REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

69[A, J, L, M].—I. M. RYSHIK & I. S. GRADSTEIN, *Summen-Produkt- und Integral-Tafeln: Tables of Series, Products, and Integrals,* VEB Deutscher Verlag der Wissenschaften, Berlin, 1957, xxiii + 438 p., 27 cm. Price DM56.

This volume of tables consists of a translation of the Russian third edition [1] into parallel German and English text. That edition has now been improved and augmented by the incorporation of corrections listed on a sheet of corrigenda accompanying the third edition, the addition of supplementary remarks in the Appendix, and the inclusion of an extensive supplementary bibliography, which consists of books and monographs pertaining to integral transforms, special functions, and indexes of mathematical tables.

The preparation of the third edition was carried out by I. S. Gradstein, following the death in the Second World War of I. M. Ryshik, who was responsible for the first two editions. A description of the contents of the first edition has appeared in a detailed review by R. C. Archibald [2].

The present book represents an extensive revision of the earlier editions. Major changes include the deletion of sections relating to the calculus of finite differences (including formulas for numerical quadrature), the addition of a completely new chapter on integral transforms, and the enlargement of the chapters on special functions.

Information on each of the special functions—in particular, elliptic, cylindrical, and spherical—is presented systematically. Such information generally includes definitions; representation by integrals, series, and products; asymptotic formulas; functional equations; special values; and theorems relating to characteristic properties.

An introductory section entitled "On the Arrangement of the Formulae" explains the arrangement of the contents of the chapters on elementary functions and on their definite integrals, and sets forth innovations in the arrangement of definite integrals, which in previous editions followed closely the classification established in the classical tables of Bierens de Haan [3].

The usefulness of this volume is enhanced by references and cross-references for the sources of most of the 5400 formulas presented. Formulas are numbered decimally within each chapter, and the chapter numbers are used for the integer part, as is customary. Furthermore, a key for the references to the literature cited on p. 434 is described in the Preface. It seems appropriate to note here that in both the Russian third edition and in this translation the list of numbered references consists of 40 items, although reference is made in several places in the book to a forty-first item and a forty-second that were apparently omitted inadvertently.

The first two editions contained a table of 10D approximations to $(2n - 1)!!/(2n)!!$ and $(2n - 1)!!/(2n)!!(2n + 1)$, for $n = 1(1)15$, and to $(2n - 1)!!/(2n + 2)!!$ and $(2n - 1)!!/(2n + 2)!!(2n + 3)$, for $n = 1(1)14$. This numerical information is now supplemented by an original table of the Lobatschefsky function $L(x)$ to 7D for $x = 0°(1°)10°$, 6D for $x = 11°(1°)30°$, and 5D for $x = 31°(1°)90°$, computed by N. V. Tomantovaia͡ under the supervision of B. L. Laptev. This function is briefly discussed (p. 296–297) in the chapter on special functions. Additional numerical data also include exact values of the first 17 Bernoulli numbers and the first 10 Euler numbers, 10D approximations to $\zeta(n)$ for $n = 2(1)11$, and Euler's constant and Catalan's constant to 16D and 9D, respectively. I have examined all these data carefully, and the errors detected, together with errors in the formulas, are enumerated separately in this issue (MTE 293).

Use of the book is facilitated by an elaborate index of special functions and notations on p. 417–422. In addition to supplementary remarks and the bibliographies already mentioned, the Appendix contains (on p. 423–429) a discussion of the variations in the notation and symbols used for special numbers and functions throughout the mathematical literature and a concise list of abbreviations (p. 432–433).

The lucid expository style employed throughout is exemplified in the Introduction. Here, a systematic summary of definitions and theorems relating to infinite products and infinite series of various types supplements the list of relevant formulas. Similar explanatory text serves as introduction to several of the subsequent chapters and their subdivisions.

Typographical errors found in the text are minor and do not detract from the intelligibility of the textual material. The typography, especially in a compilation of such a large number of formulas, is uniformly excellent, and the appearance of the book is attractive. Professor Archibald's opinion that the first edition was "undoubtedly of considerable value for any mathematician to have at hand" certainly holds true for this latest version.

J. W. W.

1. I. M. Ryžhik & I. S. Gradšteĭn, *Tablitsy Integralov, Summ, Riadov i Proizvedeniĭ*, [Tables of Integrals, Sums, Series and Products], The State Publishing House for Technical and Theoretical Literature, Moscow, 1951.

2. R. C. Archibald, **RMT** 219, *MTAC*, v. 1, 1943/45, p. 442.

3. Bierens de Haan, *Nouvelles Tables d'Intégrales Définies*, Leyden 1867. Reprinted by G. E. Stechert & Co., New York, 1939.

70[G].—Eugene Prange, *An Algorism for Factoring* $X^n - 1$ *over a Finite Field*, AFCRC-TN-59-775, U. S. Air Force, Bedford, Mass., October 1959, iii + 20 p., 27 cm.

An algorism is given for factoring $X^n - 1$ over the finite field F_q of q elements. This can be of use in constructing another finite field over F_q, in constructing a linear recursion of period n over F_q, or in constructing cyclic error-correcting group codes. The algorism has two parts: Step 1, the construction of the multiplicative identities of the minimal ideals of $F_q[X]/[X^n - 1]$; Step 2, the use of these idempotents in the construction of the irreducible factors of $X^n - 1$.

Author's Abstract

71[G].—M. Rotenberg, R. Bivins, N. Metropolis & J. K. Wooten, Jr., *The 3-j and 6-j Symbols*, The Technology Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1960, viii + 498 p., 29 cm. Price $16.00.

Wigner's 3-j symbol is closely related to the Clebsch-Gordan coefficients used in the coupling of angular momenta. If $\mathbf{J}_1$ and $\mathbf{J}_2$ are coupled to give $\mathbf{J}$, with j, j_1, j_2 as the total-angular-momentum quantum numbers and m, m_1, m_2 as the quantum numbers for the z-components, the expansion coefficient giving the coupled states in terms of the uncoupled are

$$(j_1 j_2 j m \mid j_1 m_1 j_2 m_2) = (-1)^{j_2 - j_1 - m}(2j+1)^{1/2} \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix}.$$

Here the symbol on the left is the expansion coefficient in the notation of Condon and Shortley, *Theory of Atomic Spectra*; the last symbol on the right is the Wigner 3-j symbol. The advantage of a tabulation of the 3-j symbols,

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

rather than of expansion coefficients results from the high degree of symmetry of the 3-j symbols. At most a sign change results from an interchange of columns or from changing the signs of all the m's. Thus, from these tables, which are restricted to

$$j_1 \geqq j_2 \geqq j_3 \quad \text{and} \quad m_2 \leqq 0,$$

all expansion coefficients can be obtained for any $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots, 8$.

The 6-j symbols occur in the coupling of three angular momenta $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$. One can either couple $\mathbf{J}_1$ and $\mathbf{J}_2$ first to obtain $\mathbf{J}'$, and then couple $\mathbf{J}'$ to $\mathbf{J}_3$ to obtain $\mathbf{J}$—this coupling scheme results in quantum numbers j_1, j_2, j', j_3, j, m—or one can first couple $\mathbf{J}_2$ and $\mathbf{J}_3$ to obtain $\mathbf{J}''$, and then couple $\mathbf{J}''$ to $\mathbf{J}_1$ to obtain $\mathbf{J}$—this scheme results in quantum numbers j_2, j_3, j'', j_1, j, m. The overlap integral between these two representations is given by

$$(j_1 j_2 j' j_3 j m \mid j_2 j_3 j'' j_1 j m) = (-1)^{j_1+j_2+j_3+j}[(2j'+1)(2j''+1)]^{1/2} \begin{Bmatrix} j_1 & j_2 & j' \\ j_3 & j & j'' \end{Bmatrix},$$

where the last symbol is a 6-j symbol. The 6-j symbol tabulated,

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix},$$

has sufficient symmetry that it need be listed only for

$$j_1 \geqq j_2 \geqq j_3, \quad j_1 \geqq l_1, \quad j_2 \geqq l_2, \quad j_2 \geqq l_3.$$

Within these restrictions, it is listed for all half-integral values of the j's and l's from 0 to 8.

The tables were computed on the MANIAC II at the Los Alamos Scientific Laboratory. An adequate 40-page introduction describes the symbols and their uses. Since the symbols are the square-roots of rational fractions, the squares are tabulated as powers of primes in a shorthand notation, with an asterisk used to denote the negative square root. For example, the entry $*\underline{1}5\underline{1}0,\underline{2}\underline{2}\underline{2}1$ is to be interpreted as

$$-\left[\frac{3^5 \times 7^0 \times 19^1}{2^1 \times 5^1 \times 11^2 \times 13^2 \times 17^2}\right]^{1/2}$$

George Shortley

Booz-Allen Applied Research, Inc.
Bethesda 14, Maryland

72[I].—Herbert E. Salzer & Genevieve M. Kimbro, *Tables for Bivariate Osculatory Interpolation Over a Cartesian Grid*, Convair-Astronautics, Convair Division of General Dynamics Corporation, San Diego, California, 1958, 40 p.

Formulas are developed for binary polynomials $P(x, y)$ which agree together with the partial derivatives $P_x(x, y)$ and $P_y(x, y)$, with $f \equiv f(x, y), f_x \equiv f_x(x, y)$ and $f_y \equiv f_y(x, y)$ at n specified points. They have the advantage over ordinary bivariate interpolation of packing $3n$ conditions into n points. Unlike univariate polynomial osculatory interpolation which always possesses a solution for any irregular configuration of fixed points, a binary polynomial of prescribed form may not satisfy those $3n$ conditions for *any* choice of interpolation points, or may fail for just certain *special* configurations. Explicit formulas or methods are developed for the general 2- to 5-point cases. For interpolation over any square Cartesian grid of length h, for suitable 2- to 5-point configurations of (x_i, y_i), according to the formula

$$\text{(I)} \quad \begin{aligned} f(x, y) &\equiv f(x_0 + ph, y_0 + qh) \sim P(x_0 + ph, y_0 + qh) \\ &= \sum_{i=0}^{n-1} \{A_i^{(n)}(p, q)f_i + h[B_i^{(n)}(p, q)f_{x_i} + C_i^{(n)}(p, q)f_{y_i}]\}, \end{aligned}$$

tables of exact values of $A_i^{(n)}(p, q)$, $B_i^{(n)}(p, q)$ and $C_i^{(n)}(p, q)$ are given for p and q each ranging from 0 to 1 at intervals of 0.1. A closed expression for the remainder in (I) has not been found. In its place, formulas are derived for the leading terms in the bivariate Taylor expansions for the remainders. These formulas should cut down the number of needed strips in the numerical solution of Cauchy's problem for first order partial differential equations by the method of characteristic strips.

Author's Abstract

73[K].—D. E. Barton & F. N. David, "A test for birth order effect," *Ann. Human Genetics*, v. 22, 1958, p. 250–257.

In an ordered sequence of trials it is known that there were r_1 occurrences and r_2 non-occurrences of a particular event. The question at issue has to do with the randomness of the positions of the occurrences in the sequence vs. a tendency to appear either at the ends or in the middle of the sequence. A test criterion is obtained by dividing the sequence between the Rth and the $(R + 1)$st event, where $r_1 + r_2 = 2R$ or $2R + 1$, and then assigning ranks $1, 2, \cdots$ to the events by order of position beginning at the point of division and proceeding to the left and then again starting with 1 to the right. The sum of the ranks of occurrences as assigned is the test criterion S. For $r_1 + r_2 = 4(1)16$ with $r_1 \geqq r_2$, on the null hypothesis of random position of occurrences, the exact distribution of S is tabulated for each pair (r_1, r_2) for $r_2 \geqq 2$. For $r_1 + r_2 > 16$, it is stated that the distribution of S is sufficiently closely approximated by a normal distribution.

C. C. Craig

University of Michigan
Ann Arbor, Michigan

74[K].—G. D. Berndt, "Power functions of the gamma distribution," *Ann. Math. Stat.*, v. 29, 1958, p. 302–306.

If x is a random variable from a gamma distribution with frequency function, $f_0 = f(x; \beta, \nu) = |\beta^\nu \Gamma(\nu)|^{-1} x^{\nu-1} \exp(-x/\beta)$; $\beta > 0$, $\nu > 0$ and $x \geqq 0$; the frequency function for δx with $\delta > 1$ is $f_1 = f(\delta x; \delta\beta, \nu)$. To test the null hypothesis on the mean, $H_0: \mu = \beta\nu$, against the alternate, $H_1: \mu = \delta\beta\nu$, $\delta > 1$, one may use the statistic $\alpha(x)$ with the critical region defined by $\alpha = \int_{\alpha(x)}^{\infty} f_0 \, dx$. Then the power of this test is $\pi_\delta = \int_{\alpha(x)}^{\infty} f_1 \, dx$. The mean of a random sample from a universe whose frequency law is $f_0 = f(x; \beta, \nu)$ obeys a gamma distribution with parameters β/n and $n\nu$. For $\alpha = .01, .05, .1$, charts are given for reading π_δ for $1 \leqq \delta \leqq 4$ and $\nu = \frac{1}{2}, 1(1)5, 7, 10(5)50$.

C. C. Craig

University of Michigan
Ann Arbor, Michigan

75[K].—H. Chernoff & L. E. Moses, *Elementary Decision Theory*, John Wiley & Sons, Inc., New York, 1959, xv + 364 p., 24 cm. Price $7.50.

This book is an elementary approach in the theory of statistics through the theory of the strategy of games, and as such is a refreshing change from the usual run of elementary statistics textbooks. The authors state that only an understanding of high school (U.S.) mathematics is required, which is possibly optimistic. However, it is fair to say that the mathematical content of the book is not excessive, the exposition being mostly by example.

Chapter I gives the principles of decision and an introduction to minimax. Chapter II, entitled Data Processing, turns out to be our old friends graphical representation and means and standard deviations. No mention is made of grouping corrections. There are 38 pages on probability and random variables, both continuous and discrete, followed by a brisk treatment of utility and descriptive statistics. This chapter (IV) will be rather difficult for the beginner.

The authors have now reached a stage where they can, and do, begin to discuss strategies. Chapter V, "Uncertainty due to Ignorance of the State of Nature," gives simple Bayes strategies, minimax, and expected regret. (The reviewer liked the remark, "it is difficult to visualize four-dimensional space.") Further chapters cover further Bayes strategy and the application to problems which might arise in what is termed "Classical" statistics, in testing hypotheses, and in estimation. There is a series of appendices in which some of the statements in the main body of the text are proved.

This is an interesting book and may prove useful to those who see the interpretation of numerical data as just one more decision to take; it is greatly to be doubted whether it is of general utility. In one way, however, it is unique. Fisher and Neyman in their several ways might be said to have contributed to statistical decision theory, but are not deemed worthy of reference.

F. N. David

Statistics Department
University College London
Gower Street
London WC 1, England

76[K].—R. Doornbos & H. J. Prins, "On slippage tests. I," *Indagationes Mathematicae*, v. 20, 1958, p. 38–46 (*Proc. Kon. Ned. Ak. van Wetensch.*, v. 61, Sec. A, 1958, p. 38–46); "On slippage tests. II," Ibid., p. 47–55; "On slippage tests. III," Ibid., p. 438–447.

The tables, which appear in part III, are related to two of the special cases included in this series of papers. In the first, from each of k Poisson distributions, with means μ_i, a random drawing Z_i is taken ($i = 1, 2, \cdots k$). To test the null hypothesis that $u_i = u$, $i = 1, \cdots, k$, for which the table is prepared, against the alternate that one of the u_i's is greater than the others which have equal values, the authors propose the statistic, $\max Z_i$. For $k = 2(1)10$ and the sum of the k observations, $n = 2(1)25$, values of $\max Z_i$ are given for which the significance levels are near 5% and 1%. In each case the actual significance levels are given to 3D.

In the second case, each of k objects is ranked by each of m observers. The null hypothesis under test is that each of the m rankings is independently and randomly chosen from the set of permutations of the integers $1, 2, \cdots, k$. As a test against the alternate that one of the objects has a higher probability of being ranked low while the others are ranked in random order, the proposed statistic is $\min S_i$ where S_i is the sum of ranks assigned the i-th object ($i = 1, 2, \cdots, k$). Critical values S_α of $\min S_i$ for significance levels near $\alpha = .05, .025, .01$ are tabled for $m = 3(1)9$ and $k = 2(1)10$. Again in each case true significance levels are shown to 3D.

C. C. Craig

University of Michigan
Ann Arbor, Michigan

77[K].—F. G. Foster, "Upper percentage points of the generalized beta distribution. III," *Biometrika*, v. 45, 1958, p. 492–503.

Let $\theta_{\max}$ denote the greatest root of $| \nu_2 B - (\nu_1 A + \nu_2 B)| = 0$ where A and B are independent estimates, based on ν_1 and ν_2 degrees of freedom, of a parent dispersion matrix of a four-dimensional multinormal distribution. Define

$$I_x(4; p, q) = \Pr(\theta_{\max} \leqq k)$$

with $p = \frac{1}{2}(\nu_2 - 3)$, $q = \frac{1}{2}(\nu_1 - 3)$. Employing methods similar to those used in two preceding papers [1], [2] for the two and three-dimensional cases, the author tabulates 80%, 85%, 90%, 95%, and 99% points of $I_x(4; p, q)$ to 4D for $\nu_1 = 5(2)195$ and $\nu_2 = 4(1)11$.

C. C. Craig

University of Michigan
Ann Arbor, Michigan

1. F. G. Foster & D. H. Rees, "Upper percentage points of the generalized beta distribution. I," *Biometrika*, v. 44, 1957, p. 237–247. [*MTAC*, Rev. 165, v. 12, 1958, p. 302]
2. F. G. Foster, "Upper percentage points of the generalized beta distribution. II," *Biometrika*, v. 44, 1957, p. 441–453. [*MTAC*, Rev. 167, v. 12, 1958, p. 302.]

78[K].—W. Hetz & H. Klinger, "Untersuchungen zur Frage der Verteilung von Objekten auf Plätze," *Metrika*, v. 1, 1958, p. 3–20.

For the classical distribution problem in which k indistinguishable objects are randomly distributed into n distinguishable cells (as in Maxwell-Boltzmann statistics) the authors take the number, s, of occupied cells as a statistic to test the hypothesis of uniform probability over the cells. Let $P(s \mid n, k)$ be the probability density for s. The correspondence is noted between this distribution and the results of a series of n drawings from a discrete distribution in which the random variable assumes only the values 0, 1, 2, $\cdots$, and in which the sample sum is k and the number of non-zero values is s. In developing a recursion formula for $P(s \mid n, k)$ it is shown that the uniform distribution over cells arises from the Poisson distribution, and the binomial and negative binomial distribution give particular non-uniformities. The function tabulated is $Z_{k;\alpha}$, which is defined under the hypothesis of uniformity by $\sum_{s=1}^{Z_{k;\alpha}} P(s \mid n, k) \leqq \alpha$ and $\sum_{s=1}^{Z_{k;\alpha}+1} P(s \mid n, k) > \alpha$, for $\alpha = .05, .01, .001$; $n = 3(1)20$, and ranges of k varying from (3, 15) for $n = 3$ to (2, 100) for $n = 20$.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

79[K].—A. HUITSON, "Further critical values for the sum of two variances," *Biometrika*, v. 45, 1958, p. 279–282.

Let s_i^2, $i = 1, 2$, be an estimate of the variance σ_i^2 with f_i degrees of freedom so that $f_i s_i^2/\sigma_i^2$ is distributed as χ^2 with f_i dif. To assign confidence limits to the form $\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2$, where λ_1 and λ_2 are arbitrary positive constants, the author has previously [1] tabulated upper and lower 5% and 1% critical values of

$$(\lambda_1 s_1^2 + \lambda_2 s_2^2)/(\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2).$$

The present tables are an extension, giving upper and lower $2\frac{1}{2}$% and $\frac{1}{2}$% critical values for the same function to 2D for $\lambda_1 s_1^2/(\lambda_1 s_1^2 + \lambda_2 s_2^2) = 0(.1)1$ and $f_1, f_2 = 16, 36, 144, \infty$.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. A. HUITSON, "A method of assigning confidence limits to linear combinations of variances," *Biometrika*, v. 42, 1955, p. 471–479. [*MTAC*, Rev. 19, v. 12, 1958, p. 71.]

80[K].—SOLOMON KULLBACK, *Information Theory and Statistics*, John Wiley & Sons, New York, 1959, xvii + 395 p., 24 cm. Price $12.50.

This interesting book, which discusses logarithmic measures of information and their applications to the testing of statistical hypotheses, contains three extended tables in addition to a number of shorter or more specialized ones. Table I gives $\log_e n$ and $n \log_e n$ to 10D for $n = 1(1)1000$. Table II lists values of

$$p_1 \log_e \frac{p_1}{p_2} + (1 - p_1) \log_e \frac{1 - p_1}{1 - p_2} \quad \text{to 7D for} \quad p_1, p_2 = .01(.01).05(.05).95$$

(.01).99. Table III gives 5% points for noncentral χ^2 to 4D with $2n$ degrees of freedom for $n = 1(1)7$ and noncentrality parameter β^2 for $\beta = 0(.2)5$. As it is stated, this is taken directly from an equivalent table of R. A. Fisher [1].

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. R. A. FISHER, "The general sampling distribution of the multiple correlation," *Proc. Roy. Soc., A.*, 1928, p. 654–673. See p. 665.

81[K].—G. J. LIEBERMAN, "Tables for one-sided statistical tolerance limits," *Industrial Quality Control*, v. 14, No. 10, 1958, p. 7–9.

Given a sample of n from $N(\mu, \sigma^2)$, it is desired to determine from the sample a quantity a (or b) such that with probability γ, the interval $(-\infty, a)$ (or the interval (b, ∞)) will include at least the fraction $1 - \alpha$ of the population. The tables give values of K to 3D for $n = 3(1)25(5)50$, $\gamma = .75, .9, .95, .99$, and $\alpha = .25, .1, .05, .01, .001$, such that $a = \bar{X} - Ks$ and $b = \bar{X} - Ks$, where X is the sample mean and S^2 is the usual unbiased estimate of σ^2. For more extensive tables and a more complete discussion see [1].

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. D. B. OWEN, *Tables of Factors for One-sided Tolerance Limits for a Normal Distribution*, Office of Technical Services, Dept. of Commerce, Washington, D. C., 1958. [See RMT 82.]

82[K].—D. B. OWEN, *Tables of Factors for One-sided Tolerance Limits for a Normal Distribution*, Sandia Corporation, SCR-13, 1958, 131 p., 28 cm. Obtainable from the Office of Technical Services, Dept. of Commerce, Washington 25, D. C. Price $2.75.

Given a sample of n from $N(\mu, \sigma^2)$, with $\bar{x}$ the sample mean and S^2 the usual unbiased estimate of σ^2, these tables give values of k for which

$$\Pr[\Pr(x \leqq \bar{x} + ks) \geqq P] = \gamma.$$

As stated, Table I is a reproduction of one given by Johnson & Welch [1] in which values of k are given to 3D for $\gamma = .95$, $n = 5(1)10, 17, 37, 145, \infty$ and $P = 0.7(.05).85, .875, .9, .935, .95, .96, .975, .99, .995, .996, .9975, .999, .9995$. It is also explained that Table II was obtained from Resnikoff & Lieberman's table of percentage points of the noncentral t-distribution [2] appropriately modified to give k values to 3D for $n = 3(1)25(5)50, \infty$ and $P = .75, .85, .9, .935, .96, .975, .99, .996, .9975, .999$ for $\gamma = .75, .9, .95$. For $\gamma = .99, .995$, $n = 6(1)25(5)50, \infty$, while P has the same range as before. The more extensive Table III gives values to 5D obtained by an approximative method due to Wallis [3] for $n = 2(1)200(5)400(25)1000, \infty$, $P = .7, .8, .9, .95, .99, .999$, and $\gamma = .7, .8, .9, .95, .99, .999$. For small n and the larger values of P and γ, the approximation breaks down and the entry is left blank or given with a warning sign that comparison should be made with neighboring values. (However it looks to the reviewer as if this sign has been omitted from the entries for $n = 2$, $P = .99, .999$, and $\gamma = .999$.) Finally Table IV is obtained from Bowker's table of two-sided tolerance limits [3] by an approximate procedure suggested by McClung [4] to give conservative values of k for one-sided limits. Here values are given to 3D for $n = 2(1)102(2)180(5)300(10)400(25)750(50)1000, \infty$, $P = .875, .95, .975, .995, .9995$, and $\gamma = .75, .9, .99$.

In an appendix auxiliary tables compare values in the four tables for selected values of the four parameters. The maximum difference shown between Tables I and II is .01. It is concluded that values in Table III will probably be underestimates for $\gamma \leqq .95$ and overestimates for $\gamma \geqq .99$, while in Table IV, k is probably underestimated for $P = .875$ and overestimated for the other P values. Differences shown between Table II and Table III values in a few cases exceed 20% of the presumably more accurate Table II values and differences shown between Table II and Table IV sometimes exceed 10% of the Table II values.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. N. L. JOHNSON & B. L. WELCH, "Applications of the non-central t-distribution," *Biometrika*, v. 31, 1939, p. 362–389.
2. G. J. RESNIKOFF & G. J. LIEBERMAN, *Tables of the Noncentral t-Distribution*, Stanford University Press, Stanford, Calif., 1957.
3. C. EISENHART, M. W. HASTAY & W. A. WALLIS, *Techniques of Statistical Analysis*, McGraw-Hill Book Co., New York, 1947.
4. R. M. MCCLUNG, "First aid for pet projects injured in the lab or on the range or what to do until the statistician comes," U. S. Naval Ordnance Test Station Technical Memorandum No. 1113, October 1955.

83[K].—K. V. RAMACHANDRAN, "On the Studentized smallest chi-square," Amer. Stat. Assn., *Jn.*, v. 53, 1958, p. 868–872.

Consider the F statistics, $\frac{S_i}{S} \cdot \frac{m}{t}$, $i = 1, 2, \cdots, k$, in which $S_1, S_2, \cdots, S_k$ and S are mutually independent, with each $S_{i/\sigma}2$ having a χ^2 distribution under the null hypothesis with t degrees of freedom and S/σ^2 a χ^2 distribution with m d.f. There are numerous applications of statistical methods, a few of which are discussed, in which one needs the value of V for which $\Pr \mid \frac{S_{\min}}{S} \frac{m}{t} \geqq V \mid = 1 - \alpha$. The author tabulates lower 5% points of $\frac{S_{\min}}{S} \cdot \frac{m}{t}$ for values of t, m and k as follows: For $t = 1$, $m \geqq 5$, $k = 1(1)8$ to 1S; for $t = 2$, $5 < m < 10$ and $m \geqq 12$, $k = 1(1)8$ to 3D; for $t = 3, 4, 6$, $m = 5, 6(2)12, 20, 24, \infty$, $k = 1(1)8$ to 3D; for $t = 1(1)4(2)12, 16, 20$, $m = \infty$, $k = 1(1)8$ to 3D; for $t = 1(1)4(2)12, 16, 20$, $m = 5, 6(2)12, 20, 24, \infty$, $k = 1, 2, 3$ to 3D.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

84[K].—A. E. SARHAN & B. G. GREENBERG, "Estimation of location and scale parameters by order statistics from singly and doubly censored samples. Part II.," *Ann. Math. Stat.*, v. 29, 1958, p. 79–105.

This paper, a continuation of a previous one [1], is mainly devoted to an extension of tables given in the earlier paper to cover samples $11 \leqq n \leqq 15$ and to a discussion of efficiencies of the estimators used. Samples of n are from $N(\mu, \sigma^2)$; r_1 and r_2 observations are censored in the left and right tails respectively ($r_1 r_2 \geqq 0$); and $\bar{x}$ and σ are estimated by the most efficient linear forms in the ordered uncensored observations. Table I gives the coefficients for these best linear systematic statistics to 4D for all combinations of r_1, r_2 for $n = 11(1)15$. Table II gives variances and the covariance of these estimates to 4D for $n = 11(1)15$ and all pairs of r_1, r_2 values. In Table III efficiencies of the two estimates relative to that for uncensored samples are given to 4D for the same range of values of n and r_1, r_2. For

$n = 12$ and 15, variances and efficiencies relative to best linear systematic estimates are given for alternate estimates proposed by Gupta [2] for $n > 10$, and generalized in [1] to doubly censored samples, are given to 8D and 4D respectively for all r_1, r_2. The authors state that extensions of Tables I, II, III to 8D for $16 \leqq n \leqq 20$ are available upon application.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. A. E. SARHAN & B. G. GREENBERG, "Estimation of location and scale parameters by order statistics from singly and doubly censored samples. Part I. The normal distribution up to samples of size 10," *Ann. Math. Stat.*, v. 27, 1957, p. 427–451. [*MTAC*, Review 141, v. 12, 1958, p. 289.]
2. A. K. GUPTA, "Estimation of the mean and standard deviation of a normal population from a censored sample." *Biometrika*, v. 39, 1952, p. 88–95.

85[K].—J. M. SENGUPTA & NIKHILESH BHATTACHARYA, "Tables of random normal deviates," *Sankhya*, v. 20, 1958, p. 250–286.

As explained by the editor in a foreword, this is a reissue of an original table of random normal deviates which appeared in 1934 in *Sankhya* [1]. Since errors had been discovered in the earlier tables, the new set was reconstructed by conversion of Tippett's random numbers [2] to random normal deviates, as was the case before. After the present table was prepared, in 1952, as stated by the editor, it was learned that an identical table had been constructed in 1954 at the University of California. On comparison it was found that the two tables checked perfectly. As discussed in the text, rather extensive tests of the hypothesis that the entries were random drawings from $N(0, 1)$ were applied with satisfactory results. These tables contain 10,400 3D numbers.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. P. C. MAHARANOBIS, S. S. BOSE, P. R. ROY & S. K. BANNERJEE, "Tables of random samples from a normal population," *Sankhya*, v. 1, 1934, p. 289–328.
2. L. H. C. TIPPETT, *Random Sampling Numbers*, Tracts for Computers, No. XV, Cambridge University Press, London, 1927.

86[K].—MINORU SIOTANI, "Note on the utilization of the generalized Student ratio in analysis of variance or dispersion," *Ann. Inst. Stat. Math.*, v. 9, 1958, p. 157–171.

In samples from a p-dimensional normal universe an important statistic, applications of which are discussed in this paper, is $T_0^2 = m \operatorname{tr} L^{-1}V$ in which L and V are two independent unbiased estimates of the population variance matrix with n and m degrees of freedom respectively. Tables are given for the 5% and 1% points of the distribution of T_0^2 to 2D for $m = 1(1)10(2)20$ and

$$n = 10(2)30(5)50, 60, 80, 100.$$

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

87[K].—MINORU SIOTANI & MASARU OZÄWA, "Tables for testing the homogeneity of k independent binomial experiments on a certain event based on the range," *Ann. Inst. Stat. Math.*, v. 10, 1958, p. 47–63.

Let k series of N trials each of a certain event be performed with the outcome of ν_i occurrences in the i-th series in which the fixed probability of occurrence was p_i, $i = 1, 2, \cdots, k$. To test the null hypothesis of homogeneity:

$$p_1 = p_2 = \cdots = p_k = p,$$

Siotani had previously proposed the statistic, $R_k(N, p)$, the range of the ν_i [1]. The tables in this paper give for $N = 10(1)20, 22, 25, 27, 30$; $k = 2(1)15$;

$$p = .1(.1).5;$$

$\alpha = .001, .005, .01(.01).06, .08, .1$, the greatest r_k for which

$$\Pr\{R_k(N, p) \geqq r_k\} < \alpha + .0005.$$

The cases in which for the r_k given, $\alpha < \Pr\{R_k(N, p) \geqq r_k\} < \alpha + .0005$ or

$$\alpha - .005 < \Pr\{R_k(N, p) \geqq r_k\} < \alpha$$

are indicated by attaching a $+$ or a $-$ respectively to the value of r_k.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. MINORU SIOTANI, "Order statistics for discrete case with a numerical application to the binomial distribution," *Ann. Inst. Stat. Math.*, v. 8, 1956, p. 95–104.

88[K].—P. N. SOMERVILLE, "Tables for obtaining non-parametric tolerance limits," *Ann. Math. Stat.*, v. 29, 1958, p. 599–601.

Let P be the fraction of a population having a continuous but unknown distribution function that lies between the r-th smallest and the s-th largest values in a random sample of n drawn from that population. Then for any $r, s \geqq 0$ such that $r + s = m$, Table I gives the largest value of m such that with confidence coefficient $\geqq \gamma$ we may assert that $100P\%$ of the population lies in the interval (r, s) for $\gamma = .5, .75, .9, .95, .99$ and $n = 50(5)100(10)150, 170, 200(100)1000$. Table II gives γ to 2D for the assertion that $100P\%$ of the population lies within the range, $(r, s = 1)$, in a sample of n for $P = .5, .75, .9, .95, .99$ and

$$n = 3(1)20, 25, 30(10)100.$$

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

89[K].—G. P. STECK, "A table for computing trivariate normal probabilities," *Ann. Math. Stat.*, v. 29, 1958, p. 780–800.

Let X, Y, Z be standardized random variables obeying a trivariate normal distribution law. The author finds Pr $(X \leqq h, Y \leqq k, Z \leqq m)$ in terms of three functions:

$$G(x) = (2\pi)^{-1/2}\int_{-\infty}^{x} \exp\left(-\frac{x^2}{2}\right) dx, \qquad T(h, a)$$

$$= (2\pi)^{-1} \int_{0}^{a} [\exp\{-h^2(1 + x^2)/2\}](1 + x^2)^{-1}\, dx,$$

and

$$S(h, a, b) = \int_{-\infty}^{h} T(as, b)G'(s)\, ds.$$

The T-function has been tabulated by D. B. Owen [1, 2] and a table of $S(m, a, b)$ is given in the present paper to 7D for $a = 0(.1)2(.2)5(.5)8$, $b = .1(.1)1$ and a range of values of m decreasing from $0(.1)1.5, \infty$ for $a = 0(.1)1.2$ to $0(.1).3, \infty$ for $a = 6(.5)8$. The tabulated values are believed accurate to 0.6 in the seventh decimal place. There is considerable discussion of the main problem, of properties of and relations among the functions used, and a numerical example is worked out. The method of construction of the table is given and the efficacy of linear interpolation in it is discussed.

C. C. Craig

University of Michigan
An Arbor, Michigan

1. D. B. Owen, *The Bivariate Normal Probability Distribution*, Office of Technical Services, Department of Commerce, Washington, D. C., 1957, [*MTAC* Review 134, v. 12, 1958, p. 285–286.]

2. D. B. Owen, "Tables for computing bivariate normal probabilities," *Ann. Math. Stat.*, v. 27, 1956, p. 1075–1090. [*MTAC* Review 135, v. 12, 1958, p. 286.]

90[K].—G. Taguti, "Tables of tolerance coefficients for normal populations," Union of Japanese Scientists and Engineers, *Reports of Statistical Application Research*, v. 5, 1958, p. 73–118.

The tolerance limits T_1, T_2 are to be determined so that with probability $1 - \alpha$ the interval (T_1, T_2) includes a given fraction, P, of the population. Following the method of Wald & Wolfowitz [1] for a sample from $N(\mu, \sigma^2)$, T_1 and T_2 are found by $T_1 = \hat{\mu} - k\sqrt{S_e/\nu}$ and $T_2 = \hat{\mu} + k\sqrt{S_e/\nu}$, in which $\hat{\mu}$ is an unbiased estimate of μ with variance σ^2/n and S_e is an independent error sum of squares with ν degrees of freedom. As illustrated by the author this permits useful applications in which n is not simply the sample size and $\nu = n - 1$ as is the case for the tables of Bowker [2]. The present tables give k to 3S for $P = .9, .95, .99$, $1 - \alpha = .9, .95, .99$, $n = .5(.5)2(1)10(2)20(5)30(10)60(20)100, 200, 500, 1000, \infty$ and $\nu = 1(1)20(2)30(5)100(100)1000, \infty$. The calculations were done with a slide rule and the author fears there may be errors up to one per cent. Some cursory comparisons with Bowker's tables for $\nu = n - 1$ showed frequent differences in the third significant figure.

C. C. Craig

University of Michigan
Ann Arbor, Michigan

1. A. WALD & J. WOLFOWITZ, "Tolerance limits for a normal distribution," *Ann. Math. Stat.*, v. 17, 1946, p. 208–215.
2. CHURCHILL EISENHART, M. W. HASTAY & W. A. WALLIS, *Techniques of Statistical Analysis*, McGraw-Hill Book Co., New York. 1947. (See p. 102–107.)

91[K].—R. F. TATE & R. L. GOAN, "Minimum variance unbiased estimation for the truncated Poisson distribution," *Ann. Math. Stat.*, v. 29, 1958, p. 755–765.

For a sample of n from a population with the density function, $e^{-\lambda}\lambda^x/(1 - e^{-\lambda})$, $x = 1, 2, \cdots$, i.e., a Poisson distribution truncated on the left at $x = 1$, the authors derive the minimum variance unbiased estimation of

$$\lambda : \tilde{\lambda}_0(t) = \frac{t}{n}\left(1 - \frac{\mathfrak{S}_{t-1}^{n-1}}{\mathfrak{S}_t^n}\right) = \frac{t}{n}\, C(n, t),$$

in which t is the sample sum and $\mathfrak{S}_t^n$ is a Stirling number of the second kind. Using an unpublished table of F. L. Micksa [1] of $\mathfrak{S}_t^n$ for $n = 1(1)t$, $t = 1(1)50$, this paper contains a table of $C(n, t)$ to 5D for $n = 2(1)t - 1$, $t = 3(1)50$.

C. C. CRAIG

University of Michigan
Ann Arbor, Michigan

1. FRANCIS L. MIKSA, *Stirling numbers of the second kind*, RMT 85, *MTAC* v. 9, 1955, p. 198.

92[K, P].—P. A. P. MORAN, *The Theory of Storage*, John Wiley & Sons, Inc., New York, 1960, 111 p., 19 cm. Price $2.50.

This is a book about dams. Prof. Moran is at the Australian National University at Canberra, and I imagine that dams have great practical interest there. For many years he has been interested in estimating the probability that a dam will go dry or that it will overflow. He is also interested in how one finds a program of releasing water from a dam in such a way as to optimize the operations of a hydroelectric plant.

The first chapter contains some basic information about statistics and probability. To spare 14 pages for this from a total of a mere 96 shows how necessary Prof. Moran considered it to be.

The second chapter considers various general inventory and queueing problems analogous to dam problems.

In the third chapter the author plunges into his favorite topic, dams. First he considers discrete time—he looks at his water level only once a day. Under certain conditions distributions for the amount of water can be found, but two troublesome conditions occur which limit the regions of analyticity of the distributions. One is overflow. The other is running dry. If one ignores either or both of these, then he is dealing with an imaginary "infinite dam". Some queueing is analogous to an infinite dam, since there is no law limiting the lengths of queues.

Another chapter is devoted to dams which have as input a continuous flow, and from which the release is continuous.

In practice the inputs do not satisfy the assumption of independence, dry weeks tend to come in succession, so the results of the first four chapters are of limited applicability. Monte Carlo methods get estimates of the probabilities without these restrictions, and in addition can be applied to configurations of dams completely beyond other methods of analysis. Of course Monte Carlo has disadvantages of its own. An example is given of a complex configuration for which probabilities were urgently wanted. A large retaining wall of earth was to be built. Overflow would ruin it, so a diversion tunnel was to be built large enough to insure against this contingency. During the building the tunnel is closed to permit pouring the concrete at its mouth. If water accumulates too high behind the wall there will be danger of overflow, ruining the wall. This can be prevented by opening the tunnel, ruining its outworks but preferable to damaging the main wall. The critical height changes each day as the wall is built up. What are the chances of this decision being forced?

The last few pages are devoted to ways of finding an optimum strategy for operating a hydroelectric system, or other program of releasing, replenishing, or otherwise tending the locks. The recommended solution is a method of successive approximations, which would probably be feasible only on a digital computer. The author suggests that a special analog device would be in order for the more complicated configurations.

The analogies between dams and queues or inventories are not pursued beyond the third chapter, in which it is merely mentioned. If these analogies are indeed valid they deserve more treatment. Without this treatment the title is misleading, for we find we are storing only water.

H. H. Campaigne

National Security Agency
Washington 25, District of Columbia

93[M, X].—Jakob Horn & Hans Wittich, *Gewöhnliche Differentialgleichungen*, Walter de Gruyter & Co., Berlin, 1960, 275 p., 24 cm. Price DM 32.

This book is the sixth completely revised edition of Jakob Horn's *Gewöhnliche Differentialgleichungen*, which was published first in 1905. Like the previous editions, this book is intended for mathematicians, physicists, and engineers. In the selection of the material somewhat greater emphasis has been given to subjects that lend themselves to applications. Nevertheless, this book is primarily an introduction to the theory of ordinary differential equations. The text contains existence proofs and a comparatively detailed presentation of differential equations in the complex domain.

Considerable space is devoted to special functions which arise from differential equations. Numerical and graphical methods of solution are treated in a brief chapter. Besides a thorough knowledge of differential and integral calculus on the part of the reader, a familiarity with the basic concepts of the theory of functions of a complex variable is assumed.

No problem sections appear in the book, but numerous illustrative examples are provided.

F. Theilheimer

Applied Mathematics Laboratory
David Taylor Model Basin
Washington, District of Columbia

94 [Q, S].—GARRETT BIRKHOFF AND R. E. LANGER, Editors, *Proceedings of Symposia in Applied Mathematics*, Vol. IX, "Orbit Theory," (Proceedings of the Ninth Symposium In Applied Mathematics of the American Mathematical Society, held at New York University April 4–6, 1957, cosponsored by The Office of Ordnance Research, Ordnance Corps, U. S. Army) American Mathematical Society, Providence, R. I., 1959, v + 195 p., 26 cm. Price $7.20.

The purpose of the book is, paraphrasing the words of the editors, to direct the attention of mathematicians to recent advances in celestial mechanics and, more importantly, to inform them of the problems that remain to be solved. Celestial mechanics owes its present form very largely to analysis, as it was developed in the eighteenth and nineteenth centuries. Whether modern mathematics can contribute anything important to the subject is a question that has hardly been explored, and it is high time that it should be.

Of the ten contributions by as many authors the first three deal with the motions of particles in magnetic fields, the remaining seven with motions of particles in gravitational fields. The magnetic fields considered are those in particle accelerators, in the galaxy, and about a laboratory model of the earth. The gravitational fields are principally those of the earth and of the solar system, although one paper deals generally with the field about any massive particle, and one with a general planetary system.

The various contributions are very uneven, ranging from rather trivial special applications of general formulae, through adaptations and modifications that are not trivial, to some important original contributions, both general and particular. Some authors describe what they have done themselves, some what others have done, and some what has not been done. Brouwer, Courant, and Olbert give special attention to unsolved problems; the references will be valuable to a mathematician not previously acquainted with their subjects. Herget and Eckert deal with practical computation.

G. M. CLEMENCE

U. S. Naval Observatory
Washington, D. C.

95[X].—RUDOLPH E. LANGER, Editor, *Boundary Problems in Differential Equations*, Proceedings of a symposium conducted by the Mathematics Research Center, University of Wisconsin, Madison, Wisconsin, The University of Wisconsin Press, Madison, 1960, x + 324 p., 24 cm. Price $4.00.

This volume contains the nineteen papers presented at the symposium on 'Boundary Problems in Differential Equations" conducted by the Mathematics Research Center at Madison, Wisconsin during the period April 20–22, 1959. The papers are quite varied in nature and subject matter, as is clear from the table of contents given below:

Complete Systems of Solutions for a Class of Singular Elliptic Partial Differential Equations
Peter Henrici, University of California
Application of the Theory of Monotonic Operators to Boundary Value Problems
Lothar Collatz, University of Hamburg, Germany
Upper and Lower Bounds for Quadratic Integrals and, at a Point, for Solutions of Linear Boundary Value Problems
J. B. Diaz, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland
Error Estimates for Boundary Value Problems Using Fixed-Point Theorems
Johann Schroder, University of Hamburg, Germany
On a Unified Theory of Boundary Value Problems for Elliptic-Parabolic Equations of Second Order
Gaetano Fichera, The Mathematical Institute, University of Rome, Italy
Factorization and Normalized Iterative Methods
Richard S. Varga, Westinghouse Electric Corporation, Bettis Atomic Power Division, Pittsburgh
Some Numerical Studies of Iterative Methods for Solving Elliptic Difference Equations
David Young and Louis Ehrlich, The University of Texas.
Presented by David Young
Albedo Functions for Elliptic Equations
Garrett Birkhoff, Harvard University
A Numerical Method for Analytic Continuation
Jim Douglas, Jr., The Rice Institute, Texas
Stress Distribution in an Infinite Elastic Sheet with a Doubly-Periodic Set of Equal Holes
W. T. Koiter, Technical University, Delft, Holland
Some Stress Singularities and Their Computation by Means of Integral Equations
Hans F. Bueckner, Mathematics Research Center, U. S. Army
Boundary Value Problems in Thermoelasticity
Ian N. Sneddon, The Univerity, Glasgow, Scotland
Some Numerical Experiments with Eigenvalue Problems in Ordinary Differentia Equations
Leslie Fox, University Computing Laboratory, Oxford, England
Dynamic Programming, Invariant Imbedding, and Two-Point Boundary Value Problems
Richard Bellman, The Rand Corporation, California
Remarks about the Rayleigh-Ritz Method
Richard Courant, Institute of Mathematical Sciences, New York University
Free Oscillations of a Fluid in a Container
B. Andreas Troesch, Space Technology Laboratories, Inc., California
A Variational Method for Computing the Echo Area of a Lamina
Calvin H. Wilcox, Mathematics Research Center, U. S. Army, and California Institute of Technology

Workers in Numerical Analysis will be particularly interested in the papers of Friedrichs, Garabedian, Collatz, Varga, Schroder, Young and Ehrlich, Douglas, and Fox. The first author has a very short paper in which he gives an interesting outline of a unified approach to the numerical treatment of linear partial differential equations irrespective of their type. The unified approach is said to also cover certain equations of mixed type. Unfortunately, the author did not have space to completely describe the conditions he must impose on the equations he treats.

Garabedian describes a method that has been used to calculate axially symmetric flows with free streamlines. In particular he discusses methods for calculating the contraction coefficient in the vena contracta. The method involves generalizing the differential equation governing the flow by introducing a parameter λ and studying the dependence of the solution as a function of λ.

Collatz's paper is an expository one in which he discusses various definitions of monotonic operators and applies such definitions to the determination of bounds on the solutions of various problems.

Schroder uses monotonic operators which satisfy a fixed point theorem to prove the existence of solutions to problems involving differential equations and boundary conditions. He also determines approximate solutions and error bounds by solving a so-called comparison problem.

Varga discusses a class of iterative methods for solving a system of linear equations which depend on the direct solution of matrix equations of matrices more general than tridiagonal matrices. He shows how such matrix equations can be directly and efficiently solved and, in addition, applies standard methods for accelerating convergence.

Young and Ehrlich report on numerical experiments which attempted to determine the extent to which theoretical results on the rate of convergence of the successive over-relaxation method for solving linear equations and for the Peaceman-Rachford method would apply for non-rectangular regions. The theoretical results are known for the latter method only in the rectangular case. In nearly every case it was found that the number of iterations using the Peaceman-Rachford method was less than was required using the successive over-relaxation method. However, approximately three times as much computer time is required for a double sweep of the former method as is required for a single step of the latter method.

Douglas discusses the determination of an approximation of an analytic function of a complex variable inside the disk $0 \leqq |z| \leqq 1$ when bounds on the function and its first two derivatives are known and when approximate values of the functions are known at p points equidistributed on the circle $|z| = 1$. An estimate of the error of the approximation is also obtained.

Fox discusses a method for the determination of approximate proper values and proper solutions to single or systems of ordinary differential equations for which a reasonable approximation is already known for the proper value. The method involves the introduction of parameters such as initial values of the solution at one of the boundary points and the determination of improved values for these parameters by the Newton process. The method described is not new, but the applications made by Fox to fairly difficult problems give an impressive demonstration of its power.

Space limitations prevent the reviewing of the remaining papers in this volume.

They are of high quality. The organizers of the conference are to be congratulated on the papers solicited. The University of Wisconsin Press has produced a handsome volume by a photographic process which makes a very readable page. The relatively low cost of the volume is especially noteworthy.

A. H. T.

96[X].—W. L. WILSON, JR., "Operators for solution of discrete Dirichlet and Plateau problems over a regular triangular grid," May 1959, 29 cm., 191 p. Deposited in UMT File.

These tables list to 10D coefficients of a matrix operator for conversion of boundary values over an equilateral triangle to a discrete harmonic function over a regular triangular grid of 190 points in this triangle [1]. Sixty-three boundary values are involved, of which the three at the vertices do not influence the interior values of the function. The tables are useful in the approximate numerical solution of the Laplace equation over this triangular region.

Solutions for smaller triangles have been placed in the UMT File by the same author [2].

Also included are tables giving 10D coefficients of the analog of the Douglas functional over this same grid. Specifically, these are coefficients of a quadratic form (using scalar multiplication) of vector functions from the grid points of the bounding equilateral triangle to some euclidean space such that the value of the form is the Dirichlet integral

$$D = \frac{1}{2} \int (E + G)\, d\sigma$$

where E and G are coefficients of the first fundamental form of the surface got by linear interpolation of the discrete harmonic vectors resulting from application of the operator described above to the boundary values. This is a discrete analog of the functional used by J. Douglas [3] in his solution of the Problem of Plateau; it has application in the approximate numerical solution of that problem.

C. B. TOMPKINS

University of California
Los Angeles, California

1. L. V. KANTOROVICH & K. I. KRYLOV (translated by CURTIS D. BENSTER) *Approximate Methods of Higher Analysis*, Nordhoff, Gronigen, Interscience, New York, 1958, p. 187–188.
2. W. L. WILSON, JR., "Tables of inverses to Laplacian operators over triangular grids," UMT File, MTAC, No. 58, v. XI, 1957, p. 108.
3. J. DOUGLAS, "Solution of the problem of Plateau," Amer. Math. Soc. *Trans.*, v. 33, 1931, p. 263–321.

97[Z].—JACK BONNELL DENNIS, *Mathematical Programming and Electrical Networks*, John Wiley & Sons, Inc., New York, 1959, vi + 186 p., 24 cm. Price $4.50.

As the title indicates, the purpose of this little monograph is to explore the relationships of general programming problems and corresponding electrical networks, with a view towards gaining physical insight and developing computational algorithms. The contents of the book essentially comprise the author's doctoral dissertation in the department of Electrical Engineering at M.I.T. The pages are offset reproductions of typescript. In a foreword by J. A. Stratton, it is stated that "there has long been a need for publication of research studies larger than a journal article but less ambitious than a finished book," and with this apology the present volume is put forth.

After an introductory Chapter 1, the general programming problem is presented in Chapter 2, along with discussions of convexity and concavity, the generalized method of Lagrangian multipliers due to Kuhn and Tucker, and duality. Chapter 3 consists of basic material on electrical networks containing resistors, diodes, ideal transformers, voltage sources, and current sources. The electrical network problem, which is a set of linear equations with side conditions in the form of inequalities due to the presence of diodes, is stated and shown to be equivalent to a quadratic programming problem (and its dual), viz., to find a feasible current distribution which minimizes the power absorbed by the voltage sources plus one-half the power absorbed by the resistors, with a corresponding statement for the dual. The concept of the two-terminal, or terminal-pair, network is introduced in this chapter, with a discussion of the set of solutions (ϵ, δ), where ϵ is the voltage between the terminals when current δ enters one terminal and leaves the other. The set of all (ϵ, δ) forms a "break-point curve" in the $\epsilon\delta$ plane, i.e., a non-decreasing polygonal graph.

Chapter 4 is devoted to the problem of flow in a network, which includes allocation, distribution, and assignment problems. It is shown that every flow problem can be realized by an electrical network containing only diodes, voltage sources, and current sources. Existence conditions from the theory of programming and non-redundancy assumptions are stated here in electrical network terms. Two algorithms are presented for the solution of diode-source networks, one corresponding to the primal and the other to the dual problem. They are similar to but more general than the procedure of Ford and Fulkerson for the transportation problem.

Chapters 5 and 6 treat the general linear and quadratic programming problems. A procedure is described for "tracing" the break-point curve corresponding to a pair of terminals, which is very similar to what goes on in the simplex method of Dantzig. This procedure is used with electrical models of the general quadratic (including linear) problems, and two types of algorithms for their solution are described. Chapter 7 contains a brief and incomplete discussion of the general programming problem. An algorithm is proposed which is based on the method of steepest descents. The main body of text is followed by eight appendices in which proofs are given of the main theorems of programming.

There are several typographical errors of a minor nature. The number of statements that are only partially true seems to be about par for a book on a mathematical subject written by an engineer.

J. Bram

Bureau of Ships
Navy Department
Washington 25, District of Columbia