362 | 0.15680 | 0.20856 | 0.22765 | 0.20449 |
| 12 | 0.06038 | 0.11023 | 0.13930 | 0.13881 | 0.10419 |
| 14 | 0.04348 | 0.07713 | 0.09212 | 0.08232 | 0.04723 |
| 16 | 0.03134 | 0.05394 | 0.06054 | 0.04734 | 0.01669 |
| 18 | 0.02267 | 0.03780 | 0.03961 | 0.02616 | 0.00146 |
| 20 | 0.01648 | 0.02658 | 0.02582 | 0.01359 | -0.00527 |
| <i>q</i> | $\mu = 0.6$ | $\mu = 0.7$ | $\mu = 0.8$ | $\mu = 0.9$ | $\mu = 1.0$ |
| 1 | 2.4142 | 3.1416 | 4.0172 | 5.0475 | 6.2179 |
| 2 | 1.9378 | 2.3902 | 2.8537 | 3.2814 | 3.5986 |
| 3 | 1.5235 | 1.7698 | 1.9495 | 2.0066 | 1.8751 |
| 4 | 1.1728 | 1.2712 | 1.2683 | 1.1222 | 0.80428 |
| 5 | 0.88305 | 0.88107 | 0.77178 | 0.53732 | 0.18998 |
| 6 | 0.64900 | 0.58420 | 0.42370 | 0.17408 | -0.12034 |
| 7 | 0.46430 | 0.36502 | 0.19076 | -0.03205 | -0.24102 |
| 8 | 0.32187 | 0.20848 | 0.04385 | -0.13250 | -0.25372 |
| 9 | 0.21456 | 0.10082 | -0.04135 | -0.16629 | -0.21335 |
| 10 | 0.13560 | 0.03006 | -0.08430 | -0.16155 | -0.15413 |
| 12 | 0.03941 | -0.03848 | -0.09867 | -0.10630 | -0.04642 |
| 14 | -0.00455 | -0.05363 | -0.07407 | -0.04790 | 0.01347 |
| 16 | -0.02077 | -0.04703 | -0.04423 | -0.00946 | 0.03233 |
| 18 | -0.02360 | -0.03432 | -0.02094 | 0.00930 | 0.02931 |
| 20 | -0.02082 | -0.02216 | -0.00612 | 0.01502 | 0.01890 |

TABLE 10

Values of Modified Mathieu Function $Se_4(u, q)$

| | | | | | |
|----------|-------------|-------------|-------------|-------------|-------------|
| <i>q</i> | $\mu = 0.1$ | $\mu = 0.2$ | $\mu = 0.3$ | $\mu = 0.4$ | $\mu = 0.5$ |
| 1 | 0.39463 | 0.84503 | 1.4138 | 2.1776 | 3.2347 |
| 2 | 0.37627 | 0.79817 | 1.3148 | 1.9819 | 2.8633 |
| 3 | 0.35586 | 0.74793 | 1.2133 | 1.7897 | 2.5130 |
| 4 | 0.33385 | 0.69530 | 1.1107 | 1.6029 | 2.1858 |
| 5 | 0.31081 | 0.64145 | 1.0090 | 1.4239 | 1.8836 |
| 6 | 0.28730 | 0.58752 | 0.90979 | 1.2547 | 1.6076 |
| 7 | 0.26380 | 0.53448 | 0.81449 | 1.0967 | 1.3585 |
| 8 | 0.24073 | 0.48314 | 0.72419 | 0.95107 | 1.1363 |
| 9 | 0.21842 | 0.43413 | 0.63969 | 0.81833 | 0.94041 |
| 10 | 0.19712 | 0.38791 | 0.56153 | 0.69870 | 0.76968 |
| 12 | 0.15832 | 0.30511 | 0.42530 | 0.49800 | 0.49758 |
| 14 | 0.12520 | 0.23597 | 0.31565 | 0.34487 | 0.30507 |
| 16 | 0.09787 | 0.18015 | 0.23039 | 0.23236 | 0.17521 |
| 18 | 0.07590 | 0.13626 | 0.16591 | 0.15236 | 0.09165 |
| 20 | 0.05858 | 0.10243 | 0.11819 | 0.09707 | 0.04051 |
| <i>q</i> | $\mu = 0.6$ | $\mu = 0.7$ | $\mu = 0.8$ | $\mu = 0.9$ | $\mu = 1.0$ |
| 1 | 4.7138 | 6.7818 | 9.6490 | 13.5682 | 18.8187 |
| 2 | 4.0298 | 5.5534 | 7.4914 | 9.8579 | 12.5758 |
| 3 | 3.4110 | 4.4900 | 5.7143 | 6.9759 | 8.0609 |
| 4 | 2.8560 | 3.5782 | 4.2677 | 4.7731 | 4.8735 |
| 5 | 2.3634 | 2.8050 | 3.1063 | 3.1223 | 2.6921 |
| 6 | 1.9310 | 2.1575 | 2.1891 | 1.9148 | 1.2588 |

TABLE 10 (Continued)

| | | | | | |
|----|----------|----------|----------|----------|----------|
| 7 | 1.5561 | 1.6228 | 1.4782 | 1.0577 | 0.36942 |
| 8 | 1.2350 | 1.1880 | 0.93945 | 0.47267 | -0.13593 |
| 9 | 0.96346 | 0.84048 | 0.54207 | 0.09435 | -0.38047 |
| 10 | 0.73712 | 0.56804 | 0.25870 | -0.13131 | -0.45716 |
| 12 | 0.40088 | 0.20320 | -0.05883 | -0.29067 | -0.35966 |
| 14 | 0.18811 | 0.01168 | -0.16636 | -0.25361 | -0.17308 |
| 16 | 0.06334 | -0.07176 | -0.16852 | -0.15892 | -0.02863 |
| 18 | -0.00295 | -0.09442 | -0.12925 | -0.07123 | 0.04656 |
| 20 | -0.03308 | -0.08730 | -0.08258 | -0.01164 | 0.06763 |

TABLE 11

Values of Modified Mathieu Function $Se_5(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 0.50806 | 1.1350 | 2.0261 | 3.3842 | 5.5114 |
| 2 | 0.49404 | 1.0935 | 1.9233 | 3.1498 | 5.0058 |
| 3 | 0.47893 | 1.0503 | 1.8205 | 2.9229 | 4.5311 |
| 4 | 0.46266 | 1.0055 | 1.7175 | 2.7032 | 4.0855 |
| 5 | 0.44522 | 0.95900 | 1.6143 | 2.4904 | 3.6675 |
| 6 | 0.42665 | 0.91094 | 1.5113 | 2.2847 | 3.2764 |
| 7 | 0.40710 | 0.86162 | 1.4089 | 2.0865 | 2.9116 |
| 8 | 0.38675 | 0.81145 | 1.3076 | 1.8965 | 2.5728 |
| 9 | 0.36580 | 0.76086 | 1.2082 | 1.7154 | 2.2599 |
| 10 | 0.34451 | 0.71035 | 1.1113 | 1.5437 | 1.9724 |
| 12 | 0.30177 | 0.61133 | 0.92761 | 1.2307 | 1.4718 |
| 14 | 0.26020 | 0.51760 | 0.76060 | 0.96037 | 1.0654 |
| 16 | 0.22112 | 0.43160 | 0.61301 | 0.73302 | 0.74519 |
| 18 | 0.18547 | 0.35491 | 0.48605 | 0.54707 | 0.50073 |
| 20 | 0.15383 | 0.28828 | 0.37963 | 0.39910 | 0.32040 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 8.8670 | 14.1488 | 22.4024 | 35.1601 | 54.5952 |
| 2 | 7.8187 | 12.0384 | 18.2527 | 27.1664 | 39.5007 |
| 3 | 6.8646 | 10.1820 | 14.7432 | 20.7166 | 28.0002 |
| 4 | 5.9972 | 8.5532 | 11.7889 | 15.5516 | 19.3442 |
| 5 | 5.2102 | 7.1291 | 9.3159 | 11.4525 | 12.9247 |
| 6 | 4.4984 | 5.8894 | 7.2595 | 8.2343 | 8.2493 |
| 7 | 3.8571 | 4.8161 | 5.5633 | 5.7400 | 4.9202 |
| 8 | 3.2821 | 3.8927 | 4.1773 | 3.8368 | 2.6181 |
| 9 | 2.7695 | 3.1041 | 3.0573 | 2.4121 | 1.0872 |
| 10 | 2.3154 | 2.4362 | 2.1638 | 1.3709 | 0.12471 |
| 12 | 1.5673 | 1.4110 | 0.92094 | 0.13225 | -0.69719 |
| 14 | 1.0064 | 0.72162 | 0.21470 | -0.37207 | -0.73439 |
| 16 | 0.60145 | 0.28547 | -0.13573 | -0.47604 | -0.48918 |
| 18 | 0.32205 | 0.03208 | -0.26611 | -0.39470 | -0.21558 |
| 20 | 0.13950 | -0.09651 | -0.27382 | -0.25559 | -0.01570 |

TABLE 12

Values of Modified Mathieu Function $Se_6(u, q)$

| q | $u = 0.1$ | $u = 0.2$ | $u = 0.3$ | $u = 0.4$ | $u = 0.5$ |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1 | 0.62524 | 1.4688 | 2.8234 | 5.1539 | 9.2485 |
| 2 | 0.61333 | 1.4278 | 2.7067 | 4.8536 | 8.5252 |
| 3 | 0.60091 | 1.3863 | 2.5917 | 4.5649 | 7.8454 |
| 4 | 0.58792 | 1.3442 | 2.4785 | 4.2872 | 7.2067 |
| 5 | 0.57431 | 1.3015 | 2.3667 | 4.0198 | 6.6067 |

TABLE 12 (Continued)

| | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|
| 6 | 0.56004 | 1.2581 | 2.2562 | 3.7624 | 6.0431 |
| 7 | 0.54506 | 1.2138 | 2.1469 | 3.5143 | 5.5139 |
| 8 | 0.52935 | 1.1687 | 2.0387 | 3.2753 | 5.0174 |
| 9 | 0.51290 | 1.1227 | 1.9317 | 3.0452 | 4.5517 |
| 10 | 0.49573 | 1.0760 | 1.8258 | 2.8237 | 4.1161 |
| 12 | 0.45945 | 0.98057 | 1.6184 | 2.4068 | 3.3294 |
| 14 | 0.42114 | 0.88388 | 1.4184 | 2.0256 | 2.6494 |
| 16 | 0.38169 | 0.78772 | 1.2283 | 1.6812 | 2.0693 |
| 18 | 0.34208 | 0.69405 | 1.0507 | 1.3748 | 1.5822 |
| 20 | 0.30326 | 0.60465 | 0.88754 | 1.1066 | 1.1806 |
| q | $u = 0.6$ | $u = 0.7$ | $u = 0.8$ | $u = 0.9$ | $u = 1.0$ |
| 1 | 16.4676 | 29.1545 | 51.3078 | 89.6491 | 155.255 |
| 2 | 14.7990 | 25.4199 | 43.1373 | 72.1129 | 118.287 |
| 3 | 13.2690 | 22.0897 | 36.0874 | 57.5680 | 89.0684 |
| 4 | 11.8672 | 19.1253 | 30.0221 | 45.5612 | 66.1482 |
| 5 | 10.5841 | 16.4914 | 24.8206 | 35.7016 | 48.3227 |
| 6 | 9.4108 | 14.1560 | 20.3757 | 27.6527 | 34.5956 |
| 7 | 8.3393 | 12.0899 | 16.5923 | 21.1260 | 24.1462 |
| 8 | 7.3621 | 10.2671 | 13.3865 | 15.8741 | 16.3000 |
| 9 | 6.4725 | 8.6636 | 10.6839 | 11.6855 | 10.5056 |
| 10 | 5.6645 | 7.2582 | 8.4189 | 8.3797 | 6.3134 |
| 12 | 4.2713 | 4.9650 | 4.9760 | 3.8237 | 1.3490 |
| 14 | 3.1428 | 3.2514 | 2.6685 | 1.2321 | -0.73178 |
| 16 | 2.2438 | 2.0028 | 1.1913 | -0.09221 | -1.28226 |
| 18 | 1.5421 | 1.1221 | 0.30627 | -0.63917 | -1.12283 |
| 20 | 1.0078 | 0.52675 | -0.17078 | -0.74531 | -0.72343 |