Volume XIV *1960* *Nos. 69-72*

Mathematics of Computation

A journal devoted to advances in numerical analysis, the application of computational methods, mathematical tables, high-speed calculators and other aids to computation

Formerly: Mathematical Tables and other Aids to Computation

Published Quarterly by the
National Academy of Sciences—National Research Council

Editorial Committee
Division of Mathematics
National Academy of Sciences—National Research Council
Washington, D. C.

H. Polachek, Chairman, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C.
Alan Fletcher, University of Liverpool, Liverpool 3, England
P. C Hammer, University of Wisconsin, Madison 6, Wisconsin
Eugene Isaacson, New York University, New York 3, New York
Y. L. Luke, Midwest Research Institute, Kansas City 10, Missouri
Daniel Shanks, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C.
A. H. Taub, University of Illinois, Urbana, Illinois
R. S. Varga, Case Institute of Technology, Cleveland 6, Ohio
J. W. Wrench, Jr., Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C.
D. M. Young, Computing Center, University of Texas, Austin 12, Texas

Information to Subscribers

The journal is published quarterly in one volume per year with issues numbered serially since Volume I, Number 1. Starting with January, 1959 subscriptions are $8.00 per year, single copies $2.50. Back issues are available as follows:

Volume I (1943–1945), Nos. 10 and 12 *only* are available; $1.00 per issue.
Volume II (1946–1947), Nos. 13, 14, 17, 18, 19, and 20 *only* available; $1.00 per issue.
Volume III (1948–1949), Nos. 21–28 available. $4.00 per year (four issues), $1.25 per issue.
Volume IV (1950 through 1958), all issues available; $5.00 per year, $1.50 per issue.

Microcard Edition

Volumes I–X (1943–1956), Nos. 1–56 are now available on Microcards and may be purchased from the Microcard Foundation, Box 2145, Madison 5, Wisconsin, at a cost of $20.00 for the complete set. Succeeding volumes are available on request.

Information to Contributors

All contributions intended for publication in *Mathematics of Computation* and all books for review should be addressed to H. Polachek, Technical Director, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C. The author may suggest an appropriate editor for his paper. Manuscripts should be typewritten double-spaced in the format used by the journal. Authors should submit the original and one copy, and should retain one copy.

Subscriptions, address changes, business communications and payments should be sent to:

The Printing and Publishing Office
The National Academy of Sciences
2101 Constitution Avenue
Washington 25, D. C.

PUBLISHED BY THE
NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL
Mt. Royal and Guilford Avenues, Baltimore 2, Md.
Second-class postage paid at Baltimore, Md.

Mathematics of Computation

A Quarterly Journal
Edited by

ALAN FLETCHER P. C. HAMMER EUGENE ISAACSON

Y. L. LUKE DANIEL SHANKS A. H. TAUB

R. S. VARGA J. W. WRENCH, JR. D. M. YOUNG

HARRY POLACHEK, Chairman

XIV

NOS. 69-72

1960

Formerly: Mathematical Tables and other Aids to Computation

Published by the

National Academy of Sciences—National Research Council

Washington, D. C.

TABLE OF CONTENTS

JANUARY 1960

Mathematics of Computation 1

Quadrature Formulas Involving Derivatives of the Integrand ..... PRESTON C. HAMMER & HOWARD H. WICKE 3

Tables for Use in Quadrature Formulas Involving Derivatives of the Integrand ..... GEORGE STRUBLE 8

Numerical Quadrature Over a Rectangular Domain in Two or More Dimensions, Part 1 ..... J. C. P. MILLER 13

Numerical Integration Formulas of Degree Two ..... A. H. STROUD 21

Calculation of Transient Motion of Submerged Cables ..... THOMAS S. WALTON & HARRY POLACHEK 27

Abscissas and Weights for Lobatto Quadrature of High Order ..... PHILIP RABINOWITZ 47

A Method for the Numerical Evaluation of Finite Integrals of Oscillatory Functions ..... I. M. LONGMAN 53

Products of Laguerre Polynomials ..... JOSEPH GILLIS & GEORGE WEISS 60

TECHNICAL NOTES AND SHORT PAPERS 64

The Formula-Controlled Logical Computer "Stanislaus" .. F. L. BAUER 64

Evaluation at Half Periods of Weierstrass' Elliptic Function with Rectangular Primitive Period-Parallelogram ..... CHIH-BING LING 67

A Note on the Nonexistence of Certain Projective Planes of Order Nine ..... RAYMOND B. KILLGROVE 70

A Note on Rational Approximation ..... ROBERT W. FLOYD 72

The Complete Factorization of $2^{132} + 1$ ..... K. R. ISEMANGER 73

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS 75

WAPSTRA, NIJGH & VAN LIESHOUT 1, FREVEL, TURLEY & PETERSEN 2, HOWELL 3, GAWLIK 4, LOWAN 5, DUNNETT 6, CHISTOVA 7, EMERSLEBEN 8, LOWELL 9, NUMERICAL COMPUTATION BUREAU 10, GERBES, REYNOLDS, HOES & DRANE 11, HENSMAN & JENKINS 12, HERSHEY 13, KARPOV 14, NOMURA & KATSURA 15, THORNE 16, BARTHOLOMEW & HIGGS 17, ROSE 18, WESTCOTT 19, GLASSNER 20, ARROW, KARLIN & SCARF 21, MCCRACKEN, WEISS & LEE 22, GOODIER & HODGE 23, KANTOROVICH & KRYLOV 24, ALT 25, CULBERTSON 26, DEPT. OF THE ARMY 27, GARDNER 28.

TABLE ERRATUM 96

JAHNKE & EMDE 273.

CORRIGENDA 97

FIELDER, HER MAJESTY'S NAUTICAL ALMANAC OFFICE

TABLE OF CONTENTS

APRIL 1960

Construction of Galois Fields of Characteristic Two and Irreducible Polynomials ..... J. D. SWIFT 99

Improved Methods to Calculate the Characters of the Symmetric Group ..... STIG COMÉT 104

Tables of Values of the Modified Mathieu Functions ..... E. T. KIRKPATRICK 118

Numerical Quadrature over a Rectangular Domain in Two or More Dimensions II ..... J. C. P. MILLER 130

On the Propagation of Round-Off Errors in the Numerical Integration of the Heat Equation ..... ARNOLD N. LOWAN 139

The Rational Approximation of Functions which are Formally Defined by a Power Series Expansion ..... P. WYNN 147

TECHNICAL NOTES AND SHORT PAPERS 187

On the Utility of Newton's Method for Computing Complex Roots of Equations ..... I. M. LONGMAN 187

A Problem in Abelian Groups, with Application to the Transposition of a Matrix on an Electronic Computer ..... GORDON PALL & ESTHER SEIDEN 189

The Fourier Transform of $[c^{2k} + (x - x^{-1})^{2k}]^{-1}$ Arising from Study of Tuned Circuit Spectra ..... CHARLES WALMSLEY & ARTHUR S. G. GRANT 193

On the Evaluation of Certain Complex Elliptic Integrals ..... H. A. LANG & D. F. STEVENS 195

The Numerical Evaluation of the Eighteenth Perfect Number ..... SCHEFFLER & R. ONDREJKA 199

A Note on Gaussian Twin Primes ..... DANIEL SHANKS 201

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS 204

REYNOLDS **29**, SELFRIDGE **30**, LEHMANN **31**, KARMAZINA & CHISTOVA **32**, PAXTON & ROLLIN **33**, ARMOUR RESEARCH FOUNDATION **34**, ASHBY & CATRON **35**, MILLER, GERHAUSER & MATSEN **36**, EPSTEIN **37**, LEVENS **38**, LANGDON **39**, FREUDENTHAL **40**, JEENEL **41**.

TABLE ERRATA 218

ARNDT **274**, CAHEN **275**, CHEBYSHEV; KULIK **276**, DALE **277**, DWIGHT **278**, RILEY & BILLINGS **279**, SALZER **280**, SPENCELEY & SPENCELEY **281**, WATSON **282**.

CORRIGENDA 222

LING, PARKER & NIKOLAI, ROBIN.

TABLE OF CONTENTS

JULY 1960

On the Propagation of Round-Off Errors in the Numerical Treatment of the Wave Equation ..... ARNOLD N. LOWAN 223

Rotors in Polygons and Polyhedra ..... MICHAEL GOLDBERG 229

Numerical Quadrature Over a Rectangular Domain in Two or More Dimensions III ..... J. C. P. MILLER 240

A Method for Calculating Solutions of Parabolic Equations with a Free Boundary ..... MILTON E. ROSE 249

Alternative Formulas for Osculatory and Hyperosculatory Inverse Interpolation ..... HERBERT E. SALZER 257

TECHNICAL NOTES AND SHORT PAPERS 262

Permanents of Incidence Matrices ..... PAUL J. NIKOLAI 262

On the Numerical Treatment of Heat Conduction Problems with Mixed Boundary Conditions ..... ARNOLD N. LOWAN 266

High Precision Calculation of Arcsin x, Arccos x, and Arctan x ..... I. E. PERLIN & J. R. GARRETT 270

The Calculation of Toroidal Harmonics ..... A. ROTENBERG 274

Transcendental Equation for the Schrödinger Equation ..... J. R. M. RADOK 276

A Note on Factors of $n^4 + 1$ ..... A. GLODEN 278

A Note on the Solution of Quartic Equations ..... HERBERT E. SALZER 279

A Conjugate Factor Method for the Solution of a Cubic ..... D. A. MAGULA 281

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS 284

GLODEN **42**, GANTMACHER **43**, BERKSON **44**, BROSS & KASTEN **45**, CLARK **46**, CLATWORTHY **47**, DIXON **48**, DODGE & ROMIG **49**, DUNNETT & LAMM **50**, FIELLER, HARTLEY & PEARSON **51**, HORSNELL **52**, JOHNSON **53**, MOORE **54**, NATIONAL BUREAU OF STANDARDS **55**, NIEVERGELT **56**, OWEN & MONK **57**, SARHAN & GREENBERG **58**, SATHE & KAMAT **59**, SEKAR, AGARWALA & CHAKRABORTY **60**, SHERMAN **61**, CASHWELL & EVERETT **62**, CENTRE NATIONAL D'ÉTUDES DES TÉLÉCOMMUNICATIONS **63**, ROBIN **64**, SELFRIDGE & MAXFIELD **65**, HARTREE **66**, FERGUSON & SARGENT **67**, ARBENZ **68**.

TABLE ERRATA 308

SPENCELEY, SPENCELEY & EPPERSON **283**, CAMBI **284**, MEYER ZUR CAPELLEN **285**.

NOTES 309

NEW JOURNALS

AUTOMATIC PROGRAMMING OF DIGITAL COMPUTERS—NATIONAL INFORMATION CENTRE.

CORRIGENDUM 310

HENSMAN & JENKINS.

TABLE OF CONTENTS

OCTOBER 1960

On Liouville's Function ..... R. SHERMAN LEHMAN 311

On the Conjecture of Hardy & Littlewood concerning the Number of Primes of the Form $n^2 + a$ ..... DANIEL SHANKS 321

On the Propagation of Errors in the Inversion of Certain Tridiagonal Matrices ..... ARNOLD N. LOWAN 333

Tabulation of Coefficients for Operations on Taylor Series ..... DANIEL C. FIELDER 339

Explicit Solutions of the One-dimensional Heat Equation for a Composite Wall ..... MARCIA ASCHER 346

A Method of "Alternating Corrections" for the Numerical Solution of Two-point Boundary Value Problems ..... DAVID A. POPE 354

TECHNICAL NOTES AND SHORT PAPERS 362

Note on the Asymptotic Expansion of the Modified Bessel Function of the Second Kind ..... E. DEMPSEY & G. C. BENSON 362

On the Factors of Certain Mersenne Numbers ..... JOHN BRILLHART & G. D. JOHNSON 365

Further Evaluation of Khintchine's Constant ..... JOHN W. WRENCH, JR. 370

Note on $\int_0^\infty e^{-x} J_0\left(\frac{\eta x}{\xi}\right) J_1\left(\frac{x}{\xi}\right) x^{-n}\, dx$ ..... HENRY E. FETTIS 372

Integrals of Products of Laguerre Polynomials ..... R. D. LORD 375

The Evaluation of Certain Probability Integrals ..... IRWIN GREENBERG 376

The Congruence $2^{p-1} \equiv 1 \pmod{p^2}$ for $p < 100{,}000$ ..... SIDNEY KRAVITZ 378

Values of $\frac{2}{\pi} \int_0^\infty \left(\frac{\sin t}{t}\right)^n dt$ ..... KASABURÔ HARUMI, SHIGETOSHI KATSURA, & JOHN W. WRENCH, JR. 379

Computation of Fresnel Integrals ..... J. BOERSMA 380

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS 381

RYSHIK & GRADSTEIN 69, PRANGE 70, ROTENBERG, BIVINS, METROPOLIS & WOOTEN 71, SALZER & KIMBRO 72, BARTON & DAVID 73, BERNDT 74, CHERNOFF & MOSES 75, DOORNBOS & PRINS 76, FOSTER 77, HETZ & KLINGER 78, HUITSON 79, KULLBACK 80, LIEBERMAN 81, OWEN 82, RAMACHANDRAN 83, SARHAN & GREENBERG 84, SENGUPTA & BHATTACHARYA 85, SIOTANI 86, SIOTANI & OZÄWA 87, SOMERVILLE 88, STECK 89, TAGUTI 90, TATE & GOAN 91, MORAN 92, HORN & WITTICH 93, BIRKHOFF & LANGER 94, LANGER 95, WILSON 96, DENNIS 97

TABLE ERRATA 400

BURRAU 286, FOWLE 287, HEAD & WILSON 288, HER MAJESTY'S NAUTICAL ALMANAC OFFICE 289, HER MAJESTY'S NAUTICAL ALMANAC OFFICE 290, MORSE 291, ROBIN 292, RYSHIK & GRADSTEIN 293, SILBERSTEIN 294, SPENCELEY & SPENCELEY 295

INDICES TO VOLUME XIV 405

Author Index, Papers and Technical Notes 405

Reviews by Author of Work Reviewed 406

Table Errata 409

Notes 410

Corrigenda 410

Reviews by Subject 411

CLASSIFICATION OF TABLES

A. Arithmetical Tables, Mathematical Constants

B. Powers

C. Logarithms

D. Circular Functions

E. Hyperbolic and Exponential Functions

F. Theory of Numbers

G. Higher Algebra

H. Numerical Solution of Equations

I. Finite Differences, Interpolation

J. Summation of Series

K. Statistics

L. Higher Mathematical Functions

M. Integrals

N. Interest and Investment

O. Actuarial Science

P. Engineering

Q. Astronomy

R. Geodesy

S. Physics, Geophysics, Crystallography

T. Chemistry

U. Navigation

V. Aerodynamics, Hydrodynamics, Ballistics

W. Economics and Social Sciences

X. Numerical Analysis and Applied Mathematics

Z. Calculating Machines and Mechanical Computation