29 [C, D].—LEON KENNEDY, *Handbook of Trigonometric Functions—Introducing Doversines*, Iowa State University Press, Ames, Iowa 1961, 397 p., 23.5 cm. Price $6.50.

In his preface the author states that this new set of trigonometric tables has been arranged specifically to facilitate the solution of both plane and spherical triangles. The term "doversine" has been introduced as an abbreviation for "doubled versed sine."

The introduction to these tables contains a derivation of the law of haversines for spherical triangles and of the "law of doversines" for plane triangles. Also included therein are a table of algebraic signs for all the tabulated trigonometric functions for the four quadrants; a composite graph of the cosine, versine, haversine, and doversine; a compilation of fundamental trigonometric formulas and identities; and a presentation of formulas for solving plane and spherical triangles, including the navigational triangle.

The main table, occupying 360 pages, consists of six-figure natural and logarithmic values of the six standard trigonometric functions and of the versine and doversine. These data are conveniently displayed on facing pages for each sexagesimal degree to 180 degrees at increments of a minute in the argument. Natural values and their logarithms appear in adjacent columns, printed in black and blue, respectively. Differences are not tabulated.

Following this table, there is a table of six-place mantissas of the common logarithms of the integers between 1000 and 10,000.

The numerical values presented in these original tables were computed by the author on an IBM 650 system in the Ames Laboratory at Iowa State University. The doversines and their logarithms were previously computed on the ILLIAC, and these results were compared with corresponding data obtained in the later calculation.

The reviewer has compared nearly forty percent of the tabular entries appearing in the first quarter of the main table with the corresponding entries appearing in more elaborate tables such as those of J. Peters. The reliability of these trigonometric values and their logarithms may be inferred from the fact that such careful examination revealed just one typographical error (in csc $8°48'$) and one terminal-digit error (in log sec $5°5'$). On the other hand, the following five rounding errors were discovered by the reviewer in the table of logarithms of numbers: log 499, *for* 698100, *read* 698101; log 2443, *for* 387924, *read* 387923; log 8652, *for* 937116, *read* 937117; log 8854, *for* 947139, *read* 947140; and log 8884, *for* 948608, *read* 948609.

Correspondence with the author has disclosed that these last errors are attributable to a programming error, which led to the omission of one of the tests for retention of an adequate number of terms in the evaluation of the logarithms by power series. The publishers have recently informed the reviewer that they will make appropriate corrections in the future printings.

This handbook of trigonometric functions is unique among the tables that this reviewer has examined, particularly because of its format, which is to be contrasted with the customary semiquadrantal arrangement. The tabulation of the versine and doversine clearly requires the extended range of the argument given in these tables. The author justifies the tabulation of the six standard trigonometric functions also over the first two quadrants on the basis of the resulting ease of application to the solution of triangles. The typography is generally excellent, and the arrangement of the tabular data convenient. This book should prove a useful addition to the literature of mathematical tables.

J. W. W.

30 [F].—R. Kortum & G. McNiel, *A Table of Periodic Continued Fractions*, Lockheed Aircraft Corporation, Sunnyvale, California, 1961, xv + 1484 p., 29 cm.

This huge and interesting table contains, first, the *half-period* of the regular continued fraction for the $\sqrt{D}$ for each non-square natural number D less than 10,000. For example, since

$$\sqrt{13} = 3 + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{3 + \sqrt{13}},$$

under $D = 13$ are listed the *partial quotients*: 3, 1, 1. Again, since

$$\sqrt{19} = 4 + \frac{1}{2} + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{2} + \frac{1}{4 + \sqrt{19}},$$

under $D = 19$ is the list: 4, 2, 1, 3.

Next, let

$$\sqrt{D} = q_0 + \frac{1}{q_1 +} \frac{1}{q_2 +} \cdots \quad \text{and} \quad x_i = q_i + \frac{1}{q_{i+1} +} \cdots$$

and $x_i = (\sqrt{D} + P_i)/Q_i$. Then Q_i, the *denominators of the complete quotients*, are listed in a row parallel to the q_i.

If p, the *period* of the continued fraction, is odd, as in $p = 5$ for $D = 13$, the table gives the smallest solution x, y of

$$x^2 - Dy^2 = -1.$$

If p is even, as in $p = 6$ for $D = 19$, the smallest solution is given of the so-called Pell equation:

$$x^2 - Dy^2 = +1.$$

The values of D for which p is odd are marked with an asterisk.

Finally, if $p^2/D > 1$, this ratio is given to 9 decimals. All of this was computed on an IBM 7090 in 36 minutes.

While in [1] the continued fractions for $D < 10{,}011$ have already been given, together with *both* sequences P_i and Q_i defined above, the range here of D for the solutions x, y, and for the ratio p^2/D, would appear to exceed that in any published table.

From a theoretical point of view the quantity p^2/D is of considerable interest. If we list those D where p^2/D attains a new maximum we obtain the following table:

D	p^2/D	D	p^2/D
3	1.333	631	3.651
7	2.286	919	3.917
43	2.326	1726	4.487
46	3.130	4846	4.768
211	3.204	7906	4.948
331	3.492		

Further, the authors have extended their computation to $D = 51{,}000$ and have listed all such D for which p^2/D exceeds 5. We may thus continue:

D	p^2/D	D	p^2/D
10651	5.141	19231	5.731
10774	5.257	32971	5.819
18379	5.641	48799	6.064

While these empirical data obviously suggest the possibility that

$$p = O(D \log D)^{1/2},$$

the authors refrain from such a suggestion and also from any reference to pertinent theoretical work.

Tenner's algorithm was used in the computation of the continued fractions. This requires two divisions and subtractions and one multiplication and addition in each cycle. An alternative algorithm is known that replaces one of these divisions by an addition. This latter computation would therefore be somewhat faster. But, alternatively, the redundancy in Tenner's algorithm (implicit in the fact that in one of these divisions the remainder is always zero) allows for a check at each cycle. But whether the authors utilized this check, or indeed made any check, is not indicated in their introduction.

There is a printing defect which could have been easily avoided. In a block of 10 decimal digits, if the high-order digit is a zero, it is printed as a blank. Thus, for $D = 801$, $x = 500002000001$ and $y = 17666702000$ are printed as

```
50       2000001
 1 7666702000
```

This is because the binary to decimal integer conversion subroutine which was used *deliberately* causes such suppression of high-order zeros on the assumption that they will not be preceded by a significant digit. To circumvent such suppression one can add 10^{10} to each binary number before conversion to 10 decimal digits. This *fictitious* high-order 1 is an 11th digit, and will not be printed, since only the 10 low-order digits will be converted. Nonetheless, the routine is deceived, by its presence, into thinking that the high-order zeros are not now high order. A programmer who keeps on his toes can often outwit the makers of subroutines.

In a covering letter one of the authors indicates that this table is the second [2] in a series of fourteen, or more, number-theoretic tables. While a few of these duplicate, at least in part, some known tables, the latter are often on magnetic tape, or cost money, or are otherwise inaccessible. The entire proposed series will certainly be welcome to mathematicians working in number theory.

D. S.

1. WILHELM PATZ, *Tafel der regelmässigen Kettenbrüche*, Berlin Akademie-Verlag, 1955.
2. The first is *A Table of Quadratic Residues for all Primes less than 2350*. See RMT 35, *Math. Comp.*, v. 15, 1961, p. 200.

31 [I].—HERBERT E. SALZER & CHARLES H. RICHARDS, *Tables for Non-linear Interpolation*, 11 + 500 p., 29 cm., 1961. Deposited in the UMT file.

These extensive unpublished tables present to eight decimal places the values of the functions $A(x) = x(1 - x)/2$ and $B(x) = x(1 - x)(2 - x)/6$, corresponding to $x = 0(10^{-5})1$. This subinterval of the argument is ten times smaller than that occurring in any previous table of these functions.

These tables can be used for either direct or inverse interpolation, employing either advancing or central differences. In the introductory text are listed, with appropriate error bounds, the Gregory-Newton formula and Everett's formula, for direct quadratic and cubic interpolation, and formulas for both quadratic and cubic inverse interpolation, employing advancing differences and central differences. Examples of the use of these formulas are included.

The convenience of these tables is enhanced by their compact arrangement, which is achieved by tabulating $B(1 - x)$ next to $B(x)$. This juxtaposition, in conjunction with the relation $A(1 - x) = A(x)$, permits the argument x to range from 0 to 0.50000 on the left of the tables, while the complement $1 - x$ is shown on the right.

The authors note the identity $A(x) - B(x) \equiv B(1 - x)$, which can be used as a check on interpolated values of $A(x)$, $B(x)$ and $B(1 - x)$, and also as a method of obviating interpolation for $B(1 - x)$, following interpolation for $A(x)$ and $B(x)$.

Criteria for the need of these interpolation tables are stated explicitly, with reference to both advancing and central differences.

A valuable list of references to tables treating higher-order interpolation is included.

The authors add a precautionary note that this table is a preliminary print-out, not yet fully checked.

J. W. W.

32 [I, X].—GEORGE E. FORSYTHE & WOLFGANG R. WASOW, *Finite-Difference Methods for Partial Differential Equations*, John Wiley & Sons, Inc., New York, 1960, x + 444 p., 23 cm. Price $11.50

The solution of partial differential equations by finite-difference methods constitutes one of the key areas in numerical analysis which have undergone rapid progress during the last decade. These advances have been accelerated largely by the availability of high-speed calculators. As a result, the numerical solution of many types of partial differential equations has been made feasible. This is a development of major significance in applied mathematics.

The authors of this book have made an important contribution in this area, by assembling and presenting in one volume some of the best known techniques currently being used in the solution of partial differential equations by finite-difference methods. This, I am certain, has not been an easy task, owing to the fluid state of many of the theories in this field. For the same reason it is not possible, at the present state of flux, to write a book on this subject which will successfully withstand the test of time. The authors well recognize this point when they state in their introduction: "The literature on difference methods for partial differential equations is growing rapidly. It is widely scattered and differs greatly in viewpoint and character. A definitive presentation of this field will have to wait until the present period of intense development has come to at least a temporary halt."

The book contains an introductory chapter in addition to four major chapters, as follows:

Introduction to Partial Differential Equations and Computers
1. Hyperbolic Equations in Two Independent Variables
2. Parabolic Equations
3. Elliptic Equations
4. Initial-Value Problems in More than Two Independent Variables.

Topics covered within these chapters include the concept of stability, the method of characteristics, the numerical solution of problems involving shock waves, the theory of Lax and Richtmeyer, the solution of eigenvalue problems, the Young-Frankel theory of successive over-relaxation, and the method of Peaceman and Rachford.

The phenomenon of instability, which frequently arises to plague and invalidate many solutions of partial differential equations by finite-difference methods, is discussed in detail. However, this reviewer cannot, in good conscience, agree with the method of approach used in presenting this important and fundamental concept. The authors begin their discussion by stating: "Although the stability of difference equations has been amply discussed in the literature, one rarely meets precise definitions. The subject is therefore in need of further clarification." Subsequently the authors proceed to develop their own definition of stability, which in the opinion of the reviewer, is neither precise nor especially illuminating. The definition of stability is unnecessarily complicated by its tie-in with the concept of convergence and with the "cumulative departure," whose order of magnitude can, in the words of the authors, "rarely be exactly determined."

Notwithstanding any differences of opinion concerning the method of treatment of specific topics, the book is highly recommended as an authoritative and timely exposition of some of the most significant techniques currently available for the solution of partial differential equations by finite-difference methods.

H. P.

33 [I, X].—Charles Jordan, *Calculus of Finite Differences*, Second Edition, Chelsea Publishing Co., New York, 1960, xxi + 652, 21 cm. Price $6.00.

This is a reprint of the second edition of the well-known book by Charles Jordan. The republication of this excellent text on the calculus of finite differences and its availability at a reasonable cost should be welcomed by all students of numerical analysis.

H. P.

34 [K].—W. S. CONNOR & SHIRLEY YOUNG, *Fractional Factorial Designs for Experiments with Factors at Two and Three Levels*, NBS Applied Mathematics Series, No. 58, National Bureau of Standards, Washington, D. C., 1961, v + 65 p. Price $0.40.

This publication contains a collection of fractional factorial designs for experiments in which some factors are to be studied at two levels or conditions and others at three levels. It is the sequel to two other catalogs [1], [2] of designs in the National Bureau of Standards Applied Mathematics Series that contain, respectively, plans for m factors each at two levels, and plans for n factors each at three levels. This new document gives plans for the mixed series involving $(m + n)$ factors, where the m factors each at two levels and the n factors each at three levels are given for 39 combinations (2^m3^n) of positive integer values of m and n for which $5 \leqq m + n \leqq 10$.

For each design the following are given: number of effects estimated, number of treatment combinations employed in the design, fraction of complete factorial experimental plan, analysis, and construction.

The method of construction of designs is described in Section 2. Fractions are selected so that low-order interaction effects, including main effects, are aliased with each other as little as possible. Section 3 contains a description of the mathematical model, in which it is assumed that all interactions between three or more factors are nonexistent, and a procedure for estimating the parameters contained in the model. Section 4 contains a discussion of procedures to test hypotheses and to construct confidence intervals. A worked example of 2^33^2 design is presented in Section 5.

Section 6 is devoted to six particular designs for which the interaction effects between factors at three levels are defined in a different manner from that of the other designs.

H. H. KU

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1. NATIONAL BUREAU OF STANDARDS, *Fractional Factorial Experiment Designs for Factors at Two Levels*, NBS Applied Mathematics Series, No. 48, U. S. Gov. Printing Office, Washington, D. C. 1957.
2. W. S. CONNOR & MARVIN ZELEN, *Fractional Factorial Experiment Designs for Factors at Three Levels*, NBS Applied Mathematics Series, No. 54, U. S. Gov. Printing Office, Washington, D. C., 1959.

35 [K].—N. V. SMIRNOV, Editor, *Tables for the Distribution and Density Functions of t-Distribution ("Student's" Distribution)*, Pergamon Press Ltd., New York, 1961, 130 p., 28 cm. Price $12.50.

This book, which is Volume 16 in the Mathematical Tables Series of Pergamon Press, is a translation of the Russian work issued by the V. A. Steklov Mathematical Institute of the Academy of Sciences of the U. S. S. R. There are three main tables of the (cumulative) distribution and density function of the ordinary (central) t-distribution presented to six decimal places for several ranges of the argument and parameter, followed by four auxiliary tables and one table of interpolation coefficients.

The density function of the t-distribution is

$$s_\nu(t) = K_\nu \cdot \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

where

$$K_\nu = (\pi\nu)^{-1/2}\Gamma[\tfrac{1}{2}(\nu + 1)]/\Gamma(\tfrac{1}{2}\nu);$$

and the distribution function is

$$S_\nu(t) = \int_{-\infty}^{t} s(u)\, du.$$

Table I gives, in parallel columns, the values of $S_\nu(t)$ and $s_\nu(t)$ for $t = 0.00(0.01)3.00(0.02)4.50(0.05)6.50$, with $\nu = 1(1)12$, and for almost as many values of t with $\nu = 13(1)24$. Some of the printing is imperfect, one of the values, namely, $S_{21}(1.00)$ on page 6, having its fourth decimal place almost completely missing. Fortunately, this value of t is one of those repeated at the top of the page following, so that the missing digit, 6, is available.

Table II gives the distribution function $S_\nu(t)$ for large values of the argument, namely, $t = 6.5(1.0)9.0$, and $\nu = 1(1)10$.

Table III gives $S_\nu(t)$ for large numbers of degrees of freedom for $t = 0.00(0.01)2.50(0.02)3.50(0.05)5.00$, namely, for $\nu = 25(1)35$. The first two differences of the function are also given in convenient form alongside the functional values.

Table IV lists the values of $S_\nu(t)$, according to the values of $\xi = 1000/\nu$, for $t = 0.00(0.01)2.50(0.02)5.00$ (the limit 5.00 is omitted in the Contents and Introduction) for $\xi = 30(2)0$. The values of the ξ-entry are equivalent to the values $\nu = 33\frac{1}{3}, 35\frac{5}{7}$, (etc.), 250, 500, ∞ (corresponding to the normal distribution $\Phi(t)$). This type of tabulation facilitates harmonic interpolation, which is necessary for large values of ν.

Table V gives values of the polynomials $C_i(t)$, $i = 1$ to 4, that occur in R. A. Fisher's formula for the distribution function in inverse powers of ν,

$$S_\nu(t) = \Phi(t) - \sum_{i=1}^{\infty} \frac{C_i(t)}{\nu^i},$$

for $t = 0.00(0.01)5.00$. These values, together with those of $C_5(t)$ that are given for a small number of values of t, were used in the calculation of Table III.

Table VI gives the values of K_ν and its common logarithm for $\nu = 1(1)24$. In addition, two other quantities, p_s and q_s, are tabulated for $s = 2(1)21$, for which no explanation is given in the Introduction or anywhere else in the volume. Apparently $p_s = 1/\sqrt{s}$, but the reviewer was unable to determine the meaning of q_s or the reason for tabulating these quantities.

Table VII gives the upper percentiles of the t-distribution corresponding to the probability levels Q = .4, .25, .1, .05, .025, .01, .005, .0025, .001, .0005, for ν = 1(1)30 and various other values to 10,000, and ∞.

Table VIII gives, in concise form, the coefficient $B = k(1 - k)/4$ in Bessel's quadratic interpolation formula. By showing only the values of k for which the value of B changes in the third decimal place, the table is cut down to little more than one-tenth of the size otherwise necessary.

The introductory part of the volume includes brief sections on interpolation, with numerical examples, formulas for calculating and checking the tables, and a list of eight applications. It would have been very helpful to give examples of applications listed, along the lines of the Introduction to *Biometrika Tables for Statisticians*, Vol. I, for example, and to indicate needs for tabulations of such detail and accuracy. Several key references besides the one to Fisher in *Metron* would also have been helpful.

As already indicated, the quality of printing is less than perfect. The alignment of the columns is poor, making it unnecessarily difficult to read across the page to find the entry corresponding to a t argument. It might be pointed out that provision of a column for t at the right of each page as well as the one at the left would largely have alleviated this difficulty. Other minor shortcomings are lack of a heading at the top of each page to identify the table at a glance; omission of the subscript ν in $S(t)$ at the bottom of page 7 and inconsistency in showing the argument values, as in t = 0(0.01)5.00; writing $C(t)$ for $C_5(t)$ at the top of page 8, where also the reference to values of the parameter ν is irrelevant; omission of the prime in the derivative $S'(t)$ at the beginning of Section III on page 11; and erroneously writing the t-interval for Table II as 1.0 instead of 0.1 in both the table of contents and in the title on page 69. In Table III on page 75 the subscripts on the differences Δ should be written as exponents (the bar in $\bar{\Delta}^2$ means that the second differences are all negative).

In spite of such shortcomings, this volume represents the most detailed tabulation of Student's t-distribution available and, while it is not recommended for the general practitioner, will be indispensable to statisticians and others who require finely tabulated values for theoretical or other reasons.

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36 [L].—E. N. Dekanosidze, *Tables of Lommel's Functions of Two Variables*, Pergamon Press, New York, 1960. 492 p. Price $20.00.

This is an English translation from the Russian. The original was reviewed in *MTAC*, v. 12, 1958, p. 239–240. The introduction contains several typographical errors, and these have been noted in a recent paper by J. Boersma "On the computation of Lommel's functions of two variables," in *Mathematics of Computation*, v. 16, 1962, p. 232.

Y. L. L.

37 [L].—V. N. FADDEYEVA & N. M. TERENT'EV, *Tables of Values of the Function* $w(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}} \int_0^z e^{t^2}\, dt\right)$, *for Complex Argument*, translated by D. G. FRY, Pergamon Press, New York, 1961, 280 p., 26 cm. Price $15.00.

This is a translation from the Russian edition, which appeared in 1954. The present volume is essentially a tabulation of the error function in the complex plane, and, with $z = x + iy$, $w(z) = u(x, y) + iv(x, y)$, gives 6D values of u and v. In Table 1 the range is $0 \leqq x\ 3$, $0 \leqq y \leqq 3$, with spacing in each variable of 0.02. In Table 2 the range is $3 \leqq x \leqq 5$, $0 \leqq y \leqq 3$, and $0 \leqq x \leqq 5$, $3 \leqq y \leqq 5$, with spacing in each variable of 0.1. A formula is presented so that the table is everywhere interpolable to an accuracy within two units in the last place. For interpolation about z_0, the formula uses the data at $z_0 \pm h$ and $z_0 \pm ih$, where h is the spacing. Let

$$\begin{aligned} 2\bar{\Delta}_x f(x, y) &= f(x + h, y) - f(x - h, y) \\ 2\epsilon &= (\bar{\Delta}_x u - \bar{\Delta}_y v) + i(\bar{\Delta}_y u + \bar{\Delta}_x v) \\ \Delta_x^2 f(x, y) &= f(x + h, y) - 2f(x, h) + f(x - h, y) \\ \tilde{\Delta}_x w &= \bar{\Delta}_x w - \epsilon. \end{aligned}$$

The interpolation formula reads

$$w(z) \sim w(z_0) + h\tilde{\Delta}_x w + \tfrac{1}{2}h^2\Delta_x^2 w,$$

and values of $\tilde{\Delta}_x u$, $\tilde{\Delta}_x v$, $\Delta_x^2 u$ and $\Delta_x^2 v$ are provided.

The foreword, written by V. A. Fok, enunciates applications of the tables to physical problems. Some properties of $w(z)$ are also given. The authors' introduction gives various representations of $w(z)$, including power series, asymptotic expansions, and continued fractions. The method of constructing and checking the table is discussed. To find values of $w(z)$ accurate to 6D outside the tabulated range, some approximations based on the continued fraction expansion are presented.

Other tables of the error function for complex argument have appeared since the original issue of the present tables. See, for example, *Math. Comp.*, v. 14, 1960, p. 83. In this table, as well as the one under review, z is in rectangular form. For tables of the error function with z in polar form, see *MTAC*, v. 7, 1953, p. 178 and *Math. Comp.*, v. 12, 1958, p. 304–305; v. 14, 1960, p. 84.

Y. L. L.

38 [L].—I. E. KIREEVA & K. A. KARPOV, *Tablitsy Funktsiĭ Vebera*, (*Tables of Weber Functions*), Vol. 1, Vychislitel'nyi Tsentr, Akad. Nauk SSSR, Moscow, 1959, xxiv + 340 p., 27 cm. Price 37 rubles. [An English translation by Prasenjit Basu has been published in 1961 by Pergamon Press, New York. Price $20.00].

Weber's equation

$$y'' - \left(a + \frac{z^2}{4}\right)y = 0 \tag{1}$$

is satisfied when $-a = p + \frac{1}{2}$ by Whittaker's function $D_p(z)$. If $y(a, z)$ is a solution, so also are $y(a, -z)$, $y(-a, iz)$, and $y(-a, -iz)$. The solutions for $a - 1, a, a + 1$ are connected by a linear recurrence relation.

If we replace a by $-i\alpha$, and z by $\zeta e^{i\pi/4}$, the equation becomes

$$y'' - (\alpha - \tfrac{1}{4}\zeta^2)y = 0, \tag{2}$$

which also has real solutions for real α, ζ. This has no recurrence relation connecting solutions for α, $\alpha - 1$, $\alpha + 1$, though there is a complex relation connecting those for α, $\alpha - i$, $\alpha + i$.

The equation (1) is usually taken as basic in the complex variable theory of these differential equations.

There are several special cases worth mentioning.

(i) If $p = a - \frac{1}{2}$ is an integer $\geqq 0$, $D_p(z) = e^{-z^2/4}h_n(z)$,

where
$$h_n(z) = (-1)^n e^{z^2/2} \frac{d^n}{dx^n} e^{-z^2/2}$$

is a Hermite polynomial.

(ii) If $p = -q - 1$ is an integer $\geqq 0$, a solution is $e^{z^2/4}h_n^*(z)$, in which $h_n^*(z) = (-i)^n h_n(iz)$, which is again a real polynominal.

(iii) The case of integral p also includes solutions which involve repeated integrals of $e^{\pm z^2/2}$.

(iv) When $a = -p - \frac{1}{2}$ is an integer, the solutions involve a finite series of Bessel functions of order $\frac{2k + 1}{4}$ (k an integer) and argument $\frac{z^2}{4}$. For instance, when $a = 0$, and z is real, two solutions are

$$\sqrt{\frac{z}{2}}\, K_{1/4}\left(\frac{z^2}{4}\right) \quad \text{and} \quad \sqrt{\frac{z}{2}}\left\{I_{1/4}\left(\frac{z^2}{4}\right) + I_{-1/4}\left(\frac{z^2}{4}\right)\right\}.$$

All these are relevant to equation (1), with real a and z. With equation (2) the only real case of interest is when $\alpha = 0$, and ζ is real, in which case

$$\sqrt{\zeta}\, J_{\pm 1/4}\left(\frac{\zeta^2}{4}\right)$$

are both solutions.

These cases are all considered in [1], where there are also tables for real solutions to equation (2) with $\pm\alpha = 0(1)10$.

The tables now under review break new ground. They are concerned with the case where $p = -a - \frac{1}{2}$ is real, while $z = x(1 + i)$, with x real. The reality of p means that a real recurrence relation remains in existence connecting solutions for $p - 1, p, p + 1$. The differential equation now becomes

$$\frac{d^2y}{dx^2} - \{(2p + 1)i - x^2)\}y = 0$$

and the solutions are no longer real, except when $2p + 1 = 0$.

The tables give $D_p\{x(1 + i)\} = u_p(x) + iv_p(x)$ for $\pm x = 0(.01)5$, $p = 0(.1)2$, and $\pm x = 5(.01)10$, $p = 0(.05)2$, generally to 6 decimals, except that there are only 5 decimals when $|x| \geqq 7.5$ and $p \geqq 1.8$ simultaneously.

Since p is real, the functions remain elementary when $p = 0, 1, 2$. In fact,

$$u_0(x) = \cos\frac{x^2}{2} \qquad v_0(z) = -\sin\frac{x^2}{2}$$

$$u_1(x) = x\left(\cos\frac{x^2}{2} + \sin\frac{x^2}{2}\right) \qquad v_1(z) = x\left(\cos\frac{x^2}{2} - \sin\frac{x^2}{2}\right)$$

$$u_2(x) = -\cos\frac{x^2}{2} + 2x^2\sin\frac{x^2}{2} \qquad v_2(x) = \sin\frac{x^2}{2} + 2x^2\cos\frac{x^2}{2}$$

are given in the Introduction, formula (6′).

When $p = -\frac{1}{2}$, the equation (3) becomes a real equation and has real solutions; it is then a case of (2) with $\zeta = \sqrt{2}x$. However, $D_{-1/2}\{x(1+i)\} = u_{-1/2}(x) + iv_{-1/2}(x)$ is not real, and we find

$$\begin{aligned}
u_{-1/2}(x) - iv_{-1/2}(x) &= 2^{-3/4}\sqrt{\pi x}J_{-1/4}\left(\frac{x^2}{2}\right)\\
&= \{2^{-1/4}\sqrt{\pi}/\Gamma(\tfrac{3}{4})\}\,U_1(x, 2)\\
&= 2^{-3/4}\{W(0, x\sqrt{2}) + W(0, -x\sqrt{2})\}\\
&= -2^{-5/4}\sqrt{\pi x}J_{1/4}\left(\frac{x^2}{2}\right)\\
&= -\{2^{-7/4}\sqrt{\pi}/\Gamma(\tfrac{5}{4})\}\,U_2(x, 2)\\
&= 2^{-5/4}\{W(0, x\sqrt{2}) - W(0, -x\sqrt{2})\}
\end{aligned}$$

where $W(a, x)$ is the function tabulated in [1], while $U_1(x, 2)$ and $U_2(x, 2)$ are those tabulated by Smirnov [2, p. 121].

The case $p = -\frac{1}{2}$, or $a = 0$, is in many senses the "central" case; it gives the simplest Bessel function representation, and it is rather surprising that it is not tabulated in the work under review. True, it is obtainable from other published tables, but so also are the cases $p = \frac{1}{2}, \frac{3}{2}$ as we see below; yet these two values of p occur in the tables, although the cases $p = -\frac{3}{2}, -\frac{5}{2}$, of equal complexity and involving the same Bessel functions, do not.

For $p = \frac{1}{2}, \frac{3}{2}$, consider first the derivative $\frac{d}{dx} D_{-1/2}\{x(1+i)\}$;

we have

$$\begin{aligned}
u'_{-1/2}(x) - v'_{-1/2}(x) &= -2^{-3/4}\sqrt{\pi x^3}J_{3/4}\left(\frac{x^2}{2}\right)\\
&= \{2^{-1/4}\sqrt{\pi}/\Gamma(\tfrac{3}{4})\}\,U'(x, 2)\\
&= 2^{-1/4}\{W'(0, x\sqrt{2}) - W'(0, -x\sqrt{2})\}\\
&= -2^{-5/4}\sqrt{\pi x^3}J_{-3/4}\left(\frac{x^2}{2}\right)\\
&= -\{2^{-7/4}\sqrt{\pi}/\Gamma(\tfrac{5}{4})\}\,U_2'(x, 2)\\
&= 2^{-3/4}\{W'(0, x\sqrt{2}) + W'(0, -x\sqrt{2})\}
\end{aligned}$$

where the prime indicates differentiation with respect to the argument λx of the function, concerned, whether this be $+1$, -1, $+\sqrt{2}$, or $-\sqrt{2}$; for example, $W'(0, -x\sqrt{2}) = -\sqrt{2}\frac{d}{dx}W(0, -x\sqrt{2})$.

Then

$$u_{1/2} = \frac{x}{2}(u_{-1/2} - v_{-1/2}) - \frac{1}{2}(u'_{-1/2} + v'_{-1/2})$$

$$v_{1/2} = \frac{x}{2}(u_{-1/2} + v_{-1/2}) + \frac{1}{2}(u'_{-1/2} - v'_{-1/2})$$

and

$$u_{3/2} = x(u_{1/2} - v_{1/2}) - \tfrac{1}{2}u_{-1/2}$$

$$v_{3/2} = x(u_{1/2} + v_{1/2}) - \tfrac{1}{2}v_{-1/2}.$$

None of these representations as Bessel functions is mentioned in the work reviewed.

The introduction (the reviewer has relied here on the translation by Presenjit Basu for the English edition) is an interesting one, describing properties of the function relevant to its original computation and numerical use, and including five pages of graphs and diagrams illustrating the behavior of the functions. Interpolation is considered with respect to both x and p, and the use of the recurrence relation for $p < 0$ or $p > 2$ is explained. There is no mention in the Introduction of special applications of the particular solutions given of equation (3), and none is known to the reviewer, although there are many for the solutions of equations (1) and (2), where parabolic cylinder coordinates are appropriate.

The Introduction contains also two auxiliary tables. The first (p. xx, xxi) gives $a_r = \pm p(p-1)(p-2) \cdots (p-2r+1)/2^{4r}r!$, and $b_r = \pm p(p+1) \cdots (p+2r-1)/2^{4r}r!$ for $r = 0(1)3$; these are coefficients in asymptotic expansions. The other table gives $\Gamma\left(\frac{1}{2} - \frac{p}{2}\right) \Big/ 2^{1+(p/2)}\sqrt{\pi}$ and $\Gamma\left(-\frac{p}{2}\right) \Big/ 2^{1+(p/2)}\sqrt{2\pi}$. In each case ten-figure values are given for $p = .05(.05)1.95$, $p = 1$ excluded.

The tables are arranged so that each opening gives $u_p(x)$, $v_p(x)$, $u_p(-x)$, $v_p(-x)$ for four values of p, at interval 0.1 or 0.05, and 51 values of x at interval 0.01, with extra lines (up to three) at the foot to assist in interpolation. The openings are grouped into subtables, with p running from 0 to 2 (with $p = 2$ standing alone on the last page, facing the next title), and a single block of values of x covering half a unit; there are thus 20 such subtables.

The photographic printing of typescript is reasonably good and clear, although exhibiting the familiar variation in blackness that is rather too common.

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1. J. C. P. MILLER, Editor, *Tables of Weber Parabolic Cylinder Functions*, Her Majesty's Stationery Office, London, 1955.

2. A. D. SMIRNOV, *Tablitsy Funktsiĭ Eiri i Spetsial'nykh Vyrozhdennykh Gipergeometricheskikh Funktsiĭ ...* (*Tables of Airy Functions and of Special Confluent Hypergeometric Functions* ...) Izdatel'stvo Akad. Nauk SSSR, Moscow, 1955. Edition in English, Pergamon Press, New York, 1960. See *MTAC*, v. 12, 1958, p. 84–86.

39 [L].—A. V. Luk'îanov, I. B. Teplov & M. K. Akimova, *Tablitsy solnovykh Kulonovskikh funktsiĭ (funktsiĭ Uittekera) Tables of Coulomb Wave Functions (Whittaker Functions)*, Moscow, 1961, xi + 223 p., 26 cm. Price 1.63 rubles.

These tables are intended for persons working in the field of nuclear physics; they were compiled in the Mathematics Section of the Physics Department of the Moscow State University in a joint project with the Laboratory of Nuclear Reactions, Scientific Research Institute for Nuclear Physics, the Moscow State University. The tabular values were computed on a STRELA computer in the University Computation Center.

The authors give the basic solutions of Coulomb functions as the functions $F_l(\eta, \rho)$ and $G_l(\eta, \rho)$, where $\eta = \dfrac{Zze^2}{\hbar v}$, $\rho = kr$, l is the orbital moment of the particle with only integer values, k is the wave number at infinity, ze the charge of the particle, and Ze the charge of the nucleus in whose field the particle is moving, r is the spherical coordinate, and $\hbar$ is Planck's constant divided by 2π.

Compilation of these tables was justified on the grounds that these functions are not connected with other functions by simple relationships. Available tables of Coulomb functions are essentially intended for calculating Coulomb functions at the surface of nuclei. The tables in this book are intended not only for calculating Coulomb functions on the boundary of the nuclei, but also for calculating integrals of products of radial functions encountered in the theory of direct nuclear reactions.

The tables were calculated as follows. When $\rho = 1$ the values of $F_{14}(\eta, \rho)$, $F_{15}(\eta, \rho)$, $G_0(\eta, \rho)$, $G_1(\eta, \rho)$, and their first derivatives were found by means of the series

$$F_l(\eta, \rho) = \sum_{n=l+1}^{\infty} A_n(l, \eta)\rho^n,$$

$$G_l(\eta, \rho) = F_l(\eta, \rho)K(\eta) \log \rho + \sum_{n=-l}^{\infty} B_n(l, \eta)\rho^n.$$

The coefficients $A_n(l, \eta)$, $B_n(l, \eta)$, and $K(\eta)$ can be found by formulas given in *Tables of Coulomb Wave Functions*, Applied Mathematics Series Report 17, National Bureau of Standards, 1952, and in a paper entitled "Coulomb wave functions in repulsive fields," by F. L. Yost, J. A. Wheeler, and G. Breit, in *The Physical Review*, v. 49, 1936, p. 174–189. The values of these functions for the remaining values of ρ were obtained by integrating the differential equation

$$\frac{d^2y}{d\rho^2} + \left[1 - \frac{2\eta}{\rho} - \frac{l(l+1)}{\rho^2}\right] y = 0$$

by the Runge-Kutta method programmed for automatic selection of the interval and relative accuracy of 10^{-5}, and for the remaining values of l by the recursion formula

$$\frac{1}{l+1} [\eta^2 + (l+1)^2]^{1/2} U_{l+1} + \frac{1}{l} [\eta^2 + l^2]^{1/2} U_{l-1} = (2l+1)\left[\frac{\eta}{l(l+1)} + \frac{1}{\rho}\right] U_l$$

On the basis of an error check described in the Introduction, the authors believe that the errors in the tabulated values generally do not exceed 2 to 3 units in the last decimal place, except where the function changes sign, in which case only the five tabulated decimal places are reliable. They also state that linear interpolation ensures 2 or 3 reliable decimal places, and fourth-order interpolation, 4 or 5 places. However, the accuracy of interpolation is considerably less in the region of ρ close to one, where it decreases with increasing l. In addition, 2 or 3 reliable decimal places can be obtained by linear interpolation in η, except for the region of values where $\eta > 1.5849$.

The tables for each of the functions $F_l(\eta, \rho)$, $G_l(\eta, \rho)$, and $G_l'(\eta, \rho)$ are divided into three groups corresponding to three intervals of changes in η. Part 1 (pages 2–33) gives five-place tables of values of the function $F_l(\eta, \rho)$ with $\rho = 1.0(0.2)20.0$, $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty, -0.8(0.1) - 0.4$, and $l = 0(1)15$. Part 2 (pages 36–37) gives five-place tables of $F_l(\eta, \rho)$ for the same ranges of ρ and l, $\eta = 0.50119$ (variable) 1.5849, and $\log \eta = -0.3(0.1)0.2$. Part 3 (pages 70–101) gives five-place tables of $F_l(\eta, \rho)$ for the same ranges of ρ and l, $\eta = 1.9953$ (variable) 6.3096, and $\log \eta = 0.3(0.1)0.8$.

Values of the function $G_l(\eta, \rho)$ for the same ranges of ρ and l are given as follows: on pages 104–135 for $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty, -0.8(0.1) - 0.4$; on pages 138–169 for $\eta = 0.5119$ (variable) 1.5849, $\log \eta = -0.3(0.1)0.2$; and on pages 172–203 for $\eta = 1.9953$ (variable) 6.306, $\log \eta = 0.3(0.1)0.8$. Values of the first derivative $G_l'(\eta, \rho)$ with respect to ρ, with $\rho = 1.0(0.2)20$, and $l = 0$ or 1, are given on pages 206–209, when $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty$, $-0.8(0.1) - 0.4$; on pages 212–215, for $\eta = 0.50119$ (variable) 1.5849, $\log \eta = -0.3(0.1)0.2$; and on pages 218–221 for $\eta = 1.9953$ (variable) 6.3096, and $\log \eta = 0.3(0.1)0.8$.

References cited in the Introduction include only one Soviet source and 15 English-language sources. Among the latter are the National Bureau of Standards tables (Reports 17 and 3033), Japanese tables of Whittaker functions, and three articles by C. E. Fröberg.

The tables given in this book are arranged in a convenient manner, and the clear print adds to their attractiveness. This book should be useful to persons who who are now using the National Bureau of Standards tables.

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40 [L].—M. E. Sherry, *The Zeros and Maxima of the Airy Function and Its First Derivative to* 25 *Significant Figures*, Electronics Research Directorate, Air Force Cambridge Research Center, Bedford, Mass., April 1959.

Table 1 gives to 25S the first 50 values of a_s and $Ai'(a_s)$ for which $Ai(a_s) = 0$. Similarly, Table 2 gives a_s' and $Ai(a_s')$ for which $Ai'(a_s') = 0$. These extend the precision of tables recorded in a rather recent volume edited by F. W. J. Olver (see *Math. Comp.*, v. 15, 1961, p. 214–215). Tables 3 and 4, respectively, give to 25S the first 18 coefficients in the asymptotic series of arc tan $\{Ai(x)/Bi(x)\}$ and arc tan $\{Ai'(x)/Bi'(x)\}$.

Y. L. L.

41 [I, M].—M. V. Cerrillo & W. H. Kautz, *Properties and Tables of the Extended Airy-Hardy Integrals*, MIT Technical Report 144, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, November 15, 1951.

In a previous review (*Math. Comp.*, v. 15, 1961, p. 215) I stated that tables of the Airy function $Ai(z)$ were not available for z complex in polar form. This is not correct, and I am indebted to Dr. Nelson A. Logan of Lockheed Aircraft Corporation, Sunnyvale, California, for calling my attention to the tables of Cerrillo and Kautz, here described.

Let

$$Ah_{1,3}(B) = 3^{-1/3}Ai(-3^{-1/3}B),$$

$$Ah_{2,3}(B) = \tfrac{1}{2}\,3^{-1/3}[Ai(-3^{-1/3}B) - iBi(-3^{-1/3}B)],$$

$$Ah_{3,3}(B) = \tfrac{1}{2}\,3^{-1/3}[Ai(-3^{-1/3}B) + iBi(-3^{-1/3}B)],$$

where $Ai(z)$ and $Bi(z)$ are the usual notations for the Airy integrals. Let $B = |B|e^{i\beta}$. This report gives the real and imaginary parts of $Ah_{1,3}(B)$, $Ah_{2,3}(B)$, and $Ah_{3,3}(B)$ to 7D for $|B| = 0(0.2)4$, $\beta = 0(7.5°)180°$. The functions $|Ah_{1,3}(B)|$ and arg $Ah_{1,3}(B)$ (in radians) are also tabulated to 7D for the same range, and graphs of these functions are also provided. The headings in each table should read $|B|$ for B and $|Ah_{1,3}(B)|$ for $Ah_{1,3}(B)$. Also tabulated to 7D are the values of the first 31 coefficients in the power series of $Ah_{1,3}(B)$ and the first 20 zeros of the latter for B real.

Y. L. L.

42 [L, M].—Herbert Bristol Dwight, *Tables of Integrals and Other Mathematical Data*, 4/e., The Macmillan Company, New York, 22 cm. x + 336 p. Price $3.50.

Reviews of the first two editions of these tables, published in 1943 and 1947 respectively, have appeared in *MTAC* (v. 1, p. 190–191; v. 2, p. 346).

A third edition, published in 1957, was enlarged through the addition of formulas relating to determinants, a more extensive list of derivatives of inverse trigonometric functions, a supplementary table of values of the exponential functions, and tables of natural values of the trigonometric functions (to 5D or 5S) corresponding to angles expressed degrees and hundredths.

The fourth and latest edition represents a further significant enlargement, the principal amplification being in the tabulation of definite integrals. This section now occupies 42 pages as contrasted with 11 pages in the third edition and eight pages in the first edition. The principal source of this information is cited as *Nouvelles Tables d'Intégrales Définies* by Bierens de Haan, Leyden, 1867, now readily available through republication in 1957 by Hafner Publishing Company in New York. The section devoted to elliptic functions has been extended by the inclusion of additional formulas concerning indefinite integrals expressible in terms of elliptic integrals.

Eleven references have been added to the list of 65 appearing in the third edition. A few minor changes have been made in the numerical tables; the most conspicuous is the increase in precision of Table 1060 (Some Numerical Constants), so that all the entries appear now to 10 decimal places.

Errata pointed out in the earlier reviews have been corrected. However, the correction of $K(87^\circ.6)$ in Table 1040 was not followed by appropriate changes in the column of first differences. The reviewer has compared the entries in Table 1050 with the corresponding data in the tables of Lowell [1], and thereby has detected 72 last-figure errors in Dwight's values of the Kelvin functions of zeroth order and of their first derivatives.

This useful new edition of Professor Dwight's popular tables of integrals constitutes a valuable contribution to the increasing store of such mathematical literature.

J. W. W.

1. Herman H. Lowell, *Tables of the Bessel-Kelvin Functions Ber, Bei, Ker, Kei, and their Derivatives for the Argument Range 0(0.01)107.50*, Technical Report R-32, National Aeronautics and Space Administration, Washington, D.C., 1959. (See Review 9, *Math. Comp.* v. 14, 1960, p. 81.)

43 [M].—W. F. Hughes & F. T. Dodge, *A Table of J Integrals of Hydrodynamic Lubrication Theory*. Manuscript deposited in UMT file.

This unpublished table of the numerical values, mainly to five significant figures, of the integrals $J_n = \int_0^\theta (1 - \epsilon \cos \theta)^{-n} d\theta$, corresponding to $n = 1, 2, 3$, $\epsilon = 0.1(0.1)0.9$, and $\theta = 0^\circ(5^\circ)360^\circ$, was prepared on an electronic digital computer by members of the Mechanical Engineering Department of the Carnegie Institute of Technology.

In the prefatory text the authors state that these integrals occur in the theory of the hydrodynamic lubrication of the journal bearings. The film thickness h is approximated by the formula $h = c(1 - \epsilon \cos \theta)$, in terms of the angular coordinate θ, the radial clearance c, and the ratio ϵ of the eccentricity of the journal to the radial clearance. Values of the ratio h/c to four decimal places are included in the table.

J. W. W.

44 [M].—G. Petit Bois, *Tables of Indefinite Integrals*, Dover Publications, Inc., New York, 1961, xiv + 151 p. 24 cm. Price $1.65.

This is a new printing, in an inexpensive paperback edition, of the original *Table d'Intégrales Indéfinies* published by Gauthier-Villars in Paris in 1906, and at the same time by Teubner in Leipzig under the title *Tafeln unbestimmte Integrale*.

This unabridged English translation contains 2544 indefinite integrals, systematically arranged according to integrands, as outlined in the table of contents. A preface lists the principal source books and tables. This is followed by an explanatory section devoted to notation and by a section listing 49 "transformations of integral expressions," that is, pairs of expressions possessing the same derivative.

With few exceptions, the indefinite integrals listed here involve elementary functions. Several integrals are shown to depend upon the evaluation of such functions as the sine and cosine integrals, although these are not identified as such. Examples of such higher transcendental functions, which are left in the form of integrals without comments, appear on pages 118, 122, 123, 140, and 150. Furthermore, except for a footnote on page 62, elliptic integrals are nowhere referred to in these tables.

In the foreward to their extensive collection of indefinite integrals Gröbner and Hofreiter [1] refer to these tables as one of the sources for their material. Nevertheless, the tables of Petit Bois are not comparable to these recent German tables, nor indeed to the recently enlarged compilation of Dwight [2], principally because these last two tables cover a much larger spectrum of classes of functions.

The reviewer noted only one serious error of commission, namely, on page 150 appears an evaluation of the indefinite integral of log $(a + \cos x)$ which is manifestly incorrect.

In conclusion, this reviewer considers this compilation to supplement to some degree the information presented in several more recent tables, such as those cited; nevertheless, it cannot replace them in general use.

J. W. W.

1. W. Gröbner & N. Hofreiter, *Integraltafel. Erster Teil: Unbestimmte Integrale.* Vienna and Innsbruck, Springer-Verlag, 1949.
2. H. B. Dwight, *Tables of Integrals and other Mathematical Data*, fourth edition. The Macmillan Co., New York, 1961.

45 [P, Z].—Robert S. Ledley, *Digital Computer and Control Engineering*, McGraw-Hill Book Co., New York, 1960, xvii + 835 p., 24 cm. Price $14.50.

The author's purpose in writing this book, as stated in the Preface, is "to fill the need for a comprehensive, elementary engineering textbook in the large and still growing field of digital computers and controls." Without doubt, it is comprehensive; more than enough is included in over 800 pages for a year's course or for several years of graduate work if the Additional Topics are included. The author, is justifiably proud of the 750 exercises scattered throughout the text. The exercises form a framework to hold the book together and permit some of the author's objectives to be achieved, while the additional topics provide opportunities for rich learning experiences for honors or graduate students, as well as insights for the exceptional student seeking more than grades and credits.

The book is divided into five parts, each one reasonably independent and self-contained.

Part 1, entitled Introduction to Digital Programming Systems, serves to introduce digital systems and to stimulate interest in them through examples and the theory of their applications. It may be that an unwary and not too disciplined student, not familiar with the field at all, may be frightened away by the rather sophisticated examples. Chapter I does show, however, that digital computers and controls are tools—the means to realize highly complex programs. There is not, in the first chapter, on the other hand, any hint of what these tools are or what they actually do, or how. This is probably a good technique, for now curiosity alone should lead the student to Chapter 2 to discover the secrets of such remarkably versatile hardware. This chapter, however, will probably have to be read twice to be understood by most neophytes. As all good computer people do, Professor Ledley personifies the hardware; for example, "the computer is told"; control "interprets" and "tells ... the arithmetic unit what to do," etc. Without a previous knowledge of the physical meanings and the allegorical nature of these personifications, a good number of questions may arise. Here, as throughout the text, the exercises do much to dispel such problems. The exercises are vital to the complete understanding of the material in this text, because of its often descriptive nature. Where the descriptions or analogies are weak, such as the use of the terms "read-in" and "read-out," "memorizing information," and the sentence "memory boxes are analogous to mail boxes," one must hope that the instructor can salvage the situation. Getting into an unfamiliar discipline is first of all a matter of mastering a new vocabulary; here the student will stumble a while before finding this key to new knowledge.

The input, output, and memory section of Chapter 2 is descriptive. The approach is to cite an application, show a system that can perform it, and then discuss the units making up such a system. This is logical, but the engineer will be given electrical specifications to meet, not end applications. All of the picture should be presented to him, but the greatest emphasis should be placed on where he will go—on the design of electronic circuits. This does not happen here. (An unusual feature in this book is that the history of computers is relegated to an additional topic of Chapter 2, accompanied by an interesting evolutionary tree showing computer development.) This chapter is rich in photographs, but the block diagrams are likely to carry far more meaning to the student.

In Chapter 3, after much assurance to the student that all this is really necessary, the book gets into some serious work. This chapter, concerned with coding and programming, is highly recommended. It contains numerous excellent presentations on number systems, binary arithmetic, coding, flow charting, symbolic code, multiple-address systems, decimal-coded-binary systems, more excellent exercises, and extremely interesting additional topics.

In Chapters 4 and 5, programming fundamentals and advanced programming, respectively, are both presented with clarity, and again are highly recommended, especially for prospective programmers. While it may be that so much attention to programming is discouraging to the engineer who is primarily concerned with hardware, the need for ingenuity, precision, and economy in coding is well demonstrated. This may serve to bring into sharper focus the similarity between the user's or programmer's requirements and the engineer's own disciplines.

Part 1, then, despite an uncertain start, should leave the student with few doubts about the why of computers.

Part 2, the Functional Approach to Systems Design, approaches the subject from a mathematical rather than a purely descriptive level. This Part will prove to be a trial for most engineering students, for still there is no hint of hardware. However, as another contribution to the building of a solid foundation, it, too, has great value.

Chapter 6, Fundamentals of Numerical Analysis, presents several discussions of how problems may be prepared for systematic solution by computers. The section on accuracy and errors should dispel naive notions of the infallibility of computers.

Chapter 7, entitled Searching, Sorting, Ordering, and Codifying, is another very good introductory chapter, this time to data processing. Section 7-7, Codifying, concerns itself with error correction and superimposition. The discussion of selfcorrecting and redundant codifications should prove of value in the engineer's career.

In Chapter 8 are found discussions of several special-purpose digital-computer system designs. These are all good, and again should broaden the student's outlook. With the nearly 300 pages so far devoted to introductions and elaborations, he may, however, have given up finding out just what makes these machines operate.

In Chapter 9, the system design of the Pedagac ("pedagogic automatic computer") is presented. None of the reasoning leading to this particular configuration is shown, leaving the student to accept it as a *fait accompli*. The appended Additional Topics present some fine challenges to student programmers, namely, the writing of an automonitor, a library of subroutines, and an ALGOL for Pedagac.

Part 3 deals with the Foundation for the Logical Design of Digital Circuitry. Chapter 10 provides a good, practical introduction to Boolean Algebra—a necessity for digital design. Propositional Calculus is brought in by way of background material, and is followed by a rapid, clear, and concise development of Boolean Algebra. The all-important parallel between computer logic and Boolean logic is firmly established, and a fine set of Additional Topics completes this chapter.

Chapters 11 and 12 deal with numerical representations of Boolean expressions and with many elegant computational methods of simplification and logical design, often the result of original work by the author and his associates. These are likely to be of much value to the serious computer designer. Chapter 13, entitled Boolean Matrix Equations and Fundamental Formulas, presents more simplification and computational methods. It is in this part of the book that Dr. Ledley reaches a peak of accomplishment. In Sections 11-9, 11-10, and in all of Chapters 12 and 13, it is design that is emphasized, not analysis or description. It was R. W. Johnson in the August 1961 edition of the IRE Proceedings who asked "Are electronics engineers educated?"—stressing how much analysis is taught and how little design, which is the life blood of engineering. In Part 3 of this book, Dr. Ledley teaches design. He achieves a clarity of presentation, never resorting to "mathmanship" or other "impressive confusion." The advantages of an interdisciplinary approach, at the elementary level (for example, algebra taught by a Professor of Engineering) rather than the highly sophisticated specialist trying to "write down" to students, are amply illustrated. In fact, the only quarrel is with the nomenclature, the use of the term "circuit" design when actually "logical" or "symbolic" design is meant. Chapter 14, Applications of Matrix Equations in Circuit Design, concerns itself with applying the methods and principles just learned to solving specific problems. (This chapter and its Additional Topics are unlikely material for engineering students, but are certain to have high appeal to logicians and mathematicians.)

Part 4, paradoxically named Logical Design of Digital-Computer Circuitry, reveals to the student how all the Boolean tricks he has learned may be used to group components logically to make them compute. Chapter 15 presents the logical realization of serial arithmetic. It is in this chapter that the long-postponed revelation—how does a computer actually work—is made; that is, what is it that a computer does when it computes? How is computation mechanized? The presentation is excellent.

Having exposed the "heart" of computers, the author follows through with some extremely important and interesting elaborations in Chapter 16, Parallel and Rapid Arithmetic Operations. The student will again be challenged. Equally important, but not as obvious, is the control logic of a computer, lucidly presented in Chapter 17. Chapter 18 is unfortunate in that the packaging and logical design of the Pedagac are presented in such rapid order that the "thread of continuity" aimed for by the author is violently contradicted. A complete logic for the computer is displayed, rather than being built-up or designed in a step-by-step fashion. It appears as though after the foundation was built painstakingly and in great detail, the finished product is suddenly presented, fully built.

Part 5, Electronic Design of Digital Circuits, is the only part of the book dealing with electronics, and in a somewhat superficial manner, at that. Chapter 19, which discusses Problems and Limitations in Electronic Realization, includes a good presentation of digital gating systems. Static and dynamic storage and parametric amplifier methods are described clearly. No circuits are designed in this chapter, but the important topics of clock phasing, synchronization, and circuit reliability are discussed. An interesting Boolean-probabilistic method of logical design for increased reliability is developed. The preponderant use of semiconductors in today's equipment is acknowledged by a full discussion of their physical theory in Chapter 20, Semiconductor Elements in Digital-circuit Design. Both large- and small-signal equivalent circuits for transistors are shown, and some circuit analysis is carried out. Diode gates, transistor gates, and current switching are described. A typical transistor flip-flop circuit is analyzed, and finally, too, the physical theory and parametron analogies of tunnel diodes are discussed.

Continuing at the introductory level, Chapter 21 presents a review of magnetism, and a discussion of square-hysteresis-loop cores. The sections on magnetic amplifiers, magnetic core logic, and multi-aperture cores are excellent, and will prove useful in an engineer's later work. Again, an unusual and highly desirable feature here is the rather thorough discussion of recent developments such as microwave and cryotron computer components, including exercises in their use as logical elements.

Memory and input-output methods are described in Chapter 22, which includes a section on magnetic film memories. The discussions of coincident current techniques, the several methods of recording and reading from magnetic tape and drums, analog-to-digital and digital-to-analog conversions, visual displays, page scanners and line printers, in the 30-odd pages of this chapter, provide a wealth of introductory material for an interested student. The discussion of the sampling theorem for analog-to-digital conversion must be cited as also highly desirable. The exercises for this chapter concentrate on logic and programming, and provide no further electronic circuit design experience.

Chapter 23 completes a major work of this book, the electronic design of the Pedagac. Unfortunately, this device for providing continuity serves rather to disrupt the continuity. Three widely scattered chapters present descriptions of the complete machine, that is, the system, the detailed logic, and the hardware. The final design is displayed in completed form rather than being built up step by step, and although it serves as good illustrative material for developing important concepts clearly and concretely, an opportunity for making the book truly continuous is thereby narrowly missed. This, however, is the lesser of two criticisms; the other is the extraordinarily large proportion of logic, mathematical theory, and purely descriptive matter to the material dealing peripherally with electronic design problems and their solutions.

Such might include the many input-output synchronization problems, the numerous problems of cross-talk and pick-up, the signal-to-noise ratio of input devices, the power requirements, the transmission of pulses over relatively long lines, failure localization, ground loops, and so forth, which every computer design engineer must face. Special circuitry, such as sense amplifiers, clock pulse generators, and magnetic-core switching-current pulse-generators are not mentioned, and, of course, may be found elsewhere. The point is, however, that these are the problems likely to haunt the engineer long after his Boolean expressions have been mechanized on paper. That is why this is a difficult book to evaluate. It is at once an extremely broad survey of the "state of the art," reflecting a familiarity with a good deal of the current literature and at the same time a thorough exposition of many of the basic principles and practices underlying the digital computer technology. As a textbook, it should prove valuable, for it is written to and for students, with illustrations and clarifications used profusely. This volume may also prove to be of value to those whose experience in the computer field, while long, may have been rather narrow—technicians, circuit designers, programmers, etc. One may question, however, whether this admirable work actually belongs in the publisher's Electrical and Electronic Engineering Series. For all its strong points, and they are many, the book's weakest area is precisely that of electronic or electrical engineering. It contains numerous excellent presentations of basic principles, and contributes greatly to the diffusion of knowledge in logical design methodology. These are valuable for the engineer's background and understanding, but are more likely to be used by mathematicians or logical designers. As a text in electrical and electronic engineering this otherwise remarkable book can hardly be placed alongside the books of Millman & Traub, Millman & Seely, Terman, and Skilling. It is a book that can stand on its own merits, and needs not to lean on an Electrical and Electronic Engineering Series, achieved by a title with the word Engineering in it. An alternative name for the book, such as "Elements of Digital Computer Programming and Logical Design," would not raise false hopes nor be misleading. This is perhaps a criticism of the publisher and editors and not properly of the book, but must be made, nevertheless.

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46 [Q].—DIRK BROUWER & GERALD M. CLEMENCE, *Methods of Celestial Mechanics*, Academic Press, New York, 1961, xii + 598 p., 24 cm. Price $15.50.

The title of this book appears to represent a compromise with an aspiration that must have had its origin more than a quarter of a century ago. In 1943 this reviewer overheard Bart Bok say, "I told Dirk the best thing he can do for Astronomy is to finish his book." Nearly ten years ago Brouwer was joined by the second author, with the same objective still in view. Both authors have been close collaborators in numerous astronomical endeavors since the end of World

War II. Both have also been amongst the most active research workers in celestial mechanics. This book is unequivocably the finest volume to appear in its field in the English language in the present century. The compromise referred to above is simply that it is not an exhaustive treatise. Nor is it a volume which could have appeared two decades ago. It sets forth the combined wisdom of the authors, based on their extensive experience up to the present, and includes some material (dealing with artificial satellites) not in existence two or three years ago.

The early chapters cover the usual introductory material: elliptic motion, expansions, and attraction of finite bodies. Then follow a few "practical" chapters on finite differences, numerical integration, aberration, precession, nutation, least squares, etc. The meat of the volume is in the chapters entitled General Integrals, Variation of Arbitrary Constants, Lunar Theory, Perturbations of the Coordinates, Hansen's Method, Disturbing Function, Secular Perturbations, and Canonical Variables. While the material is the same, there is nothing of the stereotyped presentation of classical treatises. It is here that the authors have given their own distinctive flavor to the work. Many features could be cited; for example, a deliberate effort to present Hansen's method in its most favorable light, and a correction method for deriving trigonometric series for the negative powers of the distance.

Whether this will prove to be a good textbook in an advanced graduate course is not easy to say without having tried it. That it will prove to be a valuable reference volume for a wide variety of workers both inside and outside this field of specialization is indubitable. For the long-term good of Celestial Mechanics this reviewer is of the opinion that an advanced text is needed which presents the material in the terminology that is familiar to present-day graduates in mathematics and physics; that is, vectors, matrices, gradients, dyadics, tensors, etc. But this is a different objective from that which the authors have set for themselves.

The volume has been meticulously prepared, both by the authors and the publisher, and competently proofread by G. Hori. If errors exist, it would be a shame, but reviewers delight in spotting what they can. To set the record straight, we note that Cowell's method (p. 186) had the incentive of its origin in the discovery of the eighth satellite of Jupiter (*Monthly Notices of the Roy. Astr. Soc.*, v. 68, p. 576). The calculations for Halley's comet came later. But such items cannot detract from the permanent value of this monumental volume.

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47 [S].—A. N. ZAIDEL', V. K. PROKOF'EV & S. M. RAISKII, *Tables of Spectrum Lines*, Pergamon Press Ltd., Oxford, 1961, xliii + 554 p., 24 cm. Price $14.00.

These tables, compiled largely from the M. I. T. Wavelength Tables, were first published in Moscow in 1952. In 1955 the tables were reprinted in Berlin as an International Edition, with introductory text in German, English, and French. The new edition appears to be identical with the Berlin edition except for rearrangement of the introductory sections.

The tables are in three parts. Part I lists the spectral wavelengths of 32 of the more common elements in order of decreasing wavelength between 8000 and 2000A. Intensities in arc, spark, and discharge tube are given, and air lines are included.

Part II lists the stronger lines and intensities of 93 elements. Excitation energies in electron volts are given for about 75 per cent of the lines. In Part III are presented several short auxiliary tables, mainly of physical properties and sensitive spectral lines, which are designed especially to aid in spectrochemical analysis. These include ionization potentials, atomic and molecular weights of the elements and their oxides, with their melting and boiling points, a table of the elements in sequence of their appearance in the spectrum of the carbon arc, a short list of sensitive lines of the elements in order of wavelengths and by elements, a table of strong lines between 2000 and 1800A, a table of iron lines suitable for intensity standards, and spectra of the hydrogen molecule and of deuterium.

A major feature of this book lies in the selection and arrangement that was made of the more important data on wavelengths and intensities. Many weaker lines, especially in the spectra of the less common elements, were omitted. The inclusion of excitation energies in Part II provides the first extensive compilation of such data in convenient form. These features and the useful auxiliary tables should be helpful particularly to the spectrochemical analyst. However, some caution should be observed in using the tables, since errors that were present in the M. I. T. tables may be found in this book; for example, 2592.627 Cu I, intensity 1000, is in error and should appear as 2392.627. The translations between languages may result in misspelling; for example, on page xx, Muir should be Moore (C. E.). However, these minor errors do not detract seriously from the general usefulness of the book.

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48 [S, V].—L. S. BARK, P. P. GANSON & N. A. MEĬSTER, *Tablit͡sy skorosti zouka v morskoĭ vode* (*Tables of the Speed of Sound in Sea Water*), Moscow, 1961, xiii + 182 p. + inserts, 26 cm. Price 1.73 rubles.

These tables of the speed of sound in sea water were computed in 1960 on a Strela-3 computer and a T-5 tabulator in the Computation Center at the request of the Institute of Oceanology supported by the Institute of Acoustics (all of the Academy of Sciences USSR).

Since the authors believe that previous tables, published by the British Admiralty [1], were based on formulas less accurate than Del Grosso's formula [2], they have based their tables on the Del Grosso formula

$$v = 1448.6 + 4.618t - 0.0523t^2 + 0.00023t^3 + 1.25(S - 35) - 0.011(S - 35)t + 0.0027 \times 10^{-5}(S - 35)t^4 - 2 \times 10^{-7}(S - 35)^4(1 + 0.577t - 0.0072t^2),$$ where t

is the temperature in degrees Centigrade and S is the salinity in parts per 1000.

The tables consist of two parts, supplemented by appendices consisting of eight nomograms of the speed of sound and by tables of corrections for depth. These appendices are inserted in a cover pocket. The tables are divided into two parts because the Del Grosso formula yields a guaranteed accuracy of 0.2 m./sec. only when the salinity is more than 19 parts per 1000. The first part of the tables (pages 1–36) gives values of the speed of sound for a range of temperatures from

$-2°$ to 33°C and salinity from 0 to 20 parts per 1000. This ensures precision in the determination of the speed of sound in water, within this range of salinity, of 0.6 m/sec without interpolation.

The second part of the tables (pages 37–180) covers the same temperature range and salinity ranging from 20 to 40 parts per 1000. The steps are, respectively, 0.05°C and 0.2 part per 1000; thus, the precision of determination of the speed of sound without interpolation is here 0.3 m/sec.

The nomograms are divided into two groups according to salinity, the first group containing Nomograms 1 to 3 for salinity ranging from 0 to 20 parts per 1000, and the second group containing Nomograms 4 to 8 for salinity from 20 to 40 parts per 1000. The range of temperature represented in these nomograms is $-2°$ to 30° Centigrade.

Since the values given in the tables and nomograms are values for zero depth in sea water, it is necessary to introduce the correction, Δv_p, for depth. These values are given for depths ranging from 0 to 10,990 meters. They are always positive, and are considered constant at 0.0161 m./sec. for depths up to 100 meters. The tables of corrections (pages x to xii, also one insert page) are based on the contents of W. D. Wilson's paper entitled "Speed of sound in sea water as a function of temperature, pressure and salinity," in the *Journal of the Acoustical Society of America*, v. 32, 1960, p. 641–644. Another basic assumption used in compiling the correction table is that a depth of 10 meters corresponds to a hydrostatic pressure of 1 kg/cm^2.

The references include three British, four Soviet, and five American authors.

The format of the tables is clear and convenient. Since only a few table and column headings are involved and the format of the tables is clear, these tables could be of value to persons who make use of similar publications in the English and German languages.

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1. *Tables of the velocity of sound in pure and sea water for use in echo sounding and sounding ranging*, Hydrographic Department of the Admiralty, London, 1927; also D. J. MATHEWS, *Tables of the velocity of sound in pure water and sea water*, 2nd ed., 1939; 3rd ed., 1944, Hydrographic Department of the Admiralty, London.
2. V. A. DEL GROSSO, *The Velocity of Sound in Sea Water of Zero Depth*, Report N 4002, Naval Research Laboratory, Washington, 1952.

49 [S, X].—RUDOLPH E. LANGER, Editor, *Partial Differential Equations and Continuum Mechanics*, The University of Wisconsin Press, Madison, 1961, xv + 397 p., 24 cm. Price $5.00.

This volume prints in full the invited papers and in abstract the contributed papers at a symposium in 1960. As to be expected in such a case, the quality of the papers is uneven. As a whole, the papers on pure analysis represent the state of the art in their field much better than those on continuum mechanics, which are also far fewer in number. The typing is as attractive as typing can be, and there are fewer misprints than usual, but the reviewer fails to see why the Department of the Army of the vaunted richest country in the world cannot afford ordinary printing as used by our poorer neighbors, or what purpose is served by publication of these papers here rather than through the normal channels of selection by the numerous journals.

Among the many excellent papers on the general theory of partial differential equations should be mentioned those of Leray, Moser, Hörmander, and Morrey. An interesting new type of problem is set up and solved by B. Frank Jones; in this problem, a coefficient function in the partial differential equation is to be determined by initial and boundary data that would lead to an overdetermined problem if the coefficient were fixed. This type of problem obviously has great use in applications to complicated systems whose constitution is only partly known; results of this kind make possible inferences about the internal nature of the system from use of observations on the boundaries.

Three papers concern directly techniques or error estimates in numerical calculation. Jim Douglas, Jr. sets up an alternating direction method for iterative solution of Laplace's equation in any number of dimensions. From the error estimates he concludes that this method is the most efficient known for Laplace's equation over a rectangular parallelepiped. The brothers Nitsche obtain error estimates for the computation of certain integrals in whose integrands occur solutions of elliptic differential equations. Poritsky discusses improvement of convergence. Weinberger obtains bounds for the square of the norm of the error in the Rayleigh-Ritz procedure, on the assumption that lower bounds for the desired proper numbers are known.

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50 [V, W, Z].—Benjamin Mittman & Andrew Ungar, *Computer Applications—1960*, The Macmillan Co., New York, 1961, vi + 193 p., 24 cm. Price $5.75.

This small volume contains papers from the 1960 Computer Applications Symposium sponsored by the Armour Research Foundation. It is divided into two sections. Part One, "Business and Management Applications," includes the use of automatic data processing systems for handling subscription lists, information retrieval in libraries, and filling mail-orders; Part Two, "Engineering and Scientific Applications," presents applications of computers in weather forecasting, the design of optical lens systems, and electronic data communications. The wide spectrum of applications described in these papers vividly demonstrates the impressive and fast-growing uses of digital computers in management, business, engineering and scientific research. A list of titles and authors follow:

Electronic Processing of 10 Million Subscription Records—B. H. Klyce

Prediction of Program Running Time as an Aid in Computer Evaluation—T. J. Tobias

A COBOL Processor for the UNIVAC 1105—J. J. Jones

The Computer in the Library—V. W. Clapp

Computer Control of Mail-Order House Operations (IBM-650 Tape RAMAC)—S. Kritzik

An Electronic Computer in Economic Research—M. H. Schwartz

A Generalized Brokerage Accounting System (RCA 501)—A. B. Goldstein
Solution of Naval Numerical Weather Problems (CDC 1604)—P. M. Wolff
Systems and Standards Preparations for a New Computer (Philco 2000)—H. S. Bright
Computer Design of Optical Lens Systems (IBM-704)—J. C. Holladay
LOGLAN and the Machine—J. C. Brown
Data Communications Between Remote Machines—V. N. Vaughn, Jr.
Some Observations on ALGOL in Use (Burroughs 220)—J. G. Herriot
The Role of a Professional Society in Program Exchange—W. M. Carlson

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51 [W].—A. CHARNES & W. W. COOPER, *Management Models and Industrial Applications of Linear Programming*, Vol. II, John Wiley & Sons, Inc., New York, 1961, xiv + 861 p., 26 cm. Price $11.75.

Those familiar with the previous work of the authors will expect, and find, a well-written work thoroughly covering the field laid out for it. The emphasis is on linear programming, with some discussion and illustration by means of simplified applications to industry. Relatively little space is devoted to the practice of formulating and constructing mathematical models.

The book being reviewed is the second of two volumes. Volume I may properly be considered an introduction to Volume II. Topics covered in the second volume include the modified simplex and dual methods (with a discussion of the evolution of computer codes using the revised simplex method), transportation type models, dyadic models and subdual methods, the development of model prototypes and compression, networks and incidence type models, game theory and linear programming, and a collection of miscellaneous topics, including integer programming. Appendices treat such topics as the double-reverse method, mixing routines, and saddle points.

The treatment is broad, fully explained and amply illustrated numerically. The exposition, in this reviewer's opinion, would have been smoother if less recourse were taken to extensive footnotes. The footnotes per chapter frequently exceed 70 in number.

Equations have been attractively set into type, but all figures and tables have been reproduced directly from typewritten copy, which detracts from the over-all quality.

One further anomaly requires mention. Included in the book is an extensive bibliography numbering over 560 citations. On the other hand, the index to Volume II appears to be a skimpy afterthought. A book as potentially useful as this deserves a far better tool.

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52 [W, Z].—ANDREAS DIERMER, *Das Wesen der automatisierten elektronischen Datenverarbeitung und ihre Bedeutung fur die Unternehmensleitung*, Walter De Gruyter & Co., Berlin, 1962, 240 p., 24 cm. Price DM 28.00.

This book belongs to the increasing number of works intended to introduce a certain group of readers to computers and their use. In one way, however, this book is rather unusual in that it is not clear, at least to this reviewer, what audience is actually being addressed. The title suggests that the work is directed towards readers in business management. In his foreword the author specifies this by stating as the aim of this book a presentation of the fundamental principles, technical aspects and use of computers in all phases of automatic business-data-processing. He qualifies this by subdividing this aim as follows: to present a thorough account of the logical and technical principles of computers in a manner understandable to business-oriented professionals; to investigate to what extent qualitatively given economic facts can be quantitized; to see which problems in the business area can be solved by the use of automatic high-speed computer systems.

With this program in mind the reader soon becomes disappointed, since the book concerns itself almost exclusively with the first point only, namely, the logical and technical fundamentals of computers. In fact, there are two parts, entitled, respectively, The Fundamentals of Automatic Electronic Data-processing, and The Methods of Automatic Electronic Data-processing as Means for the Ordering and Determination of Business Processes. The first part comprises almost 180 pages out of a total of 240. It is subdivided into three chapters which, in free translation, have the following headings: Historical Development; The Fundamentals of Electronic Data-processing; and the Interrelation of the Elements in Automatic Data-processing. Chapter 1 presents a very readable account of the history of computers and includes the European efforts in this field over the past 30 years. Chapter 2 should actually be entitled "The Building-elements of Electronic Computers." It first describes with surprising technical detail the functioning of vacuum tubes, germanium diodes, transistors, magnetic cores, etc., and then enters into a description of number systems, codes for the binary representation of decimals, etc. The chapter ends with a very general and not very enlightening philosophical discussion about the relation between mathematics and business-data-processing. Chapter 3 deals with the intercorrelation of the elements within a computer. Again the author goes into surprisingly technical details, which are rather out of place here, in view of the aim of the book and the intended readers. For example, while describing the decoding process of the instruction word, he discusses in great detail the working of a bistable multivibrator. Chapter 3 continues with a discussion of conditional-transfer instructions and fixed-point arithmetic. Floating-point representation is mentioned briefly.

The second part of the book has a very promising title, but it does not present very much substantial material. The author enters into a very general discussion of the processes fundamental to any business undertaking. Then, in equally general terms, he points to the necessity of business-data-processing, including a few words about programming problems and the Simplex method. This reviewer fails to see the aim in these discussions, which are presented in such generalities that they border on platitudes. In fact, it is here rather than in the earlier part that more specific information and greater attention to technical detail are needed in order to substantiate the numerous generalizations made.

In this reviewer's opinion, the book misses the mark and does not even meet its own aims. The largest part of the work deals with the principles of computer-building, and appears to be too technical for the business-oriented reader, while not being technical enough for the professional engineer or scientist. The rest of the book is simply too general to be useful.

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53 [X].—P. G. GUEST, *Numerical Methods of Curve Fitting*, Cambridge University Press, New York, 1961, xiv + 422 p., 23 cm. Price $15.00.

This book offers a solid treatment of curve fitting or, if preferred, regression analysis. It is intended primarily for students and graduates in Physics. Chapters 1 through 4 constitute a quick course in requisite statistical inference; Chapters 5 and 6 introduce regression theory and the fitting of a straight line; Chapters 7 through 12 cover a range of topics, including polynomial regression, standard deviations of estimates, grouping of observations, special functions regression, and multiple regression.

The emphasis is certainly on the practical side, although "it is intended that the book should cover the theoretical aspects of curve fitting and full derivations of all formulae are given." For these aspects, a knowledge of calculus is assumed.

The book is quite complete in its treatment of the problems under consideration. Calculating schemes are given (primarily for a desk machine) and great care is taken in the carrying out of specific problems drawn from physics. Chapter 12 contains a guide to the more important calculating schemes for the problems considered and provides extra illustrations of commonly occurring problems. These traits should be greatly appreciated by the worker who seeks procedural method.

Some minor comments seem appropriate. Although the accomplishment of all derivations is commendable, while the stress is laid on practicality, the usual problems arise. Thus on page 3, the statement "it will often be true that the value obtained for η does not depend on the value x of ξ" launches the reader into the notion of statistical independence. More generally, it may be expected that considerable difficulty will be met in reading through the book, unless the reader has more maturity than is indicated by the background expected. For dealing with specific sections this difficulty should be considerably reduced, especially with the aid of the guide in Chapter 12.

Some notation and terminology is disturbing to a statistician. For example, this reader would prefer some stress on the linearity of estimators in the statements of Gauss-Markov theorems.

These are small criticisms of what appears to be an excellent storehouse of information for the practical curve fitter.

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54 [X].—R. ZURMÜHL, *Praktische Mathematik für Ingenieure und Physiker*, Springer Verlag, Berlin-Wilmersdorf, 1961, xv + 548 p., 24 cm. Price DM 29.40.

This book, the third edition of Zurmuhl's *Praktische Mathematik*, is intended to introduce the reader to many of the numerical methods for the solution of those mathematical problems that are most frequently met by the engineer. A small part of the presentation is devoted to graphical procedures for the solution of problems, but the bulk of the book deals with computational methods. The tool of computation which the author has in mind is primarily the desk computer, sometimes the slide rule, and only occasionally the electronic high-speed computer.

The book assumes as prerequisite not much more than a familiarity with the contents of a good calculus course and with the elements of linear algebra and of differential equations. The computational methods presented are described in considerable detail, including a review of much of the underlying theory.

The main topics are methods of solution for equations in one unknown (especially algebraic equations), elimination and iteration procedures for systems of linear equations, the study of matrices and their eigenvalue problems, interpolation, and numerical as well as graphical integration. The chapter on statistics is greatly enlarged in the third edition. It is followed by a chapter on the presentation of arbitrary functions, which contains a section on Fourier Analysis. The last two chapters deal in considerable detail with ordinary differential equations, the first with initial-value problems, and the other with boundary-value and eigenvalue problems. Partial differential equations are considered to be outside the scope of the book.

The presentation does not claim to be comprehensive, nor is it intended to be a mere conglomeration of methods of solution for mathematical problems. While the book does not intend to prove all theorems used, it avoids the other extreme of merely enumerating results. The reader is not only presented with the details of the numerical solution of a problem, but he is also given a thorough introduction to the ideas that lead up to the method, and so he is forewarned against applying the method to cases where its validity has not been established. While the book assumes that the reader is primarily interested in the use of a desk calculator, the greater part of the book will serve well as an introduction to methods which are of great importance in the preparation of problems for an electronic high-speed digital computer.

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