REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

88[A–E, G, I, K, L, M, X].—GRANINO A. KORN & THERESA M. KORN, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill Book Co., New York, 1961, xiv + 943 p., 24 cm. Price $20.00.

Workers in every walk of mathematical life will gratefully welcome this latest addition to an illustrious series of Handbooks. In its variety and scope it may well be the largest collection of widely useful mathematical facts and data ever compiled. It admirably fills a too-long existing gap among the handbooks available to workers in technical fields. As a "tool of the trade" its price is certainly reasonable, probably offering the "lowest cost per (mathematical) fact."

"This handbook is intended, first, as a comprehensive reference collection of mathematical definitions, theorems, and formulas for scientists, engineers, and students. Subjects of both undergraduate and graduate level are included. The omission of all proofs and the concise tabular presentation of related formulas have made it possible to incorporate a relatively large amount of reference material in one volume.

"The handbook is, however, not intended for reference purposes alone; it attempts to present a connected survey of mathematical methods useful beyond specialized applications. Each chapter is arranged so as to permit rapid review of an entire mathematical subject," and chapter introductions, notes, and cross-references interrelate the many topics "for a broad view of the entire field of mathematics."

To meet the requirements of different readers the material has been arranged at three levels:

"1. The most important formulas and definitions have been collected in tables and boxed groups permitting rapid reference and review.

"2. The main text presents, in large print, a concise, connected review of each subject.

"3. More detailed discussions and advanced topics are presented in small print."

The following summary of the book's twenty-one chapters and appendices gives a brief indication of the scope of the material.

Chapters 1 through 5 review the basic college material on algebra, analytic geometry (plane and solid), elementary and advanced calculus, including Lebesgue and Stieltjes integrals, and vector analysis. Chapters 6, 7, and 8 cover curvilinear coordinates, functions of a complex variable, and Laplace and other integral transformations, respectively. Chapters 9 through 11 cover ordinary and partial differential equations (including transform methods, method of characteristics), and maxima and minima, including the calculus of variations.

Chapters 12 through 14 deal with various aspects of mathematical models. Chapter 12 introduces the elements of modern abstract language and covers concepts such as groups, fields, topological spaces, and Boolean algebras. Chapter 13 deals with matrices, and quadratic and hermitian forms. Chapter 14 treats linear vector spaces and transformations, including matrix representation, eigenvalues, and group representations. Chapter 15 handles the subject of linear integral equations, boundary-value problems, and eigenvalue problems. Chapters 16 and 17 give a good outline of the related subjects of tensor analysis and differential geometry.

Chapters 18 and 19 recognize the increasing importance of statistical methods in many fields and provide over 100 pages devoted to probability and random processes, and mathematical statistics. The material is given in appealing detail, and the modern worker has the comfort of finding succinctly in a single source many of the not always easy-to-find formulas and results on such topics as multi-dimensional distributions, limit theorems, generalized Fourier analysis including correlation and power spectra, sampling distributions, and statistical estimation and testing of hypotheses.

Chapter 20, on numerical calculations and finite differences, reviews the standard methods and has a section on difference equations. Included are numerical methods for matrix inversion, eigenvalues, interpolation and approximation, ordinary and partial differential equations, and numerical harmonic analysis, among others. Chapter 21 is essentially a brief collection of formulas on the properties of elementary and higher transcendental functions.

A significant portion (about one-sixth) of the volume consists of six appendices as follows: formulas for plane figures and solids; plane and spherical trigonometry; permutations, combinations, and related topics; tables of Fourier expansions and Laplace-transform pairs; tables of indefinite and definite integrals; and twenty numerical tables.

The book is rounded out with a glossary of symbols and notation showing where each item is explained, and a comprehensive index of almost thirty pages that enables the book to be used as a mathematical dictionary.

The painstaking care with which each subject is organized is shown by effective use of summary tables and boxes. The box is a device in common use abroad, which might well be used more widely here. A small sampling of the tables and boxes will be helpful to the prospective user and give an insight into the valuable nature of the material: a table of formulas dealing with tangents, normals, and polars for each of the four classes of conic sections; a box showing various forms for the equation of a plane, and line, in both cartesian and vector notation; a table of properties of Fourier transforms—linearity, change of scale, shift, convolution, modulation, differentiation, Parseval's theorem; tables of operations on scalar and vector point functions; tables relating to a wide variety of transformations and other properties of the various orthogonal curvilinear coordinate systems; a table of real and imaginary parts, zeros, and singularities of common functions; a graphical set of sixty conformal mappings of regions in the complex plane; a table of definitions for different types of tensors; boxes with definitions of Riemann space and associated quantities such as covariant derivatives, Christoffel three-index symbols, and the first and second fundamental quadratic differential forms of a surface; a table explaining fourteen numerical parameters describing properties of one-dimensional probability distributions; boxed formulas for moments, characteristic and other generating functions for one-dimensional distributions, and for two- and more-dimensional probability and marginal distributions; tables for the many formulas and properties of a dozen or more discrete and continuous distributions of most importance in applications, including the features "typical interpretation" and "approximations," which are rarely presented in standard treatments in such useful form; tables of formulas for tests of hypotheses and confidence intervals for normal populations; a box for the unit-step functions, with accompanying sketches, and of relations involving the delta function and its "derivatives"; and a table and sketches of various types of pulses and waveforms and their characteristics.

In addition to the appended numerical tables there are several short tables in the chapter on numerical calculations: 5- to 7-place tables for Lagrange, Newton, Stirling, Bessel, Everett, and Steffensen interpolation, and tables for abscissas and weights for Gauss and Chebyshev quadrature formulas.

While not detracting materially from the excellence of the book, mention of a few necessary corrections that were noted may be of help to the user. The pagination for Chapter 6 should be corrected in the Table of Contents as follows: Sections 6.4, 6.5, 6.6 begin on pages 170, 173, and 173, respectively; on page 112 the symbol "$\rightarrow 0$" is omitted under "lim" on the right-hand side of equation (4.6–43); on page 443, in the displayed equation at the top of the page, "$= 0$" should be replaced by "$\not\equiv 0$"; on page 489, the second line under the last box, Sec. 5.10–3 should read Sec. 16.10–3; on page 566, the typography is confusing in the last line of the table owing to wrong size of type—the mathematical expressions should read

$$\frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \quad \text{and} \quad \frac{1}{\Gamma(\alpha)} \Gamma_{x/\beta}(\alpha);$$

in the same table, the last two entries in the column "Characteristic function" are known in explicit form and should be given, namely, $F(\alpha; \alpha + \beta; it)$ (confluent hypergeometric function) and $(1 - \beta it)^{-\alpha}$; on page 570, in equation (18.8–29) the minus sign is omitted from the exponential, and in equation (18.8–30) the multiplier $(1/\pi\alpha)$ is omitted from the expression for $\phi_x(x)$; on page 626 the reference in the heading of Sec. 19.8–2 should be "Sec. 18.12–2" instead of "Sec. 18.11–2"; on page 935, the index entry "Probability distribution" might well have included a reference to Sec. 18.8, as it has eight tables showing valuable information about the most important special distributions in statistics.

It may be appropriate to mention several additional matters that may be of value in connection with any later edition. The subject of random numbers apparently is not included anywhere in the book, and it would seem that at least one of the most important modern works on this topic warrants mention either in Chapter 18, on Probability, or in Chapter 19, on Mathematical Statistics, namely, The Rand Corporation's *A Million Random Digits with 100,000 Normal Deviates*, The Free Press Publications, Glencoe, Illinois, 1955. Also, some of the older references listed at the end of Chapter 19 should be replaced by their more modern versions; for example, Arkin and Colton is in a fourth, revised edition (1955), and P. G. Hoel is in a second edition (1954). In addition, the following works might be included as being very useful for reference and application purposes:

Oscar Krisen Buros, *Statistical Methodology Reviews*, 1941–50, John Wiley & Sons, Inc., New York, 1951;

M. G. Kendall & W. R. Buckland, *A Dictionary of Statistical Terms*, Hafner Publishing Co., New York, 1957;

E. P. Adams & R. L. Hippisley, *Smithsonian Mathematical Formulae and*

Tables of Elliptic Functions, Smithsonian Miscellaneous Collections, Vol. 74, No. 1, Washington, D. C., 1922 (or later edition);

Statistics Manual, NAVORD Report 3369, Naval Ordnance Test Station, China Lake, California, 1955.

Several minor points may be noted with regard to the numerical tables in Appendix F. To the eleven numerical constants listed should be added Euler's constant, which occurs in a number of places in the text. There is space for increasing the number of decimal places shown to at least 10; this should be done to increase their usefulness. The typographical layout for several of the tables is hard on the eyes because little or no space is allowed between entries in adjacent columns. The columnar lines alone do not provide effective separation, so that the entries running across the page merge into one another. This applies to all or part of the tables for squares, integral sine and cosine, χ^2 distribution, and F distribution. This can be remedied either by use of smaller type or by printing the tables along the length rather than the width of the page, as is done with some of the other tables, resulting in much greater legibility.

Much of the material of the book is necessarily gathered from other sources. In a number of places, especially the figures, the source is cited from among the references at the end of the chapter. It would be helpful if such citation (admittedly laborious) could be done more systematically, as this could save a great deal of time and effort spent in searching through the listed references in order to follow up a particular theorem or development.

As regards the physical aspects, one would wish that a book of such utility could be constructed in such a manner as to better be able to withstand the great amount of handling it is bound to receive, perhaps by being issued in the almost indestructible form achieved by the binders used in the tax and accounting services.

Even with the minor shortcomings indicated here, this mathematical handbook is of such unique value that it can be unhesitatingly recommended for the intimate possession of everyone with a serious interest in the theory or application of virtually any aspect of mathematics.

JULIUS LIEBLEIN

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

89 [E, L].—L. N. NOSOVA, *Tablit͡sy funkt͡siĭ Tomsona i ikh͡ pervykh͡ proizvodnykh͡* (*Tables of Thomson Functions and their First Derivatives*), Izdatel'stvo Akademiĭ Nauk SSSR, Moscow, 1960, 422 p., 27 cm. Price 49 Rubles.

This new addition to the series of tables prepared at the Computation Center of the Academy of Sciences, USSR, consists of two main tables. The first of these presents values of the Thomson (or Kelvin) functions ber x, bei x, ker x, kei x, and of their first derivatives to 7S for $x = 0(.01)10$. The second principal table gives values of modified functions consisting of the Kelvin functions of the first kind (ber, bei) and their first derivatives, each multiplied by $e^{-x/\sqrt{2}}$, and the functions of the second kind (ker, kei) and their first derivatives, each multiplied by $e^{x/\sqrt{2}}$. These data are also given to 7S, for $x = 10(.01)100$. Corresponding values of $e^{x/\sqrt{2}}$ are tabulated in an adjoining column to 7S. The entries throughout appear as 7-digit integers multiplied by an appropriate power of 10.

The book closes with two smaller tables: the first consists of 7S values of exp $(m \cdot 10^{-5}/\sqrt{2})$ for $m = 1(1)1000$; the second gives exact values of $t(1 - t)/2$ for $t = 0(.001).500$, for use in interpolation with second differences, which are given throughout the main tables.

We are informed in the Preface that the underlying computations were performed on the electronic computer STRELA, using the well-known power series and asymptotic series for these functions. These details, as well as a discussion of the arrangement of the tables and their use, are presented in the Introduction. It is there stated that the tabular entries are each correct to within 0.6 of a unit in the last place shown.

The reviewer carried out a partial check of the correctness of this statement concerning the accuracy of these tables, by comparing several entries in them with corresponding data in the recent tables of Lowell [1] which give values of the Kelvin functions and their first derivatives for $x = 0(.01)107.50$ to between 9 and 14 significant figures. Comparison of the two tables revealed that the values given by Nosova for functions of the first kind and their derivatives are correct to within a unit in the last place for the range $x = 0(.01)10$. However, in the vicinity of zeros of the functions of the second kind and of their derivatives the Russian tabular values err by as much as 6 to 8 units, as, for example, kei x when $x = 8.24(.01)8.47$ and kei$'(x)$ when $x = 9.38(.01)9.43$. The reviewer has verified, moreover, that ker$'x$ is in error by 9 units when $x = 7.16$ and 7.18, and that ker$'$ 7.17 is too low by 86 units, which is explainable by virtue of the fact that this last tabular entry is only one-tenth its neighbors, and all three are subject to a nearly constant absolute error.

This same difficulty in attaining the stated accuracy occurs in the second table in the book under review. Egregious examples of just a few of the large relative errors that were discovered occur in $e^{x/\sqrt{2}}$ kei x when $x = 97.19$ (tabular value too low by 335 units), in $e^{-x/\sqrt{2}}$ bei x when $x = 98.30$ (too high by 1027 units), and in $e^{x/\sqrt{2}}$ ker x when $x = 99.41$ (too high by 363 units). It should be stated here that the table of $e^{x/\sqrt{2}}$ was also checked at several places, and no errors were found.

The arrangement of the material is very convenient, all functions of a given argument being found on facing pages. It is indeed unfortunate that the attractiveness and convenience of these tables could not have been matched by acceptable accuracy. This accuracy could have been attained throughout by use of computer routines employing double-precision arithmetic, such as were used by Lowell.

J. W. W.

1. Herman H. Lowell, *Tables of the Bessel-Kelvin Functions Ber, Bei, Ker, Kei, and their Derivatives for the Argument Range 0(.01)107.50*, Technical Report R-32, National Aeronautics and Space Administration, Washington, D. C., 1959. See *Math. Comp.* v. 14, 1960, p. 81 (Review 9).

90[H, S, X].—A. L. Loeb, J. Th. G. Overbeek & P. H. Wiersema, *The Electrical Double Layer Around a Spherical Colloid Particle*, The Technology Press of M.I.T., Cambridge, Mass., 1961, 375 p., 26 cm. Price $10.00.

These tables give the numerical solution of the Poisson-Boltzmann equation

$$r^{-2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = -\frac{4\pi e}{\epsilon}\left[n_+ z_+ \exp\left(-\frac{z_+ e\psi}{kT}\right) - n_- z_- \exp\frac{z_- e\psi}{kT}\right]$$

where ψ is the electric potential at radius r from the center of a charged, spherical colloidal particle in an electrolyte; the local charge distributions and the free energy are also given. The electrolyte is characterized by the following parameters: ϵ, the dielectric constant; z_+, the valence of the positive ions; z_-, the valence of the negative ions; n_+, the concentration of positive ions far from the particle; n_-, the concentration of negative ions far from the particle; T, the absolute temperature.

For the numerical computations reduced variables are introduced. In these new variables the Poisson-Boltzmann equation becomes

$$\frac{d^2y}{dx^2} = \frac{\exp(z_- y) - \exp(-z_+ y)}{2z_- x^4}$$

and the boundary conditions are $y = 0$ at $x = 0$ and $y = y_0$ at $x = x_0$, where y is the reduced potential and x is the new independent variable. The local charge distributions and the free energy are represented by $I_+(x)$, $I_-(x)$, and $F(x)$ where:

$$I_+(x) = x^2 \int_0^x \frac{1 - e^{-z_+ y}}{2z_- \tau^4}\, d\tau;$$

$$I_-(x) = x^2 \int_0^x \frac{e^{z_- y} - 1}{2z_- \tau^4}\, d\tau;$$

$$F(x) = x^2 \int_0^x \left[\frac{1}{2}\left(\frac{dy}{d\tau}\right)^2 + \frac{z_+(e^{z_- y} - 1) - z_-(1 - e^{-z_+ y})}{2z_+ z_-^2 \tau^4}\right] d\tau.$$

The quantities x, $y(x)$, $I_+(x)$, $I_-(x)$, and $F(x)$ are tabulated for a variety of values of z_+, z_- (1,1; 2,1; 3,1; 1,2; 1,3) and of $1/x_0$ (from 0.1 to 20 in varying steps) and of y_0 (from 0.5 to 16 in varying steps). The values of y, I_+, I_- and F are said to be accurate to four significant figures, except for a few cases where there is an error in the third figure.

The tables include a forty-page discussion of the equation to be solved, the numerical methods and the results.

These computations are said to be more extensive and more accurate than similar computations performed by N. E. Hoskin, *Trans. Faraday Soc.*, v. 49, 1953, p. 147.

Lloyd D. Fosdick

Digital Computer Laboratory
University of Illinois
Urbana, Illinois

91[H, X].—Lothar Collatz, *The Numerical Treatment of Differential Equations*, Third Edition, Translated by P. G. Williams from a supplemented version of the second German edition, Springer-Verlag, Berlin, 1960, xv + 568 p., 24 cm. Price DM 98.

The translation of Professor Collatz's book into English will be welcomed by all those people who are in any way concerned with the numerical solution of differential equations. No other single book on this subject contains such a vast amount of material. The following list of the chapter headings gives some idea of the range of topics covered.

I. Mathematical Preliminaries and Some General Principles

II. Initial-Value Problems in Ordinary Differential Equations
III. Boundary-Value Problems in Ordinary Differential Equations
IV. Initial and Initial-Boundary-Value Problems in Partial Differential Equations
V. Boundary-Value Problems in Partial Differential Equations
VI. Integral and Functional Equations

The appendix contains a number of tables giving the various numerical methods in handy tabular form for both ordinary and partial differential equations.

This book is a translation of the second German edition with some differences. As the author states, "It differs in detail from the second edition in that throughout the book a large number of minor improvements, alterations and additions have been made, and numerous further references to the literature included; also new worked examples have been incorporated."

The book is large but by no means covers the subject completely, as the author is careful to point out in the preface. Professor Collatz also disclaims any attempt to make general critical comparisons of the various methods presented. This is to be regretted, since it decreases the value of the book to those persons who would be most likely to refer to it, namely the neophytes in this numerical field. Along this same line of criticism there is no mention made of the use of computers, either analog or digital, for the numerical solution of differential equations. This gives a new book a distinctly old-fashioned flavor. A specific example may be cited to illustrate the criticism. In Chapter II the author states that the Runge-Kutta and Adams methods are stable with respect to small random errors. However, he does not warn the reader as to which of the well-known methods are unstable. Thus, the uninitiated might be tempted to code an unstable method as a subroutine for a computer.

On the positive side, the book can be recommended for its vast coverage, its many worked examples, and its close attention to error estimates. The translation is smooth and the printing is excellent.

RICHARD C. ROBERTS

U. S. Naval Ordnance Laboratory
White Oak, Silver Spring, Maryland

92[H, X].—L. DERWIDUÉ, *Introduction à l'Algèbre Supérieure et au Calcul Numérique Algébrique*, Masson et Cie, Paris, 1957, 432 p., 25 cm. Price 6000 fr.

This author presents an interesting combination of pure theory and computational methods for linear and non-linear algebra—that is, linear equations, eigenvalues, roots of algebraic equations, etc. He concludes with an introduction to abstract algebra.

The numerical methods are developed for use with desk computers and are amply illustrated throughout the text. A listing of the chapter headings will indicate the scope of the work.

I. Mécanisation du calcul algébrique. Nombres complexes.
II. Les déterminants et les systèmes d'équations linéaires.
III. Théorie générale des polynômes et des fractions d'une indéterminée.
IV. Elimination et systèmes d'équations algébriques.
V. Résolution numérique des équations.

VI. Substitutions linéaires, formes quadratiques et transformations rationnelles.
VII. Calcul matriciel
VIII. Equations dont les racines sont dans un cercle ou un demi-plan. Critères de stabilité.
IX. Notion sur les groupes et sur l'algèbre abstraite.
Appendice Sur les determinants de Hurwitz et la séparation des racines complexes des équations à coefficients réels.

E. I.

93[H, X].—MINORU URABE, HIROKI YANAGIWARA & YOSHITANE SHINOHARA, "Periodic solutions of van der Pol's equation with damping coefficient $\lambda = 2 \sim 10$," reprinted from the *J. Sci. Hiroshima Univ., Ser. A*, v. 23, No. 3, March 1960.

The periodic solution of van der Pol's equation

$$\frac{d\chi^2}{dt^2} - \lambda(1 - \chi^2)\frac{d\chi}{dt} + \chi = 0$$

is tabulated for $\lambda = 2, 3, 4, 5, 6, 8, 10$. For each λ a four-decimal-place listing of the function $\chi(t)$ and the function

$$y(t) = \begin{cases} \dfrac{d\chi}{dt} & \text{for} \quad \lambda \leq 4 \\[2ex] \dfrac{d\chi}{dt}\Big/\lambda & \text{for} \quad \lambda \geq 5 \end{cases}$$

is given for the range $T_1(a) \leqq t \leqq T_2(a)$, where a is the initial positive amplitude of the periodic solution normalized so that at $t = 0, \dfrac{d\chi}{dt} = 0$; and where $T_2(a)$ is the smallest positive time at which $\chi = 0$, while $T_1(a)$ is the largest negative time at which $\chi = 0$.

Since the periodic solution corresponds to a closed curve in the (χ, y) plane which is symmetric with respect to the origin, the above tabulation is sufficient.

For $\lambda \geq 5$, an additional three-decimal-place tabulation of $\dfrac{d\chi}{dt}$ is given.

The interval size in t depends on λ and on the value of t as in the following table:

λ	$t > 0$	$t < 0$
2	.05	.025
3, ⋯ , 8	.025	.0125
10	0.0(.0125)0.2 0.2(.025)9.0 9.0(.0125)9.25	.00625

Each table includes a listing of the same quantities at $t = T_1(a)$ and $t = T_2(a)$. Furthermore, four-decimal values of the amplitude a, the period $\omega =$

$2[T_2(a) - T_1(a)]$, and the characteristic exponent h are given for each λ. If we set

$$h(t) = \lambda \int_0^t (1 - \chi^2)\, dt, \qquad \text{then} \quad h = h(\omega)/\omega.$$

For each λ there is a plot of the hodograph $\left(\chi, \frac{d\chi}{dt}\right)$ and the curve $\chi(t)$ (including $\lambda = 0, 1$). An additional graph depicts a, ω, and h as functions of λ in the interval [0, 10].

E. I.

94[M, P, S].—R. L. MURRAY & L. A. MINK, *Tables of Series Coefficients for Burnup Functions*, Bulletin No. 71, Department of Engineering Research, N. C. State College, Raleigh, N. C., May 1959, 82 p., 28 cm. Price $1.50.

In a certain model, calculation of nuclear reactor properties under long-term operation requires the evaluation of

$$A_0 = \bar{\varphi}^{(l)} \left[\frac{1}{\delta_z\, \pi/2} \int_0^{\delta_z \pi/2} (\cos x)^l\, dx\right]\left[\frac{2}{(\delta_r\, j_0)^2} \int_0^{\delta_r j_0} x[J_0(x)]^l\, dx\right],$$

$$a_0 = \frac{\overline{\varphi^{(l+1)}}}{\overline{\varphi^{(l)}}}, \quad A_1 = \frac{1}{2}\left[\frac{\overline{\varphi^{(l+2)}}}{\overline{\varphi^{(l)}}} - a_0^{\,2}\right]$$

and some other combinations of $\overline{\varphi^{(l)}}$. Here j_0 is the smallest positive zero of $J_0(x)$. All functions are tabulated for the range $l = 0(1)4$, δ_i, $\delta z = 0.50(0.05)0.60(0.02)1.0$. A_0 and a_0 are given to 7D; A_1 to 6D, and the remaining functions not listed here are given with less accuracy. Similar tables are given for the function

$$A_0 = \frac{3}{(\delta\pi)^3} \int_0^{\delta\pi} x^2 \left(\frac{\sin x}{x}\right)^l dx.$$

The method of computation is not explained, nor is j_0 defined. We infer the definition of j_0 from physical considerations corroborated by numerical evaluation. Spot checks indicate the entries are accurate to the number of places given.

Y. L.

95[W, X].—RUSSELL L. ACKOFF, Editor, *Progress in Operations Research*, Vol. 1, John Wiley & Sons, Inc., New York, 1961, 505 p., 23 cm. Price $11.50.

Each chapter of this book is written by different authors. It treats recent progress in some of the methodological fields of operations research, such as linear programming, in an outstanding manner, scantily discussing progress in others, such as queuing theory.

The introductory chapter, written by the editor of the book, is excellent. He points out that in other well-established scientific fields one is not as concerned about definitions as those in operations research have been, that this field is now accepted and has acquired the confidence of workers in other fields, and that, as a result, there is less craving for definitions.

An interesting chapter by Churchman on contributions to decision and value theory then follows. Hanssmann's chapter on inventory theory leaves much to be desired and is not saved even by attempting to justify the presentation in an operations research framework. The use of parallel and series stations leaves many problems untouched. The stochastic nature of the field does not come through satisfactorily. A chapter on mathematical programming follows, in which Arnoff and Sengupta give a superb account of progress in programming except for non-linear programming, in which there have been several recent contributions, such as that of Zoutendijk [1]. A remark at the top of page 176 regarding the unavailability of work on sensitivity is inaccurate. This reviewer has proved in the 1959 paper referred to on page 209 that at a solution vertex the objective function (in the customary notation) has the following sensitivity to a_{ij}

$$\frac{\partial V}{\partial a_{ij}} = - x_j^0 y_i^0 = \frac{\partial V}{\partial b_i} \frac{\partial V}{\partial c_j}$$

where x_j^0 and y_i^0 are the solutions to the primal and the dual, respectively. A readable and very useful account of dynamic programming, including adaptive processes, is then given by Dreyfus. Chapter 6 by Morse deals with Markov and queuing processes. Sisson studies sequencing theory in chapter 7, and a variety of very useful replacement models, developed by a number of individuals, are treated by Dean in the next chapter. In another chapter, Morgenthaler describes simulation and Monte Carlo in a manner which provides useful guide-lines for application. Thomas, well known for his contributions to game theory, treats the subject in chapter 10 in an interesting style which utilizes historical ideas on the subject. The presence of these two chapters clarifies in the mind of the reader differences between simulation and gaming. Magee and Ernst examine the future of operations research in chapter 11 and point out the need of quantitative models of human behavior, marketing, interaction of men with men and with machines, organization, and information; and they call for a better grasp of risk and competition. They point out that operations research is far from mature and has promise. This book is recommended reading.

THOMAS L. SAATY

Office of Naval Research
Navy Department
Washington, D. C.

1. G. ZOUTENDIJK, "Maximizing a function in a convex region," *J. Roy. Statist. Soc.*, v. 21B, 1959, p. 338–355.

96[X].—THOMAS L. SAATY, *Mathematical Methods of Operations Research*, McGraw-Hill Book Co., Inc., New York, 1959, xi + 421 p., 24 cm. Price $10.00.

This book is about some selected mathematical methods of operations research, but it offers both more and less than what its title may suggest to some readers. Though full of mathematical results, this book is not a cycle of "lemma, theorem, proof, and corollary." Though it has many problems, and though Saaty is deeply concerned with principles of solution, this book is not a "problem manual." Despite the fact that there are many illustrative applications, this is far from being either a "cook book" or a collection of case studies.

For the unifying thread of the book, one must look to the exuberant creativity of Saaty himself. He sets the tone of the book in the preface with the statement that begins, "We wish to warn the reader at the outset that even though we may attempt partly to inform him and partly to stimulate him . . ." It is the effort to stimulate the reader, to jolt him from his mental ruts, that is most striking. Saaty continually reminds the reader of the elemental role of creativity, that formal proof is not all of mathematics, nor is mathematics all of operations research. Operations research, incidentally, according to Saaty's preferred definition, "is the art of giving bad answers to problems to which otherwise worse answers are given."

The preface states nominal reader requirements of "a course in calculus, with some elements of advanced calculus and rudimentary knowledge of matrix theory," but it also admits to "compromise in the presentation" as a response to an anticipated "variety of background among the readers." As the development unfolds, it becomes clear that Saaty hesitates neither to emphasize an elementary result that he finds suggestive nor to introduce a more advanced result that he finds particularly intriguing.

The outline of the book is much what one might expect from the title. The introductory first chapter presents history and concepts of operations research. Then, Part 1 gives three chapters, also largely introductory in nature, that range rather widely over topics related to the scientific method as an approach to truth, to mathematics and logic as approaches to validity, and to some elementary classical methods useful in the formulation of mathematical models. As the first of the two major sections, Part 2 has three chapters on the subjects, respectively, of optimization, linear and quadratic programming, and the theory of games. In Part 3, the other major section, there are four chapters that dispose of basic probability, applications of probability, fundamental statistics, and queuing theory. The lone chapter of Part 4 concludes the book with an unusual essay on creativity. Each of the four parts gives a collection of interesting problems, and each chapter ends with a set of valuable references.

The logic of the outline does not always withstand successfully Saaty's attempts to stimulate the reader. Sometimes Saaty seems to give way to his own effervescence, getting ahead of his story by appealing to concepts or methods that are stated clearly only in a later chapter. The resulting unevenness of presentation would be a fault in a conventional text with more staid objectives. But conventional standards hardly apply to a book that draws upon such diverse topics as James Thurber's rooster, the trial of Madeline Smith for the murder of her lover, "The Critique of Pure Reason," "The Prince" of Machiavelli, and "The Three Princes of Serendip." Even the typographical errors, which are of at least the usual frequency, may be viewed, generously, as useful stimuli to the reader's alertness.

There is only one reference to computers ("Computers are often used in gaming methods . . . They have the advantage of speed and economy and can be controlled and relied upon."). There are several passages that touch on topics of numerical analysis. What the book conveys in fullest measure, however, is the spirit of mathematics and a passion for the creative solution of problems, wherever they may arise.

Clayton Thomas

U. S. Air Force
Washington 25, D. C.

97[X, Z].—UNESCO, *Information Processing, Proceedings of the International Conference on Information Processing*, R. Oldenbourg, München & Butterworths, London, 1960. Distributed by International Documents Service, Columbia University Press, 520 p., 30 cm. Price $25.00.

This volume publishes the proceedings of the first full-scale international conference on information processing by the use of modern digital high-speed calculators. The conference was sponsored by Unesco, and was held at the Sorbonne and at Unesco House in Paris during 15–20 June, 1959. It was attended by nearly 2000 participants from 37 countries. Fifty-nine papers were presented at eleven plenary sessions; in addition, approximately sixty short lectures were given at twelve symposia. The volume is divided in seven main chapters covering the following areas:

Chapter I.—Methods of Digital Computing
Chapter II.—Common Symbolic Language for Computers
Chapter III.—Automatic Translation of Languages
Chapter IV.—Pattern Recognition and Machine Learning
Chapter V.—Logical Design of Computers
Chapter VI.—Special Session on Computer Techniques of the Future
Chapter VII.—Miscellaneous Topics

In addition, there are published introductory or closing remarks by the Editor, S. de Picciotto; Réne Maheu, Director General of Unesco; Howard H. Aiken, President of the Conference; André Danjon, President of the Association Française de Calcul; Pierre Auger, Secretary General of the Conference; and Hughes Vinel, representative of the Department of State of France in the field of scientific research.

The list of authors and participants includes some of the best-known names in the field of high-speed calculators and their application. Thus, although the papers are not uniformly excellent, there is no doubt that the material contained in the volume constitutes the most comprehensive compilation of knowledge in certain areas of information processing available at the time of the meeting. The talks are especially lucid, and the discussions extremely helpful in clarifying many points. Also, the summaries, presented by the chairmen of the various sessions, are well done and easily readable. Of special interest are the frank and clear exchanges of technical information between the U. S. specialists in the field of MT (machine translation) and their counterparts in the USSR. D. Panov sets the tone for these meetings in his introductory statement, which is punctuated with humorous remarks (almost "wisecracking" in character) in the best American style. Thus he divides the work in the field of MT in four stages, (1) "Talking" about future accomplishments, (2) "Complacency," when the algorithm has finally been constructed and we say how good it would be if only it worked, (3) "Enthusiasm," when the algorithm works but still has many difficulties, and (4) "The final stage—more talking."

Because of the large number of technical papers, and the wealth of material contained in the volume, it is not possible to give special attention to any number of selected papers in this review. The accession of this historic volume is a "must" for every modern technical library.

The sponsorship of this conference by the United Nations and the success which it enjoyed tend to emphasize the significance of this field of research in the modern world of great technological advances. There is no doubt that it will serve as the forerunner for many other such meetings to be held in the future.

H. P.

98[Z].—Ivan Flores, *Computer Logic: The Functional Design of Digital Computers*, Prentice-Hall, Inc., New Jersey, 1960, xii + 458 p., 24 cm. Price $12.00.

The author has attempted to write an all embracing book on computers for the reader who has some scientific training, but little engineering or mathematical training. In the reviewer's opinion, the author "talks down" to his reader and interlaces his discourse with many irrelevant comments; e.g., "The concept of the individual is one of far reaching consequences. Would you expect this idea to have ramifications in biology, law and ethics? . . ." (from section 7.1 of Chapter 7, Number Systems and Counting).

The author's style is unfortunate in another respect: he often uses a term or an expression before he has defined it. In some cases the definition can only be found in the glossary at the end of the book. The reader is not helped by the fact that certain terms used in the text and partially defined therein are not listed in the index; e.g., the term "overflow".

It is unfortunate that the book is not error free. Thus the unwary reader will have difficulty with the following passage from page 100 where the base 12 numbers are being discussed: "To represent the quantity thirty-two, we would count out our bundle of a dozen twice and have seven (sic) units left over. Thus thirty-two will be represented . . . (by) . . . $(27)_{12}$."

The first half of the book is intended to give "a bird's-eye view from the air . . . to see how the computer fits into the over-all system of scientific investigation and business enterprise." The second half discusses the logical design of various basic units of a computer and the synthesis of larger functional units of such a machine.

The sequence in which the author takes up various topics seems somewhat strange. Thus the chapter on Number Systems and Counting follows the one on Machine Arithmetic. In the latter chapter, addition is done by use of addition tables but the reasons for using these instead of counting techniques is never discussed.

The list of chapter headings follows:

Chapter One	Introduction
Chapter Two	First Principles and Definitions
Chapter Three	Specifying the Computer For the Problem
Chapter Four	The Flow and Control of Information
Chapter Five	Coding
Chapter Six	Machine Arithmetic
Chapter Seven	Number Systems and Counting
Chapter Eight	Machine Languages
Chapter Nine	Logic
Chapter Ten	Logical Construction
Chapter Eleven	Functional Units
Chapter Twelve	The Logic of Arithmetic
Chapter Thirteen	Memory Devices and Their Logic
Chapter Fourteen	The Control Unit
Chapter Fifteen	Input and Output Equipment
Chapter Sixteen	A Problem

A. H. T.

99[Z].—FRITZ REUTTER, *Die nomographische Darstellung von Funktionen einer komplexen Veränderlichen und damit in Zusammenhang stehende Fragen der praktischen Mathematik*, DK 518.3:517.53. Forschungsberichte des Landes Nordrhein-Westfalen, herausgegeben durch das Kultusministerium, Nr. 912, Westdeutscher Verlag, Köln and Opladen, 1960, 123 p., 30 cm. Price DM 35.40.

This volume is primarily a collection of material given by the author in a series of previously published papers. The first part of the book deals with methods to represent the analytic function $w(z) = u(x,y) + i\,v(x,y)$ by a nomogram with two parallel nonlinear scales and two "sliding" curves. For example, a reading-line tangent to the curves $u(x,y) = c_1$ and $v(x,y) = c_2$, where c_1 and c_2 are constants, intersects the parallel scales in points from which x and y may be read. Variations of this technique are also considered. For instance, the parallel scales may represent u and v while the sliding curves represent x and y.

Examples include the elementary functions $w = z^3$, $z = w^{1/3}$, $w = \sin z$ and $w = \ln z$. Considerable attention is devoted to the Jacobian and Weierstrassian elliptic functions. The 30 illustrative nomograms which make up the second part of the volume are very neatly drawn and easily read. The author also includes on each figure at least two numerical problems so that the reader can quickly become familiar with their use.

Y. L. L.

100[Z].—MARSHAL H. WRUBEL, *A Primer of Programming for Digital Computers*, McGraw-Hill Book Co., Inc., New York, 1959, xv + 230 p., 24 cm. Price $7.50.

As a good professor presents a new topic through ideas which are already familiar to his students, Professor Wrubel, in the introductory chapter of *A Primer of Programming for Digital Computers*, smoothly leads his reader to a basic understanding of the electronic digital computer and of the nature of programming. The potential programmer is cautioned not to attribute superhuman powers to the computer, and he is advised to consider carefully the value of a computer solution to each individual problem.

Written for "scientists, engineers and their students who are planning to use computers as tools of research," the primer is divided into two sections: Elementary Programming and Advanced Programming. The first section instructs the novice in the elements of programming as illustrated by the Bell Laboratories interpretive language. The second section trains the more experienced programmer in advanced techniques and in the use of a machine (IBM 650) language. Chapter subheadings in both sections follow the same sequence, enabling the reader to easily apply his basic knowledge to the more complex programming concepts.

In his approach to the elements of programming Professor Wrubel defines and builds on each component from the basic digit through the word and the language, until he completes his construction of the computer program. Arithmetic, logical, and input-output control instructions are taught with clarity. Taking advantage of his readers' familiarity with mathematical notation, the author employs symbolism freely in explaining such aspects of programming as conditional transfers, address modification and looping, and subroutines. Professor Wrubel also describes and stresses the importance of drawing the flow diagram, of code-checking, and of preparing the written report of each problem.

The presentation of the Bell code and of the elementary programming concepts is lucid, and provides adequate information to enable the novice to program his problem. In this part of the text, particularly, most of the reader's questions are anticipated, and it is obvious that the author speaks with a great deal of practical experience. The chapter of instructions for problem testing probably should be supplemented by IBM publications related to this phase of programming.

In the final chapter of this section, Professor Wrubel speaks briefly on the subject of automatic programming, and then immediately re-introduces the elementary concepts in the language of FORTRANSIT, an automatic programming system for the IBM 650. This chapter might have been more meaningful to the reader had the author's commentary on the general nature of automatic programming been more fully developed.

The Advanced Programming section of the text treats the basic functions of the computer through the machine's own code of instructions and through SOAP, the Symbolic Optimal Assembly Program. The reader becomes acquainted with the use of error conditions for program control, with double-precision arithmetic, with the scaling of variables and constants, and with many more advanced topics.

The primer includes many practice problems and a useful glossary of programming terms.

Although the primer's discussions are primarily concerned with programming for the IBM 650, they cover the concepts of programming in such a way as to be valuable to all newcomers to the field.

Rita Horbett Burns

Research Computing Center
Indiana University
Bloomington, Indiana