

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

- 1 [C, D, E, L, X].—C. W. CLENSHAW, *Chebyshev Series for Mathematical Functions*, Mathematical Tables, v. 5, National Physics Laboratory, Her Majesty's Stationery Office, London, 1962, iv + 36 p., 27.5 cm. Price 12s.6d.

In an introductory section the author states the two-fold purpose of this volume, namely, to present tables, mostly to 20 decimal places, of the coefficients in the Chebyshev expansions of a number of the more common mathematical functions, and to set forth the basic techniques for evaluating and manipulating such series.

The remaining text consists of sections devoted to: the fundamental properties of Chebyshev polynomials (with particular reference to a discussion by Lanczos [1]); the alternative methods used in the calculation of Chebyshev coefficients (use of the orthogonal properties of integration and of summation, rearrangement of series, and solution of differential equations); a description of the tables and their preparation, mainly by solving the appropriate differential equations; the application of the tables and the use of Chebyshev series; and a critical comparison of Chebyshev series with alternative forms of storing a table-equivalent in a computer (these forms include explicit polynomials, "best" polynomials, and rational functions).

A bibliography of 28 books and papers is included, followed by two appendixes: one on the spelling of "Chebyshev," and the other on the normalization of Chebyshev polynomials.

The body of this work consists of seventeen tables of Chebyshev coefficients, given to 20 decimal places except for the last table, which gives 14 places. The functions considered include the trigonometric functions sine, cosine, and tangent; the inverse functions  $\sin^{-1}x$ ,  $\tan^{-1}x$ ; the exponential function; the logarithmic function  $\ln(1+x)$ ; the inverse hyperbolic sine; the Gamma function; the error function; the exponential integral; and both regular and modified Bessel functions of orders 0 and 1, together with auxiliary functions.

The numerous summation checks of the Chebyshev coefficients that are included in the tables inspire confidence in the accuracy of these extensive results. This reviewer has, moreover, found that Clenshaw's values of  $J_{2k}(n\pi/2)$  agree with similar data of Owen R. Mock deposited in the UMT file (see Review 118, *MTAC*, v. 9, 1955, p. 223).

This excellent set of tables appears to be the most extensive and elaborate compilation of such coefficients that has yet been published.

J. W. W.

1. NATIONAL BUREAU OF STANDARDS, *Tables of Chebyshev Polynomials*, Applied Mathematics Series No. 9, U. S. Government Printing Office, Washington, D. C., 1952.

- 2 [F].—J. C. P. MILLER, *Table of Least Primitive Roots*, 3000-page manuscript in possession of the author at The University Mathematical Laboratory, Cambridge, England; one copy thereof deposited with the Royal Society of London;

a second copy temporarily in the custody of Professor D. H. Lehmer, University of California, Berkeley 4, California, prior to deposit in UMT File.

This extensive manuscript table lists for each of the 150,000 primes  $p$  from 3 to 2,015,179, inclusive, the following four integers:  $g$ , the least positive primitive root;  $h$ , the least positive prime primitive root;  $g'$ , where  $-g'$  is the negative primitive of least modulus; and  $h'$ , where  $-h'$  is the negative prime primitive root of least modulus.

When  $p$  is of the form  $4k + 1$ ,  $g'$  and  $h'$  are equal to  $g$  and  $h$ , respectively, and are not explicitly tabulated; furthermore,  $h$  is not tabulated for such primes unless it differs from  $g$ . On the other hand, for primes of the form  $4k - 1$ , all four data are shown explicitly.

The primes  $p$  are listed 50 to a page, the last three figures of each appearing in the argument column; to these figures there must be added the multiple of a thousand listed at the top of the column, with an increment of a thousand if the thousands digit has changed since the top of the column.

The author has informed this reviewer that this table was computed in about 50 hours on EDSAC 2 by means of a program prepared by M. J. Ecclestone. As the result of built-in program checks and visual inspection of all the printed output, this table is considered to be very reliable.

J. W. W.

**3 [F. G, I, X].**—JOHN TODD, Editor, *A Survey of Numerical Analysis*, McGraw-Hill Book Co., 1962, 23.5 cm., xvi + 589 p. Price. \$12.50

In 1957 the National Bureau of Standards, with support from the National Science Foundation, offered a course designed for mature mathematicians and aimed at arousing their interest in the practice and theory of computing. This book is made up of lectures prepared for this course, except for two of the chapters which were prepared in collaboration with lecturers who spoke at a similar course given in 1959.

Chapters include one on "motivation," and topics range from simple interpolation, through matrices and differential equations, to the application of functional analysis to numerical analysis, with added chapters on discrete problems, number theory, and linear estimations. Contributors are John Todd, Olga Taussky, Morris Newman, Harvey Cohn, Philip Davis, Marvin Marcus, Henry A. Antosiewicz, Walter Gautschi, David Young, Hans Bückner, Werner Rheinboldt, Marshall Hall, and Marvin Zelen.

Although there are substantial differences among chapters as to depth and sophistication, and the coverage is not uniform, nevertheless there is much more coherence and coordination than one might expect, and the coverage is quite extensive. Rather long lists of references are included in each chapter, although usually the authors made little effort to bring them up to date (i.e., later than 1957). It is regrettable that publication was so long delayed, but the book is a valuable contribution, nevertheless.

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EDITORIAL NOTE: A list of corrections to this volume has been compiled by the publisher.

- 4 [H].—HENRY C. THACHER, JR., *Real Roots of the Equation  $x \tan y + \tanh y = 0$* , ms. of six leaves deposited in UMT File.

The author describes in detail the procedure he followed in the preparation of these original manuscript tables, which give to six decimal places the first two real roots of the transcendental equation  $x \tan y + \tanh y = 0$  for  $x = 0(.05)1$  and for  $\pm x^{-1} = 1(-.05)0$ , respectively, and the first three real roots of that equation for  $-x = 0(.05)1$ , together with an auxiliary table to facilitate interpolation.

Such data are used, according to the author, in the application of one theory of the convective heat transfer between parallel plates. (Reference is made to A. F. Lietzke, *Theoretical and Experimental Investigation of Heat Transfer by Laminar Natural Convection between Parallel Plates*, NACA Report 1223, 1955.)

J. W. W.

- 5 [I, K, P].—B. M. BROWN, *The Mathematical Theory of Linear Systems*, John Wiley & Sons, Inc., New York, 1961, x + 267 p., 9 cm. Price \$8.00.

For its size this book is amazingly broad and thorough in its coverage of linear systems. Since the author is British, however, the manner of expression and sometimes the notation may seem strange to the American reader.

After a general discussion of linear differential equations, operational methods of solution are introduced. The unilateral Laplace transform is then used to solve equations with given initial conditions. Fourier series and integrals and the bilateral Laplace transform are then brought in, along with impulse, step, and ramp functions.

Linear systems are then discussed with the aid of block diagrams and simple physical examples, which are solved by various mathematical methods, including the weighting function. Feedback is then introduced, along with the stability criteria of Routh, Hurwitz, and Nyquist. The graphical methods using  $M$ -circles and root loci are summarized.

Statistical methods are used to introduce the concepts of correlation functions and spectral density, which are then applied to system optimization both with and without constraints.

After a short treatment of difference equations, the  $Z$ -transform is developed, followed by a treatment of sampling servos. Other topics treated include time-variant systems, multivariable systems, and interpolation systems.

The several appendices contain mathematical background materials and proofs. The book is probably too condensed to be a good college text, but it presents a good logical development and can serve as a valuable reference.

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- 6 [I, X].—D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, "Sur une classe de nombres se rattachant aux nombres de Stirling," *Publ. Fac. Élect. Univ. Belgrade* (Série: Math. et Phys.), No. 60, 1961, 63 p. (French with Serbian summary.)

The two tables given in this publication are extensions of earlier ones by the same authors (see *Math. Comp.*, v. 15, 1961, p. 107; v. 16, 1962, p. 252).

Table I (p. 17–44) gives exact values of  $(-)^m C_m^k$  for  $k = 0(1)59$  and  $m = 51(1)60$ , with  $k < m$ . The values are integers having between 67 and 111 digits.

Table II (p. 45–62) gives exact values of the Stirling numbers of the first kind,  $(-)^m S_n^{n-m}$ , for  $m = 7(1)59$  and  $n = 51(1)60$ , with  $m < n$ , and also for  $m = 60$ ,  $n = 61$ . The values are integers having between 18 and 82 digits. Various values of  $S_n^{n-m}$  given in the tables were checked in laboratories at Liverpool, Rome, and Hamburg. In addition, the value of  $S_{70}^{10}$  was computed at Liverpool and Rome, and the 98-digit result is given separately on p. 8.

A. F.

7 [J].—I. J. SCHWATT, *An Introduction to the Operations with Series*, Chelsea Publishing Company, New York, 1961, x + 287 p., 21 cm. Price \$3.95.

The present, second edition is a reprint, with corrections, of the first edition of 1924, and this date must be kept in mind. Even so, the volume has several weaknesses, as considerable information on transcendental functions and related topics available in the literature of 1924 apparently was not known by the author, or if known, there are no references. For example, *Modern Analysis* by E. T. Whittaker and G. N. Watson, and *A Textbook of Algebra* by G. Chrystal are not mentioned. The preface states that the “book had its inception in the author’s efforts to obtain the value for the sum of the series of powers of natural numbers, in an explicit form and without the use of Bernoulli numbers. This problem led to the study of the higher derivatives of functions of functions, which in turn required certain principles in operations with series, which had to be established. By means of these and other principles, methods for the expansion of certain functions and the summation of various types of series were devised and other topics developed.” The volume is replete with numerous examples, and a number of ingenious devices are used. In the following we give some of the highlights of each chapter, along with comments which should enhance the usefulness of the volume.

Chapter I deals with derivatives and expansions in powers of  $x$  of  $(\sum_{m=0}^r a_m x^m)^p$ . Power series for  $\sin^p x$ ,  $\cos^p x$ ,  $\tan^p x$ , and their reciprocals are given in Ch. II for  $p = 1$ , and in Ch. IV for a general integer  $p$ . That the coefficients of powers of  $x$  in the expansions of  $\tan x$  and  $\sec x$  are related to the Bernoulli and Euler numbers, respectively, is not mentioned in Ch. II. These numbers are studied in Ch. XV. The operator  $\delta^n$ , where  $\delta = x d/dx$  is the subject of Ch. V. The operator is used to find the differential equation satisfied by certain series. If the solution to the differential equation can be found in a simple form, then the given series is summed. We recognize that the differential equation satisfied by the generalized hypergeometric function  ${}_pF_q$  can be easily expressed with the operator  $\delta$ . (See A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions*, v. 1, McGraw-Hill, 1953.) Indeed, many of the problems treated are of hypergeometric type. For example,  $S = \sum_{n=1}^{\infty} (-)^{n-1} n^p x^n = \delta^p (x/x + 1)$ . Extension of the results in Ch. V are given in Ch. X, and that technique is applied to sum trigonometrical series in Ch. XII. In Ch. VI, derivatives of continued products  $\prod_{k=1}^n f(x, k)$  are treated for  $f(x, k) = \sin kx$ ,  $x + k$ , and  $1 - x^k$ . Chapter VII generalizes some of the results in Chs. I, IV, and V. Separation of fractions into partial fractions is treated in Ch. VIII, and these results are used in Ch. IX to

evaluate  $\int \frac{x^m dx}{x^n \pm 1}$  where  $m$  and  $n$  are integers. The author studies

$$S = \sum_{n=0}^{\infty} \prod_{k=0}^n \left( \frac{a+k}{b+k} \right) r^n,$$

and shows that it can be written as a finite sum if  $b - a$  is a positive integer. The value of  $S$  for  $r = 1$  is determined. In the course of this discussion, the beta integral is evaluated. The author does not seem to recognize that  $S = (a/b) {}_2F_1(1, a+1; b+1; r)$  where  ${}_2F_1$  is the Gaussian hypergeometric function and

$$S = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^a (1-t)^{b-a-1} (1-tr)^{-1} dt,$$

provided  $R(b+1) > R(a+1) > 0$  and  $|\arg(1-r)| < \pi$ . Chapter XI takes up the separation in partial fractions of trigonometric expressions such as  $\cos^p x / \cos nx$ ,  $p < n$ .

Chapter XIII is concerned with the evaluation of definite integrals such as  $\int_a^b x^{-1} e^x dx$ . In an aside, the author says (p. 244), "If a personal reference be permitted, the author has spent considerable effort in trying to express in terms of elementary functions  $S = \sum_{n=1}^{\infty} \frac{x^n}{n!n}$  and the solution of  $x d^2 S/dx^2 + (1-x) dS/dx = 1$  which is satisfied by  $S$ . It is hoped that mathematicians will feel induced to take up this and similar problems in the operation with series which are waiting solution and which have such an important bearing on mathematical analysis." Now  $S = -\gamma - \ln x - \int_{-x}^{\infty} t^{-1} e^{-t} dt$ , and Liouville showed that this could not be expressed in terms of elementary functions. In this connection, see *Integration in Finite Terms*, by J. F. Ritt, Columbia Univ. Press, 1948, p. 49.

The study of sums of some conditionally convergent series under rearrangement is taken up in Ch. XIV. Examples include  $\sum_{k=0}^{\infty} (-)^k (b + kh)^{-1}$  rearranged so that  $m$  positive terms are followed by  $n$  negative terms.

In conclusion, the volume contains many results of interest in applied work, but the reader is cautioned to keep in mind the developments of the last four decades.

Y. L. L.

**8 [K].**—E. L. PETERSON, *Statistical Analysis and Optimization of Systems*, John Wiley & Sons, 1961, xi + 190 p., 23 cm. Price \$9.75.

The title of the book may suggest that this book is an extensive treatise on statistical analysis and optimization of systems. However, this is not the case here. The book treats mainly time-varying linear systems such as missiles and fire control systems. The first three chapters and the first two sections of Chapter 4 (Chapter 1, Linear System Theory; Chapter 2, Statistics of Random Variable; Chapter 3, Response to Distributed Inputs; and Chapter 4, Systems Analysis and Design—General Approach and the Adjoint Method of Analysis) are not too rigorous summaries of some transform techniques, stability theory, theory of probabilities and some filtering and prediction theory, as can be found in a standard textbook such

as J. H. Laning, Jr. & R. H. Battin, *Random Process in Automatic Control*, McGraw-Hill, 1956. The merit of the book lies mainly in Chapters 5 and 6, Optimum Systems and Applications in Optimal Systems, where such things as determination of the optimum impulse response and the optimum system block diagram are discussed with the usual mean-square error criterion. Applications are taken from problems in fire control, tracking, prediction, guidance, and navigation. Publications by M. Shinbrot are referred to quite often.

In these chapters, analysis and synthesis processes of control systems are illustrated from a very rough, sketchy beginning to a more and more precise determination of various system parameters. The book occupies a middle position in the spectrum of technical books, in that it is not recommended for beginners and yet it is not very useful for people actually working in this area. However, students in control systems can benefit from such step-by-step explanations of synthesis processes.

On the whole, the reviewer feels that the book has not come up to the expectation raised in the reader's mind when he looks at the chapter headings.

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9 [L].—V. S. AIZENSHTAT, V. I. KRYLOV & A. S. METLESKII, *Tables for Calculating Laplace Transforms and Integrals of the form  $\int_0^\infty x^s e^x f(x) dx$* , Izdatel'stov Akad. Nauk SSSR, Minsk, 1962, 378 p. (Russian).

This book gives tables of Gaussian quadrature formulas for approximate evaluation of the integrals in the title. These formulas have the form

$$\int_0^\infty x^s e^{-x} f(x) dx = \sum_{k=1}^n A_k f(x_k)$$

where the  $A_k$  and  $x_k$  depend on the parameter  $s$  and the value of  $n$ . The  $A_k$  and  $x_k$  are chosen so that the approximation is exact whenever  $f(x)$  is a polynomial of degree  $\leq 2n - 1$ . This means that the  $x_k$  are the zeros of the  $n$ th degree generalized Laguerre polynomial  $L_n^{(s)}(x)$ , which satisfies the orthogonality condition

$$\int_0^\infty x_r^{s-r} L_n^{(s)}(x) P(x) dx = 0,$$

where  $P(x)$  is an arbitrary polynomial of degree  $\leq n - 1$ .

The first 22 pages of the book discuss properties of these formulas and give some examples of their use. The remainder of the book contains the formulas and is divided into three tables. Table 1 gives formulas for  $s = -0.90(0.02)0.00$ ; Table 2, formulas for  $s = 0.55(0.05)3.00$ ; and Table 3, formulas for  $s = -\frac{3}{4}, -\frac{1}{4}, n + \frac{k}{3}$ , where  $n = -1(1)2$ ,  $k = 1, 2$ . For each value of  $s$ , the numbers  $A_k$ ,  $x_k$ , and  $A_k e^{x_k}$  are given to 8 significant figures for  $n = 1(1)15$ .

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- 10 [L].—BURTON D. FRIED & SAMUEL D. CONTE, *The Plasma Dispersion Function: The Hilbert Transform of the Gaussian*, Academic Press, New York and London, 1961, 419 p., 26 cm. Price \$12.00.

The title of the book is deceptive, as the transcendent considered is

$$Z(\xi) = 2ie^{-\xi^2} \int_{-\infty}^{i\xi} e^{-t^2} dt, \xi = x + iy,$$

which is essentially the error function of complex argument. The related function  $w(z) = i\pi^{1/2}Z(z)$  has been previously tabulated [1]. The present volume gives tables for  $Z$  and  $Z'$  mostly to 6S (see the remarks below) for the range  $x = 0(0.1)10$ ,  $y = -10(0.1)10$ . The authors seem unaware of other tables of the error function for complex argument [2, 3, 4, 5].

Some properties of  $Z(\xi)$  and figures which show its behavior are presented. The method of computation is described. We have noted two typographical errors.

On p. 3, line 3, read  $Y(x) \equiv x^{-1} \exp(-x^2) \int_0^x \exp(t^2) dt$ , and on p. 6, in the continued fraction representation for  $Z(\xi)$ , for  $\xi^2 + 5/2$ , read  $-\xi^2 + 5/2$ . Also on p. 6 the sign of  $a_{n+1}$  in the difference equations for  $A_n$  and  $B_n$  should be negative. If  $1 < |y| \leq 10$ ,  $0 \leq x \leq 10$ , a well-known continued fraction representation based on the asymptotic expansion of  $Z$  was employed. This representation converges in the entire complex plane except for points on the real axis. Near the real axis, the number of terms required for convergence increases, and maintenance of accuracy was difficult, owing to underflow and overflow. Accordingly, in the region  $|y| \leq 1$ ,  $0 \leq x \leq 10$ , the entries were found by numerical integration of the differential equation satisfied by  $Z(\xi)$ . For  $y = 0$ ,  $5 < x < 10$ , the entries for  $\text{Im}(Z)$  and  $\text{Im}(Z')$  become very small, and significance is gradually lost. At  $x = 9$ , the first four figures are significant, but thereafter the entries are not reliable. In this region, one should use  $\text{Im}(Z(x)) = \pi^{1/2}e^{-x^2}$ .

It should be noted that even though the continued fraction convergents do not converge on the real axis, they may still be used for computational purposes in the same way that one uses an asymptotic expansion. In fact, the second-order convergent  $-\xi(\xi^2 - 5/2)/(\xi^4 - 3\xi^2 + 3/4)$  approximates  $Z(\xi)$  to almost 6 decimal places if  $x \geq 0$ ,  $y \geq 0$  and  $|\xi| \geq 5$ . Thus, use of the above approximation, together with the formula connecting  $Z(\xi)$  and  $Z(\bar{\xi})$ , would have considerably reduced the bulk of the present tables.

Y. L. L.

1. V. N. FADDEEVA & N. M. TERENT'EV, *Tablitsy značeniĭ funktsii*

$$w(z) = e^{-z^2} \left( 1 + 2i\pi^{-1/2} \int_0^z e^{t^2} dt \right)$$

ot kompleksnogo argumenta, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1954. Also available as *Tables of Values of the Function  $w(z) = e^{-z^2} \left( 1 + 2i\pi^{-1/2} \int_0^z e^{t^2} dt \right)$* , Pergamon Press, 1961. See *Math. Comp.*, v. 16, 1962, p. 384-387.

2. P. C. CLEMMOW & C. M. MUNFORD, "A Table of  $(\pi/2)^{1/2}e^{1/2i\pi\rho^2} \int_{\rho}^{\infty} e^{-1/2i\pi\lambda^2} d\lambda$  for complex values of  $\rho$ ," *Philos. Trans. Roy. Soc. London*, ser. A, v. 243, 1952, p. 189-211. See *MTAC*, v. 7, 1953, p. 178.

3. K. A. KARPOV, *Tablitsy funktsii*  $w(z) = e^{-z^2} \int_0^z e^{z^2} dx$  v kompleksnoy oblasti, Insdat. Akad. Nauk SSSR, Moscow, 1954. See *MTAC*, v. 12, 1958, p. 304-305.

4. K. A. KARPOV, *Tablitsy funktsii*  $F(z) = \int_0^z e^{z^2} dx$  v kompleksnoy oblasti, Izdat. Akad. Nauk SSSR, Moscow, 1958. See *Math. Comp.*, v. 14, 1960, p. 84.

5. R. HENSMAN & D. P. JENKINS, "Tables of  $(2/\pi)e^{iz^2} \int_z^\infty e^{-t^2}$  for complex  $z$ ," UMT file, *Math. Comp.*, v. 14, 1960, p. 83.

11 [L].—FRITZ OBERHETTINGER & T. P. HIGGENS, *Tables of Lebedev, Mehler, and Generalized Mehler Transforms*, Math. Note No. 246, Boeing Scientific Research Laboratories, Seattle, 1961, 48 p., 21.5 cm.

The transform pairs tabulated are:

A. (Lebedev)

$$g(y) = \int_0^\infty f(x) K_{ix}(y) dx,$$

$$f(x) = 2\pi^{-2} x \sinh \pi x \int_0^\infty y^{-1} K_{ix}(y) g(y) dy$$

where  $K_\nu(x)$  is the modified Bessel function of the second kind.

B, C. (Mehler, Generalized Mehler)

$$g(y) = \int_0^\infty f(x) P_{ix-1/2}^k(y) dx$$

$$f(x) = \pi^{-1} x \sinh \pi x \Gamma(\tfrac{1}{2} - k + ix) \Gamma(\tfrac{1}{2} - k - ix) \int_1^\infty g(y) P_{ix-1/2}^k(y) dy,$$

where  $P_{ix-1/2}^k(y)$  is the Legendre function. The Mehler transform is the case  $k = 0$ . Furthermore,  $k = \frac{1}{2}$  and  $k = -\frac{1}{2}$  give rise to Fourier cosine and sine transforms, respectively.

Most of the results given here are new. A list of Lebedev transforms is available in *Tables of Integral Transforms* by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, McGraw-Hill, 1954, v. 2, Ch. 12, but the present compilation is much more extensive. Only a few entries of the Mehler transform are given in the above reference.

The transforms are useful to solve certain boundary-value problems of the wave or heat conduction equation involving wedge or conically shaped boundaries, and a number of references to physical problems are given in the bibliography. To facilitate use of the tables, definitions of higher transcendental functions which enter into the transforms are provided in a separate section.

Y. L. L.

12 [W].—F. P. FOWLER, JR., *Basic Mathematics for Administration*, John Wiley & Sons, Inc., New York, 1962, xvii + 339, 23.5 cm. Price \$7.95.

This book presents a general survey of basic mathematics used in the development of modern decision-making techniques. The authors give a background sketch

of a wide spectrum of topics: mathematical logic, theory of equations, matrix algebra, linear programming, differential calculus, integral calculus, probability, and mathematical models. In general, no rigorous proofs of theorems are given. Mathematical concepts are developed in an intuitive fashion, with some attention to the underlying assumptions made in the application of particular "optimization" procedures.

The chapter on probability contains an informative and readable exposition of the rudimentary principles, and provides the beginner with background material for the understanding and practical assessment of "management games" as a decision-making tool.

The overly simplified and sketchy treatment of linear programming was disappointing to this reviewer in view of the fact that the necessary mathematical tools for a more detailed discussion had been developed in the preceding chapters. The examples illustrated in the text deal with graphical solutions of integer linear programming problems, and the reader is given the impression that the Simplex method without modification may be used to solve general integer linear programming problems. A more serious shortcoming is the erroneous procedure described for converting a linear programming problem containing a system of inequalities to an equivalent one containing a system of equations. The example

$$\begin{array}{ll}
 \text{Maximize: } \pi = 800X_1 + 1200X_2 & \\
 \text{Subject to: } 20X_1 + 35X_2 \leq 280 & \\
 6000X_1 + 5000X_2 \leq 60000 & \\
 1000X_1 + 500X_2 \leq 9000 & \\
 500X_1 + 500X_2 \leq 6000 & \\
 4000X_2 \leq 24000 & \\
 X_1 \geq 3 & \\
 X_2 \geq 2 &
 \end{array}$$

is transformed to the following "equivalent" system by the introduction of "slack" variables  $X_3, X_4, X_5, X_6, X_7$  and "artificial" variables  $X_8, X_9$ :

$$\begin{array}{ll}
 \text{Maximize: } \pi = 800X_1 + 1200X_2 + MX_3 + MX_4 + MX_5 + MX_6 + MX_7 & \\
 \text{Subject to:} &
 \end{array}$$

$$\begin{array}{rcl}
 20X_1 + 35X_2 + X_3 & = & 280 \\
 6000X_1 + 5000X_2 + X_4 & = & 60000 \\
 1000X_1 + 500X_2 + X_5 & = & 9000 \\
 500X_1 + 500X_2 + X_6 & = & 6000 \\
 4000X_2 + X_7 & = & 24000 \\
 X_1 - X_8 & = & 3 \\
 X_2 - X_9 & = & 2
 \end{array}$$

where  $M$  represents a very large negative number. The coefficients of the "slack" variables in the transformed objective function should be zero; however, the authors claim that the "slack" variables represent imaginary units, and consequently a large negative number should be used to insure that the "slack" variables reduce to zero in the final solution. Furthermore, the author's definitions of "slack" and "artificial" variables are not in accordance with standard usage. The term

"slack" variable is usually applied to a variable which is introduced to transform an inequality to an equation, while an "artificial" variable is usually applied to a variable which is introduced to provide a basis-variable in the process of obtaining an initial solution.

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- 13 [W].—E. S. VENTZEL, *Lectures on Game Theory*, Gordon and Breach, New York, 1961, 22 cm., 78 p. Price \$4.50.

The 78 pages of this book cover an elementary exposition of game theory in eight chapters touching on the object of the theory of games, the minimax principle, pure and mixed strategies, elementary methods of solution, general methods of solution of finite games (for example, linear programming), approximate methods and methods of solving a few infinite games. The book may give a good idea of the subject to the non-mathematician, particularly since it concentrates on elementary applied illustrations of game theory.

THOMAS L. SAATY

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- 14 [W, Z].—MARTIN GREENBERGER, Editor, *Management and the Computer of the Future*, John Wiley & Sons, Inc., and The M.I.T. Press, New York, 1962, xxvi + 340 p., 21 cm. Price \$6.00.

This volume contains the proceedings of a series of eight lectures on the subject, Management and The Computer of the Future, sponsored by the School of Industrial Management of the Massachusetts Institute of Technology during the spring of 1961 in celebration of MIT's centennial. At each session the main speaker presented a paper, which was followed by prepared remarks by two discussants. After additional brief remarks by the speaker the meeting was opened for general discussion. The list of participants includes some of the best known experts in the field of computers, admixed with a sprinkling of "amateurs" and prominent names outside the field. The following are the topics covered at the individual sessions:

1. Scientists and Decision Making—C. P. Snow, Speaker; E. E. Morison and N. Wiener, Discussants; H. W. Johnson, Moderator.
2. Managerial Decision Making—J. W. Forrester, Speaker; C. C. Holt and R. A. Howard, Discussants; R. C. Sprague, Moderator.
3. Simulation of Human Thinking—H. A. Simon, Speaker; A. Newell, Coauthor; M. L. Minsky and G. A. Miller, Discussants; S. S. Alexander, Moderator.
4. A Library of 2000 A.D.—J. G. Kemeny, Speaker; R. M. Fano and G. W. King, Discussants; W. N. Locke, Moderator.
5. The Computer in the University—A. J. Perlis, Speaker; P. Elias and J. C. R. Licklider, Discussants; D. G. Marquis, Moderator.
6. Time-Sharing Computer Systems—J. McCarthy, Speaker; J. W. Mauchly and G. M. Amdahl, Discussants; E. R. Piore, Moderator.
7. A New Concept in Programming—G. W. Brown, Speaker; G. M. Hopper and D. Sayre, Discussants; P. M. Morse, Moderator.

8. What Computers Should Be Doing—J. R. Pierce, Speaker; C. E. Shannon and W. A. Rosenblith, Discussants; V. Bush, Moderator.

The lectures and discussions were generally held at a non-technical level. Thus the book constitutes an informative and authoritative compilation of the prevailing views concerning the impact which the development of computers may have upon the future—written in a style which can easily be understood by the scientist, administrator or layman. The large variety of subjects covered, the many excellent presentations and the spirited discussions evoked by the speakers all contribute to the enjoyment and value which the reader may derive from the book. It is difficult for the reader not to become personally engulfed in the diverse points of view expounded concerning the future impact of computers. He may find himself becoming an ardent Luddite (named for Ned Lud, a poor nineteenth century English stocking weaver who, finding his meager livelihood threatened by automation, went out and destroyed the looms in his city), or a partisan defender of computer supremacy. Notwithstanding the preeminent position held by many of the contributors in the field of computer technology, the reader is not likely to find any earth-shaking pronouncements or discoveries—for, as pointed out by Martin Greenberger, the editor, in the matter of predicting the future of so revolutionary a device the layman or “amateur” can successfully hold his own with the so-called expert. This was shown to be the case during many of the discussions.

H. P.

15 [X, Z].—FRANZ, L. ALT, Editor, *Advances in Computers*, Volume 2, Academic Press, 1961, xiii + 434 p., 23.5 cm. Price \$14.00. [For a review of Volume 1, see *Math. Comp.*, v. 15, 1961, p. 220–221.]

In view of the extent and variable quality of the literature in our field nowadays, periodical critical surveys of topics of current interest by experts are especially welcome. The present volume contains five such articles, all provided with extensive bibliographies:

“A survey of numerical methods for parabolic equations,” by Jim Douglas, Jr., p. 1–54;

“Advances in orthonormalizing computation,” by Philip J. Davis and Philip Rabinowitz, p. 55–133;

“Microelectronics using electron-beam-activated machining techniques,” by Kenneth R. Shoulder, p. 135–293;

“Recent developments in linear programming,” by Saul I. Gass, p. 295–377;

“The theory of automata, a survey,” by Robert McNaughton, p. 379–421.

The article by Shoulder—the one most closely related to the title of this series—is concerned with a research program leading to the fabrication of machines several generations in the future. The other articles are in or near the field of numerical analysis, but there must be very few, apart from the editor, who could appreciate both these and that of Shoulder.

The title of this series suggests that we might expect to see evaluations of procedures in practice on the machine, as well as on paper. Those given by Douglas are largely qualitative, but Davis and Rabinowitz denote more than half their article to the results of numerical experiments—material of this kind is invaluable to the conscientious practical computer. It is surprising that the method of ortho-

normalization has not been exploited more widely; it has a wide range of application, and many of its strengths and weaknesses on the computer have been examined by Davis and his colleagues. About half of Gass's article is devoted to an account of machine codes for linear programming and related problems, mostly from this country but some from England and France. The sizes of problems acceptable are given, and time estimates are often given. There is a preliminary report on SCEMP: "*Standardized Computational Experiments in Mathematical Programming.*"

McNaughton's point of view is that the theory of automata is a theory that is best understood in its own terms; its proper relationship to computer work is one of a pure science to an applied science. He restricts his survey to the study of general various behavioral descriptions of automata.

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