33[A–E, G–N, X, Z].—A. Fletcher, J. C. P. Miller, L. Rosenhead, & L. J. Comrie, *An Index of Mathematical Tables*, Second edition (in two volumes), Addison-Wesley Publishing Company, Reading, Massachusetts, 1962, xi + iv + 994 p. (consecutively numbered), 25.5 cm. Price $42.00. [Published in England by Blackwell Scientific Publications, Oxford (for Scientific Computing Service, London).]

The continuing rapid increase in table-making, as one consequence of the extraordinary growth of automation, is graphically reflected in the comparative sizes of the first and second editions of this authoritative index of mathematical tables.

For example, the first main division, Part I: Index according to Functions (p. 21–608), although arranged as in the first edition, with 24 constituent sections, has been expanded from 355 to 588 pages. Furthermore, a large amount of new material has been added, constituting more than one hundred new articles in this division of the book. Representative of these numerous additions are the articles and subsections devoted to: harmonic means (2.47); coefficients in powers of Euler's product (3.7); symmetric functions (3.8); sums over lattice points (4.69); zeros of special sets of polynomials (5.67); segments of the Hilbert matrix (5.68); logarithms to base 2 (6.39); quantities related to the five convex regular polyhedra (7.88); repeated exponentials (10.39); natural logarithms of logarithms, exponential and hyperbolic functions (11.7); functions arising in problems of elasticity (12.95); exponential, sine and cosine integrals of complex argument (13.8); Sievert's integral (13.98); psi function of complex argument (14.66); tables for probit analysis (15.38); integrals related to order statistics (15.519); associated Legendre functions of non-integral degree (16.595); scattering functions for spherical particles (17.54); Airy functions of complex argument (20.25); transition curves (20.65); Fresnel integrals of complex argument (20.69); series involving Bessel functions (20.7); hyperelliptic integrals (21.9); the generalized zeta function (22.15); the Dirichlet L-functions (22.17); Mathieu functions of imaginary argument (22.235); Coulomb wave functions (22.59); Bose-Einstein functions (22.68); van der Pol's equation (22.895); miscellaneous indefinite integrals, infinite series, ordinary and partial differential equations (22.94–22.97); solutions of integral and integro-differential equations (22.98); partial derivatives at lattice points (23.49); Sard's quadrature formulas (23.512); osculatory quadrature formulas (23.67); and summation of slowly convergent series (23.695).

The references comprising the second main division, Part II: Bibliography (p. 609–780) have increased proportionately in number, from approximately 2000 to more than 4470, corresponding to an increment of 100 pages over the space devoted to that division in the first edition. References are made to publications dated as recently as 1960 and 1961, but the number of such is scanty, and the great majority of references are dated 1958 or earlier.

An innovation is the inclusion in the present edition of a division, Part III, entitled Errors (p. 781–932), which presents information available to the authors concerning errors in mathematical tables up to 1954, when this part of the *Index* was sent to press. The first source of this information was the relevant material amassed at the Scientific Computing Service under the direction of Dr. Comrie. Following his death in 1950, the remaining authors of this edition performed a further examination of tables for errors. The remaining main source was the notices of table errata appearing regularly in *Mathematical Tables and other Aids to Computation.* The authors considered this division of the book subsidiary to their main objective, which was to discover what tables of a given function exist. Incomplete as Part III must necessarily be, inasmuch as errors are continually being discovered in tables, nevertheless, this division of the book should prove extremely valuable to users of tables.

The elaborate Introduction (p. 1–18) is most informative, and should be read carefully by all users of this index. Included therein are introductory historical remarks, a large amount of bibliographic information, and a detailed description of the arrangement of the material in the *Index.* Reference is made to previous general indexes, such as those by Davis, Davis & Fisher, Schütte, Lebedev & Fedorova, and Buronova. Collections of mathematical formulas are cited; these include compilations by Láska, Adams & Hippisley, Silberstein, Kamke, Tölke, Magnus & Oberhettinger, Erdélyi, and Ryshik & Gradstein. This information is supplemented by enumerations of: handbooks for physicists and engineers; tables of integrals; lists of Fourier transforms, Laplace transforms, and Mellin transforms; textbooks and treatises on numerical methods and related theory; publications relating to the numerical solution of differential equations and of integral equations; manuals on calculating machines and instruments; books on nomography; astronomical tables; nautical tables; financial tables; statistical tables; and standard references on probability and statistics. Tables in the theory of numbers are not cited to any appreciable extent, for it was felt by the authors that adequate coverage has been provided by relevant indexes prepared by A. Cayley and D. H. Lehmer and by notes and papers appearing regularly in *MTAC* and *Math. Comp.*

The book closes with Part IV: Index to Introduction and Part I (p. 933–994). One reference omitted is that to transport integrals (subsection 22.65).

It is difficult in this limited space to convey an adequate idea of the wealth of information available in this edition. The new *Index* certainly is pre-eminent among the books of its kind, and is a worthy successor to the first edition, which was elaborately reviewed in this journal by the late R. C. Archibald (*MTAC*, v. 2, 1946, p. 13–18).

In a work of this size errors must almost inevitably appear; a number of these are listed in the appropriate section of this issue of *Math. Comp.* These are relatively inconsequential flaws, and this monumental work can be most highly recommended as an indispensable accession to the library of every computation laboratory, and should be readily available to teachers, students, and practitioners in the field of numerical mathematics.

J. W. W.

34[A–F, K–N, Q].—Samuel M. Selby, Robert C. Weast, Robert S. Shankland, Charles D. Hodgman, Editors, *Handbook of Mathematical Tables*, Chemical

Rubber Publishing Company, Cleveland, Ohio, 1962, x + 579 p., 23 cm. Price $7.50.

The present volume is a further enlargement, reformating, and repricing of the *CRC Standard Mathematical Tables* [1], which were themselves an outgrowth of the *Mathematical Tables from Handbook of Chemistry and Physics.*

The larger pages and type size in the new volume are a distinct improvement. The new tables (or sections) in the hundred or so extra pages include the following:

1. Six-place common logarithms (18 pages)
2. Table of Random Units (8 pages)
3. Legendre Function formulas (3 pages)
4. Surface Zonal Harmonics, $P_n(x)$ (10 pages)
5. Definitions of Concepts in Set Theory, Groups, Fields, etc. (7 pages)
6. Planetary Elliptic Orbit Theory (9 pages)
7. Binomial and Poisson Distributions (16 pages)
8. Dictionary of Curves and Surfaces (8 pages).

There exists, as in the older editions [1], a certain amount of disorder in the sequencing of the tables. Although the table of Factors and Primes is no longer found between the Elliptic Integrals and the table of Indefinite Integrals, as it was in the 10th edition of the CRC, some new disorder has entered. Thus, tables of factorials occur on page 209 and much later on page 260, the degree-radian tables are not adjacent to the trigonometric, the algebraic formulas and the (often duplicating) "miscellaneous" algebraic formulas are separated by the Planetary Orbits, and the Probable Error and χ^2 tables are far removed from other statistical tables.

Although some of the tables, such as the 23-page table of $\sin^2 \theta$, $\cos^2 \theta$, and $\sin \theta \cos \theta$, may strike certain readers as not being of urgent utility, the collection as a whole is certainly very useful.

D. S.

1. *CRC Standard Mathematical Tables*, Tenth & Eleventh Editions, RMT **61**, *MTAC*, v. 12, 1958, p. 146.

35[D].—Herbert E. Salzer & Norman Levine, *Table of Sines and Cosines to Ten Decimal Places at Thousandths of a Degree*, Pergamon Press, New York, 1962, xiv + 900 p. (unnumbered), 22 cm. Price $10.00.

This extensive table is a compilation of electronically computed 10D values of sines and cosines, without differences, arranged semiquadrantally in adjacent columns, at an increment of 0.001° in the argument. Each entry is printed *in extenso*, thereby obviating the necessity of searching generally elsewhere on the page for the leading three digits, as in other large tables of this kind.

In the introductory text Dr. Salzer shows in detail that linear interpolation in this table yields results correct to within a unit in the tenth decimal place. He cites the 15D table of sines and cosines published by the National Bureau of Standards [1], corresponding to an interval of 0.01°, which permits the attainment of only 8D accuracy when linear interpolation is used. Reference is also made to a table of Peters [2], which includes also the tangent and cotangent; it is arranged according to the subdivision 0.001°, but is limited to seven decimal places.

These limitations in earlier tables constitute the stated justification for the preparation of the present table. It seems appropriate to note here that a proposal for a similar table was made by O. Kohl [3] in 1953, using basic data computed by Peters.

The user of this table will undoubtedly read with profit the detailed discussion of both direct and inverse linear interpolation, including the use of both Lagrange's formula and Taylor's theorem, which is supplemented by a total of ten numerical illustrations. On the other hand, the user will vainly search in this book for a description of the procedures followed in the calculation and checking of the tabular data. Furthermore, he will probably be somewhat disconcerted to discover at the beginning of the table an inserted slip advertising several errors, the most conspicuous occurring in sin 30°!

Through correspondence with Dr. Salzer this reviewer learned the following details relating to the preparation of this table. Dr. Levine used a computer program based on Maclaurin series to obtain 15D values of sine and cosine at multiples of 0.001°, which were rounded to 10D on the computer and stored on tape preliminary to printout. A similar table was subsequently calculated by Ward Hardman on another electronic computer, using a different double-precision program, involving the use of key values in conjunction with the appropriate addition formulas. Proofreading of both versions of the table was performed by Dr. Salzer, who thereby found no error in the duplicate table of Hardman.

The isolated error in sin 30° was apparently caused by an error in the routine for converting the computer output from binary to decimal form. The error in sin 38.441° noted on the errata slip is clearly attributable to a typographical imperfection, whereas the error noted in sin 42.055° was caused by a careless hand-correction of a partially obliterated digit when this table was printed in Poland. Neither of these last two errors appeared in the original computer output.

A number of additional examples of annoying typographical imperfections are to be found, notably in cos 2.268° and sin 38.438°, where individual digits are nearly obliterated. Despite these defects, this unique table should be very useful and reliable, after the necessary emendations have been made. Especially welcome would be a second printing, of improved quality, incorporating the known corrections.

J. W. W.

1. National Bureau of Standards, *Table of Sines and Cosines to Fifteen Decimal Places at Hundredths of a Degree*, Applied Mathematics Series, No. 5, U. S. Government Printing Office, Washington, D.C., 1949.

2. J. Peters, *Seven-Place Values of Trigonometric Functions for Each Thousandth of a Degree*, Van Nostrand, New York, 1942.

3. A. Fletcher, J. C. P. Miller, L. Rosenhead, & L. J. Comrie, *An Index of Mathematical Tables*, Second Edition, Addison-Wesley, Reading, Massachusetts, 1962. (See Vol. I, Art. 7.2, p. 173.)

36[E].—H. C. Spicer, *Tables of the Ascending Exponential Function* e^x, U. S. Geological Survey, Washington 25, D. C. Deposited in UMT File.

This manuscript is in the form of original computation sheets. It contains the values of e^x with x ranging in value as follows: [0(0.0001)1] 21D; [1(0.001)6.963] 24D; [6.96(0.01)15.80] 24D.

On each sheet the column indicated as x, the argument, is followed immediately on the same line with the 25-decimal-place value of e^x. The four sets of values just beneath the tabular e^x are to be disregarded, as they were obtained as parts of the computational procedure. The vertical lines drawn in divide the decimal part of the table into groups of five places.

Values indicated by a check have been recalculated either by another computer or by comparison with previous tabulations.

AUTHOR'S SUMMARY

37[E].—H. C. SPICER, *Tables of the Descending Exponential Function* e^{-x}, U. S. Geological Survey, Washington 25, D. C. Deposited in UMT File.

This manuscript is in the form of original computation sheets. It contains the values for e^{-x} with x ranging in value as follows: [0(0.0001)1] 25D; [1(0.001) 3.923] 25D; [3.923(0.01)10] 25D.

On each sheet the column indicated as x, the argument, is followed immediately on the same line with the 25-decimal-place value of e^{-x}. All of the values tabulated between two tabular values of e^{-x} are not to be used, as they were obtained as parts of the computational procedure.

The values indicated by C. K. at each 0.0005 mid-value are check values obtained by an additional computation. The difference between the two values is only indicated for the digits at the end of the value.

The values indicated by T. V. are comparison values from previous tabulations. The difference, as before, is only indicated for the end digits.

AUTHOR'S SUMMARY

38[F].—ROBERT SPIRA, *Tables Related to* $x^2 + y^2$ *and* $x^4 + y^4$. Five large manuscripts deposited in UMT files.

The following three tables have been computed:

1. All representations of $p^k = a_i^2 + b_i^2$, where p is a prime $\equiv 1 \pmod 4$ and $p < 1000$. The k's are such that max $(a_i, b_i) < 2^{35}$. The factorizations of a_i and b_i are also given.

2. All representations of $n = a^2 + b^2$ for $n < 122{,}500$. Also given are the factorizations of n, a, and b. The table continues to $n = 127{,}493$ but is not complete here, since a and b are always less than 350. Francis L. Miksa [1] has previously given the representations of the odd $N < 100{,}000$; as he explains in his introduction, the even N are easily derived from these. Miksa did not give the factorizations of n. It is not clear why Spira factors a and b also.

3. All representations of $n = a^4 + b^4$ for a and $b \leqq 350$. The table is thus complete for $n < 351^4 = 15{,}178{,}486{,}401$ but continues up to $n = 350^4 + 350^4$. Also given are the factorizations of n, a, and b.

This last table was searched for solutions of

$$U^4 + V^4 = W^4 + T^4,$$

and only the three known solutions, for U, V, W, and $T \leqq 350$, were found. This confirms the result of Leech [2]. The author adds that there is no solution of $U^5 + V^5 = W^5 + T^5$ for U, V, W, and $T \leqq 110$.

The calculations were done using a sorting routine on an IBM 704 in the University of California Computer Center.

D. S.

1. FRANCIS L. MIKSA, *Table of quadratic partitions* $x^2 + y^2 = N$, RMT **83**, *MTAC*, v. 9, 1955, p. 198.
2. JOHN LEECH, "Some solutions of Diophantine equations," *Proc. Cambridge Philos. Soc.*, v. 53, 1957, p. 778–780.

39[F].—DAVID C. MAPES, *Fast Method for Computing the Number of Primes less than a Given Limit*, Lawrence Radiation Laboratory Report UCRL-6920, May 1962, Livermore, California. Table of 20 pages deposited in UMT File.

This report is the original writeup of [1]. The table in [1] gives $\pi(x)$, $Li(x)$, $R(x)$, $L(x) - \pi(x)$ and $R(x) - \pi(x)$ for $x = 10^7(10^7)10^9$, where $\pi(x)$ is the number of primes $\leqq x$, and $Li(x)$ and $R(x)$ are Chebyshev's and Riemann's approximation formulas. The table here gives the same quantities for $x = 10^6(10^6)10^9$. It thus has greater "continuity," but not enough to trace the course of $\pi(x)$ unequivocally.

For example, Rosser and Schoenfeld [2] have recently proved that $\pi(x) < Li(x)$ for $x \leqq 10^8$. While it is highly probable that this inequality continues to $x = 10^9$, the gaps here, of $\Delta x = 10^6$, would appear to preclude a *rigorous* proof at this time. Study of the table, however, shows no value of x for which $\pi(x)$ approaches $Li(x)$ sufficiently close to arouse much suspicion. The relevant function is

$$PI(x) = \frac{Li(x) - \pi(x)}{\sqrt{x}} \log x,$$

and for $313 \leqq x \leqq 10^8$, Appel and Rosser [3] showed a minimum value of $PI(x)$, equal to 0.526, at $x = 30{,}909{,}673$. Here (and also in [1]) one finds values of 0.615 and 0.543 at $x = 110 \cdot 10^6$ and $180 \cdot 10^6$, respectively. It is thus likely that a value of $PI(x)$ less than 0.526 can be found in the neighborhood of these x (especially the second), but it is unlikely that $PI(x)$ becomes negative there. The relevant theory [4] is made difficult by incomplete knowledge of the zeta function. In the second half of the table, $x > 500 \cdot 10^6$, no close approaches at all are noted, and $Li(x) - \pi(x)$ exceeds 1000 there, except for $x = 501 \cdot 10^6$, $604 \cdot 10^6$, and $605 \cdot 10^6$.

The low values of $PI(x)$ are always associated with the condition $\pi(x) > R(x)$. The largest value of $R(x) - \pi(x)$ shown here is $+914$, for $x = 905 \cdot 10^6$.

D. S.

1. DAVID C. MAPES, "Fast method for computing the number of primes less than a given limit, "*Math. Comp.*, v. 17, 1963, p. 179–185.
2. J. BARKLEY ROSSER AND LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, p. 64–94.
3. KENNETH I. APPEL AND J. BARKLEY ROSSER, *Table for Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.*, v. 16, 1962, p. 500–501.
4. A. E. INGHAM, *The Distribution of Prime Numbers*, Cambridge Tract No. 30, Cambridge University Press, 1932.

40[F].—J. BARKLEY ROSSER & LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, Tables I-IV on p. 90–93.

The four number-theoretic tables reviewed here were presented by the authors in connection with their proofs of numerous inequalities concerning the distribution of primes. These inequalities include

$$\frac{x}{\log x}\left(1 + \frac{1}{2 \log x}\right) < \pi(x) < \frac{x}{\log x}\left(1 + \frac{3}{2 \log x}\right) \qquad (59 \leqq x),$$

$$\frac{x}{\log x - \frac{1}{2}} < \pi(x) < \frac{x}{\log x - \frac{3}{2}} \qquad (67 \leqq x),$$

$$n\,(\log n + \log\log n - \tfrac{3}{2}) < p_n < n\,(\log n + \log\log n - \tfrac{1}{2}) \qquad (20 \leqq x),$$

and

$$li(x) - li(\sqrt{x}) < \pi(x) < li(x) \qquad (11 \leqq x \leqq 10^8).$$

Table II is an excerpt from a table computed several years earlier by Rosser and R. J. Walker on an IBM 650. It lists the four functions, $\theta(x) = \sum_{p \leqq x} \log p$, $\sum_{p \leqq x} p^{-1}$, $\sum_{p \leqq x} p^{-1} \log p$, and $\prod_{p \leqq x} p/(p - 1)$ to 10D for $x = 500(500)16{,}000$. Here the sums and the product are taken over the primes not exceeding x. For larger values of x see [1].

Table III lists $\psi(n) - \theta(n)$ to 15D for each of the 84 values of n equal to a prime power, $p^a (a > 1)$, which is less than $50{,}653 = 37^3$. The function $\psi(x) - \theta(x)$ remains constant between such prime-powers, and at such numbers the function increases by a jump equal to $\log p$.

Table I (which, for convenience, we describe out of order) is concerned with bounds on $\psi(x)$. The table lists 120 pairs of numbers, ϵ and b, such that

$$(1 - \epsilon)x < \psi(x) < (1 + \epsilon)x \quad \text{for} \quad e^b < x.$$

For example, $\epsilon = 4.0977 \cdot 10^{-6}$ for $b = 4900$.

Finally, Table IV lists $-\zeta'(n)$, $-\zeta'(n)/\zeta(n)$, and $\sum_p p^{-n} \log p$ to 17D for $n = 2(1)29$. Here ζ is the Riemann zeta function. This table was computed with the Euler-Maclaurin formula on an electronic computer. The values obtained were checked by a different computation and agree with Walther's 7D table of $-\zeta'(n)/\zeta(n)$ [2], and Gauss's 10D value of $-\zeta'(2)$ [3]. The purpose in computing Table IV was to use it in evaluating the limit:

$$\sum_{p \leqq x} p^{-1} \log p - \log x \to -1.33258227573322087.$$

However, Table IV certainly has other uses; for example, the reviewer has recently used it in [4].

D. S.

1. Kenneth I. Appel and J. Barkley Rosser, *Table for Estimating Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.* v. 16, 1962, p. 500–501.

2. A. Walther, "Anschauliches zur Riemannschen Zetafunktion," *Acta. Math.*, v. 48, 1926, p. 393–400.

3. C. F. Gauss, *Recherches Arithmétiques*, Blanchard, Paris, 1953, p. 370.

4. Daniel Shanks, "The second-order term in the asymptotic expansion of $B(x)$," *Notices*, Amer. Math. Soc., v. 10, 1963, p. 261, Abstract 599–46. For errata see *ibid.*, p. 377.

41[F, G, X].—Gabor Szegö et al, Editors, *Studies in Mathematical Analysis and Related Topics—Essays in Honor of George Pólya*, Stanford University Press, Stanford, 1962, xxi + 447 p., 25 cm. Price $10.00.

This substantial volume, consisting primarily of sixty new research papers by leading mathematicians, was published on December 13, 1962, Professor Pólya's seventy-fifth birthday. The topics are of a great variety and include analysis, topology, algebra, number theory, and applied mathematics. While many of the papers begin with an opening paragraph that mentions some related work of Pólya, they have no other common theme.

There is included a list of Pólya's 217 papers (up to 1961) and his six books. We learn that *How To Solve It* has been translated into Arabic, Croatian, French, German, Hebrew, Hungarian, Japanese, and Russian (so far).

Even the table of contents is too long to reproduce here, and we merely list the many authors. They form, to misuse a definition of Alexander Weinstein, a *distinguished sequence*: L. V. Ahlfors, N. C. Ankeny, H. Behnke, S. Bergman, A. S. Besicovitch, R. P. Boas, Jr., A. Brauer, R. Brauer, H. S. M. Coxeter, H. Cramér, H. Davenport, B. Eckmann, A. Edrei, A. Erdélyi, P. Erdös, W. H. J. Fuchs, T. Ganea, P. R. Garabedian, H. Hadwiger, W. K. Hayman, J. Hersch, E. Hille, P. J. Hilton, J. L. Hodges, Jr., A. Huber, A. E. Ingham, M. Kac, J. Karamata, S. Karlin, J. Korevaar, C. Lanczos, P. D. Lax, E. L. Lehmann, D. H. Lehmer, E. Lehmer, P. Lévy, J. E. Littlewood, C. Loewner, E. Makai, S. Mandelbrojt, N. Minorsky, Z. Nehari, J. Neyman, L. E. Payne, M. Plancherel, J. Popken, H. Rademacher, A. Rényi, J. Robinson, R. M. Robinson, W. W. Rogosinski, P. C. Rosenbloom, H. L. Royden, G. Scheja, M. Schiffer, I. J. Schoenberg, L. Schwartz, E. L. Scott, J. Siciak, D. C. Spencer, J. J. Stoker, J. Surányi, G. Szegö, E. C. Titchmarsh, P. Turán, J. G. Van der Corput, J. L. Walsh, H. F. Weinberger, A. Weinstein, and A. Zygmund.

D. S.

42[G].—L. E. Fuller, *Basic Matrix Theory*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962, ix + 245 p., 23 cm. Price $9.45.

The preface addresses this book to the "person who needs to use matrices as a tool." The writing style is conversational but careful. Illustrative examples are given in unusual detail. Procedures are outlined in stepwise fashion, and potential pitfalls in the application of an algorithm are red-flagged.

The attitude toward rigor is suggested by the following: many definitions are stated formally; several properties are enunciated; but nowhere are there any so-called theorems. Nevertheless, in the casual style, much worthwhile information appears, information which other authors might label as theorems. Many of these propositions are proved or made convincingly plausible. Other results whose proof would be long and/or deep are simply asserted.

Considerable attention is paid to canonical forms. The algorithms for finding these are elaborately discussed. Deeper questions concerning whether the claimed canonical forms are really entitled to be accorded such a title are quietly suppressed —an action carefully designed to keep the mathematics at "as simple a level as possible."

The first four chapters develop basic notions about matrices, vectors, and determinants; elementary row or column transformations are emphasized. The next three chapters stress computational methods; techniques for finding characteristic roots and characteristic vectors are presented; methods discussed for matrix inversion or for solution of a system of equations include those of Crout, Doolittle and Gauss-Seidel, as well as partitioning, iteration, and relaxation. The final chapter is devoted to bilinear, quadratic, and Hermitian forms. Each chapter has exercises, mostly of a practice nature.

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43[G, H, X].—Richard S. Varga, *Matrix Iterative Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963, xiii + 322 p., 23.5 cm. Price $10.00.

This book is concerned with the solution of large systems of linear algebraic equations by means of iteration methods. In particular, the aim is to develop a theory of cyclic interative methods which is particularly useful for dealing with (and which was, in fact, motivated by) systems of equations arising from discrete approximations to elliptic and parabolic boundary-value problems. Such problems give rise to systems often consisting of thousands of linear equations whose associated square matrices have mostly zero elements and whose non-zero entries occur in some regular pattern. With present-day high-speed computing machines, many problems of the above-mentioned type have been successfully treated, using methods described in this book.

Chapter I introduces some fundamental concepts in matrix theory. In particular, some eigenvalue inequalities are given and the concept of irreducibility is introduced. The very useful notion of a directed graph is given and used as a geometric description of irreducibility.

The second chapter treats the Perron-Frobenius theory of non-negative matrices, which forms the basis for some of the later material. Cyclic and primitive matrices are discussed, and graph theory is again shown to be a helpful tool.

The next chapter deals with three basic iterative methods: Jacobi, Gauss-Seidel, and Successive Overrelaxation (SOR). Rates of convergence are discussed, and in this connection the comparison theorem of Stein-Rosenberg is given. The theorem of Ostrowski-Reich, which provides necessary and sufficient conditions for convergence of SOR for Hermitian matrices, is presented.

Matrices whose inverses have only non-negative entries are discussed, and iterative methods obtained from "regular splittings" of matrices are studied.

The SOR method is discussed further in Chapter IV, particularly in connection with the theoretical determination of the optimum relaxation factor for p-cyclic matrices. This is an extension of the important work of D. Young (1950) in the case $p = 2$. Further extensions due to Kahan (1958) and Varga (1959) are given.

Chapter V describes the so-called Chebyshev semi-iterative method. The connection between this method and SOR is discussed, as well as the relation between the rates of convergence.

The next chapter is concerned with the derivation of difference equations corresponding to elliptic boundary-value problems. Several points of view are presented. The equations derived here are meant to motivate the matrix analysis, and hence no convergence proofs are given.

The widely used alternating direction implicit (ADI) methods of Peaceman and Rachford (1955) and Douglas and Rachford (1956) are presented in Chapter VII. An analysis of these methods as applied to a certain system arising from a discrete approximation to the Dirichlet problem is given. Some experiments comparing ADI with SOR are discussed at the end of the chapter.

Chapter VIII deals with matrix problems resulting from discrete approximations in parabolic problems. Various iterative methods are applied.

The final chapter is concerned with estimation of acceleration parameters. Numerical examples are given in Appendices A and B.

Professor Varga's book is very clearly written and contains a large amount of

There is included a list of Pólya's 217 papers (up to 1961) and his six books. We learn that *How To Solve It* has been translated into Arabic, Croatian, French, German, Hebrew, Hungarian, Japanese, and Russian (so far).

Even the table of contents is too long to reproduce here, and we merely list the many authors. They form, to misuse a definition of Alexander Weinstein, a *distinguished sequence*: L. V. Ahlfors, N. C. Ankeny, H. Behnke, S. Bergman, A. S. Besicovitch, R. P. Boas, Jr., A. Brauer, R. Brauer, H. S. M. Coxeter, H. Cramér, H. Davenport, B. Eckmann, A. Edrei, A. Erdélyi, P. Erdös, W. H. J. Fuchs, T. Ganea, P. R. Garabedian, H. Hadwiger, W. K. Hayman, J. Hersch, E. Hille, P. J. Hilton, J. L. Hodges, Jr., A. Huber, A. E. Ingham, M. Kac, J. Karamata, S. Karlin, J. Korevaar, C. Lanczos, P. D. Lax, E. L. Lehmann, D. H. Lehmer, E. Lehmer, P. Lévy, J. E. Littlewood, C. Loewner, E. Makai, S. Mandelbrojt, N. Minorsky, Z. Nehari, J. Neyman, L. E. Payne, M. Plancherel, J. Popken, H. Rademacher, A. Rényi, J. Robinson, R. M. Robinson, W. W. Rogosinski, P. C. Rosenbloom, H. L. Royden, G. Scheja, M. Schiffer, I. J. Schoenberg, L. Schwartz, E. L. Scott, J. Siciak, D. C. Spencer, J. J. Stoker, J. Surányi, G. Szegö, E. C. Titchmarsh, P. Turán, J. G. Van der Corput, J. L. Walsh, H. F. Weinberger, A. Weinstein, and A. Zygmund.

D. S.

42[G].—L. E. Fuller, *Basic Matrix Theory*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962, ix + 245 p., 23 cm. Price $9.45.

The preface addresses this book to the "person who needs to use matrices as a tool." The writing style is conversational but careful. Illustrative examples are given in unusual detail. Procedures are outlined in stepwise fashion, and potential pitfalls in the application of an algorithm are red-flagged.

The attitude toward rigor is suggested by the following: many definitions are stated formally; several properties are enunciated; but nowhere are there any so-called theorems. Nevertheless, in the casual style, much worthwhile information appears, information which other authors might label as theorems. Many of these propositions are proved or made convincingly plausible. Other results whose proof would be long and/or deep are simply asserted.

Considerable attention is paid to canonical forms. The algorithms for finding these are elaborately discussed. Deeper questions concerning whether the claimed canonical forms are really entitled to be accorded such a title are quietly suppressed —an action carefully designed to keep the mathematics at "as simple a level as possible."

The first four chapters develop basic notions about matrices, vectors, and determinants; elementary row or column transformations are emphasized. The next three chapters stress computational methods; techniques for finding characteristic roots and characteristic vectors are presented; methods discussed for matrix inversion or for solution of a system of equations include those of Crout, Doolittle and Gauss-Seidel, as well as partitioning, iteration, and relaxation. The final chapter is devoted to bilinear, quadratic, and Hermitian forms. Each chapter has exercises, mostly of a practice nature.

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significance points for the 2×3 contingency table for $A = 3(1)20$ at levels of significance $\alpha = 0.05, 0.025, 0.01$, and 0.001, respectively, when $a = [\frac{3}{2}A]$, which by symmetry includes all significant combinations if the categories of Success and Failure are interchanged.

AUTHORS' SUMMARY

46[K].—SAMUEL S. WILKS, *Mathematical Statistics*, John Wiley & Sons, New York, 1962, 23.5 cm., xvi + 644 p. Price $15.00.

This long awaited book by Professor Wilks is written primarily as a text on the graduate mathematics level. A brief summary of its contents will best indicate the wide extent of the material it covers. The first part of the book is concerned with standard topics. Chapter 1 gives a condensed treatment of probability measures and spaces, followed by chapters on distribution functions, expected values and moments, special distributions, characteristic functions, and limit theorems. Each of these subjects is covered in considerable detail for a general-purpose mathematical statistics text. For instance, the chapter on mean values and moments is composed of 24 pages of tightly packed exposition on this topic for one-dimensional random variables, two-dimensional random variables, k-dimensional random variables, linear functions of a random variable, conditional random variables and, finally, least-squares regression.

The book then has chapters on sampling theory, asymptotic sampling theory, linear estimation, nonparametric estimation, and parametric estimation. Separate chapters treat the testing of parametric and nonparametric hypotheses. At the end there are introductory chapters on sequential analysis, decision functions, time series, and multivariate theory.

The outstanding feature of this book is its extensive and detailed, yet unified, coverage of material. It gives enough topics for a full-year graduate course, with much to spare. In this way the book fills a real need for instructors who want a more modern treatment and selection of topics, and less measure theory, than Harald Cramér's 1946 classic has, but who still want a graduate text that gives a thorough grounding in fundamentals along with plenty of "elbow room".

However, this bounty comes at a price, namely, the neglect of discussion on the statistical aspects of the mathematical theory. The author himself states in the preface that, in order to give a proper mathematical treatment, no attempt was made to discuss the statistical methodology for which the mathematics is being developed. Hence, this book is better described as a treatise on the "mathematics of statistics" rather than "mathematical statistics". There is a lack of motivation and emphasis, and the novice mathematical statistician will not know where he is going or why.

For example, in the chapter on special distributions ten pages are devoted to introducing the beta distribution and its multivariate form, the Dirichlet, while a total of only five pages are used for the χ^2-, the t-, and the F-distributions. Without any guiding discussion on their relative importance and usefulness in later work, the student is likely to be misled and will spend a disproportionate amount of energy on the beta and Dirichlet distributions. But here we also see an example of the value of the book; the section on the beta and Dirichlet distributions is preliminary to an unusually thorough treatment of order statistics.

Although it is supposed to be self-contained in the sense that no prior knowledge of mathematical statistics is necessary to follow the mathematical argumentation, this text will present insurmountable obstacles for anyone who is not already well acquainted with statistical method and practice, or who is not being guided by a skillful teacher. Moreover, the mathematical argumentation is itself condensed and frequently difficult, so that there will be further difficulty for all but the most mathematically mature readers.

This book is well referenced with frequent comments about related research and extensions of topics under discussion. An especially convenient feature is the inclusion of "backward" references in the bibliography, that is, after each book or paper the text page number on which it was referenced is listed. Many excellent problems are included after every chapter. Some awkwardness results from the author's attempt to preserve consistent notation throughout the book. In particular, no notational distinction is made between a random variable and the corresponding real variable, and reading some sections, such as those on conditional random variables, becomes a guessing game for anyone who does not know the material beforehand. Also regrettable is Professor Wilks' decision not to use matrix notation for the multivariate work.

There are quite a few mistakes in the text; some places the argumentation even goes astray, so it is hoped that corrigenda will be forthcoming, and eventually a second edition.

In short, this book is a comprehensive text written uncompromisingly for the graduate student of mathematical statistics. It is exceptionally useful because of its detailed coverage of topics, but it needs to be supplemented either by a teacher who provides direction and motivation, or by previous experience in the field. The book will be widely used as a text for graduate mathematical statistics courses for students with strong backgrounds in mathematics and some undergraduate training in statistics. It will also be an invaluable reference text for mathematical statisticians.

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47[K, W].—R. L. Ackoff, with the collaboration of S. K. Gupta & J. S. Minas, *Scientific Method: Optimizing Applied Research Decisions*, John Wiley & Sons, New York, 1962, xii + 464 p., 23.5 cm. Price $10.25.

This book is a study "in the large" of scientific activity and is intended to aid the scientist in the evaluation of his own research procedures by making use of Decision Theory.

The book consists of fifteen chapters and two appendices and a large number of references. The first chapter gives a delightful discussion of science and its methods. The discussion of optimal solutions to problems in the second chapter has a broad philosophical scope, using ideas from statistics, decision theory, and game theory.

The formulation of the problem discussion in the third chapter introduces much of the author's knowledge and practice of operations research, and is a valuable contribution, as are the chapters on Models, Defining and Measurement. The next four chapters are on Sampling, Estimation, Testing Hypotheses, and Experimentation and Correlation. The next chapter is on Deriving Solutions from Models, in which the author briefly mentions analytical and numerical methods, simulation, Monte Carlo Methods, statistical techniques, and operational gaming with its limitations. In Chapter 12 the author returns to optimization in experimental form, for example, through simulation. This is followed by chapters on Testing and Controlling the Model and Solution; on Implementation and Organization of Research, in which he points out that a solution, partly because of prestige of science, already involves the scientist as having made a recommendation; and finally on The Ideals of Science and Society: An Epilogue.

The wisdom of the author as a director of research scientists from different fields of specialization tremendously enriches the last two chapters. The book in its vastness should provide source material for evaluation of research and inspiration to those desiring a clear perspective of scientific activity.

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48[K, W].—Harvey M. Wagner, *Statistical Management of Inventory Systems*, John Wiley & Sons, Inc., New York, 1962, xiv + 235 p., 23 cm. Price $8.95.

Inventory models have probably been the subject of more mathematical analysis in the operations research literature than any other type of application, with the possible exception of queueing models.

In a well organized and cogently written monograph, the author has extended the mathematical analysis of the inventory problem to that of instituting appropriate management controls.

The book is divided into four chapters. The first and smallest chapter provides an introductory framework for the main text. Chapter 2, by far the longest, is an excellent exposition of the (s, S) model. An (s, S) inventory policy is one which prescribes two numbers, s and S, which might be termed the reorder point and the maximum stock level. Specifically, if the stock on hand and on order has fallen to a level $x \leqq s$, then an amount $S - x$ is ordered to return to the level S. The purpose of Chapter 2 is to provide the mathematical tools to analyze control mechanisms.

Chapter 3 is the heart of the contribution made by the monograph. The author analyzes various statistical indices which may serve as controls for management. He contrasts barometer controls of the form $B = \theta$ (index number-target), where $\theta > 0$, with quota controls, which implies some limit on the index number itself. The barometer control implies a system of rewards and punishments based on the value of B. Quota control, as defined by Wagner, is dichotomous in the sense that if some limit were violated, it is presumed that the policy has not been followed.

A basis for the author's choice of the barometer control is consistency; which refers to the control system's encouragement given to the observance of the standards of an operating (s, S) policy. "A consistent control scheme is one in which the probability of exceeding the index limit is greater when violations of standards are present than when they are not." Chapter 4 extends the results of the previous chapter to cases where the distribution of demand varies.

The various chapters are fully illustrated by numerical examples, many of which are based on Monte Carlo runs of 10,000 periods of operation. The book is a worthy member of Wiley's publications in Operations Research.

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49[L].—GARY D. BERNARD & AKIRA ISHIMARU, *Tables of the Anger and Lommel-Weber Functions*, Technical Report No. 53, AFCRL 796, University of Washington Press, Seattle, 1962, ix + 65 p., 28 cm. Price $2.00.

These important tables result from work on electromagnetic theory. They were computed on an IBM 709 at the Pacific Northwest Research Computing Laboratory of the University of Washington, with support from the Boeing Company, Seattle and the Air Force Cambridge Research Laboratories, Bedford, Mass. The functions tabulated are the Anger functions.

$$J_\nu(x) = \frac{1}{\pi} \int_0^\pi \cos(\nu\theta - x \sin\theta)\, d\theta$$

and the Lommel-Weber functions

$$E_\nu(x) = \frac{1}{\pi} \int_0^\pi \sin(\nu\theta - x \sin\theta)\, d\theta.$$

When ν is an integer n, the Anger function reduces to the Bessel function $J_n(x)$.

Both functions are tabulated to 5D, without differences, for $\nu = -10(0.1)10$, $x = 0(0.1)10$. Tables for negative x are unnecessary, since changing the sign of both ν and x leaves J unchanged and merely changes the sign of E. There are graphs of both functions against ν and contour maps of both functions in the (ν, x) plane. An appendix contains an IBM 709 FORTRAN program for computing the functions.

Previous tables of the Anger functions (other than the Bessel functions for integral ν) are exceedingly slight. Rather more has been done on the Lommel-Weber functions; see FMRC *Index* [1]. The concise and handy tables of Bernard and Ishimaru now establish both functions firmly in the repertoire of numerically available functions.

Precision is stated to be ± 1 in about the last (fifth) decimal place. If this is taken to mean that the tabular values are always within about one final unit of the true values, the statement appears to be true, but does less than justice to the accuracy of the tables. With perfect rounding of the normal kind, tabular values lie within half a final unit of the true values, and it might be thought that the distribution of the rounding errors in the present tables has twice the perfect scatter. As far as one can judge, this is not so.

Only a small fraction of the tabular values can be compared with values already available, but the chief comparisons which are possible have been carried out by the reviewer, in order to test his hypothesis that the vast majority of the tabular values for $\nu \geqq -\frac{1}{2}$ are correctly rounded according to a different convention. This is that positive values are rounded upwards, and negative values are rounded (numerically) downwards; in other words, that in both cases the tabular values are algebraically greater than the true values, and that the part of the tables mentioned ($\nu \geqq -\frac{1}{2}$) has a positive bias of about half a final unit. If the tabular values conformed perfectly to this convention, the part of the tables mentioned would be just as accurate, properly interpreted, as tables perfectly rounded on the normal convention. Actually, the tabular values checked (which may not be representative, having been selected for comparability with values already available) differ from the true values by between 1 and about $1\frac{1}{4}$ final units in a very small percentage of cases; they are thus comparable in accuracy with values in a normally rounded table "imperfect" to the extent of having a very small percentage of rounding errors lying between $\frac{1}{2}$ and about $\frac{3}{4}$ of a final unit. It is because of the positive bias that $E_{\frac{1}{2}}(0)$ and $E_{-\frac{1}{2}}(0)$, which equal $\pm 2/\pi$, appear as $+0.63662$ and -0.63661 respectively; on the normal convention, the latter is one final unit out, the true digits to 8D being 63661 977.

As far as the Anger functions are concerned, the 1111 values of $J_\nu(x)$ for $\nu = 0(1)10$, $x = 0(0.1)10$ have been read against the 10D British Association values [2] of Bessel functions. There are ten cases in which the tabular value differs from the true value by more than one final unit. These are for:

ν	0	0	0	1	3	3	4	4	5	6
x	8.5	9.4	9.9	9.2	9.8	9.9	9.1	9.5	9.4	9.3

The largest of these discrepancies is for $J_3(9.9)$, where the B.A. tables give $+0.03431\ 83264$ and the present tables have $+0.03433$, so that the difference is less than 1.17 final units. This is so slight an excess over unity that it does not seem worth while to set out details for the other nine cases. There are also 32 cases in which the tabular value is correctly rounded by the ordinary convention, instead of by the hypothetical one. Four of these cases are for $x \geqq 8.2$, and the other 28 are for small x, where the rounding of the very small values of J is sometimes to 0.00000 and sometimes to 0.00001; the latter is given for $J_9(0) = 0$. The six 6D values of $J_\nu(\nu)$ for $\nu = 0(0.1)0.5$ given in Brauer & Brauer [3] make possible five additional comparisons, valuable because they are for non-integral ν; the case $J_0(0) = 1$ has already been included above. The five values for $\nu = 0.1(0.1)0.5$ are all positive, and all are rounded upwards in the present tables.

It must be added that the values of $J_{-n}(x)$ given for $n = 2(2)10$ are those given for $J_n(x)$, and for $n = 1(2)9$ are those given for $J_n(x)$ with the signs changed. Thus the bias of the rounding is reversed for odd negative integral ν, but not for even negative integral ν. This shows that the rounding hypothesis being tested would fail if it were extended to include $\nu = -1$ (but it will be seen to be valid for Lommel-Weber functions at $\nu = -\frac{1}{2}$).

As far as the Lommel-Weber functions are concerned, 439 different values of $E\nu(x)$ were read against values of

$$E_\nu(\nu),\ E_{\nu-1}(\nu);\qquad E_0(x),\ E_1(x);\qquad E_{\frac{1}{2}}(x),\ E_{-\frac{1}{2}}(x)$$

given in the British Association Reports for 1923, 1924, and 1925, respectively [4]. All these B.A. values are to 6D, but with indication of halves of a final unit. We omit 24 cases in which the B.A. value ends in an unqualified zero (and hence is useless for determining the hypothetical rounding, just as a final 5 would be useless for determining the normal rounding). In the remaining 415 comparisons, there are five discrepancies greater than one final unit at:

ν	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
x	8.4	9.9	9.0	9.6	9.9

The greatest discrepancy is at $\nu = 1$, $x = 9.9$, where the B.A. value (for $E = -\Omega$) is $-0.251012\frac{1}{2}$, and the present tables have -0.25100, a difference of about $1\frac{1}{4}$ final units. The accuracy of the B.A. tables appears to be excellent, but a 6D table only partially investigated cannot provide quite the check that a good 10D table does. Nevertheless, this provides further reason to think that about one per cent of the Bernard and Ishimaru values differ from the true values by more than one final unit. There are also four cases in which the rounding is correct by the normal rule, instead of by the hypothetical rule. Of the five additional 6D values of $E_\nu(\nu)$ given in Brauer & Brauer [3], that for $\nu = 0.1$ ends in zero and so is useless for testing the hypothesis; those for $\nu = 0.2(0.1)0.5$ are all positive and all rounded upwards in Bernard & Ishimaru, so that they conform to the hypothesis.

The discussion given above is unavoidably partial and incomplete for $\nu \geqq -\frac{1}{2}$, while for $\nu < -\frac{1}{2}$ it merely shows that the hypothesis needs modifying, without discovering how it should be modified. One would welcome some statement by the authors on a subject which might on occasion be of great interest to users of the tables. Lacking information, users will presumably have either to delve into analytical details and the FORTRAN program, or to accept rounding uncertainties of a size which an authoritative statement might almost halve. The work involved in the discussion, tentative as it is, has been felt to be worthwhile, because those who are interested in special higher mathematical functions are likely to rank the tables of Bernard and Ishimaru among the most important produced in that field since automatic computers began to contribute.

A. F.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, second ed., vol. 1, 1962, p. 458. Blackwell, Oxford, England (for scientific Computing Service, London); American ed., Addison-Wesley.
2. British Association for the Advancement of Science, *Mathematical Tables*, vol. 10, 1952, p. 180, Cambridge Univ. Press.
3. P. Brauer & E. Brauer, *Z. Angew. Math. Mech.*, vol. 21, 1941, p. 177–182, especially p. 180–181.
4. British Association for the Advancement of Science, *Reports* for 1923, p. 293; for 1924, p. 280; for 1925, p. 244. London.

50[L].—E. Paran & B. J. Kagle, *Tables of Legendre Polynomials of the First and Second Kind*, Research Report 62-129-103-R1, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, Sept. 7, 1962, i + 202 p., 28 cm.

These are tables of the functions $P_n(x)$ and $Q_n(x)$, in the usual notation. Since $P_n(x)$ and $Q_n(x) - P_n(x) \tanh^{-1} x$ are polynomials, $Q_n(x)$ is not, so that the word "functions" could profitably replace "polynomials" in the title. $P_n(x)$ is tabulated for $x = 0.001(0.001)1$ and $Q_n(x)$ for $x = 0.001(0.001)0.999$, in both cases for $n = 0(1)27$ and to 6S without differences. At $x = 1$, $Q_n(x)$ is infinite, and the numbers given in the tables are limiting values of $Q_n(x) - P_n(x) \tanh^{-1}x$, although this is not explained in the very brief accompanying text. It is a pity that the values of $P_n(0)$ and $Q_n(0)$ are not given. The tables were calculated on an IBM 7090 computer, and it is stated that the accuracy of the values is about ± 2 in the last significant figure.

The limit $n = 27$ is considerably higher than for previous tables of $P_n(x)$ and especially $Q_n(x)$, though $P_n(\cos\theta)$ has been tabulated for still higher values of n.

Most of the values of $P_n(x)$ up to $n = 16$ may be checked against a 6D table of Tallqvist [1]; this is insufficient near zeros, where Paran & Kagle keep 6S. Other checking tables also exist. A few random comparisons suggest good accuracy in the present table.

In 1945 the Admiralty Computing Service issued a useful little 5D table of $Q_n(x)$, $n = 0(1)7$, $x = 0(0.01)1$, mainly copied from Vandrey with corrections. The reviewer has compared all 792 common values with Paran & Kagle, and a few slight corrections to the ACS table are given in the appropriate section of this issue (p. 335). The table of Paran & Kagle has most often one extra figure, so that the comparison checks it only very partially indeed, but here again one has the impression of good accuracy in the Paran & Kagle values. If these values contain any errors as large as two final units, perhaps they occur for the higher values of n.

This is a "working table" rather than a definitive one, with properly rounded values, but it is important enough to make one glad that it has been classified as for unlimited circulation.

A. F.

1. H. Tallqvist, *Sechsstellige Tafeln der 16 ersten Kugelfunktionen $P_n(x)$*, *Acta. Soc. Sci. Fenn.*, Nova Ser. A, Tom II, No. 4, 43 p., Helsingfors, 1937.

51[L, M].—Yudell L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Company, New York, 1962, xv + 419 p., 23 cm. Price $12.50.

This book professes to deal with definite and indefinite integrals involving Bessel functions (and related functions) and purports to provide the applied mathematician with the basic information relating to such integrals; but, in fact, it does more than it promises. In addition to information relating to Bessel functions, it gives much useful information about other special functions, and the reader learns a great deal about the evaluation, convergent expansion, and asymptotic expansion of integrals involving functions of the hypergeometric type. Within the field of integrals involving Bessel functions, special emphasis is placed on indefinite integrals, since these are somewhat scantily treated in other well-known and easily accessible works of reference where definite integrals are more adequately covered.

Chapter I is preparatory and contains information and useful collections of formulas regarding the gamma function, generalized hypergeometric series, and Bessel functions; in the latter case including polynomial approximations useful for the numerical computation of these functions. A brief list of tables of Bessel functions is appended.

Chapter II is devoted to the integral

$$Wi_{\mu,\nu}(z) = \int_0^z t^\mu W_\nu(t)\,dt \qquad (1)$$

in which W is J, Y, $H^{(1,\,2)}$, I, or K. Since the organization of this chapter is typical of the organization of several other chapters, it is worth considering it in some detail. First the connections between the various Bessel functions, the differential equation satisfied by them, and their power series expansions are used to obtain the corresponding formulas for Ji, Yi, etc. Special cases (ν a non-negative integer) are also considered, and the expansions of

$$\int_z^\infty t^\mu W_\nu(t)\, dt$$

for small z are given in cases when the integrand is not integrable at $t = 0$. Next follow expansions in series of Bessel functions, and asymptotic expansions for large z. These lead to the values of related infinite integrals. Approximations of Bessel functions in terms of trigonometric functions are used to obtain similar approximations for Wi; and polynomial approximations to some Wi, with tabulated values of the numerical coefficients, and error bounds for stated intervals, are also given. The chapter concludes with a list of available numerical tables of Ji, Yi, Ii, Ki. Detailed derivations are not given, but usually there is a reference or enough information to enable a competent analyst to verify the results.

In Chapter III Lommel's functions $S_{\mu,\nu}(z)$ and $s_{\mu,\,\nu}(z)$ are introduced. For these, and for the Struve functions and Anger-Weber functions, a detailed list of formulas is given. These functions are used to provide alternative expressions for some Wi and related integrals. Somewhat surprisingly, there is a collection of formulas facilitating the expansion of polynomials in a Fourier-Bessel series.

Chapter IV concerns integrals of the form

$$\int^z e^{\epsilon t} t^\mu W_\nu(t)\, dt, \tag{2}$$

where $\epsilon = \pm i$ if W is a Bessel function, and $\epsilon = \pm 1$ if W is a modified Bessel function; the functions corresponding, in this case, to Lommel's functions are also introduced. The organization is similar to that of Chapters II and III.

Chapter V is very brief; it gives five reduction formulas for the integral

$$\int^z e^{-pt} t^\mu W_\nu(\lambda t)\, dt, \tag{3}$$

and then the explicit values of some two dozen special integrals of this form. The latter are also special cases of the integrals of the earlier chapters.

Chapter VI treats Airy functions and their integrals, and Chapter VII treats the incomplete gamma functions, their special cases (error functions, exponential integrals, etc.), and their integrals, in a similar fashion.

Repeated integrals of fractional order, both Riemann-Liouville and Weyl integrals, of Bessel functions form the subject of Chapter VIII. In Chapter IX we find integrals of the form (1), (2), or (3) in which, however, W_ν is a Struve function (rather than a Bessel function). Chapter X is devoted to the integrals

$$\int_0^z e^{it} J_0(\lambda t)\, dt, \qquad \int_0^z e^{it} Y_0(\lambda t)\, dt$$

("Schwarz functions") and to their generalization

$$\int_0^1 e^{i\omega t}(1 - t)^\delta t^\mu J_\nu(\beta t)\, dt.$$

Chapter XI lists a great many indefinite integrals whose integrands contain a product of two Bessel functions, or a product of a Bessel function and a Struve function. The integrals are evaluated by utilizing the differential equations satisfied by these functions. There is also a brief section on an integral over a product of three Bessel functions.

The last chapter on indefinite integrals, Chapter XII, contains a miscellany of integrals that do not fit into the classification of earlier chapters. Some samples are:

$$e^{-y} \int_0^x e^{-t} I_0(2(yt)^{1/2})\, dt; \qquad \int^z e^{-a}(\alpha t^2 + \beta)^{-2} t^\nu J_{\nu-1}(S)\, dt,$$

where $2\alpha(\alpha t^2 + \beta) = \gamma(t^2 - 1)$, $S(\alpha t^2 + \beta) = \gamma t$, and α, β, γ are independent of t; and similar and related integrals.

Many definite integrals can be obtained from the indefinite integrals of the earlier chapters; others, which cannot be so obtained are collected in Chapter XIII. In view of existing collections, the author emphasizes the numerical and analytical results obtained since about 1945 and 1950, respectively, but the more important earlier results are also included. Some of the groups of definite integrals listed here are: integrals expressing orthogonal properties (in Fourier-Bessel and Neumann series); convolution integrals (including Sonine's and related integrals); Lommel's functions of two variables; Hankel's, Weber's, Weber-Schafheitlin's, Sonine-Gegenbauer's, and related infinite integrals; infinite integrals involving products of Bessel functions; integrals with respect to the order. In many of these cases references to numerical tables are given in addition to analytical results. There is also a brief section on dual and triple integral equations.

Chapter XIV (38 p.) contains a useful collection of numerical tables of Bessel functions (10 tables) and their integrals (2 tables); these are extracted from published works.

A bibliography of 18 pages, an index of notation, author index, and subject index complete the volume.

The volume is reproduced by photo offset from typed copy. Both the typing and the reproduction are excellent.

The value of such a compilation depends essentially on how practical the grouping of integrals will prove in actual use, how easy it is (for a non-expert) to find a given integral, and how well the author succeeded in keeping down the number of (inevitable) misprints. Meanwhile, the first impression is decidedly favourable, and there is every prospect of the book becoming a valuable work of reference.

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52[L, M].—H. C. Spicer, *Tables of the Inverse Probability Integral*

$$P = \frac{2}{\sqrt{\pi}} \int_0^\beta e^{-\beta^2}\, d\beta,$$

U. S. Geological Survey, Washington 25, D. C. Deposited in the UMT File.

This manuscript is in the form of original computation sheets. It contains inverse values for $P = \frac{2}{\sqrt{\pi}} \int_0^{\beta} e^{-\beta^2} \, d\beta$: with P ranging in value as indicated: [0(0.0001) 0.9] 9D; [0.9(0.00001)0.99997]9D.

On each sheet the column indicated by P, the argument, is followed immediately on the same line with the 9-decimal-place value for beta. The values following the beta values on the intervening lines, indicated as $\Delta_1 P$, are the differences between successive pairs of beta values. Only the last decimal place is tabulated in the argument except at the 0.0010's values, where all are given. The digits in the values of beta to the right of the decimal point are omitted when they continue downward the same in the tabulation, except at the 0.0010's values, where all are given.

AUTHOR'S SUMMARY

53[L, S].—JOSEPH HILSENRATH & GUY G. ZIEGLER, *Tables of Einstein Functions—Vibrational Contributions to the Thermodynamic Functions*, National Bureau of Standards, Monograph 49, U. S. Government Printing Office, Washington, D. C., 1962, vii + 258 p., 26 cm. Price $2.75.

It is a well-known theorem of statistical mechanics that a harmonic oscillator (or any degree of freedom of a complex molecule quantized in the same way) contributes the following to the (Gibbs) free energy, enthalpy, entropy, and heat capacity:

$$-(F^\circ - E_0^\circ)/RT = -\ln(1 - e^{-x})$$

$$(H^\circ - E_0^\circ)/RT = xe^{-x}(1 - e^{-x})^{-1}$$

$$S^\circ/R = xe^{-x}(1 - e^{-x})^{-1} - \ln(1 - e^{-x})$$

$$C^\circ_p/R = x^2e^{-x}(1 - e^{-x})^{-2}$$

where $x = hc\nu/kT$. This book contains first a table of the above dimensionless quantities for $x = 0.0010$ (.001) 0.1500 (.001) 4.00 (.01) 10.00 (.2) 16.0. A second table gives $-(F^\circ - E_0^\circ)/T$, S°, and C°_p (all in calories/mole-deg) for $T = 273.15°$K, 298.15°K, 400°K and thence by 100° intervals to 5000°K. For each temperature the frequency ν (in cm^{-1}) runs from 100 to 4000 in steps of 10. All results are stated to 5 decimal places, and the accuracy is claimed to be better than one-half a unit in the last place.

These tables would be far more useful if harmonic oscillators were more common components of molecules and crystals. Unfortunately, most vibratory degrees of freedom are not very harmonic, and accurate computations of thermodynamic properties require corrections for anharmonicity. These corrections cannot easily be applied to the final thermodynamic functions, but rather tend to require a complete new calculation.

In this reviewer's opinion the need for tables like these is passing. With computers now very generally available, the essential content of these tables could have been stored much more conveniently in the form of a set of subroutines in standard machine languages like FORTRAN. This would make the results available where they are most needed: as inputs to machine calculations for specific molecules and crystals.

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EDITOR'S NOTE: The author has notified us that he has recomputed the tables over certain ranges of the variable in multiple precision and has found numerous last-place errors on pages 3 and 4 of $x = .0010$ to $.0100$. An Errata sheet has been prepared and is available on request.

54[M].—L. S. PONTRYAGIN, *Ordinary Differential Equations*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962, vi + 298 p., 23 cm. Price $7.50.

From the publisher's preface: "This book constitutes a mildly radical departure from the usual one-semester first course in differential equations."

From the author's preface: "The most important and interesting applications of ordinary differential equations to engineering are found in the theory of oscillations and in the theory of automatic control. These applications were chosen to serve as guides in the selection of material."

One could attempt to characterize Pontryagin's "mildly radical departure" as a combination of more modern theory and more realistic application. There is a long chapter on stability theory, Lyapunov's theorem, limit cycles, and periodic solutions. While an earlier and even longer chapter has a title that is "classical" enough, namely, "Linear Equations with Constant Coefficients," the strong geometric emphasis, and the many diagrams of phase trajectories, nodes, saddle points, etc., are again distinctly modern in character.

It seems likely that the book will not only be successful in itself, but will also markedly influence the content of future textbooks. Although American authors are unlikely to put quite as much stress on Vyshnegradskiy's theory of the centrifugal governor and Andronov's analysis of the vacuum-tube oscillator, the approach used here will probably be widely followed.

Educational prognostications aside, the book can be recommended to those who learned differential equations the "old way" and who wish an introduction to newer technique and content. The book is interesting, and individual in style. Who but Pontryagin would combine "The breakdown in performance of governors in the middle of the 19th century is explained by the fact that, due to the development of engineering, all four quantities appearing in (15) were subjected to changes which served to diminish the stability" (page 220) with "Such cases can be easily imagined; for example, N can be the perfect set of Cantor" (page 233)?

There is a supplementary chapter on relevant matrix theory. There are no exercises.

D. S.

55[M, X].—ATHANASIOS PAPOULIS, *The Fourier Integral and its Applications*, McGraw-Hill Book Company, Inc., New York, 1962, ix + 318 p., 23 cm. Price $10.75.

This book treats what has long been known as operational calculus from the point of view of the Fourier integral theorem and the Fourier transform rather than from the point of view of the Laplace transform. The book consists of three parts and two appendices. In the first part, in addition to the Fourier integral theorem, the convolution theorem, Parseval's formula etc., an elementary discussion of "singularity functions" (the δ-function and its derivatives) is given and the Poisson sum formula is derived. The last chapter of the first part treats numerical techniques and the uncertainty principle which states a relationship between a function and its Fourier transform. The second part of the book treats linear systems, low-pass and bandpass filters and spectrum analyzers. The third part shows the connection between the Laplace and Fourier transforms and discusses Hilbert and Wiener-Lee transforms. The last part treats positive functions and limit theorems, generalized harmonic analysis, correlation, and power spectra. Each part is followed by a collection of about twelve problems, to many of which the solutions are given. The first appendix treats the δ-function as a distribution function, and the second gives a summary account of the theory of analytic functions, ending with an account of the saddle-point method.

The book contains a great amount of useful information and is written in a readable, lively manner. The level of mathematical sophistication is lower than we like to see, the order of integration in a repeated infinite integral, for example, being cheerfully inverted without any mention of the restrictions this places upon the integrand. Doubtless the author feels that these matters properly belong to a parallel class in advanced calculus. If students of the book cover these finer points in such a class, or by private study, they should find the book very informing and rewarding.

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56[T, Z].—DONALD N. HANSON, JOHN H. DUFFIN, & GRAHAM F. SOMERVILLE, *Computation of Multistage Separation Processes*, Rheinhold Publishing Corp., New York, 1962, viii + 361 p., 22 cm. Price $8.75.

In this book the authors present a discussion of the mathematics of multistage separation processes with application to vapor-liquid systems and liquid-liquid extraction. A large part of the book is devoted to a series of computer programs (written in Fortran) to solve a wide range of separation problems, including multiple-feed and multiple-product processes in distillation, absorption, stripping, and extraction. The authors state that the programs have been extensively checked in typical chemical engineering problems.

The general presentation of the mathematical background material seems to be very concise and clear. The description of the various computer programs is very good, although an acquaintance with Fortran would be very desirable for anyone actually planning to use the routines. It should be noted that most of the programs would have to be somewhat modified to run under the monitor systems used in most IBM-7090 computer installations. The changes necessary would be mostly in the read-write statements and in the avoidance of the use of sense switches.

In general, the book should prove to be a valuable contribution to the literature of separation processes, both as a textbook for an advanced course and as part of the working library of engineers concerned with problems in this area.

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57[V].—Caius Jacob, *Introduction Mathématique à la Méchanique des Fluides*, Gauthier-Villars, Paris, 1959, 1286 p., 23 cm. Price Lei 66,50.

This work is a translation of a book originally published in Romanian in 1952.

For the present French edition, the author has added additional material and references to bring it up to date. The book is very large and covers many topics quite extensively. Some idea of this may be gained from looking at a listing of the Table of Contents:

The main emphasis of the book is on mathematical methods. The author states that he has chosen the topics for the book on the basis of their mathematical interest and importance and no attempt has been made at complete coverage of the field of fluid mechanics. The coverage of classical fluid dynamics of perfect fluids is quite extensive and reasonably complete. Viscous fluids are less thoroughly covered, and there is no discussion of such important problems as boundary-layer theory. The book should serve as an excellent reference volume, since there are extensive references to the literature as well as a clearly written and understandable text.

The principal criticism of this book is that there is no index either of authors cited or of subject matter covered. The table of contents, although extensive does not fulfill this need. Nevertheless, this book should prove to be a very valuable reference work.

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58[W].—L. R. Ford, Jr. & D. R. Fulkerson, *Flows in Networks*, Princeton University Press, Princeton, N. J., 1962, xii + 194 p., 23 cm. Price $6.00.

This small volume presents a unified treatment of network flow methods for solving a class of linear programming problems subsumed under the phrase "transportation-type models." The problems treated by the authors are limited to the case for which the assumption of integral constants in the constraints implies the existence of an integral solution. Numerous examples are provided to elucidate the various techniques. Extensive bibliographies at the end of each chapter greatly enhance the value of this book.

An elegant proof of the max-flow min-cut theorem is developed that yields an efficient computational method for determining a maximal steady-state flow in a network subject to capacity limitations on arcs. This theorem is also utilized to (a) develop necessary and sufficient conditions for the existence of network flows that satisfy linear inequalities of various kinds, (b) solve combinatorial problems involving linear graphs, partially ordered sets, set representatives, and zero-one matrices, and (c) study multi-terminal maximal flows in undirected networks.

Adaptations of the maximal flow algorithm are presented for obtaining network flows that minimize cost, subject to various kinds of constraints on arcs. Included in this category are:

(a) A specialized algorithm for the standard Hitchcock transportation problem,

(b) A general algorithm for the trans-shipment problem with capacity constraints on arcs,

(c) A shortest chain algorithm for an arbitrary network,

(d) A method for constructing minimal-cost feasible circulations in a network having lower bounds as well as capacities on arcs.

Two interesting and useful applications of the theory developed for minimal cost flows are discussed; namely, (a) constructing a maximal dynamic flow in a given time interval for an arbitrary network in which each arc has a traversal time as well as a capacity, and (b) constructing a cost curve which depicts the minimal project cost as a function of the completion date of the project.

The authors have made a commendable contribution to the development of computationally efficient methods for the application of network flow theory to operations research and combinatorial mathematics. Their lucid discussions of the basic principles should make this book a valuable reference for those who are interested in the solution of management control problems which can be depicted by arrow diagrams of a network-analogue variety.

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59[W].—Western Data Processing Center, University of California, Los Angeles, *Contributions to Scientific Research in Management*, [Proceedings of the Scientific Program following the Dedication of the Center January 29–30, 1959], Division of Research, Graduate School of Business Administration, University of California, Los Angeles, 1961, ix + 172 p., 26 cm. Price $2.50.

This volume is comprised of three chapters, which contain the proceedings of the symposium listed in the title. Of the four papers in the first chapter, two (Arrow, Dorfman) are of primarily theoretical content; each of these essentially develops a program for future research and exhibits methods of investigation and nature of results by carrying the reader through the program's first step. One (Lazarsfeld) reports on a specific application, while Daly reports on developments in his Bureau during the decade since 1950.

In the second chapter, Marschak's paper is of theoretical content, Shubik's and the Cyert-March paper point out areas for research, Ashenhurst tells those in need of a computer some of the things they may find useful to know.

In the last chapter all four papers report on specific applications. Rowe's paper is also of methodological interest (in developing sequential decision rules); Hoggatt gets striking results from a simple model; Eisenpress reports on a computer program for estimating the parameters of a single equation in a linear system of structural equations using Anderson and Rubin's method; the Schutz-Gross paper reports on a sociometric test.

The table of contents follows:

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60[X].—L. Zipperer, *Tables for the Harmonic Analysis and Synthesis of Periodic Functions*, Physica Verlag, Wurzburg, Germany, 1961, 4 p. + 24 tables, 30 cm. Price 9.75 DM.

A periodic function $f(x)$ with period 2π may be represented by the series $y = \sum_{k=0}^{n} a_k \cos kx + \sum_{k=1}^{n} b_k \sin kx$. Divide the range into $2n$ equal parts and let y_s, $s = 1, 2, \cdots 2n$ be the functional values at these points. Then it is well known that the coefficients a_k and b_k can be simply expressed in terms of the functional values. For $n = 12$, formats are provided to facilitate the computation of the coefficients on a desk computer. Layover formats are also provided to evaluate y_s for a given set of a_k's and b_k's, and so evaluation of Fourier series is afforded. The text is in both German and English.

Y. L. L.

61[Z].—John E. Coulson, *Programmed Learning and Computer-Based Instruction*, John Wiley and Sons, Inc., New York and London, 1962, xv + 291 p., 23.5 cm. Price $6.75.

This book consists of the proceedings of a conference, held late in 1961, on applications of digital computers to automated instruction. As one of several books published in the past two years about the "teaching machine" business, it illustrates a fundamental characteristic about the current accelerated interest in programmed learning: many people and institutions have rushed into the field with very little useful corresponding output. The 1961 conference, cosponsored by the Office of Naval Research and the System Development Corporation, was a serious attempt to assess the place of digital computer in the context of programmed learning. Like most of the noncomputerized work on automated instruction, that reported here doesn't come off very well. Of the twenty-three papers presented, fewer than one-third report actual experience with computers as teaching machines. Of these, four deserve mention, and they should be read carefully by anyone with more than a passing interest in the subject. Here I refer to the chapters by Coulson (on the elegantly appointed experimental facility known as CLASS), Bitzer et al. (on PLATO II, the University of Illinois automated teaching facility), Licklider (on experiments using computer-generated displays in teaching German, and the graphical presentation of certain mathematical functions), and Uttal (on experience with three courses taught automatically, including one involving instruction in a psychomotor skill). Perhaps the shortcomings of the remainder of the book—tediousness, speculation, no quantitative evidence on effectiveness of the techniques—are representative of most meetings of this type, involving too many participants, too few people with anything of substance to offer, and too much enthusiasm for a promising but little understood area of human behavior. The meeting sponsors should, however, be commended for their recognition of the importance of this application of computer technology. The Second Conference, which I hope will not be held until some of the noise dies down and is replaced by competent experimental test of the unverified claims of the 1961 meeting, will be worth looking forward to.

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62[Z].—J. A. P. Hall, Editor, *Computers in Education*, Pergamon Press, New York, 1962, xvi + 122 p., 23 cm. Price $7.50.

In September 1959, Hatfield Technical College—now Hatfield College of Technology in Hertfordshire County, England—set up a department of mathematics, which at the same time was expected to organize and run a computing laboratory. In order to gain and exchange ideas with other colleges on "the purpose, equipment and use of a computing laboratory in a technical college, with particular reference to the effect of computing machines on mathematics today," the department organized a two-day conference on May 27 and 28, 1960. Of the 22 papers read at that conference 20 have been collected into this book, most of them in a revised and updated version. In addition, there is an "Introduction" and a "Conclusion," both written by the editor, J. A. P. Hall.

The papers can be divided into essentially three groups. One group, comprising nine papers, is concerned with the educational aspects of computation. The second group, also of nine papers, could be entitled "the organization of a computing laboratory and the selection of its equipment." Finally, the third group contains two papers, a general lecture on "Applied Mathematics and Computing Machines" and a list of "Bibliographies for Numerical Analysis."

In general, the presentation reflects very strongly the special problems of the small institutes of technology in England. There is apparently no program in Britain similar to that of the National Science Foundation in this country, which provides a certain source of funds for the establishment of computing facilities in such colleges. As a result, a number of the papers discuss the advantages and disadvantages of self-built computers, joint computer installations with local industries or even the local community, and, of course, of self-supporting centers, selling time to outside users. At the same time, the over-all discussion is centered around small computers, and this includes all the emotional arguments in favor of the "simple", and "robust" machine which gives the student the proper "feel" for debugging, and so on. Evidently related to this problem is the comparatively strong case made for desk-calculator laboratories; this includes a very thorough article on the selection of the appropriate desk calculator.

Throughout all the papers there is agreement between the conferees that "computation", as they call it, is now a vital part of engineering education. Accordingly, they recommend that every technical college should have a computing laboratory. And this is unquestionably a very modern and excellent recommendation. However, there seemed to be a general understanding here that computation stands primarily for numerical analysis, and to some extent for statistics. This is particularly evident in the papers recommending the inclusion of computation in the requirements of the Diploma in Technology and Degree Courses and in those of the National Certificate and Diploma Courses, as well as in the papers outlining various computer-related courses for colleges. Somehow, all through the book, computation is viewed as the laboratory science of mathematics; in fact, in the Conclusion that is stated in just this way. In line with this, no mention is made of non-numerical problems, not to speak of a recognition of computer science as a field in its own right. Neither is there any mention made of computer use in such areas as industrial management or industrial control, while operations research receives only fleeting mention in some articles.

Summing it up, we might say that this book essentially makes a case for more and stronger education in applied and numerical mathematics, education which must include the direct and intensive use of computers and modern computing methods. In fact, in his paper Sir Graham Sutton makes a statement which might almost be said to capsule the general tone of the entire conference:

"It is sometimes said that the applied mathematician has not the care for rigor that characterizes the work of the pure mathematician. To some extent this is true, but it is not an excuse for mediocrity or slapdash methods. In many ways the computer is a dangerous instrument, and there is every need for the best brains in its use. It is up to the teachers to see that this need is met." Indeed, and that is not just a British problem!

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63[Z].—ROBERT S. LEDLEY, *Programming and Utilizing Digital Computers*, McGraw-Hill Book Company, Inc., New York, 1962, xxi + 568 p., 23 cm. Price $12.50.

There is a long list of books on the current market which provide an introduction to computer programming and the application of computers. These books proceed along a variety of lines: in addition to those concerned with programming one specific computer, there are those which introduce the details of programming from a more general viewpoint by discussing a fictitious machine. Still others deal with a specific area of computer applications and discuss programming problems in connection with these applications. This particular book by R. S. Ledley contains elements of all of these types, and yet it is fundamentally different from all of them in a number of ways. Two very striking aspects are a well-written introduction to automatic programming, including a chapter each on ALGOL and COBOL, and the almost startlingly far-ranging list of topics discussed in the book.

In his introduction the author specifies his aims as follows: "This book was written as a college text in programming digital computers; it can be used on various levels, ranging from sophomore to first year graduate. This book is intended to fill the great need for an up-to-date, comprehensive text to provide an introduction to the many aspects of the digital computer-programming field . . . of course, an introductory exposition of a field as large and rapidly advancing as this can never hope to treat all subjects exhaustively . . . hence, each chapter is designed primarily to introduce the student to certain fundamental concepts and techniques of development."

The book comprises three roughly equal parts, entitled, respectively, Machine Languages, Automatic-Programming Techniques, and Data-Processing Techniques.

The first part begins with some descriptions of computer applications, including such topics as: process control, simulations, and aids to medical diagnosis. Then follows the usual block-diagram and functional description of computers. Chapter 2 contains an introduction to number systems, flow-charting, and the principles of machine languages. These principles are explained using a four-address instruction format. Chapter 3 then reduces the four-address format successively to three-, two-, and single-address instruction systems; it also discusses such topics as indirect and relative addressing. Chapter 4 is entitled Programming for Special-Purpose Digital Computers. This is one of the topics usually not covered in textbooks of this type. The chapter contains some general discussion of digital differential analyzers, real-time control computers, information retrieval computers, and other similar topics.

With Chapter 5 we enter into Part 2 of the book. This chapter introduces various topics of automatic programming such as interpretive routines, including even a brief discussion of threaded-lists, and an introduction to algebraic compiling routines and the associated translation problems. Chapter 6 is an introduction to ALGOL; the Backus normal notation is used to define the language. This chapter appears to be quite condensed and, as a result, seems to lack the instructional value of, for example, H. Bottenbruch's "primer" on ALGOL 60. (See *Journal of the ACM*, v. 9, 1962, p. 161–221.) Chapter 7 presents an introduction to the essential concepts of COBOL 1961. Chapter 8, entitled Programming to Achieve Intelligence, is a quite ambitious chapter, as a list of subtitles will indicate: list processing, automatic programming-language translation and examples of translating ALGOL codes, mathematical optimization including dynamic programming, programming for medical diagnosis, proving geometric theorems and trigonometric identities, and abstraction and creativity.

Part 3 begins with Chapter 9, which gives a brief introduction to the fundamentals of numerical analysis, ranging, nevertheless, from simultaneous linear equations to the solution of Laplace's equation by difference techniques. Chapter 10 gives an equally condensed introduction to Boolean algebra. The last chapter, Chapter 11, discusses search and sorting techniques as well as some problems of codifying information.

As can be seen from this partial enumeration of topics, the author certainly introduces a wide variety of subjects. He definitely does this in a very lucid way and his selection of topics is up-to-date. However, one might perhaps wish that the author had gone into greater detail in certain sections of the book, compensating for this by leaving other sections out. As it stands, many sections are so short that they appear to be superficial, even though the author has tried to overcome this by giving extensive lists of relevant references. Nevertheless, the emphasis on automatic programming techniques and the discussion of modern problems in the computer sciences make this a very attractive book, which probably will find widespread use in introductory computer courses.

WERNER C. RHEINBOLDT

64[Z].—DANIEL D. MCCRACKEN, *A Guide to ALGOL Programming*, John Wiley & Sons, Inc., New York, 1962, viii + 106 p., 27 cm. Price $3.95.

This is not a reference work on the syntax and semantics of ALGOL, but a highly practical text suitable for a beginner. It assumes no prior knowledge of computers or programming nor any mathematical sophistication.

Eight Chapters are contained within 93 pages. New concepts are introduced gradually. The if-statement appears in Chapter 3, the for-statement in Chapter 4, and procedures in chapter 7. Each chapter is followed by a set of exercises, the answers to some of which are given in the back of the book. Flow charts are used effectively to help illustrate the algorithms. Several chapters are concluded with case studies: examples of programs which can occur in practice. The final chapter deals with an input-output scheme which the author admits is not ALGOL, and the reviewer hopes will never be.

This text is on the other end of the scale from that of Naur, which was evidently written for the experienced programmer. It is probably more suitable for the beginner than those of Bottenbruch or Dijkstra. However, the reader is not warned that ALGOL has imperfections, nor is he given an indication of the precise manner in which the syntax is defined. The author gets into trouble by not making clear the distinction between procedure-identifier and function-designator. Thus the statement (on page 76) that a function name "must never appear anywhere but on the left-hand side of an assignment statement", will probably convince many readers falsely that the recursive factorial procedure on page 79 is incorrect.

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65[Z].—James A. Saxon & William S. Plette, *Programming the IBM 1401: A Self-Instructional Programmed Manual*, Prentice-Hall, Inc., New Jersey, 1962, xv + 208 p., 23 cm. Price $9.00.

The sub-title of this book; namely, *A Self-Instructional Programmed Manual*, describes the special feature of its design. It is a text on the 1401 designed for study without the aid of a teacher. In order to accomplish this purpose, frequent problems are provided, with the answers given on the immediately succeeding pages. The answers are accompanied by comments which help to clarify any errors that may have been made by the student.

This general technique seems very useful, and can certainly assist in the initial training of 1401 programmers, with a reduction in the time required by an instructor.

Considering the space demands of this special method of presentation, the text provides a good coverage of the fundamentals of programming. Machine language, flow charting, symbolic coding, assembly programs, input and output, editing features, and subroutines are only briefly mentioned.

With the completion of this text, the student has acquired a good start in 1401 programming.

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66[Z].—Rajko Tomovic & Walter J. Karplus, *High-Speed Analog Computers*, John Wiley and Sons, Inc., New York, 1962, xi + 255 p., 22 cm. Price $9.95.

This book presents the material on electronic devices and circuits which combine to constitute the repetitive type of analog computers (where a solution can be displayed on a cathode ray tube) and on applications of such computers to engineering problems. Professor Tomovic is associated with the University of Belgrade, Yugoslavia. He has written a book (entitled *Calculateurs Analogiques Répétitifs*, published in Paris in 1958) from which the present volume was derived. The co-author, Professor Karplus of the University of California, author of other books on analog computers, is well known in this field. This book is apparently the contributions of the two professors while Professor Tomovic was a Visiting Professor at the University of California in 1961.

This book is divided into three parts of about equal length: Part I on Theory, part II on Equipment, and part III on Application; in addition, the first chapter is an introduction.

Part I consists of Chapters 2, 3 and 4, respectively entitled Analytical Foundations, Error Analysis of Analog Computers, and Scale Factors. In Chapter 1, the authors describe a system of differential equations (expressed in a matrix form in the case of linear equations), which is shown to be realizable by a set of multiple-input operational amplifiers. The multiloop feedbacks from the set of operational amplifiers obviate the necessity of iterations in a repetitive analog computer. This is the fundamental principle employed in the book. In Chapter 3, the relation between the sensitivity of computer solutions to errors and the stability of solution is presented as well as the error coefficients developed on a theoretical level. Chapter 4 discusses the scale factor, the interrelation of scale factors, and the effect of drift on scale factoring.

Part II consists of Chapters 5, 6, 7, and 8, respectively entitled Linear Elements, Nonlinear Operations, Output Equipment, and Auxiliary Devices. Chapter 5 discusses essentially the operational amplifier and dynamic memory (an integrating operational amplifier used for memory purpose). In Chapter 6, the nonlinear elements include hyperbolic electrostatic-field multipliers, Hall-effect multipliers, photoformers, diode function generators, so-called universal nonlinear operators, and function generators of two variables. Chapter 7 describes an "electronic graph paper" for display on a cathode-ray tube and servo-driven plotter. Auxiliary devices of Chapter 8 cover pulse generators and power supply.

Part III consists of Chapters 9, 10, 11, and 12, concerned, respectively, with ordinary differential equations, partial differential equations, integral equations, and miscellaneous applications. The topics in these chapters include Fourier analysis, two-point boundary-value problems, Monte-Carlo method, analog iterative solution of integral equations, roots of a polynomial, conformal mapping, and system analysis by statistical techniques.

This volume fills the need for a book on repetitive analog computers. It includes material published in Europe (including Russia) and in America, and gives many interesting ideas on analog computers. It is a contribution and a welcome addition to the literature on analog computers.

Yoahan Chu

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