REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

34[D, E, L, S, X].—NELSON A. LOGAN, *General Research in Diffraction Theory*, Volumes I and II, Missiles and Space Division, Lockheed Aircraft Corp., Sunnyvale, California. Reports LMSD-288087 and LMSD-288088, December 1959, xxiii + 345 p. and xviii + 268 p., 28 cm. Deposited in UMT File.

Volume I consists of a study of the theory and applications of a class of integrals defined by

$$A_m{}^n(\xi) = \sum_{s=1}^{\infty} \alpha_s{}^n[Ai'(-\alpha_s)]^{-m}e^{-(\sqrt{3}-i)\alpha_s\xi/2},$$

$$B_m{}^n(\xi) = \sum_{s=1}^{\infty} \beta_s{}^{n-1}[Ai(-\beta_s)]^{-m}e^{-(\sqrt{3}-i)\beta_s\xi/2}$$

where α_s, β_s denote the roots defined by $Ai(-\alpha_s) = 0$, $Ai'(-\beta_s) = 0$, and $Ai'(-\alpha_s)$, $Ai(-\beta_s)$ are the turning values of the Airy integral. This representation is useful only for $\xi > 0$. Alternative representations useful for $\xi \rightarrow 0$ are developed for the case $A_0{}^n$ and $B_0{}^n$. For $m = 1$ the functions are entire functions of ξ, and tables are given for the coefficients of the Taylor series of $A_1{}^n$ and $B_1{}^n$. These coefficients are evaluated by using the Euler summation scheme to sum the divergent series obtained by setting $\xi = 0$, $m = 1$ in the above representations. When $m = 2$ it is necessary to extract some terms which are singular at $\xi = 0$. The remaining parts of $A_2{}^n$ and $B_2{}^n$ are shown to be entire functions. Tables for the coefficients in the Taylor series for these non-singular parts are found by using the Euler-Maclaurin summation formula to sum the divergent series which are obtained by setting $\xi = 0$, $m = 2$ in the above representations. For $\xi \rightarrow -\infty$, asymptotic expansions are obtained for the cases $m = 1$ and $m = 2$. Tables are given for the coefficients in these expansions.

Volume II consists of a set of 26 tables and 17 figures that provide a supplement to the theoretical analysis contained in Volume I. Tables A, B, and C contain special tables of exponential and trigonometric functions which will facilitate computation with residue series and asymptotic expansions of the diffraction integrals. The functions tabulated in the remaining tables can be used to study diffraction effects when (a) source and receiver are on the surface, (b) source (or receiver) is on the surface and the receiver (or source) is at a great distance, and (c) both source and receiver are at a great distance.

AUTHOR'S SUMMARY

35[F].—R. KORTUM & G. MCNIEL, *A Table of Quadratic Residues for all Primes less than 2350*, LMSD 703111, October 1960, Lockheed Missiles and Space Division, Sunnyvale, California, iii + 3 + 378 unnumbered pages, 28 cm.

This large report, bound with a plastic spiral, lists all 187,255 quadratic residues of the 347 primes from 3 to 2347. The tables were computed on an IBM 7090 in about ten minutes. Presumably most of this time was spent in binary-decimal conversion and in writing on tape. The original printing was done on a high-speed, wire matrix printer and is readable, but certainly not elegant. In compensation, the tables are very easy to use, since the spiral binding allows the pages to lie flat.

The tables also give $N(p)$, the number of positive non-residues $<\frac{1}{2}p$. In the introduction it is pointed out that for primes of the form $4m + 3$ we have

$$[\tfrac{1}{2}(p - 1)]! \equiv (-1)^{N(p)} \pmod p.$$

It is also indicated that for all such primes (but we must add >3) the class number, $h(-p)$, is given by

$$h(-p) = -\frac{1}{p}\sum_{a=1}^{p-1}\left(\frac{a}{p}\right)a.$$

The much more easily computed formula [1],

$$h(-p) = \frac{p - 1 - 4N(p)}{4 - 2\,(2/p)},$$

is not mentioned. The introduction also states that it can be "found" in the table that $N(p) = m$ for all primes of the form $4m + 1$. But surely one does not need the table to be convinced of this simple theorem. The quantity which is really useful for those primes is $2\sum_{a=1}^{m}(a/p)$, and not the redundant $N(p)$.

D. S.

1. E. Landau, *Aus der elementaren Zahlentheorie*, Chelsea Publishing Co., New York, 1946, p. 128.

36[G X].—V. N. Faddeeva, *Computational Methods of Linear Algebra*, Translated by Curtis D. Benster, Dover Publications, Inc., New York, 1959, x + 252 p., 21 cm. Price $1.95.

The first chapter of this book forms a clear and well-written introduction to the elementary parts of linear algebra. The second chapter deals with numerical methods for the solution of systems of linear equations and the inversion of matrices, and the third with methods for computing characteristic roots and vectors of a matrix. Most of the important material in these domains is to be found here, and many numerical examples which illustrate the algorithms and point out their merits and deficiencies are given.

The discussion is directed principally to the hand computer, and machine computation in the modern sense is hardly present, but the book must be regarded as a valuable guide for the worker in the general area of linear computation.

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37[G, X].—M. Midhat J. Gazalé, *Les Structures de Commutation à m Valeurs et Les Calculatrices Numériques*, Collection de Logique Mathématique, Série A, Monographies Réunies par Mme. P. Fevrier, Gauthier-Villars, Paris, 1959, 78 p., 24 cm. Price 16 NF.

The theme of this pamphlet is sets of operations which are complete in the sense that "conjunction" and "negation" (or "exclusive or," "conjunction" and "1," or the "Sheffer stroke") are complete. By an operation we mean a (single-valued) function whose domain is S^n for some set S and positive integer n. A set F of operations on S is complete if any operation f on S (of any number of arguments) can be constructed from F by composition (substitution) and identification of variables.

The first three chapters, which are introductory, include, among other things, a discussion of how constructing f from F corresponds to constructing a network "realizing" f from elements (primitive networks) which realize the elements of F. Chapter IV is preparatory to Chapter V, where the first significant theorem appears. This is to the effect that sum modulo p (p a prime), product modulo p, and the constant functions are complete. Alternatively, every n-ary operation on $0, 1, 2, \cdots, p - 1$ is representable by a polynomial in n variables over the field of integers, modulo p. The author fails to note, however, that for any finite field, any operation on the p^n field elements is representable by a polynomial over the field. As a matter of fact, essentially the same argument the author gives for p elements is applicable to the more general situation.

Chapter VI deals with a theorem of Webb to the effect that the binary operation W defined over $0, 1, \cdots, m - 1$ by $W(x, y) = 0$ if $x \neq y$ and by $W(x, y) = x + 1$, mod m, if $x = y$ is complete. The author gives a formulation of the theorem which does not make use of the additive structure on the set, and gives a proof of it.

The last chapter (VII) generalizes a theorem of E. L. Post to the effect that if R is a permutation of E, the integers mod m, then the pair of operations $\otimes_R$, P_R is complete, where $R(i) \otimes_R R(j) = R$ (min i, j) and $P_R(R(i)) = R(i + 1)$, for all $i, j \varepsilon E$.

The following misprints were discovered:

p. 39 line 10, $E = (0, 1 \cdots, n - 1)$ *should read* $E = (0, 1, \cdots, p - 1)$;
p. 40 (63), *read* A_{rq} *for* Arq;
p. 40 line 6 from bottom, *read* M_{rt} *for* Mrt;
p. 51 line 2 from bottom, *read* p^n terms *for* p terms;
p. 55, 56; each recurrence of bg *should read* b_g;
p. 59 last line, $a = \delta ab$ *should read* $q = \delta ab$.

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38[I].—A. O. GELFOND, *Differenzenrechnung*, Deutscher Verlag, Berlin, 1958, viii + 336 p., 23 cm. Price DM 40.

This is a translation of the Russian edition (1952) which is a revised and extended version of an earlier book (1936). It is mainly concerned with problems in the complex domain, and some material, traditional in courses on Finite Differences, is omitted. The techniques used are those of classical analysis. There are occasional sets of problems, and some very interesting worked examples.

The book is divided into three large chapters (1, 2, 5) and two smaller ones (3, 4). Chapter One deals with the problem of interpolation ("construct an (approximate) expression for a function, given its values at a discrete set of points"), and includes an account of the Chebyshev theory.

Chapter Two deals with the Newton Series, first for equally spaced nodes, then for more general cases. The chapter concludes with applications of interpolation-theoretic methods to number-theoretic problems, in particular, to a proof of the theorem that α and $\beta = e^{\alpha}$ cannot both be algebraic, except for $\alpha = 0$.

The early part of Chapter Five is concerned with conventional material, including, for example, Ostrowski's proof of Hölder's result that $\Gamma(z)$ does not satisfy an algebraic differential equation; the latter part is concerned with work of the author (1951) on linear differential equations of infinite order, with constant coefficients.

Chapter Three is concerned with earlier (1937) research of the author on the construction of (entire) functions given their values at a series of points a_n, $a_n \rightarrow \infty$, and with related problems, e.g., the uniqueness of such functions.

Chapter Four contains standard material on the Summation Problem and the theory of Bernoulli numbers and polynomials; it includes, e.g., a proper account of the Euler-MacLaurin Summation Formula.

The book is clearly and precisely written. It can be recommended as an excellent source for many of the basic theorems in numerical analysis, and is a very suitable complement to such books as Natanson [1], which is largely concerned with the real domain.

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1. I. P. NATANSON, *Konstrucktive Funktionentheorie*, Akad. Verlag, Berlin, 1955 [*MTAC* Review 9, v. 13, 1959, p. 64–67.]

39[I, X].—J. KUNTZMANN, *Méthodes Numériques*, Dunod, Paris, 1959, xvi + 252 p., 25 cm. Price NF 36.00.

The author (who is a professor of applied mathematics of the Faculty of Sciences at Grenoble) admits his concern over the lack of a suitable textbook in numerical mathematics written in French. Rather than translate a foreign (to him) work, he decides to write a new book.

For various reasons he decides to limit his book almost exclusively to interpolation. The usual interpolation formulas (Newton-Gregory, Stirling, Gauss, Bessel, Everett, and Lagrange) are included for equally-spaced abscissas and also for divided differences as appropriate.

For the most part, approximation by the standard sets of polynomials (Legendre, Chebyshev, etc.) is avoided, but Bernoulli polynomials and Bernoulli numbers are discussed.

More general formulas for which the given data might be either values of the function or values of certain derivatives are discussed. Numerical integration is avoided, but interpolation for functions of two or more variables as well as of a complex variable is included. The last two chapters deal with the theory of interpolation for linear sums of special functions (exponentials, trigonometric sums, etc.)

Since the book was written to fulfill a need in France, and since there is no co.

responding need in the United States, the reviewer feels that the book will have limited appeal to American numerical analysts.

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40[K].—W. S. CONNOR & MARVIN ZELEN, "Fractional factorial experiment designs for factors at three levels," *Nat. Bur. Standards Appl. Math. Ser.*, No. 54, May 1959, v + 37 p., 26 cm. For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price $.30.

This is a sequel to NBS Applied Mathematics Series, No. 48 [1] which contains plans for fractional factorial designs for factors at two levels. The present compilation lists fractional factorial designs for factors at three levels as follows: for 1/3 replications, 2 for 4 factors and 3 each for 5, 6, and 7 factors; for 1/9 replications, 3 each for 6, 7, and 8 factors; for 1/27 replications, 3 each for 7, 8, and 9 factors; for 1/81 replications, 3 each for 8 and 9 factors; and for 1/243 replications, 3 each for 9 and 10 factors. For the same replication and number of factors the designs differ by the size of the blocks into which the treatment combinations are arranged. No main effects are confounded with other main effects or with two-factor interactions. Measurable two-factor interactions when the design is used as completely randomized or when treatments are grouped into blocks are listed. In addition, interactions confounded with blocks are given. Text material discusses the plan of the designs, loss of information, and the analysis of this type of designs.

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1. NBS APPLIED MATHEMATICS SERIES, No. 48, *Fractional Factorial Experiment Designs for Factors at Two Levels*, U. S. Gov. Printing Office, Washington, D. C., 1957 (*MTAC* Review 7, v. 12, 1958, p. 66).

41[K].—EDWIN L. CROW & ROBERT S. GARDNER, "Confidence limits for the expectation of a Poisson variable," *Biometrika*, v. 46, 1959, p. 441–453.

For any m, the inequalities $\sum_{i=c_1}^{c_2} p_i(m) \geqq 1 - \epsilon$, $\sum_{i=c_1+1}^{c_2+1} p_i(m) < 1 - \epsilon$, $\sum_{i=a+1}^{a+c_2-c_1} p_i(m) < 1 - \epsilon$ for all a, where $p_i(m) = e^{-m}m^i/i!$, define $c_1(m)$ and $c_2(m)$ uniquely. Define $m_L(c)$ to be the smallest m for which $c_2(m) = c$, and $m_U(c)$ to be the greatest m for which $c_1(m) = c$.

Table 1, p. 448–453, gives m_L and m_U to 2D for $\epsilon = .2, .1, .05, .01, .001$, and $c = 0(1)300$. The table was computed to 3D on an unspecified electronic computer; when the computed third place was a 5, the 5 was retained in the printed table.

Table 2, p. 444, compares the present confidence limits with the system δ_1 of Pearson & Hartley [1] and the system δ_3 of Sterne [2]; table 3, p. 445, compares them with the approximate formulas of Hald [3]; table 4, p. 446, compares them with the mean randomized confidence intervals of Stevens [4].

Reprints may be purchased from the Biometrika Office, University College, London, W.C. 1, under the title "Tables of confidence limits for the expectation of a

Poisson variable." Price: Two Shillings and Sixpence. Order *New Statistical Tables*, No. 28.

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1. E. S. PEARSON & H. O. HARTLEY, *Biometrika Tables for Statisticians*, Cambridge Univ. Press, 1954, p. 204–205. [*MTAC*, v. 9, 1955, p. 205–211].
2. T. E. STERNE, "Some remarks on confidence or fiducial limits," *Biometrika*, v. 41, 1954, p. 275–278. [*MTAC*, v. 9, 1955, p. 216].
3. ANDERS HALD, *Statistical Theory with Engineering Applications*, John Wiley & Sons, Inc., New York, 1952, p. 718.
4. W. L. STEVENS, "Fiducial limits of the parameter of a discontinuous distribution," *Biometrika*, v. 37, 1950, p. 117–139.

—F. G. FOSTER & D. H. REES, *Tables of the Upper Percentage Points of the Generalized Beta Distribution*, New Statistical Tables Series No. 26. Reprinted from *Biometrika*, vols. 44 & 45. Obtainable from the Biometrika Office, University College, London, W.C. 1, 30 p., 27 cm. Price five shillings.

In sampling from a k-variate normal population, let A and B be independent estimates, based on ν_1 and ν_2 degrees of freedom, respectively, of the population variance-covariance matrix. Let $\theta_1 \leqq \cdots \leqq \theta_k$ be the roots of the determinantal equation $| \theta\nu_1 A + (\theta - 1)\nu_2 B | = 0$. Then the distribution of θ_k is given by

$$I_x(k; p, q) = K \int_0^x d\theta_k \int_0^{\theta_k} d\theta_{k-1} \cdots \int_0^{\theta_2} d\theta_1 \prod_{i=1}^{k} \theta_i^{p-1}(1 - \theta_i)^{q-1} \prod_{i>j} (\theta_r - \theta_j)$$

where $p = \frac{1}{2}(| \nu_2 - k | + 1)$, $q = \frac{1}{2}(\nu_1 - k + 1)$. $I_x(1; p, q)$ is simply the incomplete-beta-function ratio $I_x(p, q)$. Foster & Rees argue that the 'generalized beta distribution' is *a* (not *the*) natural generalization of the Beta distribution from univariate to multivariate analysis of variance; for other generalizations see [1], [2], [3], [4].

The tables under review constitute a compilation of tables previously published in three papers by Foster and Rees [5], [6], [7]. Tabulated therein to 4D are values of the root of the equation

$$I_x(k; p, q) = P$$

for $P = .8(.05)\ .95,\ .99$;

$$k = 2,\ p = \tfrac{1}{2},\ 1(1)\ 10,\ q = 2\ (1)\ (20)\ (5)\ 50,\ 60,\ 80;$$
$$k = 3, 4,\ p = \tfrac{1}{2}(\tfrac{1}{2})\ 4,\ q = 1\ (1)\ 96.$$

Two- to four-point Lagrangian interpolation in p and q is recommended; no specific accuracy is guaranteed.

The computations for $k = 2$ were carried out on the N.R.D.C. Elliott 401 computer at Rothamsted; for $k = 3, 4$, on the DEUCE computer of the English Electric Company. Tables for $P = .95, .99$ and $k = 2(1)6$ have been given by Pillai [8], [9].

Two examples are given of the application of the tables to the analysis of dispersion of means, and one example is given of such application to the analysis of dispersion of regressions.

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1. S. S. WILKS, "Certain generalizations in the analysis of variance," *Biometrika*, v. 24, 1932, p. 471–494.
2. E. S. PEARSON & S. S. WILKS, "Methods of statistical analysis appropriate for k samples of two variables," *Biometrika*, v. 25, 1933, p. 353-378.
3. STATISTICAL RESEARCH GROUP, COLUMBIA UNIVERSITY, *Selected Techniques of Statistical Analysis*, McGraw-Hill Book Co., New York, 1947, chap. 3, p. 111–184.
4. J. NEYMAN, Editor, *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Univ. California Press, Berkeley & Los Angeles, 1951, p. 23–41.
5. F. G. FOSTER & D. H. REES, "Upper percentage points of the generalized beta distribution, I," *Biometrika*, v. 44, 1957, p. 237–247. [*MTAC*, Rev. 165, v. 12, 1958, p. 302]
6. F. G. FOSTER, "Upper percentage points of the generalized beta distribution. II," *Biometrika*, v. 44, 1957, p. 441–453 [[*MTAC*, Rev. 167, v. 12, 1958, p. 302]
7. F. G. FOSTER, "Upper percentage points of the generalized beta distribution, III," *Biometrika*, v. 45, 1958, p. 492–503. [*Math. Comp.*, Rev. 77, v. 14, 1960, p. 386]
8. K. C. S. PILLAI, *Concise Tables for Statisticians*, Statistical Center, Univ. of the Philippines, Manila, 1957.
9. K. C. S. PILLAI & C. G. BANTEGUI, "On the distribution of the largest of six roots of a matrix in multivariate analysis," *Biometrika*, v. 46, 1959, p. 237–240.

43[K].—TOSIO KITAGAWA & MICHIWO MITOME, *Tables for the Design of Factorial Experiments*, Dover Publications, Inc., New York, 1955 (printed in Japan; originally published by the Baifukan Company of Japan as part 3 of the work with the same title), vii + 253 p., 26 cm. Price $8.00.

These tables consist of the actual tables that appeared in the original 1953 publication in Japanese. An introduction to design principles and an explanation of the mathematical principles, parts 1 and 2 of the first publication, have been omitted. Readers are now referred to Kitagawa's *Lectures on the Design of Experiments* for this information and presumably for some help in the use of these tables.

The American publisher's jacket states that "this book contains tables for the design of factorial experiments and covers Latin squares and cubes, factorial design, fractional replication in factorial design, factorial designs with split-plot confounding, factorial designs confounded in quasi-Latin squares, lattice designs, balanced incomplete block designs, and Youden's squares." The table of contents gives more detail under each of the eight main headings just listed, except for the last two. For example, orthogonal squares and cubes are listed, the 2^n series of factorial arrangements goes up through 2^9, mixtures of factorials such as $a^n b^m$ mostly for $m = 1$ are listed, and the factorial replicates cover the 2^n for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ replicates plus the $\frac{1}{3}$ replicate for 3^n. Perhaps it should be noted that tables such as these are not really "for the design of experiments"; the function of the tables is to help select a layout or make easy the randomization of the layout after the design has been selected.

An examination of these tables shows that four Japanese pages have been cut out with scissors, and four English pages pasted in their place. The jacket further describes these tables as a "New revised edition. Explanatory notes." The author's preface does not describe what this reviewer would call a 'revised edition' and the explanatory notes consist of only one page. Since Kitagawa's *Lectures on Design of Experiments* may not be readily available to some users of these tables, other references might have been cited, e.g., O. Kempthorne, *Design and Analysis of Experiments*, W. G. Cochran and G. M. Cox, *Experimental Designs*, and O. L. Davies, *Design and Analysis of Industrial Experiments*.

These tables are excellently and clearly printed. After one becomes acquainted with their structure and arrangement the tables should prove useful on many occasions to those persons engaged in the design of experiments in any field. One unique feature of these tables deserves notice. A complete listing of all 576 configurations of the 4x4 Latin square is given. Continuation of this procedure for larger squares would have produced a bulky volume. One wonders about the special utility of 4x4 Latin squares which merited this complete listing.

There are two comments that must be made about these tables. The first comment is a criticism on the failure to include a table of random numbers within the volume. This reviewer's first act in using these tables will be to insert a small table of random numbers in both the front and rear of the volume. A table of experimental designs cannot be used without a random number table. As a consultant, when I pick up 'my tables,' I want to be sure that both items are with me.

The second comment follows from the first. A preliminary section on randomization procedures and choice of specific layout for each design should have been included. If omitted, specific references to such instructions in the Fisher & Yates tables or in O. Kempthorne's book should have been given. In this reviewer's experience both minor and major errors in designs have occurred because of a lack of clear understanding of proper randomization procedures.

Finally, one may remark that these tables would have been much improved by the inclusion of some explanatory materials, and references for each design included. For statisticians, R. A. Fisher & F. Yates, *Statistical Tables for Biological Agricultural and Medical Research*, Oliver & Boyd Ltd., Edinburgh (Fifth edition 1957), and E. S. Pearson & H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. I, Cambridge, published for the Biometrika Trustees of the University Press (2nd printing, 1956) have set a high standard in this respect. The continued rapid development in the field of experimental design makes it difficult to keep tables of this type up to date. It is hoped that a really revised edition will soon appear. Designs for response surface investigation and new fractional factorial arrangements need to be readily available.

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44[K].—Ingram Olkin, Sudhish G. Ghurye, Wassily Hoeffding, William G. Madow, & Henry B. Mann, Editors, *Contributions to Probability and Statistics, Essays in Honor of Harold Hotelling*, Stanford University Press, 1960, x + 517 p., 24 cm. Price $6.50.

This volume contains a collection of forty-two essays on probability and mathematical statistics in honor of Professor Harold Hotelling on his sixty-fifth birthday. The list of contributors, limited to those who have been closely associated with

Professor Hotelling, looks nevertheless like an up-to-date "Who's Who" in the subject field. This fact alone pays an appropriate tribute to his influence and leadership.

The first two essays, fitting to the occasion, deal with Hotelling the man, and as a leader and teacher in the field of mathematical statistics. The third one is a reprint of Hotelling's own excellent paper on "Teaching of Statistics," and the fourth one is a bibliography of his work. A total of ninety papers, not including reviews, were credited to him between 1925 and 1959—a truly impressive record of accomplishment.

The remaining thirty-eight research papers cover a wide spectrum of topics. There are seven papers on design and analysis of experiments, and about the same number in non-parametric statistics and also multivariate problems. Investigations into power, optimality, consistency, and robustness of tests, distribution theorems, and stochastic processes make up the bulk of the remaining papers. There is one paper on the inversion of partitioned matrices (Greenberg and Sarhan) and one on the numerical convergence of iterative processes (Moriguti).

Since a listing of titles and authors takes about two pages, a detailed review of this diversified volume is an impossible task within the space allotted. If one paper has to be singled out as truly outstanding among the thirty-eight, I believe most people would agree to the choice of John Tukey's "A Survey of Sampling from Contaminated Distributions," which investigates the robustness of efficiency of competitive estimators. In the paper the author considers two normal populations which have the same mean but whose standard deviations are in the ratio 3:1. One of the questions asked was: "What fraction of the wider normal population must be added to the narrower one in order for the mean deviation to be as good a large sample measure of scale as the standard deviation?" The answer, given two pages later, turns out to be a shockingly low .008. Tukey then suggests that "Problems of robustness of efficiency are probably as important as problems of robustness of validity, and, because of their relatively undeveloped stage, deserve even more attention from statisticians." No doubt this suggestion will be heeded.

A list of titles and authors follows. Texts which are accompanied by tables are marked with an asterisk. The tables in paper No. 20 are separately described in the review immediately following. All the other tables are of illustrative nature, with limited selections of entries, and will not be discussed here.

Part I. An Appreciation

1. Harold Hotelling—William G. Madow
2. Harold Hotelling—A Leader in Mathematical Statistics—Jerzy Neyman
3. The Teaching of Statistics—Harold Hotelling
4. Bibliography of Harold Hotelling

Part II: Contributions to Probability and Statistics

5. Some Remarks on the Design and Analysis of Factorial Experiments—R. L. Anderson
6. A Limitation of the Optimum Property of the Sequential Probability Ratio Test—T. W. Anderson and Milton Friedman
7. Decision Theory and the Choice of a Level of Significance for the t-Test—Kenneth J. Arrow

8. Simultaneous Comparison of the Optimum and Sign Tests of a Normal Mean—R. R. Bahadur
9. Some Stochastic Models in Ecology and Epidemiology—M. S. Bartlett
10. Random Orderings and Stochastic Theories of Responses—H. D. Block and J. Marschak
11. On a Method of Constructing Steiner's Triple Systems—R. C. Bose*
12. A Representation of Hotelling's T^2 and Anderson's Classification Statistic W in Terms of Simple Statistics—Albert H. Bowker
13. Euler Squares—Kenneth A. Bush
14. A Compromise Between Bias and Variance in the Use of Nonrepresentative Samples—Herman Chernoff
15. Construction of Fractional Factorial Designs of the Mixed $2^m 3^n$ Series—W. S. Connor
16. Application of Boundary Theory to Sums of Independent Random Variables—J. L. Doob, J. L. Snell, and R. E. Williamson
17. Some k-Sample Rank-order Tests—Meyer Dwass
18. Characterization of Some Location and Scale Parameter Families of Distributions—S. G. Ghurye
19. Generalization of Some Results for Inversion of Partitioned Matrices—B. G. Greenberg and A. E. Sarhan
20. Selecting a Subset Containing the Best of Several Binomial Populations—Shanti S. Gupta and Milton Sobel*
21. Consistency of Maximum Likelihood Estimation of Discrete Distributions—J. Hannan
22. An Upper Bound for the Variance of Kendall's "Tau" and of Related Statistics—Wassily Hoeffding
23. On the Amount of Information Contained in a σ-Field—Gopinath Kallianpur
24. The Evergreen Correlation Coefficient—M. G. Kendall
25. Robust Tests for Equality of Variances—Howard Levene*
26. Intrablock and Interblock Estimates—Henry B. Mann and M. V. Menon
27. A Bivariate Chebyshev Inequality for Symmetric Convex Polygons—Albert W. Marshall and Ingram Olkin
28. Notes on the Numerical Convergence of Iterative Processes—Sigeiti Moriguti
29. Prediction in Future Samples—George E. Nicholson, Jr.*
30. Ranking in Triple Comparisons—R. N. Pendergrass and R. A. Bradley*
31. A Statistical Screening Problem—Herbert Robbins
32. On the Power of Some Rank-order Two-sample Tests—Joan Raup Rosenblatt
33. Some Non-parametric Analogs of "Normal" ANOVA, MANOVA, and of Studies in "Normal" Association—S. N. Roy and V. P. Bhapkar
34. Relations Between Certain Incomplete Block Designs—S. S. Shrikhande
35. Infinitesimal Renewal Processes—Walter L. Smith
36. Classification Procedures Based on Dichotomous Response Vectors—Herbert Solomon
37. Multiple Regression—Charles Stein
38. An Optimum Replicated Two-sample Test Using Ranks—Milton E. Terry*
39. A Survey of Sampling from Contaminated Distributions—John W. Tukey
40. Multidimensional Statistical Scatter—S. S. Wilks

41. Convergence of the Empiric Distribution Function on Half-Spaces—J. Wolfowitz
42. Analysis of Two-factor Classifications With Respect to Life Tests—M. Zelen.*

The five editors are to be congratulated for assembling and presenting this volume in an excellent manner.

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National Bureau of Standards
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45[K].—SHANTI S. GUPTA & MILTON SOBEL, "Selecting a subset containing the best of several binomial populations," p. 224–248, *Contributions to Probability and Statistics, Essays in Honor of Harold Hotelling*, edited by Olkin et al., Stanford University Press, 1960. [See preceding review.]

Given k binomial populations with unknown probabilities of success p_1, p_2, $\cdots p_k$, a procedure R is studied by the authors which selects a subset that guarantees with preassigned probability P^* that, regardless of the true unknown parameter values, it will include the best population; i.e., the one with the highest parameter value. Procedure R for equal sample sizes is given as follows. Retain in the selected subset only those populations for which $x_i \geqq x_{\max} - d$, where $d = d(n, k, P^*)$ is a non-negative integer, and x_i denotes number of successes based on n observations from the ith population. Table 2 gives the values of d for $k = 2(1)20, 20(5)50$; $n = 1(1)20, 20(5)50, 50(10)100, 100(25)200, 200(50)500$; $P^* = .75, .90, .95, .99$ (a trial and error procedure R is given for large, unequal sample sizes).

Table 3 gives the expected proportion of populations retained in the selected subset by procedure R (for the special case $p_1 = p_2 = \cdots = p_{k-1} = p$, $p_k = p + \delta$, $0 \leqq \delta \leqq 1$, $0 \leqq p \leqq 1 - \delta$) for $n = 5(5)25$; $p^* = .75, .90, .95$; $\delta = .00, .10, .25, .50$; and $p + \delta = .50, .75, .95, 1.00$.

H. H. KU

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46[K].—MAURICE HENRI QUENOUILLE, "Tables of random observations from standard distributions," *Biometrika*, v. 46, 1959, p. 178–202. EGON SHARPE PEARSON, "Note on Mr. Quenouille's Edgeworth Type A transformation," *Biometrika*, v. 46, 1959, p. 203–204.

Quenouille offers a random sample of 1000 each from the normal distribution and seven specified non-normal distributions. While a sample of 1000 is too small for much serious Monte Carlo work, the method of construction of the present tables, where the normal sample uniquely and monotonely determines the 7 non-normal samples, makes it suitable for pilot studies of the sensitivity of statistical procedures to departures from normality.

Specifically, let x_1 be a unit normal deviate from the tables of Wold [1]. Define

$$y = (2\pi)^{-1/2} \int_{-\infty}^{x_1} \exp\left(-\tfrac{1}{2}x^2\right) dx,$$
$$x_2 = 3^{1/2}[2y - 1],$$

$$
\begin{aligned}
x_3 &= 0.46271\, e^{x_1} - 0.76287, \\
x_4 &= -1 - \log_e (1 - y), \\
&\begin{cases} 124416\, x_5 = -9552 + 127225\, x_1 + 7824\, x_1^2 - 40\, x_1^3 + 576\, x_1^4 - 252\, x_1^5, & x_1 > -2.5, \\ x_5 = -1.86,\ x_1 \leqq -2.5, & \end{cases} \\
1536\, x_6 &= 1411\, x_1 + 56\, x_1^3 - 3\, x_1^5, \\
124416\, x_7 &= -12144 + 122878\, x_1 + 14304\, x_1^2 - 1066\, x_1^3 - 720\, x_1^4 + 261\, x_1^5, \\
&\begin{cases} x_8 = -2^{-1/2} \log_e [2 - 2y], & x_1 > 0 \\ x_8 = 2^{1/2} \log_e 2y & x_1 \leqq 0. \end{cases}
\end{aligned}
$$

Then, for $i = 1(1)8$, $E(x_i) = 0$, $E(x_i^2) = 1$. Here x_2 is a rectangular random variate; x_3, a log-normal variate; x_4, a one-tailed exponential variate; x_8, a two-tailed exponential variate; x_5, x_6, x_7 are Cornish-Fisher expansions with specified κ_3 and κ_4. A short table on p. 179 shows that the specifications are not met precisely; Pearson's note shows that this failure is negligible for samples of 1000.

The main table, p. 183–202, gives 1000 values of x_i, $i = 1(1)8$, to 2 D, with Σx and Σx^2 in blocks of 50. Auxiliary tables on p. 180–182 give the first and second sample moments of the x_i; their theoretical κ_3, κ_4, κ_5, κ_6; frequency distributions of the 8 samples; x_5, x_6, x_7 to 3 D for $x_1 = -3.2(.1) + 3.2$. The italic headlines on p. 181–182 should be interchanged.

It is not clear why random normal numbers were used as the basis for this table rather than random rectangular numbers, nor why the 2 D deviates of Wold [1] were chosen over the 3 D deviates of Rand Corp. [2].

Reprints may be purchased from the Biometrika Office, University College, London, W.C. 1, under the title "Tables of 1000 standardized random deviates from certain non-normal distributions." Price: Two Shillings and Sixpence. Order New Statistical Tables, No. 27.

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1. Herman A. O. Wold, *Random Normal Deviates*. Tracts for Computers, no. 25, Cambridge Univ. Press, 1954.
2. The RAND Corporation, *A Million Random Digits With 100,000 Normal Deviates*. The Free Press, Glencoe, Illinois, 1955. [*MTAC*, v. 10, 1956, p. 39–43].

47[K].—Alfred Weissberg & Glenn H. Beatty, *Tables of Tolerance-Limit Factors for Normal Distributions*, Battelle Memorial Institute, 1959, 42 p., 28 cm. Available from the Battelle Publications Office, 505 King Avenue, Columbus 1, Ohio.

The abstract of the booklet reads as follows: "Tables of factors for use in computing two-sided tolerance limits for the normal distribution are presented. In contrast to previous tabulations of the tolerance-limit factor K, we tabulate the factors $r(N, P)$ and $u(f, \gamma)$, whose product is equal to K. This results in greatly increased compactness and flexibility. The mathematical development is discussed, including methods used to compute the tabulated values and a study of the accuracy of the basic approximation. A number of possible applications are discussed and examples given."

Since the mean μ and the standard deviation σ are frequently unknown, the tolerance limits must be computed on the basis of a sample estimate $\bar{x}$ of the mean and an estimate s of the standard deviation. The tolerance limits treated in the booklet have the form $x \pm Ks$, where the factor K (the product of the tabulated entries $r(N, P)$ and $u(f, \gamma)$) accounts for sampling errors in $\bar{x}$ and s as well as for the population fraction P.

Six levels of probability for P and γ are used (.50, .75, .90, .95, .99, .999). The values of N used are given by

$$N = 1(1)300(10)1000(1000)10000, \infty.$$

The values of f used are given by

$$f = 1(1)1000(1000)10000, \infty.$$

Values of $r(N, P)$ and $u(f, \gamma)$ are given to four decimal places, which means that most of the tabular entries have five significant figures.

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48[K].—E. J. WILLIAMS, *Regression Analysis*, John Wiley & Sons, Inc., New York, 1959, ix + 214 p., 24 cm. Price $7.50.

This useful volume is a monograph devoted to the exposition of the practical aspects of "regression analysis." These so-called regression analysis techniques are based on the method of least squares and are equivalent to analysis of variance procedures. The author discusses many different techniques, some containing much novelty. All are accompanied by illustrations using actual data drawn mainly from the biological sciences. The book contains a great deal of interesting discussion and advice on the proper and practical applications of the methods.

No attention is devoted to the planning of experiments; the book is only concerned with the analysis of data. Although nearly all the techniques involve the solution of simultaneous equations, there is little discussion of numerical techniques, except to recommend the "Crout" method.

The author makes much use of statements about parameters which are termed fiducial statements. This reviewer feels these are confidence statements. In explaining the meaning of fiducial statements the author writes (p. 91), ". . . a fiducial statement about a parameter is, broadly speaking, a statement that the parameter lies in a certain range or takes a certain set of values. The statement is either true or false in any particular instance, but it is made according to a rule which ensures that such statements, *when applied in repeated sampling*, have a given probability (say 0.95 or 0.99) of being correct."

The various techniques are presented without theory, as "to have done so would have made the book unnecessarily long." Without the accompanying theoretical material, this book is simply a handbook of regression methods. It is for this reason that the reviewer feels the book will be more useful to applied statisticians than to the author's intended audience, i.e., research workers in the experimental sciences.

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49[L].—G. F. MILLER, *Tables of Generalized Exponential Integrals*, National Physical Laboratory Mathematical Tables, Vol. 3, British Information Services, New York, 1960, iii + 43 p., 28 cm. Price $1.43 postpaid.

According to the author, the tables under review were prepared to meet the requirements of quantum chemists concerned with the evaluation of molecular integrals, who frequently have found the tables computed by the New York Mathematical Tables Project and edited by G. Placzek [1] inadequate for this purpose.

Actually tabulated in the present work is the auxiliary function

$$F_n(x) = (x + n)e^x E_n(x),$$

where $E_n(x)$ represents the generalized exponential integral, defined by the equation $E_n(x) = \int_1^\infty e^{-xt}t^{-n}\, dt$. Three tables of $F_n(x)$ to 8D are provided. The first table covers the range $x = 0(0.01)1$ for $n = 1(1)8$; the second, the range $x = 0(0.1)20$ for $n = 1(1)24$; and the last, the range $1/x = 0(0.001)0.05$ for $n = 1(1)24$. Modified second (and occasionally fourth) central differences are provided throughout for use with Everett's interpolation formula. For details of methods of interpolation and tables of interpolation coefficients the table-user is referred to the first two volumes of this series of tables [2], [3].

It is stated that the total error in an unrounded interpolated value of $F_n(x)$ derived from the present tables need never exceed $1\frac{1}{2}$ units in the eighth decimal place if the tabulated differences are used. Furthermore, the values of $F_n(x)$ here tabulated are guaranteed to be accurate to within 0.6 unit in the last place.

The tables are preceded by an Introduction containing a brief account of pertinent literature, followed by a section devoted to a description of the tables and a justification for the tabulation of $F_n(x)$ in preference to $E_n(x)$. The properties of the generalized exponential integral, many of them reproduced from Placzek [1], are enumerated in a third section. The fourth section of the text is devoted to a careful description of the several procedures followed in the preparation of the tables. An excellent set of references is appended to this introductory textual material.

The typography is uniformly excellent, and the format of the tables is conducive to their easy use. The only defect observed was a systematic error in the heading of Table 2 on pages 24 through 37, where this heading erroneously appears as Table 3.

J. W. W.

1. NATIONAL RESEARCH COUNCIL OF CANADA, Division of Atomic Energy, Report MT-1, *The Functions* $E_n(x) = \int_1^\infty e^{-xu}u^{-n}\, du$, Chalk River, Ontario, December 1946. Reproduced in

Nat. Bur. Standards Appl. Math. Ser. No. 37, *Tables of Functions and of Zeros of Functions*, 1954, p. 57–111. See RMT **392**, *MTAC*, v. 2, 1946–47, p. 272; and RMT **104**, *MTAC*, v. 10, 1956, p. 249–250.
2. L. Fox, *The Use and Construction of Mathematical Tables*, National Physical Laboratory Mathematical Tables, v. 1, London, 1956. See RMT **8**, *MTAC*, v. 13, 1959, p. 61–64.
3. L. Fox, *Tables of Everett Interpolation Coefficients*, National Physical Laboratory Mathematical Tables, v. 2, London, 1958.

50[L].—F. W. J. Olver, Editor, *Bessel Functions*, Part III, *Zeros and Associated Values*, Royal Society Mathematical Tables No. 7, Cambridge University Press, New York, 1960, lx + 79 p., 29 cm. Price $9.50.

The present volume is a step towards the completion of a program for the tabulation of Bessel functions initiated by the British Association Mathematical Tables Committee, and continued since 1948 by the Royal Society Mathematical Tables Committee. Part I of this series, *Bessel Functions, Functions of Order Zero and Unity* appeared in 1937, and Part II, *Bessel Functions, Functions of Positive Integer Order* appeared in 1952 (see *MTAC* v. 7, 1953, p. 97–98). Recall that Part I contains a section on the zeros of $J_n(z)$, $Y_n(z)$, $n = 0, 1$, but Part II is without a section devoted to zeros.

Part III, the present work, deals with the evaluation of zeros of the Bessel functions $J_\nu(z)$ and $Y_\nu(z)$ for general ν and z. Tables are also provided as described later in this review. A history of the project is given in the "Introduction and Acknowledgements," by C. W. Jones and F. W. J. Olver. A chapter on "Definitions, Formulae and Methods" by the above authors is a valuable compendium of techniques for the enumeration of zeros and associated functions. In particular, it is an excellent guide if zeros are required of other transcendental functions which satisfy second-order linear differential equations. Several methods of computation are outlined. For instance, the method of McMahon is useful for ν fixed and z large, while the inverse interpolation approach of Miller and Jones presupposes a tabulation of the functions themselves. Between the regions covered by these techniques is a gap which increases with increasing ν. The gap is bridged by application of Olver's important contributions on uniform asymptotic expansions of Bessel functions.

The section "Description of the Tables, Their Use and Preparation" is by the editor. A short description of the tables follows. Table I gives zeros $j_{n,s}$ of $J_n(x)$, $y_{n,s}$ of $Y_n(x)$, and the values of $J_n'(j_{n,s})$, $Y_n'(y_{n,s})$. Table II gives zeros $j'_{n,s}$ of $J_n'(x)$, $y'_{n,s}$ of $Y_n'(x)$, and the values of $J_n(j'_{n,s})$, $Y_n(y'_{n,s})$. Table III gives zeros $a'_{m,s}$, $b'_{m,s}$ of the derivatives $j_m'(x)$, $y_m'(x)$ of the spherical Bessel functions $j_m(x) = (\pi/2x)^{1/2}J_{m+1/2}(x)$, $y_m(x) = (\pi/2x)^{1/2}Y_{m+1/2}(x)$, and the values of $j_m(a'_{m,s})$, $y_m(b'_{m,s})$. The ranges covered are

$$n = 0(\tfrac{1}{2})20\tfrac{1}{2}, \qquad s = 1(1)50, \quad \text{Tables I and II;}$$

$$m = 0(1)20, \qquad s = 1(1)50, \quad \text{Table III.}$$

All entries are to eight decimals, and in no case should the end-figure error exceed 0.55 of a unit in the eighth decimal.

The coefficients in the uniform asymptotic expansions (previously mentioned) which are used to evaluate items in Tables I–III for n (or m) large are given in Table IV. The expansions for the Bessel functions of Tables I–III also depend on zeros and associated values of certain Airy functions and their derivatives. These data are recorded in Table V. Further description of the contents of Tables IV–V requires much more space and so is omitted here. Suffice it to say that with the aid of these tables, the entries in Tables I–III can with a few exceptions be evaluated to at least eight significant figures for $20 \leqq n < \infty$, $1 \leqq s < \infty$.

There is a good set of references. The printing and typography are excellent, and the present volume upholds the eminent tradition of British table-makers.

Y. L. L.

51[L, V].—J. W. Miles, "The hydrodynamic stability of a thin film of liquid in uniform shearing motion," *J. Fluid Mech.* 8, Pt. 4, 1960, p. 593–610. (Tables were computed by David Giedt.)

Let

$$\mathfrak{F}(z) = [1 - F(z)]^{-1} = w\,[A_i'(-w)]^{-1}\left[\frac{1}{3} + \int_0^w A_i(-t)\,dt\right], \qquad w = ze^{i\pi/6}.$$

$$\mathfrak{F}'(z) = z^{-1}\mathfrak{F}(z) + we^{i\pi/6}[A_i'(-w)]^{-1}A_i(-w)[1 - \mathfrak{F}(z)].$$

$$\mathfrak{F}^{(k)}(z) = \mathfrak{F}_r^{(k)}(z) + i\mathfrak{F}_i^{(k)}(z), \qquad k = 0, 1; \qquad F(z) = Fr(z) + iF_i(z).$$

The paper contains tables of $\mathfrak{F}(z)$, $\mathfrak{F}'(z)$, $F(z)$ and $z^3F_i(z)$ for $z = -6(.1)10$, 4S. The tables were obtained on an automatic computer by numerical integration of an appropriate differential equation. It can be seen from the above that the tables depend on values of the Airy integral $A_i(z)$, its derivative and integral along the rays $\pi/6$ and $-5\pi/6$ in the complex plane. Tables of $A_i(z)$ and its derivative are now available for complex z in rectangular form, but not in polar form. Also, tables of $\int_0^z A_i(\pm t)\,dt$ are available for z real. Thus, the given tables depend on values of some basic functions which, if available, would cut new ground. Unfortunately, the basic items were swallowed up in the automatic computation of $F(z)$. We have here a poor example of table making,—a practice which should not be emulated.

Y. L. L.

52[P].—Helmut Hotes (Compiler), *Wasserdampftafel der Allgemeinen Elektricitäts-Gesellschaft*, R. Oldenbourg, Munich, 1960, 48 p., 30 cm. DM 16 (Paperback).

There are two tables in this collection. Table I is a four-place table giving the temperature, the specific volume, the specific enthalpy, and the specific entropy as functions of the absolute pressure p. The last three dependent variables are given both for the fluid state and the gaseous state. The variable p ranges from 0.010 to 225,650 atmospheres, and the interval varies from 0.001 to 2000. Table II gives the specific volume, specific enthalpy and specific entropy as functions of temperature for constant pressure. Here p has the values 1, 5, 10 (10) to 400 atmospheres, and t varies from 0 (10) to 330 degrees centigrade.

The tables were calculated by expressing each of the dependent variables as polynomials in the pressure with coefficients as functions of the temperature or in some cases functions of the temperature and pressure. The error bounds given by such approximations to the specific volume and specific enthalpy as functions of pressure and temperature are included in the collection.

A. H. T.

53[P, X, Z].—WARD C. SANGREN, *Digital Computers and Nuclear Reactor Calculations*, John Wiley & Sons, New York, 1960, xi + 208 p., 24 cm. Price $8.50.

As the author states in his preface, the primary objective of this book is to present to nuclear engineers and scientists an introduction to high speed reactor calculations. Since the appearance of the basic reference, *The Elements of Nuclear Reactor Theory* by Glasstone and Edlund, Van Nostrand, 1952, the entire complexion of actual reactor design calculations has changed as a result of the growth in speed and size of computing machines, and reactor design calculations represent today a significant part of scientific computing time on modern computers.

The outline of the book by chapters is

Chapter 1. Introduction
Chapter 2. Digital Computers
Chapter 3. Programming
Chapter 4. Numerical Analysis
Chapter 5. A Code for Fission-Product Poisoning
Chapter 6. Diffusion and Age-Diffusion Calculations
Chapter 7. Transport Equation—Monte Carlo
Chapter 8. Additional Reactor Calculations

In Chapter 1, the author reviews the tremendous parallel growth of high speed computing machines and nuclear reactors, and their present interplay. In Chapter 2, an introduction and description of present day computers is given. In Chapter 3, programming for computers is introduced. After some preliminary remarks (no proofs) about interpolation, numerical integration, matrices, etc., items which can be found in many well-known texts on elementary numerical analysis, the author treats in Chapter 4 the more relevant problem of the numerical approximation of partial differential equations by difference equations, and their solution by means of iterative methods. Also, the treatment of interface conditions, which arise naturally in heterogeneous reactors, is given.

In Chapter 5, a simple code for fission-product poisoning is followed from the physical and mathematical definitions through to the construction of a program in the Bell (Wolontis) system.

In Chapter 6, the longest chapter, the author describes diffusion calculations, extending from steady-state criticality problems for reactors to the solution of two- and three-dimensional multigroup diffusion equations. In Chapter 7, the S_n method of Carlson is described, along with the use of Monte Carlo methods for solving problems such as those encountered in shielding calculations.

In his primary aim, the author does succeed. Nevertheless, the reviewer, being quite familiar with this area, was most critical with respect to the age of the references, as most of the technical papers referred to had appeared prior to 1957. As no serious attempt was made to fill the gap between these earlier developments and the developments which have taken place in the reactor field in the last few years, many statements in the book are either somewhat obsolete or misleading. For example, the numerical inversion of tridiagonal matrix equations on page 74 by an algorithm is not stated to be simply Gauss elimination applied to the matrix problem, and in fact the author states that this "method" has not appeared in textbooks as yet. The iterative methods of Young-Frankel, and Peaceman-Rachford are each discussed twice, (p. 84 and p. 144) and not one of the four definitions is completely accurate. The book is, however, the only existing bridge between *The Elements of Nuclear Reactor Theory* and present computational technique in the reactor field.

R. S. V.

54[S, W].—Ronald A. Howard, *Dynamic Programming and Markov Processes*, Technology Press & Wiley, New York, viii + 136 p., 23 cm. Price $5.75.

Consider a physical system S represented at any time t by a state vector $x(t)$. The classical description of the unfolding of the system over time uses an equation of the form $x(t) = F(x(s), s \leqq t)$, where F is a prescribed operation upon the function $x(s)$ for $s \leqq t$. In certain simple cases, this reduces to the usual vector differential equation $dx/dt = g(x)$, $x(0) = c$.

For a variety of reasons, it is sometimes preferable to renounce a deterministic description and to introduce stochastic variables. If we take $x(t)$ to be a vector whose i-th component is now the probability that the system is in state i at time t, and allow only discrete values of time, we can in many cases describe the behavior of the system over time quite simply by means of the equation $x(t + 1) = Ax(t)$. Here $A = (a_{ij})$, $i, j = 1, 2, \cdots, N$, is a transition matrix whose element a_{ij} is the probability that a system in state j at time t will be found in state i at time $t + 1$. Processes of this type are called Markov processes and are fundamental in modern mathematical physics.

So far we have assumed that the observer plays no role in the process. Let us now assume that in some fashion or other the observer has the power to choose the transition matrix A at each stage of the process. We call a process of this type a *Markovian decision process*. It is a special, and quite important, type of dynamic programming process; cf. Chapter XI of R. Bellman, *Dynamic Programming*, Princeton University Press, 1957.

Let us suppose that at any stage of the process, we have a choice of one of a set of matrices, $A(q) = (a_{ij}(q))$. Associated with each choice of q and initial state i is an expected single-stage return $b_i(q)$. We wish to determine a sequence of choices which will maximize the expected return from n stages of the process. Denoting the maximum expected return from an n-stage process by $f_i(n)$, the principle of optimality yields the functional equation

$$f_i(n) = \max_q \left[b_i(q) + \sum_{j=1}^{N} a_{ij}(q) f_j(n-1)\right].$$

In this form, the determination of optimal policies and the maximum returns is easily accomplished by means of digital computers; see, for example S. Dreyfus, *J. Oper. Soc. of Great Britain*, 1958. Problems leading to similar equations, resolved in similar fashion, arise in the study of equipment replacement and in continuous form in the "optimal inventory" problem; see Chapter Five of the book mentioned above and K. D. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production*, Stanford University Press, 1959.

As in the case of the ordinary Markov process, a question of great significance is that of determining the asymptotic behavior as $n \rightarrow \infty$. It is reasonable to suspect, from the nature of the underlying decision process, that a certain steady-state behavior exists as $n \rightarrow \infty$. This can be established in a number of cases.

The author does not discuss these matters at all. This is unfortunate, since there is little value to steady-state analysis unless one shows that the dynamic process asymptotically approaches the steady-state process as the length of the processes increases. Furthermore, it is essential to indicate the rate of approach.

The author sets himself the task of determining steady-state policies under the assumption of their existence. Granted the existence of a "steady state," the functions $f_i(n)$ have the asymptotic form $nc + b_i + o(1)$ as $n \rightarrow \infty$, where c is independent of i. The recurrence relations then yield a system of equations for c and the b_i.

This system can be studied by means of linear programming as a number of authors have realized; see, for example, A. S. Manne, "Linear programming and sequential decisions," *Management Science*, vol. 6, 1960, p. 259–268.

Howard uses a different technique based upon the method of successive approximations, in this case an approximation in policy space. It is a very effective technique, as the author shows, by means of a number of interesting examples drawn from questions of the routing of taxicabs, the auto replacement problem, and the managing of a baseball team.

The book is well-written and attractively printed. It is heartily recommended for anyone interested in the fields of operations research, mathematical economics, or in the mathematical theory of Markov processes.

RICHARD BELLMAN

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(Reprinted from Quarterly of Applied Mathematics)

55[V, X].—GERTRUDE BLANCH, KARL GOTTFRIED GUDERLEY & EMMA MARIAN VALENTINE, *Tables Related to Axial Symmetric Transonic Flow Patterns*, WADC Technical Report 59–710, 1960, Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C., xlviii + 108 p., 27 cm.

The equations of motion of a compressible fluid are non-linear and are generally difficult to handle. In certain cases, such as in the flow past slender bodies of revolution, the equations can be approximated by much simpler ones. For subsonic and supersonic flow these approximating equations are linear. When the flow velocity is nearly equal to one, the approximate equation for the disturbance potential takes the non-linear form

$$-\Phi_x \Phi_{xx} + \Phi_{yy} + \frac{\Phi_y}{y} + \frac{1}{y^2}\Phi_{\omega\omega} = 0$$

when x, y, ω are cylindrical coordinates. K. G. Guderley and his colleague H. Yoshihara have studied the flow past slender bodies at Mach numbers close to one in a series of papers and in a book by Guderley, *Theorie schallnaher Strömungen*, Springer-Verlag, 1957.

The basic technique applied to axially symmetric flows is to find a basic solution Φ^B of the form

$$\Phi^B = y^{3n-2}f(\zeta)$$

where $\zeta = x/y^n$ and n is a constant. The variable f then satisfies the ordinary differential equation

$$(f' - n^2\zeta^2)f'' + (5n^2 - 4n)\zeta f' - (3n - 2)^2 f = 0.$$

Further solutions necessary to satisfy particular boundary conditions are then found by perturbing the basic solution by the function $\bar{\Phi}$, i.e.,

$$\Phi = \Phi^B + \bar{\Phi}(x, y, \omega).$$

Then $\bar{\Phi}$ is assumed to satisfy the linear equation

$$-\Phi_x{}^B\bar{\Phi}_{xx} - \Phi^B_{xx}\bar{\Phi}_x + \bar{\Phi}_{yy} + \frac{\bar{\Phi}_y}{y} + \frac{\bar{\Phi}_{\omega\omega}}{y^2} = 0.$$

Particular solutions of the equation are then found in the form

$$\bar{\Phi} = y^\nu g(\zeta) \cos m\omega; \qquad m = 0, 1, 2, \cdots$$

Then $g(\zeta)$ satisfies the ordinary differential equation

$$(f' - n^2\zeta^2)g'' + [f'' + (2\nu n - n^2)\zeta]g' + (m^2 - \nu^2)g = 0.$$

The solution of this equation leads to an eigenvalue problem with the eigenvalue ν.

The present report tabulates the functions f and g together with their derivatives and some other related functions. In Table 1 appear 6D values of $f(\zeta)$ and $df/d\zeta$ for $\zeta = -7.5(.1) - 3(.02)1$; in Table 2 similar information appears for $g(\zeta)$ and $dg/d\zeta$. These tabular data are given for several values of the eigenvalue ν.

A rather complete discussion of the mathematical problems involved is given in the introduction. The eigenvalues are found using a contour integration technique. It is stated that the numerical calculations are performed on an ERA 1103, with considerable pains taken to insure accuracy. The entries are stated to be correct to within one unit in the last place.

The tables should be quite useful to anyone interested in the study of special cases of transonic flow.

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56[W].—H. L. Toothman, *A Table of Probability Distributions useful in War Games and other Competitive Situations*, NRL Report 5480, U. S. Naval Research Laboratory, Washington, D. C., May 16, 1960, i + 91 p., 27 cm.

A player makes a maximum of $(2r - 1)$ plays. On odd-numbered plays he scores 1 with probability p_1 and 0 with probability $1 - p_1$; on even-numbered plays he is eliminated from subsequent play with probability p_2. The probability that he will score exactly n is

$$S_n = \binom{r}{n} p_1{}^n(1 - p_1)^{r-n}(1 - p_2)^{r-1} + \sum_{k=n}^{r-1} \binom{k}{n} p_1{}^k(1 - p_1)^{k-n}p_2(1 - p_2)^{k-1}.$$

The probability, U_t, that m independent players score a total of exactly t is the coefficient of x^t in $(\Sigma S_n x^n)^m$.

The table on p. 6–91 gives U_t to 4D for $m = 1(1)4, r = 1(1)4, t = 0(1)mr$. For $p_1 = .01(.01).06(.02).22, .25, p_2 = 0(.05).2(.1).9$; for $p_1 = .3(.05).95, p_2 = 0(.01).02(.02).12, .15(.05).9$. U_t was computed to 9D or better on the NAREC, and each value was rounded to 4D individually; i.e., the U_t were not forced to sum to 1. Quadratic interpolation in p_1 or p_2 is stated to yield a maximum error of .0016.

The typography (photo-offset reproduction of Flexowriter script) is adequate but undistinguished; all decimal points are omitted from the body of the table.

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57[W, X].—S. Vajda, *An Introduction to Linear Programming and the Theory of Games*, John Wiley & Sons, Inc., New York, 1960, 76 p., 22 cm. Price $2.25.

This book introduces the basic mathematical ideas of linear programming and game theory (mostly matrix games) in a form suitable for anyone who has had a little analytic geometry (and is not frightened by subscripts and double subscripts). Part I, on linear programming, begins with two examples, the second of which is a transportation problem, and then describes the simplex method of solving the transportation problem. Then comes the graphical representation of the general linear programming problem, followed by the general simplex method and a discussion of such complications as finding a first feasible solution, multiple solutions, and degeneracy. The chapter closes with the duality theorem.

Part II, on games, begins with two examples of matrix games, the second of which admits no saddle point, and introduces the concepts of mixed strategy and value. This is followed by a discussion of games in extensive form, and their normalization. A section on graphical representation is followed by the description of the equivalent linear program, and the Shapley-Snow "algorithm" is offered as an alternative method of calculating equilibrium strategies and value. Next the concept of equilibrium point in non-zero sum games is discussed, followed by three examples of infinite games. The book closes with an appendix proving the main theorem of matrix games along Ville's lines.

This book, compiled from lecture notes of short courses offered by the author, is suitable as a text for a short course for students with slight mathematical preparation.

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58[W, Z].—Franz L. Alt, Editor, *Advances in Computers*, Vol. 1, Academic Press, Inc., New York, 1960, x + 316 p., 24 cm. Price $10.00.

Advances in Computers is a useful addition to the rapidly growing literature on modern high-speed computers and their application. It is intended by the editor to be the first volume in a series which will contain monographs by specialists in various areas of work in this field. These are to be written in non-technical language, so as to be easily understood by specialists in areas other than those of the writer.

The present volume contains six articles by well-qualified authors in six significant and interesting areas of work related to computers. Four summarize progress to date in the application of computers to weather prediction, translation of languages, playing games, and recognition of spoken words. Two are related to techniques used in computer programming and design. The titles and authors are:

1. General-Purpose Programming for Business Application—Calvin E. Gotlieb
2. Numerical Weather Prediction—Norman A. Phillips
3. The Present Status of Automatic Translation of Languages—Yehoshua Bar-Hillel
4. Programming Computers to Play Games—Arthur L. Samuel
5. Machine Recognition of Spoken Words—Richard Fatehchand
6. Binary Arithmetic—George W. Reitwiesner.

Since most of the areas of work covered by the papers in this volume are in a rapid state of flux, the assignment to write survey papers in these areas, undertaken by the authors, is a most difficult one. Each author has proceeded to carry out this assignment in his own characteristic manner. Thus, Gotlieb attempts to present a factual summary of some of the programming procedures used at present in processing data for business applications; whereas, Yehoshua Bar-Hillel presents a critical evaluation of the various efforts conducted in the field of automatic translation of languages—at times, highly critical. A large part of the material covered is admittedly subjective, and bears the imprint of the writers' points of view and contributions. Nevertheless, the six papers in this volume constitute authoritative surveys of the areas of work discussed. Together with the bibliographies given at the end of each paper, these articles will be valuable to the new researcher in the fields covered, as well as to the interested layman who wishes to familiarize himself with the exciting advances in computer technology.

H. P.

59[Z].—Andrew D. Booth, *Automation and Computing*, The Macmillan Co., New York, 1959, 158 p., 21 cm. Price $5.00.

This book is intended mainly for the educated layman. In it the author attempts "to bridge the gap between the superficial accounts of electronic computers and automation . . . and the specialists' monographs. . . ." He has given an admirably written and lucid account of digital and analogue computers. His three chapters on the logical design of digital computers, the physical basis of this design, and programming for digital computers are very clear and informative, though concise.

The three chapters on automation in clerical work, control of continuous processes, and automatic machine tools and assembly processes are not as well done as the first three. The well-educated layman will have to expend a great deal of effort in order to follow the discussion in these chapters.

The last two chapters entitled "Strategic and Economic Planning" and "Non-numerical Applications of Computing Machines" are very brief. The former is much too short to give the reader more than a glimmer of what is involved in game theory. The last chapter furnishes a well-written introduction to methods for non-numerical applications of computers, but, because it is so short, leaves the reader wishing the author had devoted more space to this subject. This reader would have preferred to have the author do this and omit some of his pronouncements on government (for example, the discussion on page 20 beginning with "Democratic government, too, is an example of Man in decay, . . .").

There are a few typographical errors in the book. The most disturbing one appears on page 36 where the binary addition table has the entry

$$1 + 1 = 1 \text{ (carry 1)}.$$

A. H. T.

60[Z].—Robert H. Gregory & Richard L. Van Horn, *Automatic Data-Processing Systems*, Wadsworth Publishing Co., San Francisco, 1960, xii + 705 p., 23 cm. Price $11.65.

This introductory book on automatic data-processing systems (ADPS) is a revision of a text which was used in management development courses sponsored by the Army Ordnance Corps. The affirmative objective is to instruct, enlighten, and inform management on the developments, techniques and applications of methods in management science, mathematics, and large-scale computing for the solution of today's complex business problems.

The book is divided into seven parts and three appendices. In Part One, "Orientation," the principles of basic computer programming are elucidated by means of a hypothetical computer which embodies an instruction repertoire of several existing machines. Various numerical and alphanumerical coding systems for storing data on punched cards, punched paper tapes, and magnetic tapes are also discussed here.

Part Two, "Automatic Equipment," deals with input-output hardware, storage devices, arithmetic and control units. The section concludes with a synopsis of the salient characteristics of approximately twenty computing systems: speed, storage, instruction repertoires, tapes, and peripheral equipment.

Advanced programming techniques and systems provide the subject matter of Part Three, "Programming and Processing Procedures." In this section the authors present a synthesis of the pros and cons of automatic programming and integrated data processing, two important and topical subjects.

The role of the data-processing unit in "management information systems" is the theme of Part Four, "Principles of Processing Systems." Several methods are suggested for selecting from a welter of available facts the pertinent information for effective executive decision-making. The reporting-by-exception principle is described in detail. Since the efficacy of the final system design is inextricably related to economic considerations, the authors analyze the major factors for determining the cost of obtaining and processing data, and explore the concept of the "value" of information in relation to its cost. The last chapter in Part Four outlines the broad principles underlying systems analysis and design.

Factors that affect the organizational structure of data processing are subject to examination in Part Five, "Systems Design." In particular, considerable attention is devoted to problems associated with centralized data processing and decentralized management control. General tools for systems analysis and specific data-processing techniques are also described here.

Part Six, "Equipment Acquisition and Utilization," presents in a nontechnical manner a methodical approach for evaluating, selecting, installing, and implementing automatic data-processing systems for business problems. Considerable space is devoted to the preparation of feasibility studies, application studies, and equipment acquisition proposals. This is followed by a detailed discussion of the problems entailed in the installation of new equipment.

The concluding portion, Part Seven, "System Re-examination and Prospective Developments," touches on a variety of mathematical techniques for the solution of management problems and concludes with a discussion on anticipated future developments in automatic data processing.

Three appendices are:

I. History of Computation and Data-Processing Devices
II. Questions and Problems
III. Glossary of Automatic Data-Processing Terminology

Although the treatment of the basic principles of computer programming illuminates many of the complex and important aspects of business data processing, the authors give little heed to the practical requirements for large-scale production system runs. Such concepts and techniques as rerun procedures, interior tape labels, alternation of servos, and programming methods for effective utilization of buffering are not even mentioned, while the subjects of editing, flow charting of instruction routines, and sorting techniques for large tape files are glossed over. But on the whole, the informative and lucid presentation of the general principles of automatic data processing from the standpoint of business systems will provide management personnel with a short, intensive, and enlightened education on electronic computers and their impact on business data processing.

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61[Z].—ANTHONY G. OETTINGER, *Automatic Language Translation*, Harvard University Press, Cambridge, 1960, xix + 380 p., 24 cm. Price $10.00.

Automatic Language Translation by Anthony G. Oettinger is the eighth in a series of Harvard Monographs in Applied Science. "These monographs are devoted primarily to reports of University research in the applied physical sciences, with special emphasis on topics that involve intellectual borrowing among the academic disciplines." Professor Oettinger's monograph is devoted to the lexical and technical aspects of automatic language translation, with particular emphasis on Russian-to-English translation.

The contents of this work can quickly be conveyed by the titles of its chapters. Chapter 1, "Automatic Information-Processing Machines," discusses the organization, elements of programming, and the characteristics of information-storage media. Chapter 2, "The Structure of Signs," differentiates the notions of use, mention, and representation of signs; mathematical transformations; and mathematical models. Chapter 3, "Flow Charts and Automatic Coding," treats the use of flow charts, addressing, algorithms, and programs. Chapter 4, "The Problem of Translation," is a general discussion of the possibilities and types of automatic translation, grammar, and interlingual correspondence, and syntactic and semantic problems. Chapter 5, "Entry Keys for an Automatic Dictionary," discusses inflection, paradigms, affixes, stems, and inflection algorithms for Russian. Chapter 6, "Morphological and Functional Classification of Russian Words," consists of a detailed account of Oettinger's morphological and functional system for Russian words; nominal forms, adjectival forms, and verbal forms; and an appendix that gives synoptic classification tables. Chapter 7, "Dictionary Compilation," describes the structure of the Harvard automatic dictionary by giving the structure of items and files, methods of detecting and correcting mistakes in transcription and classification, and English correspondence and grammatical codes. Chapter 8, "Dictionary Operation," describes the function of the Harvard automatic dictionary, lookup procedures, and word-by-word translation; and is followed by an appendix that presents an edited trot, the transcription of the edited trot, and an example of conventional translation. Chapter 9, "Problems in Dictionary Compilation and Operation," discusses the problems of paradigm homography, stem homography, "short" words, and a detection and correction of mistakes in dictionary compilation. Chapter 10, "From Automatic Dictionary to Automatic Translator," presents the author's views on how the Harvard automatic dictionary might lead to a complete system of automatic translation.

Since this is the first book published in America devoted to automatic translation of languages, it is a landmark. Several cautions should be mentioned, however, for those who are not familiar with the state of progress in machine translation. First, this book is not a work devoted to the general problem of translating one natural language to another. It is highly specialized, since it treats only the Russian-to-English translation problem. Second, much of the book is devoted to the very detailed description of the particular automatic dictionary compiled at Harvard University. This description does not permit conclusions to be drawn "automatically" about dictionary compilation at other machine translation research centers. Third, all detailed computer descriptions are in terms of the Sperry-Rand UNIVAC I computer, whereas almost all other machine translation programs in the United States are written for IBM 704 or 709 computers.

Nevertheless, Professor Oettinger is to be congratulated for presenting the first detailed, and scientifically accurate description of any machine translation project in the U.S., if not in the world. As such, this book will be of interest to computer scientists, mathematicians, linguists, and to others interested in acquiring knowledge about this important subfield of modern linguistic analysis. As this reviewer has often emphasized, the gains to be made in linguistic analysis will overshadow those which have been made in numerical analysis.

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